Introduction to works of Takuro Mochizuki

Pierre Deligne

T $\mathbb{R}$-v.s. $\qquad T^\vee_{\mathbb{C}} \qquad \Lambda^n T^\vee_{\mathbb{C}}$

$\mathbb{C}$-str. on T $\qquad (T^\vee_{\mathbb{C}})^{1,0}, \ (T^\vee_{\mathbb{C}})^{0,1} \qquad \bigoplus_{p+q=n} \Lambda^p T^{\vee 1,0} \otimes \Lambda^q T^{\vee 0,1} = (p,q)$

$z^{-1} \qquad \bar{z}^{-1} \qquad z^{-p}\bar{z}^{-q}$

X $\mathbb{C}$-mfld. $\qquad \Omega^n = \bigoplus_{p+q=n} \Omega^{p,q}$

$d = d' + d''$

V C^∞- v.b. on X $\quad , \ \nabla : V \to \Omega^1(V) \ , \ \nabla(fv) = df \cdot v + f \nabla v$

$\Omega^n(V) \to \Omega^{n+1}(V) \qquad \underline{\nabla^2 = 0 : \text{flat}}$

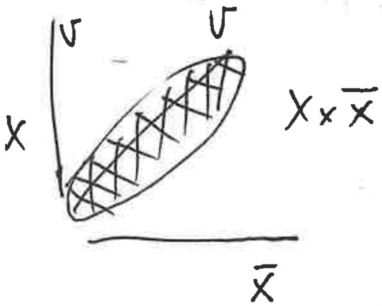
$\nabla(\alpha \cdot v) = d\alpha \cdot v \pm \alpha \cdot \nabla v$

$\nabla', \nabla'' : \quad V \to \Omega^{1,0}(V) \ / \ \Omega^{0,1}(V)$

$\Omega^n(V) \to \Omega^{n+1}(V)$

X $\mathbb{R}$-var.

$\underline{\nabla''^2 = 0 : \text{cplx str. on } V}$



X, V, $\underbrace{\partial', \partial'', u\theta, u^{-1}\bar{\theta}}$ $\quad u \in S^1$

$\quad\quad\quad\quad\quad$ 1,0 $\quad$ 0,1 $\quad$ 1,0 $\quad$ 0,1

$\bar{X}$, $\bar{V}$

$a, b \in \mathbb{C}$ $\quad \left(a(\partial' + \bar{\theta}) + b(\partial'' + \theta)\right)^2 = 0$

$a = b = 1$, $\quad V$ flat bundle

a $\quad b = 1$ $\quad \left((a\partial' + \theta) + (\partial'' + a\bar{\theta})\right)^2 = 0$

$\partial' + \frac{\theta}{a}$ holomorphic conn' $\quad V_a$, ∇_a $\quad$ a-connection

$$\nabla_a(fv) = a\,df\,v + f\,\nabla_a v.$$

$a = 0$: $\theta^2 = 0$

Higgs bundle

$X \times \mathbb{C}_a$

ϕ hermitian V, $\quad \partial' + \partial''$ respect ϕ

$$\partial'\phi(v, w) = \phi(\partial' v, w) + \phi(v, \partial'' w)$$

∂'' $\quad\quad \partial''$ $\quad\quad \partial'$

$\left.\begin{matrix} d \\ \phi \end{matrix}\right)$

θ, $\bar{\theta}$ adjoint

$V \rightsquigarrow \overline{V^\vee}$

$$0 = \phi(\theta v, w) - \phi(v, \bar{\theta} w)$$

∇ $\quad \nabla^u$ $\quad d\phi(v, w)$

$= \phi(\nabla v, w) + \phi(\bar{v}, \nabla^* w')$

$\frac{\nabla + \nabla^*}{2}$ $\quad \partial' + \partial''$

$\frac{\nabla - \nabla^*}{2}$ $\quad \theta - \bar{\theta}$

X curve. V, ∇ locally, V

ϕ $\phi(x)$

space of hermitian forms : Riemannian symmetric space

$$\delta \int \langle d\phi, d\phi \rangle = 0$$

X Kähler

$$H(X, \overset{\mathbb{C}}{\mathbb{R}}) = H(X, \Omega_X^*)$$

$\mathbb{C}^* : T_X$ $d'\alpha = d''\alpha = 0$

$d'd''$?

$$H^* = \bigoplus H^{p,q}$$

$H = H_{\mathbb{Z}}$, $H_{\mathbb{C}} = \bigoplus H^{p,q}$, $H^{p,q} = \overline{H^{q,p}}$

$i \in \mathbb{C}^\times$ acts by $i^{-p} i^{q} = i^{q-p}$

$\mathrm{Im}(\phi)$ 2-form of type $(1,1)$

$\phi(x, x)$ real

$\eta \in H^2(X)$ $\eta^i : H^{n-i} \xrightarrow{\sim} H^{n+i}$

$\mathbb{C}$ $\mathbb{C}\eta$ $\mathbb{C}\eta^2$ $\mathbb{C}\eta^n$

H^1 $H^1\eta$ $H^1\eta^\circ$

$\otimes$ $\otimes\eta$ $*$

$*$

$\phi(x, C\bar{y})$ has a sign

$\psi \qquad \psi(x, c\bar{y}), h > 0$

$$\begin{array}{c} Y \\ \downarrow \\ X \end{array}$$

local system of $H^i(Y_x)$, $H^i_x = \bigoplus H^{p,q}$, ψ

$\bigoplus_{p \geq a} H^{p,q}$ varies hol.

$\bigoplus_{q \geq b} H^{pq}$ varies anti-hol.

d_V^{pq} $\qquad d_V^{pq} \xrightarrow{\partial' + \partial''} V^{p,q}$, $\xrightarrow{\theta} V^{p-1,q+1}$, $\searrow V^{p+1,q-1}$

$$\boxed{\partial' + \partial'' + \theta + \bar{\theta}}$$

$\psi(x, \bar{y}) \qquad \bigoplus H^{pq}, \psi^{pq} \qquad \pm \psi^{pq}$

$$\begin{array}{ccccc} H^{pq} & \xleftarrow{\theta} & H & \xrightarrow{\bar{\theta}} & H \\ \| u & & \| 1 & & \| u^{-1} \\ H & \xleftarrow{u\theta} & H & \xrightarrow{u^{-1}\bar{\theta}} & H \end{array} \qquad u^2$$

$V \hookrightarrow \bar{V}, \bar{\nabla}$ $\qquad \Omega^*(V)$

X $\qquad$ reg sing @ ∞

$\qquad X$