a) $Y = \frac{1}{2}X^2; \quad X \sim U(-2,2)$ (1)

$E(Y) = \frac{1}{2} E(X^2)$

$Var(Y) = \frac{1}{4} Var(X^2)$

$= \frac{1}{4} \left\{ E(X^4) - E(X^2)^2 \right\}$

Hay que calcular $E(X^2)$ y $E(X^4)$

$E(X^2) = \int_{-2}^{2} \frac{x^2}{4} dx = \frac{1}{4} \cdot 2 \int_0^2 x^2 dx$

$= \frac{1}{2} \frac{1}{3} x^3 \Big|_0^2 = \frac{2^3}{2 \cdot 3} = \frac{4}{3}$

$E(X^4) = \frac{1}{4} \cdot 2 \int_0^2 x^4 dx$

$= \frac{1}{2} \frac{1}{5} x^5 \Big|_0^2 = \frac{2^5}{2 \cdot 5} = \frac{16}{5}$

$\Rightarrow E(Y) = \frac{1}{2} \cdot \frac{4}{3} = 2/3$

$Var(Y) = \frac{1}{4} \left\{ \frac{16}{5} - \frac{16}{9} \right\} = \frac{16}{45}$

1b) $y = x^2/2$ no es biunívoca en $(-2, 2)$, pero lo es en $(-2,0)$ y $(0,2)$, en $\{0\}$ es $P(X=0)=0$ ②

Luego, hay dos inversas

$x_1 = +\sqrt{2y}$ y $x_2 = -\sqrt{2y}$

$$\Rightarrow f_Y(y) = \begin{cases} \left|\frac{\partial \sqrt{2y}}{\partial y}\right| f_X(+\sqrt{2y}) + \left|\frac{\partial(-\sqrt{2y})}{\partial y}\right| f_X(-\sqrt{2y}), & \text{si } 0<y<2 \\ 0 & \sim \end{cases}$$

$$= \begin{cases} \left|\frac{1}{\sqrt{2y}}\right| \frac{1}{4} + \left|-\frac{1}{\sqrt{2y}}\right| \cdot \frac{1}{4} & \text{si } 0<y<2 \\ 0 & \sim \end{cases}$$

$$= \begin{cases} \frac{1}{2\sqrt{2y}} & \text{si } 0<y<2 \\ 0 & \sim \end{cases}$$

2a) $X \sim N(1,4)$

$\Rightarrow Z = \frac{X-1}{2} \sim N(0,1)$

$\Rightarrow P(-1 < X < 3) = P(-1 < Z < 1)$

$= F_Z(1) - F_Z(-1)$

$= \Phi(1) - \Phi(-1)$

Pero, $\Phi(-1) = 1 - \Phi(1)$

$\Rightarrow P(-1 < X < 3) = 2\Phi(1) - 1$

2b) De los datos,

$M_X(t) = e^{t + 2t^2}$

$M_Y(t) = E(e^{tY}) = E(e^{t\frac{X-1}{2}})$

$= E(e^{tX/2} e^{-t/2})$

$= E(e^{tX/2}) e^{-t/2}$

$= M_X(t/2) e^{-t/2}$

$M_Y(t) = e^{t/2 + 2(t/2)^2} e^{-t/2}$ ④

$= e^{2t^2/4} = e^{t^2/2}$

$=$ f g m de $Y \sim N(0,1)$

3a) De la tabla:

$x \backslash y$	-1	0	1	$P(X=x)$
-1	1/9	1/9	1/9	3/9
0	1/9	1/9	1/9	3/9
1	1/9	1/9	1/9	3/9
$P(Y=y)$	3/9	3/9	3/9	

$P(X=x, Y=y)$

marg de X

marg de Y

3b) X e Y no son indep, por ejemplo:

$P(X=-1, Y=-1) = 1/9 \neq P(X=-1)\,P(Y=-1)$

4a) $f_X(x) = \int^{x}$

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4a) $f_X(x) = \int_0^\infty 4xe^{-(x^2+y^2)}dy$

$= 2xe^{-x^2}\int_0^\infty 2ye^{-y^2}dy$

$= 2xe^{-x^2}\, e^{-y^2}\Big|_0^\infty$

$= \begin{cases} 2xe^{-x^2} & \text{si } x>0 \\ 0 & \text{si no} \end{cases}$

Similarmente,

$f_Y(y) = \begin{cases} 2ye^{-y^2} & \text{si } y>0 \\ 0 & \text{si no} \end{cases}$

4b) Claramente,

$f_{X,Y}(x,y) = f_X(x)\, f_Y(y) \quad \forall (x,y)$

$\Leftrightarrow$ X e Y son v.a.'s indep.