

FACHHOCHSCHULE VORARLBERG

MASTER IN MECHATRONICS

HIGHER MATHEMATICS III

Homework

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1 First Homework

1.1 Linear Least Squares

1.1.1 Script Applied Numerical Computing - Exercises 8.1

Formulate the following problems as least-squares problems. For each problem, give a matrix A and a vector b , such that the problem can be expressed as

$$\text{minimize } \|Ax - b\|^2.$$

(You do not have to solve the problems.)

(a) Minimize
following Equation

$$x_1^2 + 2 * x_2^2 + 3 * x_3^2 + (x_1 - x_2 + x_3 - 1)^2 + (-x_1 - 4 * x_2 + 2)^2 \quad (1.1)$$

According to the lecture the equation

$$\|Ax - b\|^2 \quad (1.2)$$

can be written as

$$\left\| \begin{pmatrix} a^1 \\ a^2 \\ a^3 \\ \vdots \\ a^m \end{pmatrix} * \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} - \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{pmatrix} \right\|^2 = \quad (1.3)$$

$$= (a_{11} * x_1 + a_{12} * x_2 + a_{13} * x_3 - b_1)^2 + (a_{21} * x_1 + a_{22} * x_2 + a_{23} * x_3 - b_2)^2 + \dots \quad (1.4)$$

To find the single parts of the A matrix and the b vector a comparison between the terms of equation 1.1 and the terms of equation 1.4.

$$x_1^2 = (a_{11} * x_1 + a_{12} * x_2 + a_{13} * x_3 - b_1)^2$$

$$a_{11} = 1, a_{12} = 0, a_{13} = 0, b_1 = 0$$

Comparison of the second term from equation 1.1

$$a_{21} = 0, a_{22} = \sqrt{2}, a_{23} = 0, b_2 = 0$$

same steps for the rest of the terms leads to following matrix and vector:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{3} \\ 1 & -1 & 1 \\ -1 & -4 & 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 2 \end{pmatrix}$$

(b) Minimize

following Equation

$$(-6 * x_2 + 4)^2 + (-4 * x_1 + 3 * x_2 - 1)^2 + (x_1 + 8 * x_2 - 3)^2 \quad (1.5)$$

To minimize the equation a factorization with the following equation is necessary:

$$(a_{11} * x_1 + a_{12} * x_2 + a_{13} * x_3 - b_1)^2 + (a_{21} * x_1 + a_{22} * x_2 + a_{23} * x_3 - b_2)^2 + \dots$$

That leads to a Matrix A and a b of:

$$A = \begin{pmatrix} 0 & -6 & 0 \\ -4 & 3 & 0 \\ 1 & 8 & 0 \end{pmatrix} \quad b = \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$$

(c) Minimize

following Equation

$$2 * (-6 * x_2 + 4)^2 + 3 * (-4 * x_1 + 3 * x_2 - 1)^2 + 4 * (x_1 + 8 * x_2 - 3)^2 \quad (1.6)$$

The equation can be rewritten as

$$(-6 * \sqrt{2} * x_2 + \sqrt{2} * 4)^2 + (-4 * \sqrt{3} * x_1 + 3 * \sqrt{3} * x_2 - \sqrt{3})^2 + (2 * x_1 + 16 * x_2 - 6)^2$$

To minimize the equation a factorization with the following equation is necessary:

$$(a_{11} * x_1 + a_{12} * x_2 - b_1)^2 + (a_{21} * x_1 + a_{22} * x_2 - b_2)^2 + \dots$$

That leads to a Matrix A and a b of:

$$A = \begin{pmatrix} 0 & -6 * \sqrt{2} \\ -4 * \sqrt{3} & 3 * \sqrt{3} \\ 2 & 16 \end{pmatrix} \quad b = \begin{pmatrix} -\sqrt{2} * 4 \\ \sqrt{3} \\ 6 \end{pmatrix}$$