Exercise: Collective Dynamics of Human Mobility

Summer term 2024 - TU Dresden

When you come to a fork in the road, take it.

Yogi Berra

Optimal transport networks

The optimal structure of transportation networks follows from their intended function: the networks should enable fast and efficient transport with minimal investment in the infrastructure. However, depending on the mode of transport, 'fast and efficient' may mean different things. For example, travel time along a road or highway network is typically approximately proportional to the distance traveled. On the other hand, travel time in train or aviation networks significantly depends on the number of connections required to reach the destination due to the waiting times between connecting trains or flights.

Gastner and Newman devised a simple, abstract model to interpolate between the different network structures emerging from these conditions connecting a given set of points V:

- The network $G(E,V)$ should minimize the total cost

  $$C_{\text{tot}} = C_{\text{infra}} + \gamma\, C_{\text{trans}}$$

  given by the weighted sum of infrastructure costs C_{infra} and travel costs C_{trans}. The parameter γ describes the importance of transportation costs relative to infrastructure costs.

- The infrastructure cost is proportional to the total length of the network, measured as the sum of over the geodesic distances $d\left(e_{ij}\right) = d_{ji}$ of all edges $e_{ij} = \{j, i\} \in E$ in the network

  $$C_{\text{infra}} = \sum_{e_{ij} \in E} d(e_{ij})$$

- To represent both the distance traveled along the network and the number edges in the transportation cost, we introduce the effective length $\tilde{d}(e_{ij})$ of an edge e_{ij} as

  $$\tilde{d}(e_{ij}) = (1-\delta)\, d(e_{ij}) + \delta$$

  The parameter δ interpolates between the geodesic distance $d(e_{ij})$ for $\delta = 0$ and simply counting the number of edges for $\delta = 1$.

  The transportation cost for a trip from node i to j are then given as the shortest path distance $\tilde{d}_{ij} = \sum_{e \in \Pi_{ji}} \tilde{d}(e)$ along the shortest path Π_{ji} in the network in terms of these effective distances. Finally, the total transportation cost in the network is given by the weighted sum over all shortest path distances

  $$C_{\text{trans}} = \frac{1}{2} \sum_{i,j \in V} P_i\, P_j\, \tilde{d}_{ji}\,.$$

  where P_i and P_j denote the importance of node i and j, respectively.

As an example, we generate transportation networks $G(V,E)$ based on the $N = 40$ largest German cities and take their importance P_i for $i \in V$ to be proportional to the population of the city. The distance, location, and population data for the cities is available in OPAL.

Task

Implement the model and generate network structures for different values of $\delta \in \{0, 1/3, 2/3, 1\}$.

Hand in:

- figures illustrating the resulting network configurations for $\delta \in \{0, 1/3, 2/3, 1\}$.

No more than 2 pages. Include short captions to explain your figures and tables.

Hints and tips

- You can find more information on this model and the expected results in

  Gastner and Newman,
  Optimal Design of Spatial Distribution Networks,
  Phys. Rev. E, 74(1):016117, 2006

- The data provided in OPAL is already normalized. The direct spatial distances between the cities are normalized such that the average nearest-neighbor distance is 1. The population of the cities is normalized such that the total population is $\sum_{i \in V} P_i = 1$.

  With this normalization, C_{infra} is of the order of the number of cities N and C_{trans} is of the order of unity. To observe an interesting trade-off between infrastructure and transportation costs, choose $\gamma \approx N$ to make both contributions roughly equal in size. You should get a reasonable result using $\gamma = 200$.

- The search space of possible networks is very large. Use simulated annealing to find efficient structures: interpret the total costs of the network as the energy of the system with an effective temperature T and sample network structures with the Metropolis-Monte-Carlo algorithm. Then slowly decrease the temperature T to find structures that minimize the costs.

  In each step, select a pair of cities uniformly at random to add an edge to the network (or remove it from the network if the edge already exists). Accept the change, if either (i) the total cost of the network reduces, $C_{\text{tot,new}} - C_{\text{tot,old}} < 0$, or (ii) with a probability $\exp\left[-\frac{C_{\text{tot,new}} - C_{\text{tot,old}}}{T}\right] < 1$. Repeat this procedure and gradually reduce the temperature (or repeatedly decrease and increase the temperature in case you get stuck in a local minimum of the cost), keeping track of the minimal cost network you find. Make sure to run this process for sufficiently many steps, sampling each possible edge multiple times to sufficiently cover the search space.
  You should get a reasonable result using temperatures $T \in [0.01, 10]$ and sampling N^2 edges for each value of the temperature. You may want to run multiple shorter attempts instead of one long one to cover more of the search space and find better solutions.

  Alternatively, you can use the same idea with a fixed temperature $T = 0$ starting from a complete graph. Randomly sample all edges and remove them, if the total cost of the network decreases. Repeat the process multiple times to sufficiently sample the search space since you are very likely to get stuck in a local minimum.

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