Faisceaux pervers

Noobs

August 11, 2023

> I give you the mausoleum of all hope and desire...I give it to you not that you may remember time, but that you might forget it now and then for a moment and not spend all of your breath trying to conquer it. Because no battle is ever won he said. They are not even fought. The field only reveals to man his own folly and despair, and victory is an illusion of philosophers and fools.
>
> William Faulkner, *The Sound and the Fury*

Abstract

We are trying to understand the basic theory of *faisceaux pervers* [BBD82], with applications to algebraic geometry and representation theory.

1 Schedule

Time & Venue: Every Tuesday evening of the fall semester of 2023 in the Building of Math Department.

Prerequisite: homological algebra at the level of [GM13], first three chapters of [Har13].

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Lecture -14 Introduction.

- Decomposition in the derived category, Leray–Hirsch and Blowup;
- Deligne's smooth decomposition theorem;
- Artin vanishing Theorem;
- Intersection cohomology;
- Statement of decomposition theorem.

Speaker: 张鼎新

Lecture -13 Six functor formalism.

- Stratification, local systems, constructible sheaves;
- Verdier duality;
- Six functor formalism.

Speaker: 付艺渲

Lecture -12 Perverse t-structure (BBD chaper 2).

- Introduction: Intersection homology and Deligne's formula;
- Definition of t-structure, properties;
- Definition of perverse sheaves.

Speaker: 张鼎新

Lecture -11 Perverse t-structure (BBD chapter 4).

- Affine morphisms;
- Adjunctions;
- Simple objects.

Speaker: 张鼎新

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> The niche D where M dwells, may be recovered from M.
>
> Sasha Beilinson

Lecture -10 How to glue perverse sheaves: vanishing cycle, after A Beilinson [Bei06a], [Bei06c], [Bei06b].

Speaker: 李心宇

Lecture -9 Vanishing cycle (cont'd).

Speaker: 李心宇

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Lecture -8 Proof of BBD.

Speaker: 张鼎新

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Lecture -7 How to glue perverse sheaves: Fourier transform, after Kazhdan–Laumon, Polishchuk–Bezrukavnikov [KL88], [Pol01].

Speaker: 李心宇

Lecture -6 Fourier transform (cont'd).

Speaker: 李心宇

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Lecture -5 Göttsche's formula, cf. [GS93], [Nak99].

Speaker: 陈宇浩

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Lecture -4 Semismall maps, cf. [dCM02].

Speaker: 许福临

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Lecture -3 Affine Grassmannian.

Speaker: 李心宇

Lecture -2 How to convolute perverse sheaves: geometric Satake, after V Drinfeld, V Ginzburg, Mirković–Vilonen [Gin95], [MV00], [MV07].

Speaker: 李心宇

Lecture -1 Geometric Satake (cont'd).

Speaker: 李心宇

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Lecture 0 Perverse coherent sheaves, after P Deligne, Arinkin–Bezrukavnikov [AB09].

Speaker: 许福临

References

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[MV00] Ivan Mirković and Kari Vilonen. Perverse Sheaves on affine Grassmannians and Langlands Duality. *Mathematical Research Letters*, 7(1):13–24, 2000.

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[Nak99] Hiraku Nakajima. *Lectures on Hilbert schemes of points on surfaces*. Number 18. American Mathematical Soc., 1999.

[Pol01] A Polishchuk. Gluing of perverse sheaves on the basic affine space. *Selecta Mathematica, New Series*, 7(1):83–147, 2001.