Introduction

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$\left\{ \begin{array}{c} \text{automorphic representations} \\ \pi \quad \text{of} \quad G \end{array} \right\}$ $\longleftarrow \longrightarrow$ $\left\{ \begin{array}{c} \check{G}\text{-valued Galois rep'n} \\ \rho: \mathrm{Gal}(\bar{F}|F) \to \check{G} \end{array} \right\}$

period integrals

numbers

L- functions $\quad L(s, \rho, V)$

$\chi:\ _{F^\times}\backslash \mathbb{A}^\times \longrightarrow \mathbb{C}^\times$

$\int_{F\backslash \mathbb{A}/\mathcal{O}} |x|^{s-\frac{1}{2}}$

$L(s;\chi)$

Generalization

$\int_{G J\ GL_n\ {}_{M_n(F)}\backslash {}^{M_n(\mathbb{A})}/_{M_n(\mathcal{O})}} \varphi(g)\left|\det(g)\right|^{s-\frac{1}{2}} \sim L(s,\rho,\text{std})$

RS $\quad GL_n \times GL_m$

$\quad m < n$

$\int_{GL_m(F)\backslash GL_m(\mathbb{A})/GL_m(\mathcal{O})} \varphi\begin{pmatrix} g & \\ & I_{n-m} \end{pmatrix} \varphi'(g)|\det g|^{s-\frac{1}{2}}$

$\sim \qquad L(s, \rho_n \otimes \rho_m)$

$\quad m = n \quad \int \varphi(g)\, \varphi'(g)\, \text{Eis}$

$\sim$

$$\left\{ \begin{array}{l} (G, M) \quad \text{— Hamiltonian } G\text{-space} \\ \quad \overset{\shortparallel}{} \quad \text{-- period} \quad \text{for auto. rep. of } G \\ \text{eg. } T^*X \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{l} (\check{G}, \check{M}) \quad L\text{-function} \\ \text{for } \check{G}\text{-valued Galois rep'n} \end{array} \right\}$$

Basic example: $X = G/H$

(G, M)- period $\displaystyle\int_{H(F)\backslash H(A)/H(O)} \varphi$

GJ period
$$G = GL_n \times GL_n$$
$$M = T^*X, \quad X = M_n$$

GJ L-func.
$$X^\vee = \left(GL_n \times GL_n \right) \underset{\Delta GL_n}{\times} \underline{\mathbb{A}^n}$$
$$T^*X^\vee = M^\vee$$

RS $m < n$

$$X = GL_n \times GL_m / GL_m$$

$m = n$

$$X = \left(GL_n \times GL_n \right) \underset{\Delta GL_n}{\times} \mathbb{A}^n \qquad\qquad X^\vee = M^n$$