Introduction to the VOA/SCFT correspondence

Chris Beem

Lecture 1.

4d $\mathcal{N}=2$ SCFTs $\rightsquigarrow$ VOAs

(abstract OPE algebra)

$$\mathcal{H}[S^3] \cong \hat{V}^{loc} \quad \text{(completed) space of local ops.}$$

$\circlearrowleft$

$$\mathfrak{g} = su(2,2|2)$$

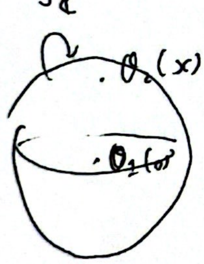
$$\mathfrak{g}_{\mathbb{C}} \cong sl(4|2) \supset sl(2)_z \times sl(2)_{\bar{z}} \times gl(1)_{w,\bar{w}} \times gl(1)_r \times sl(2)_R$$

$$\left(\mathbb{R}^4 \cong \mathbb{C}_{z,\bar{z}} \times \mathbb{C}_{w,\bar{w}}\right)$$

$$V^{loc} \cong \bigoplus_{n=1}^{\infty} \mathrm{Irrep}[h_i, \bar{h}_i, \overset{j_1-j_2}{\overset{\shortparallel}{l_i}}, r_i, R_i]$$

$\mathfrak{g}_{\mathbb{C}}$

OPE :



$$= \sum_{k=1}^{\infty} c_{12}^{k}(x)\, \overset{\mathfrak{g}_{\mathbb{C}}}{\curvearrowright}\mathcal{O}_k(0)$$

"$\mathfrak{g}_{\mathbb{C}}$ - eq. OPE algebra"

$$\mathfrak{g} \supset \mathfrak{g}_{odd} \supset \mathrm{Nilp}(\mathfrak{g}_{odd}) = \{ d \in \mathfrak{g}_{odd} : d^2 = \tfrac{1}{2}\{d,d\} = 0 \}$$

Let $d \in \mathrm{Nilp}(\mathfrak{g}_{odd})$ be homogeneous w.r.t. $h \in \mathfrak{g}_{even}$

$(\hat{V}_{loc}, d)$ is dg-OPE algebra.

Let $\mathfrak{c} = \mathrm{Comm}(d, \mathfrak{g})$,

$\mathfrak{e} = \mathrm{ad}_{\mathfrak{g}}(d) = [\mathfrak{g}, d]$ (d-exact sym)

$\rightsquigarrow$ $H^\bullet_d(\mathcal{V}^{loc})$ will be a $\mathfrak{c}$-equiv., $\mathfrak{e}$-inv., OPE alg.

For $\mathcal{N}=2$ SCFTs, $\mathfrak{g}_{odd} = \mathrm{span}\{ Q^I_\alpha, \widetilde{Q}_{I\dot\alpha}, S^\alpha_I, \widetilde{S}^{I\dot\alpha} \}$

$$I = 1,2, \quad \alpha = \pm, \quad \dot\alpha = \dot\pm$$

$\exists$ two vectors
$$d = Q^1_- + \widetilde{S}^{1\dot-}$$
$$\check{d} = S_1^- - \widetilde{Q}^1_{\dot-}$$
$\in \mathrm{Nilp}(\mathfrak{g}_{\mathbb{C}})$ graded by r s.t.

$$\mathfrak{c}[\overset{(\vee)}{d}] \supset \mathfrak{sl}(2)_z \qquad \mathrm{span}\{ \widehat{L}_{\pm1,0} = \bar{L}_{\pm1,0} \pm R_{\pm1,0} \}$$

$$\mathfrak{e}[\overset{(\vee)}{d}] \supset \widehat{\mathfrak{sl}(2)}_{\bar z} = \mathrm{diag}\,\overbrace{(\mathfrak{sl}(2)_{\bar z} \times \mathfrak{sl}(2)_R)}$$

$$\supset \mathrm{diag}\,\underbrace{(\mathfrak{gl}(1)_{w,\bar w} \times \mathfrak{gl}(1)_r)}_{Z = r - \ell}$$

$$[\partial_w \pm \partial_{\bar w}, \overset{(\vee)}{d}] \neq 0$$

$H^\bullet_d(\mathcal{V}^{loc}) \cong H^\bullet_{\check d}(\mathcal{V}^{loc})$ inherit structure of $\mathfrak{sl}(2)_z$-equiv. $\widehat{\mathfrak{sl}(2)}_{\bar z}$-inv. OPE alg

$\rightsquigarrow$ (quasi-conformal) vertex algebra

Vector space cohomology: can choose harmonic reps

$$\Delta_d = \{d, d^+\} = \Delta_{Q^1_-} + \Delta_{\widetilde{S}^{1\dot-}} = \{\check d, \check d^+\} = \Delta_{\check d}$$

$$\Delta_{Q^1_-} = \hat{L}_0 - Z, \qquad \Delta_{\tilde{S}^{\dot{-}}_1} = \hat{L}_0 + Z$$

$$\hat{L}_0 = Z = 0 \quad \text{Schur operators}$$

$$\begin{cases} \bar{h} = R \\ \ell = r \end{cases} \Rightarrow \quad h = \frac{E + (j_1 + j_2)}{2} = E - R = R + (j_1 + j_2) \in \frac{1}{2}\mathbb{N}$$

Vertex alg. is triply graded

- h conformal grading
- $r = \cdot$ cohomological grading
- R "extra" grading

Vertex algebra structure

$\checkmark$ $sl(2)_R$ h.w. states

Schematically: $\partial_z^N (\mathcal{O}_{prim})^{(1,\ldots,1)}_{(+,\ldots+)(\dot{+},\ldots\dot{+})}$

$$z = x^{+\dot{+}}, \qquad \bar{z} = x^{-\dot{-}}$$

$$\mathcal{O}(z) = \Big[\!\Big[e^{zL_{-1} + \bar{z}\hat{L}_{-1}} \, \mathcal{O}(0) \, e^{-zL_{-1} - \bar{z}\hat{L}_{-1}} \Big]\!\Big]_{d^{(v)}}, \qquad \hat{L}_{-1} = \bar{L}_1 + R_-$$

$$= \Big[\!\Big[\mathcal{O}^{h.w.}(z,\bar{z}) + \bar{z}\,[R^-, \mathcal{O}^{h.w.}(z,\bar{z})] + \cdots + \frac{\bar{z}^{2R}}{(2R)!}[(R^-)^{2R}, \mathcal{O}^{h.w.}(z,\bar{z})] \Big]\!\Big]_{d^{(v)}}$$

$$\Rightarrow \mathcal{O}_1(z)\,\mathcal{O}_2(0) \sim \sum_k \frac{C_{12}^k \, \mathcal{O}_k(0)}{z^{h_1+h_2-h_k}} + d^{(v)}\text{-exact.}$$

R-filtration. $\mathcal{G}_p V_h^{\text{Schur}} = \bigoplus_{R \le p} V_{h,R}^{\text{Schur}}$

$\mathbb{C}^{\delta_{0,h}} \cong \mathcal{G}_0 V_h^{\text{Schur}} \subset \mathcal{G}_1 V_h^{\text{Schur}} \subset \mathcal{G}_2 \cdots$

$$\left.\begin{array}{l} \text{if } a \in \mathcal{G}_p V,\ b \in \mathcal{G}_q V,\ a(z)b \in \mathcal{G}_{p+q} V((z)) \\ a(z)b|_{\text{sing}} \in \mathcal{G}_{p+q-1} V[z^{-1}] \end{array}\right] \text{Good increasing filtration (H. Li)}$$

Consequence: $\mathrm{gr}_{\mathcal{G}} V = \bigoplus_p \dfrac{\mathcal{G}_p V}{\mathcal{G}_{p-1} V}$ has str. of vertex poisson algebra.

(from 4d: alg. of local ops in holo-top twist)

$$d_{HT} = Q_-^1 + \tilde{Q}_{\dot{-}}^1$$

Free field theories

$q(z_1) = [\![Q]\!]$

$\tilde{q}(z_2) = [\![\tilde{Q}]\!]$

hypermultiplet : $(Q, \tilde{Q})$ cx. scalars $E = 1,\ r = 0,\ R = 1/2$

vector multiplet

$q(z_1)\tilde{q}(z_2) \sim \frac{1}{z_1 - z_2}$ symplectic bosons

$(\lambda_+^1, \tilde{\lambda}_{\dot{+}}^1)$

$E = \frac{3}{2},\ r = \pm 1.\ R = \frac{1}{2}.$

$\eta_+(z_1) = [\![\lambda]\!]$

$\eta_-(z_2) = [\![\tilde{\lambda}]\!]$

$\eta^+(z_1)\eta^-(z_2) \sim \frac{1}{(z_1 - z_2)^2}$ symplectic fermions

free fields $R = 1/2$

Conserved currents $R = 1$

$R = 1 \cdot j_1 = j_2 = r = 0$

$$\mathcal{M}^{A=1,\dim G} \overset{\mathbb{Q}\tilde{\mathbb{Q}}}{\rightsquigarrow} J_\mu^{A=1,\dim G}$$

$k_{2d} = -\frac{1}{2}k_{4d}$

$$[\![\mathcal{M}^A]\!] = J^A, \quad J^A(z)J^B(w) \sim \frac{-\frac{1}{2}k_{4d}}{(z-w)^2} + i\frac{f^{AB}{}_C J^C(w)}{z-w} + \text{reg.}$$

$$R=1,\ j_1=j_2=1/2,\ r=0,\ J_{+\dot{+}}^{(R)1} \overset{\mathbb{Q}\tilde{\mathbb{Q}}}{\rightsquigarrow} T_{\mu\nu}$$

$$\Rightarrow T(z)T(w) \sim \frac{-6c_{4d}}{(z-w)^4} + \frac{2T}{(z-w)^2} + \frac{T'}{z-w} + \text{reg}$$

$$c_{2d} = -12c_{4d}$$

(A_1, A_{2n}) Argyres - Douglas theory $\longleftrightarrow$ $(2, 2n+3)$ Virasoro VOA, rat'l

(A_1, D_{2n+1}) " " $\longleftrightarrow$ $V_{-2+\frac{2}{2n+1}}(sl_2)$, not rat'l

$SU(2)$ $N_f = 4$ gauge theory $\longleftrightarrow$ $V_{-2}(so(8))$, not ~~ad~~ admissible

$SU(2)$ $\mathcal{N}=4$ SYM $\longleftrightarrow$ Small $\mathcal{N}=4$ Super-Virasoro VOA, $c=-9$

Lecture 2. Geometric aspects of SCFT / VOA

$\mathcal{T} \rightsquigarrow \mathcal{M}_H[\mathcal{T}]$ Higgs branch

(preserves $U(1)_r$ symmetry)

- hyperkähler cone
- holo. symp. variety
- symplectic singularity

$R_H = \mathbb{C}[\mathcal{M}_H]$ is realised through (some) Schur op's:

$$\mathbb{C}[\mathcal{M}_H] \cong \bigoplus_{R=0}^{\infty} (V_{\text{Schur}})^{0 \leftarrow U(1)_r}_{R,R} \leftarrow \text{scalar Schur ops.}$$

$\mathcal{T}$	$\mathcal{M}_H[\mathcal{T}]$	R_H
free hyper	$\mathbb{C}^2$	$\mathbb{C}[q,\tilde{q}]$
(A_1, D_{2n+1})	$\mathbb{C}^2/(\mathbb{Z}/2) \simeq \mathcal{N}ilp(sl_2)$	$\mathbb{C}[j^1, j^2, j^3] \big/ (\sum_a j^a j^a = 0)$
$SU(2)$ $N=4$ SYM	"	"
$SU(2)$ $N_f = 4$	$\overline{\mathbb{O}_{min}(so(8))}$	$Sym(so(8)) / I_{\text{Joseph}}$

R_H encoded in $(\mathbb{V}, \mathcal{G}_\bullet)$

(Recall $\mathcal{G}_p \mathbb{V}_h = \bigoplus_{R \le p} (V_{\text{Schur}})_{R,h}$, $gr_{\mathcal{G}} \mathbb{V} = \bigoplus_{R,h} \dfrac{\mathcal{G}_R \mathbb{V}_h}{\mathcal{G}_{R-1} \mathbb{V}_h} = \bigoplus_{R,h} (gr_{\mathcal{G}} \mathbb{V})_{R,h}$)

$$R_H = \bigoplus_R (gr_{\mathcal{G}} V)_{R,R} \qquad (\text{recall } R \le h)$$

as Poisson algebra

$$\tilde{a} \in \frac{\mathcal{G}_{h_1} \mathbb{V}_{h_1}}{\mathcal{G}_{h_1 - 1} \mathbb{V}_{h_1}} \longrightarrow a \in \mathcal{G}_{h_1} \mathbb{V}_{h_1} \text{ as rep.}$$

$$\tilde{b} \in \frac{\mathcal{G}_{h_2} \mathbb{V}_{h_2}}{\mathcal{G}_{h_2 - 1} \mathbb{V}_{h_2}} \longrightarrow b \in \mathcal{G}_{h_2} \mathbb{V}_{h_2} \text{ as rep.}$$

$$a(z)\, b(w) \sim \cdots + \frac{c(w)}{z-w} + \text{reg.}$$

$$c \in \mathcal{G}_{h_1 + h_2 - 1} \mathbb{V}_{h_1 + h_2 - 1} \longrightarrow \tilde{c} \in \cdots$$

$$\{\tilde{a}, \tilde{b}\} := \tilde{c}$$

Li's canonical filtration (descending)

$$F_p V = \text{span}\left\{ a^1_{-h_{a_1}-n_1}\, a^2_{-h_{a_2}-n_2} \cdots a^k_{-h_{a_k}-n_k}\, \Omega : \sum_{i=1}^{k} n_i \geq p \right\}$$

(counts derivatives up to upward error)

Weight based filtration (ascending)

given strong gen's $\{b_1, \dots\}$

$$\tilde{F}_p V = \text{span}\left\{ b^1_{-h_{b_1}-n_1} \cdots b^k_{-h_{b_k}-n_k}\, \Omega : \sum_{i=1}^{k} h_{b_i} \leq p \right\}$$

$F_p \cap V_h \cong \tilde{F}_{h-p} \cap V_h$, so $gr_F V \cong gr_{\tilde{F}} V$ as VPA

Canonical Poisson alg:

$$R_V := \frac{F_0(V)}{F_1(V)} = \frac{V}{C_2(V)} = \bigoplus_h \frac{\tilde{F}_h(V_h)}{\tilde{F}_{h-1}(V_h)}$$

$X_V = \text{Spec}^{max}(R_V)$ associated variety

$\tilde{X}_V = \text{Spec}(R_V)$ associated scheme

If $\dim(\tilde{X}_V) = \dim(X_V) = 0 \Rightarrow C_2$-cofinite.

If X_V is symplectic $\Rightarrow$ quasi-lisse

One remark: $\tilde{F}_p \mathbb{V} \subset G_p \mathbb{V}$. R-filtration is "coarser" than w.b. filtration

define $\mathbb{V}_+ = \bigoplus_h G_{h-1} \mathbb{V}_h \Rightarrow R_H = \mathbb{V}/\mathbb{V}_+$

Note: $C_2(V) \subset V_+$.

$$R_H = V/V_+ = \frac{V/C_2(V)}{V_+/C_2(V)} = \frac{R_V}{I}, \quad I \text{ Poisson ideal of } R_V$$

$$R_V \twoheadrightarrow R_H$$

$$X_V \hookleftarrow M_H$$

<u>Conjecture</u>: $I = \mathrm{nil}(R_V)$, i.e. $X_V = M_H$.

$\Rightarrow$ all VOAs from 4d are quasi-lisse

<u>Thm</u> [Arakawa, Kawasetsu] For V quasi-lisse,

- V has finite # of simple ordinary modules
- characters of ord. modules satisfy a common monic, weight zero MLDE.

Quasi-lisse $\Rightarrow$ $N_T = T^k + \rho^{\in C_2(V)}$

$$\Rightarrow 0 = \mathrm{STr}_M\left(o(N_T)\, q^{L_0 - c/24}\right)$$

$\Rightarrow$ recursion.

- Naive order of MLDE is k, can be higher
- can be lower if consider non-monic MLDEs.
- $\chi_M(q)$ need not span modular vector.

(A_1, D_{2n+1}) AD $\longleftrightarrow$ $V_{-2+\frac{2}{2n+1}}(sl_2) \longleftarrow \{J^{A=1,2,3}\}$

null states of form

$$J^B (J_A J^A)^n + \overset{C_2(V)}{\rho} = 0, \qquad B = 1,2,3$$

R_V: $\mathbb{C}[j^1, j^2, j^3] / \langle j^B (j^A j_A)^n = 0 \rangle$

(note: $(j^A j_A)^{n+1} = 0$)

$(R_V)_{red} \cong \mathbb{C}[j^1, j^2, j^3] / \langle j^A j_A = 0 \rangle = \mathbb{C}[\mathbb{C}^2 /\!/ \mathbb{Z}/2]$

R_H: $T \sim (J^A J_A)$, let t = image of T in $gr_g V$.

in $gr_g V$
$$\left\{ \begin{array}{c} j^A j_A = 0 \\ t (j^A j_A)^n = 0 \\ t^{n+1} = 0 \end{array} \right\} \xrightarrow[t=0]{RH} \mathbb{C}[j^A] / \langle j^A j_A = 0 \rangle$$

Lecture 3

Free field realizations

$$V_{-\frac{4}{3}}(sl_2) \underset{\text{Adamović}}{\longleftrightarrow} \Pi = \bigoplus_{n=-\infty}^{\infty} (V_{\partial\varphi} \otimes V_{\partial\delta}) e^{n(\delta+\varphi)} \cong D_k^{ch}(\mathbb{C}^\times)$$

w/ $\langle \varphi, \varphi \rangle = -\langle \delta, \delta \rangle = \frac{2}{k}$

$J^+ = e^{\delta + \varphi}$

$J^3 = \frac{k}{2} \partial \varphi$

$J^- = -\left(\left(\frac{k}{2} \partial\delta\right)^2 - \frac{k(k+1)}{2} \partial^2 \delta \right) e^{-\delta - \varphi}$

works for $k \in \{-\frac{4}{3}, -\frac{1}{2}, -2\}$.

J^+, J^3, J^- generate $V_k(sl_2)$

$X_{V_k(sl_2)} \cong \mathbb{C}^2/\mathbb{Z}_2 \cong \mathrm{Nil}(sl_2)$

Conj: can realize R- filtration in FFR as a (not conformal) weight based filtn.

w/ $R[\partial\varphi] = 1$

$R[e^{\delta+\varphi}] = 1$

$R[(\partial\varphi + \partial\delta)] = 0.$ $\Rightarrow J^+J^- + J^3J^3 \in G_1 V$ filtered piece

$$\mathbb{C}^2/\mathbb{Z}_2 \cong \mathbb{C}[j^\pm, j^3] / \langle j^+j^- + {j^3}^2 = 0\rangle \qquad \{j^3, j^\pm\} = \pm j^\pm$$

$$\{j^+, j^-\} = 2j^3$$

$\bigcup$ Zariski open $(j^+ \neq 0)$

$$\mathcal{U} \cong \mathbb{C}[j^+, (j^+)^{-1}, j^3] \quad , \quad \{j^3, j^+\} = j^+$$

$$\cong \mathbb{C}[x, x^{-1}, p] \qquad \{p, x\} = 1$$

$$p = j^3/j^+$$

$$\cong T^\vee \mathbb{C}^*$$

residual deg. of freedom $\Big(\underbrace{\text{free vectors} \atop \text{interacting SCFT w/ } \mathcal{M}_H = pt}_{T_{IR}}\Big)$

$$\begin{tikzcd}
T^\vee((\mathbb{C}^\times)^m \times \mathbb{C}^n) \arrow[r, hook, "\text{sympl.}"] & \mathcal{M}_H \\
\mathbb{C}[\cdots] \arrow[r, leftrightarrow] \arrow[d, dashed, leftrightarrow, "X_V"'] & R_H \\
\left(\mathbb{V}[T_{IR}] \otimes\right) \mathcal{D}^{ch}((\mathbb{C}^\times)^m \times \mathbb{C}^n) & \mathbb{V}[T] \arrow[l] \arrow[u, dashed]
\end{tikzcd}$$

$$\Pi^{\otimes m} \otimes (Sb)^{\otimes n}$$

symplectic bosons

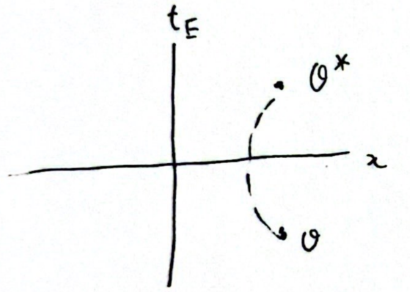
$$
\begin{tikzcd}
(A_1, D_{2n+1}) \arrow[r, "T_{IR}"] \arrow[d, "\mathrm{IV}"'] & (A_1, A_{2n-2}) \arrow[d, "\mathrm{V}"] \\
sl(2)_{-2+\frac{2}{2n+1}} \arrow[r, "\text{DS reduction}"] & (2, 2n+1) \text{ Virasoro}
\end{tikzcd}
$$

$$sl(2)_{-2+\frac{2}{2n+1}} \hookrightarrow \Pi \otimes Vir_{(2,2n+1)}$$

<u>Unitarity in 4d</u>

$*: V^{loc} \to V^{loc}$ (antilinear inv.)

$$\mathcal{O}(\underline{x}, t_E) \to \mathcal{O}^*(\underline{x}, -t_E)$$



$$\langle \mathcal{O}^*(\underline{x}, t_E)\, \mathcal{O}(\underline{x}, t_E) \rangle > 0$$

for Schur ops $\mathcal{O}_{E,R,r,j_1,j_2} \xrightarrow{*} \mathcal{O}^*_{E,-R,-r,j_2,j_1}$

$$E = R + j_1 + j_2 \qquad E \neq (-R) + (j_2) + (j_1)$$

$$r = j_1 - j_2 \qquad (-r) = (j_2) - (j_1)$$

<u>Def.</u> $P: V^{loc} \to V^{loc}$

$$\mathcal{O}(0) \to e^{-\pi i R_2}\, \mathcal{O}^*(0)\, e^{\pi i R_2}$$

Then $\rho : V^{\mathrm{Schur}} \to V^{\mathrm{Schur}}$

$$\rho^2 = (-1)^{2R} = R\text{- parity} \quad (\text{know this})$$

$$\rho \circ d = d^\vee \circ \rho$$

ρ defines a vertex algebra automorphism

$$\rho(a(z)b) = \rho\Big(\sum_n z^n a_{-h_a-n} b\Big)$$

$$= \sum_n \bar{z}^n \underline{\rho(a_{-h_a-n} b)}$$

$$= \rho(a)_{-h_a-n}\rho(b)$$

ρ defined on strong gen's

Positivity ?

def $(\, , \,) : \mathbb{V}_{q.p} \times \mathbb{V}_{q.p} \to \mathbb{C}$

$$(a,b) = \langle a(i)\, \rho(b)(-i) \rangle$$

this obeys $\overline{(a,b)} = (-1)^{2R_a}(b,a)$

def. $\sigma : \mathbb{V} \to \mathbb{V}$

$$a \mapsto (-1)^{2R_a} a \qquad (\text{requires } G.)$$

def $(\!(\, , \,)\!) : \mathbb{V}_{q.p} \times \mathbb{V}_{q.p} \to \mathbb{C}$

$$(\!(a, b)\!) = (a, \sigma(b))$$

Compute $(\!(a,a)\!) = \langle a(2i)\, (\rho\circ\sigma\, a)(0) \rangle > 0.$

n

let $\langle \mathcal{O}(z)\, \rho(\mathcal{O})(0) \rangle \sim \frac{K_{\mathcal{O}}}{z^{2h_{\mathcal{O}}}}$

then $K_{\mathcal{O}} \overset{+}{\propto} (i)^{2h_{\mathcal{O}} - 2R_{\mathcal{O}}} = (i)^{2(j_1 + j_2)} \overset{\text{bosons}}{=} (-1)^{h-R}$

Recap: $(\mathbb{W}^{\circ}, \mathcal{G}_{\bullet}, \rho)$

$\rho^2 = (-1)^{2R}$ (tracks $h - r \bmod \mathbb{Z}$)

$\mathcal{G}_{\bullet}$ good inc. filtr. on $\mathbb{W}_{R,\text{odd}}$ & $\mathbb{W}_{R,\text{even}}$

$\mathcal{G}_{\bullet}$ is non-deg. w.r.t. $(\ ,\ )$ $\Rightarrow$ R-grading

$(1, 1)$ pos. def.

Suppose $\mathbb{W} \simeq Vir_c$

$T(z)$: $h = 2$

$R = 1$

$\rho(T) = T$

$$\langle T(z) T(0) \rangle = \frac{c_{2d}/2}{z^4} < 0$$

$$T^2_{(4)} = (TT) - \frac{3}{10} T'' : \quad h = 4$$

$R \leq 2$

$(R = 2)$

$$\left(T^2_{(4)}, T^2_{(4)}\right) = \frac{c(22 + 5c)}{1088} \geq 0 \Rightarrow c \leq -\frac{22}{5}$$

Assume $\mathcal{G}_p V = \text{span}\{L_{-2-n}, L_{-2-n_k}\,\Omega \;\; k \leq p\}$

$$T_6^{(2)} = T''T + \frac{5}{168} T''''$$

$$T_6^{(3)} = T^3 + \cdots$$

$$\langle T^3_{(6)} T^3_{(6)} \rangle = \frac{c(c-\frac{1}{2})(c+\frac{22}{5})(c+\frac{68}{7})}{(c+24/25)} \leq 0$$

$$c = -\frac{22}{5} \quad \text{or} \quad c \leq -\frac{68}{7} = c(2,7)$$

given $\mathcal{G}_\bullet \Rightarrow$ # of states @ fixed level w/ diff. R-charges

$\Rightarrow$ predict <u>sign</u> of Kac determinant

$\to$ for $c \neq c_{2,2n+3}$, $\text{Vir}(\mathcal{G}_\bullet)$ <u>disallowed</u>.