Singular support and the determinant of cohomology

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$X/\mathbb{F}_q$ proj. curve, $\mathcal{F}$ constr. sheaf over X ($\mathbb{Z}/\ell$-vec. sp.)

$$\det(-F^*, H^*(X,\mathcal{F})) = \prod_{x\in X} \varepsilon(\mathcal{F},\nu)_x$$, ν nonzero meromorphic 1-form on X

$$\det H^*(X,\mathcal{F}) \overset{?}{=} \bigotimes_{x\in X} \varepsilon(\mathcal{F},\nu)_x$$

$X/\mathbb{C}$ $\supset$ S finite subset, (V,∇) on the complement of S, $\det H_{DR}(X\setminus S,(V,\nabla)) = \bigotimes \varepsilon_{DR}(-)$

$\widetilde{X}_S \to S$ real blow up

V^∇

$$\begin{tikzcd} & \widetilde{X}_S \arrow[d] \\ X\setminus S \arrow[ur, "\tilde{j}"] \arrow[r, "j"'] & X \end{tikzcd}$$

$\tilde{j}_* V^\nabla \overset{\text{tame}}{\subset} \tilde{j}_* V^\nabla$

$\uparrow$

moderate growth

$$\begin{array}{ccc} \det H_{DR}(X\setminus S,(V,\nabla)) & = & \bigotimes \varepsilon_{DR}(-) \\ \wr & & \wr \\ \det H_B(\quad) & = & \bigotimes \varepsilon_B(-) \end{array}$$

Global period = $\prod$ local periods

X real analytic mfd, cpt, $\mathcal{F}$ perfect constr. cplx on X of R-modules

$\det(R\Gamma(X,\mathcal{F})) \in \mathcal{L}=\mathcal{L}_R$ graded lines Picard groupoid. $\deg \det R\Gamma = \chi(X,\mathcal{F})$

$T^*X \supset SS(\mathcal{F})$ conical Lagrangian subset

$CC(\mathcal{F})$

<u>D-K</u>: $\chi(X, \mathcal{F}) = (CC(\mathcal{F}), X)$

zero section

<u>Idea</u>: $\det R\Gamma(X, \mathcal{F}) = (\widetilde{CC}(\mathcal{F}), X)$

<u>More precise setting</u>: $Q = Q_R$ K-theory spectrum of R

$Q = (Q_{(0)}, Q_{(1)}, \cdots)$, $Q_{(n)} \simeq \Omega Q_{(n+1)}$

pointed spaces

Homotopy points of Q = those of $Q_{(0)}$.

They form a Picard groupoid ΠQ.

C perfect R-cpx $\longmapsto [C] \in Q$ homotopy point

Filt. on C, $[C] = \sum [gr_i C]$

$$\begin{array}{ccc} \det : Q & \longrightarrow & \mathcal{L} \\ \Psi & & \Psi \\ [C] & \longmapsto & \det(C) \end{array}$$

<u>Want</u>: "compute" $[R\Gamma(X, \mathcal{F})] \in Q$

$X \xrightarrow{\pi} pt$ $\qquad Q_X = \pi^* Q$

$\qquad\;\; Q$ spectrum $\qquad Q_X^! := \pi^! Q$

$C(X, \mathbb{Q})$ $\qquad \mathbb{Q}_{(0)} \times X \to \Omega(\mathbb{Q}_{(1)} \times X) \to \cdots$

$\underset{\text{smash prod.}}{\uparrow}$

$\shortparallel$

$\varinjlim \Omega^i(\mathbb{Q}_{(i)} \times X)$

$$\begin{array}{l} X \\ \cup \\ U \longmapsto C(X,\mathbb{Q})/C(X\setminus U, \mathbb{Q}) \end{array}$$

C means homology

<u>Construction</u>: $X, \mathcal{F} \longmapsto$

a) $\langle \mathcal{F} \rangle \in \Gamma(X, \mathbb{Q}^!_X)$ $\quad (= C(X,\mathbb{Q})$ if X cpt$)$ $\quad (\mathbb{Q} = \mathbb{Q}_{\mathbb{R}})$

$\pi : T^*X \longrightarrow X$, $p^* : \Gamma(X, \mathbb{Q}^!_X) \xrightarrow{\sim} \Gamma(T^*X, \pi^*\mathbb{Q}^!)$

$$\begin{array}{ccc} \Psi & & \\ \langle \mathcal{F} \rangle & \longmapsto & p^*\langle \mathcal{F} \rangle \end{array}$$

b) $\tau_{\mathcal{F}}$: trivilization of $p^*\langle \mathcal{F} \rangle \big|_{T^*X \setminus SS(\mathcal{F})}$

$$\begin{array}{c} \Pi \\ \Gamma(T^*X \setminus SS(\mathcal{F}), \pi^*\mathbb{Q}^!) \end{array}$$

$\mathrm{tr} : \Gamma(X, \mathbb{Q}^!) \to \mathbb{Q}$

$\mathrm{tr}\langle \mathcal{F} \rangle = [R\Gamma(X, \mathcal{F})]$

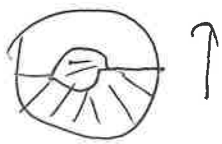
<u>Application</u>: ν cont. 1-form $\quad X \underset{\pi}{\overset{\nu}{\rightleftarrows}} T^*X$, $\quad S = \nu^{-1}SS(\mathcal{F})$

$\langle \mathcal{F} \rangle = \nu^*\pi^*\langle \mathcal{F} \rangle \in \Gamma(X, \mathbb{Q}^!_X)$

$\nu^*(\tau_{\mathcal{F}})$ trivialization on $X \setminus S$

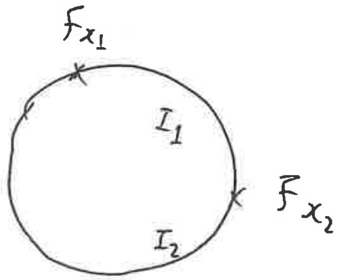
$$(\langle \mathcal{F} \rangle, \nu^*(\tau_{\mathcal{F}})) \in \Gamma(S, \mathbb{Q}^!_S) = C(S, \mathbb{Q})$$
$$\shortparallel$$
$$\Sigma_S(\mathcal{F}, \nu)$$

$$[R\Gamma(X, F)] = \mathrm{tr}\, \xi(F, \nu)$$



$X = S^1$

$\nu = d\theta$



$$F_{x_i} \to F_{I_i},\ F_{I_{i-1}}$$