Martes 24/Mayo/2016.-

Ayudantía
Intro. a Estadística.

1. $X_1, \dots, X_n \sim$ Uniforme $[0, \theta]$

a) $\mathcal{L}(x_1, \dots, x_n | \theta) = \prod_{i=1}^{n} \frac{1}{\theta} \cdot I_{(0 \leq x_i \leq \theta)}$

$$= \frac{1}{\theta^n} \, I(\max\{x_i\} \leq \theta)$$

Luego $Y_1 = \max\{x_1, \dots, x_n\}$ es el E.M.V. de esta muestra.

b) $\mathbb{E}(x_i) = \frac{\theta + 0}{2} = \frac{\theta}{2} = \overline{X} \Rightarrow \hat{\theta} = 2\overline{X}$

Luego $Y_2 = 2\overline{X}$ es el E.M.

c) $EMC(\hat{\theta}) = Var(\hat{\theta}) + Sesgo^2(\theta)$
$Sesgo(\theta) = \mathbb{E}(\hat{\theta}) - \theta$

Para $\hat{\theta}_{E.M.V.}$ y $F_x(y) = \frac{y}{\theta}$

buscamos $F_Y(y) = \mathbb{P}(Y \leq y)$

$Y = \max\{X_1, \dots, X_n\}$ $= \mathbb{P}(\max\{x_1, \dots, x_n\} \leq y)$ $f_x(y) =$

$= \mathbb{P}(X_1 \leq y, X_2 \leq y, \dots, X_n \leq y)$

$= \prod_{i=1}^{n} \mathbb{P}(X_i \leq y)$

$= (\mathbb{P}(X_i \leq y))^n$

$= (F_X(y))^n$

$\Rightarrow f_Y(y) = n \cdot (F_X(y))^{n-1} \cdot f_X(y) = n \cdot \left(\frac{y}{\theta}\right)^{n-1} \cdot \frac{1}{\theta}$

$f_Y(y) = \frac{d F_Y(y)}{dy} = n \cdot \left(\frac{y}{\theta}\right)^{n-1} \cdot \frac{1}{\theta} = \frac{n \cdot y^{n-1}}{\theta^n} \Rightarrow \mathbb{E}(Y) = \frac{n \cdot \theta}{n+1}$

$$\mathrm{Var}(Y) = \frac{n\theta^2}{n+2} - \frac{n^2\theta^2}{(n+1)^2} \quad \text{(por hint)}$$

$$= \frac{n\theta^2(n^2+2n+1) - n^2\theta^2(n+2)}{(n+1)^2\cdot(n+2)}$$

$$= \frac{n^3\cancel{\theta^2} + \cancel{2n^2\theta^2} + n\theta^2 - n^3\cancel{\theta^2} - \cancel{2n^2\theta^2}}{\text{''}} = \frac{n\theta^2}{(n+1)^2(n+2)}$$

$\therefore$ ~~EM~~ $$ECM(\theta_{E.M.V}) = \frac{n\theta^2}{(n+2)(n+1)^2} + \left(\frac{n\theta}{n+1} - \theta\right)^2$$

$$+ \left(\frac{n\theta - \theta n - \theta}{n+1}\right)^2$$

$$+ \frac{\theta^2}{(n+1)^2}$$

$$= \frac{n\theta^2 + \theta^2(n+2)}{(n+2)(n+1)^2}$$

$$= \frac{2n\cdot\theta^2 + 2\theta^2}{(n+2)(n+1)^2} = \frac{2\theta^2\cancel{(n+1)}}{(n+2)(n+1)^2} = \frac{2\theta^2}{n^2+3n+2}$$

Y para $\hat{\theta}_{E.M} = 2\bar{X}$

$$EMC(\hat{\theta}_{EM}) = \mathrm{Var}(2\bar{X}) + (\mathbb{E}(2\bar{X}) - \theta)^2$$

$$= 4\cdot \mathrm{Var}(\bar{X}) + (2\,\mathbb{E}(\bar{X}) - \theta)^2$$

$X \sim \text{Uniforme}(0,\theta)$

$\mathbb{E}(X) = \theta/2$

$\mathrm{Var}(X) = \theta^2/12$

independencia $$= \frac{4}{n^2}\cdot n\,\mathrm{Var}(X_i) + \left(\frac{2}{n}\,\mathbb{E}(\Sigma X_i) - \theta\right)^2$$

$$= \frac{4}{n^{\cancel{2}}}\,\cancel{n}\,\frac{\theta^2}{\cancel{12}\,3} + \left(\frac{2}{\cancel{n}}\,\cancel{n}\cdot\frac{\theta}{\cancel{2}} - \theta\right)^2$$

$$= \frac{\theta^2}{3n} + 0^2$$

$\hookrightarrow$ insesgado.

$$= \frac{\theta^2}{3n}$$

Elijo el que se va + rápido a 0: $\hat{\theta}_{E.M.V.}$