

Ayudantía 02 MATH 67. 24-03-2022

Problema 3.

a) Demuestre, usando completación de cuadrados, que

$$x^2 - \frac{9x}{5} - \frac{2}{5} \geq 0 \Leftrightarrow x \leq -\frac{1}{5} \vee x \geq 2.$$

Solución:

Notemos que  $\left(\frac{1}{2} \cdot \frac{9}{5}\right)^2 = \left(\frac{-9}{10}\right)^2 = \frac{81}{100}$

de modo que  $\left(x - \frac{9}{10}\right)^2 = x^2 - \frac{9x}{5} + \frac{81}{100}$

$$\Leftrightarrow \left(x - \frac{9}{10}\right)^2 - \frac{81}{100} - \frac{2}{5} = x^2 - \frac{9x}{5} - \frac{2}{5}.$$

$$= \left(x - \frac{9}{10}\right)^2 - \frac{121}{100}$$

$$= \left(x - \frac{9}{10}\right)^2 - \left(\frac{11}{10}\right)^2$$

$$= \left[\left(x - \frac{9}{10}\right) + \frac{11}{10}\right] \cdot \left[\left(x - \frac{9}{10}\right) - \frac{11}{10}\right]$$

$$= \left[x + \frac{1}{5}\right] \cdot [x - 2]$$

$$^{\circ} \quad x^2 - \frac{9x}{5} - \frac{2}{5} \geq 0$$

$$\Leftrightarrow (x + \frac{1}{5})(x - 2) \geq 0$$

$$\Leftrightarrow (x + \frac{1}{5} \geq 0 \wedge x - 2 \geq 0)$$

$$\vee (x + \frac{1}{5} \leq 0 \wedge x - 2 \leq 0)$$

$$\Leftrightarrow (x \geq -\frac{1}{5} \wedge x \geq 2)$$

$$\vee (x \leq -\frac{1}{5} \wedge x \leq 2)$$

$$\Leftrightarrow (x \geq 2) \vee (x \leq -\frac{1}{5}). //$$

b) Resuelva la ecuación

$$\sqrt{x^2 - \frac{9x}{5} - \frac{2}{5}} > \sqrt{\frac{3-9x}{5}}$$

Constraints: Es necesario que  $x^2 - \frac{9x}{5} - \frac{2}{5} \geq 0$   
 y  $\frac{3-9x}{5} \geq 0$ , de modo que las raíces estén bien definidas.

$$\text{Es decir } (x \leq -\frac{1}{5} \vee x \geq 2) \wedge (\frac{1}{3} \geq x)$$

$$\Leftrightarrow x \leq -\frac{1}{5}$$

$$\text{Luego: } \sqrt{x^2 - \frac{9x}{5} - \frac{2}{5}} > \sqrt{\frac{3-9x}{5}} \quad (\geq 0)$$

$$\Leftrightarrow x^2 - \frac{9x}{5} - \frac{2}{5} > \frac{3-9x}{5} \quad \wedge \quad x \leq -\frac{1}{5}$$

$$\Leftrightarrow x^2 > 1 \quad \wedge \quad x \leq -\frac{1}{5}$$

$$\Rightarrow |x| > 1 \quad \wedge \quad x \leq -\frac{1}{5}$$

$$\Rightarrow -x > 1 \quad \wedge \quad x \leq -\frac{1}{5}$$

$$\Rightarrow -1 > x \quad \wedge \quad x \leq -\frac{1}{5}$$

$$\Rightarrow x < -1.$$

$$\Rightarrow x \in (-\infty, -1).$$