CLASE 11 : GRÁFICA DE FUNCIONES
FUNCIÓN CUADRÁTICA

• Ej: Esbozar la gráfica de

$$g(x) = 3 - \sqrt{4-2x}$$

Sol:

$$g(x) = A f(w(x-h)) + C$$

$$\begin{aligned} g(x) &= 3 - \sqrt{4-2x} \\ &= 3 - \sqrt{2(2-x)} \\ &= 3 - \sqrt{-2(x-2)} \end{aligned}$$

Orden de las operaciones:

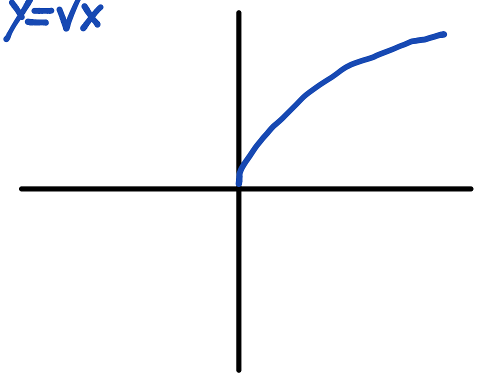
1.- Reflexiones

2.- Compresiones/elongaciones

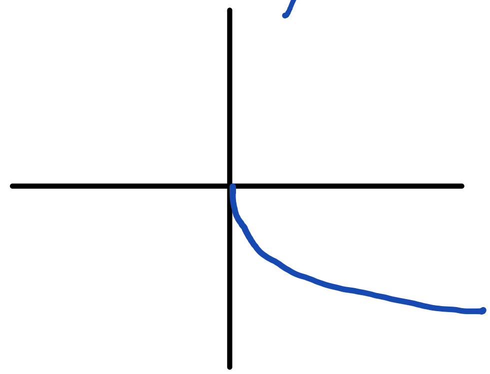
3.- Traslaciones

$g(x) = 3 - \sqrt{-2(x-2)}$

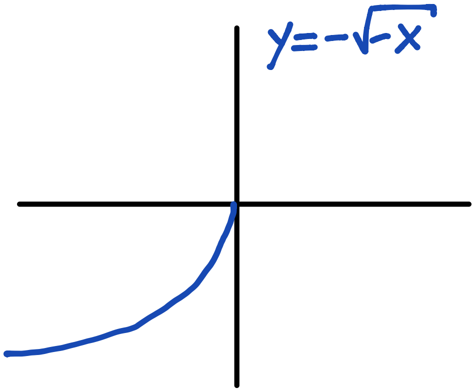
$y=\sqrt{x}$



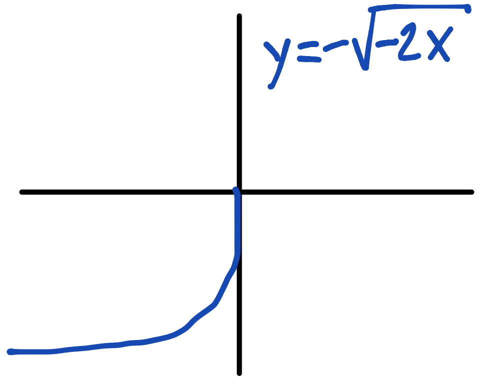
$y=-\sqrt{x}$



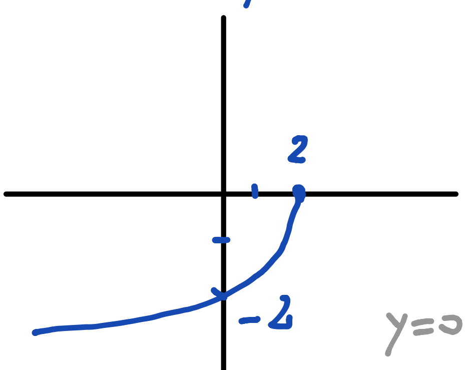
$y=-\sqrt{-x}$



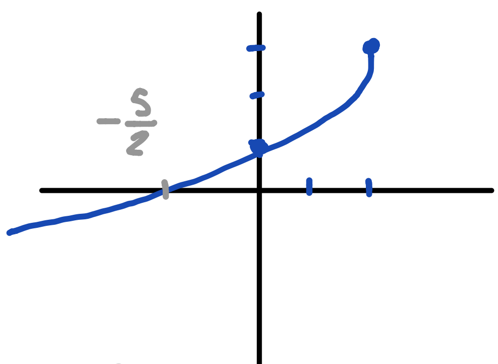
$y=-\sqrt{-2x}$



$y=-\sqrt{-2(x-2)}$



$y=-\sqrt{-2(x-2)}+3$



$y=0$

$\sqrt{-2(x-2)}=3$ $\qquad$ $x-2=-\frac{9}{2}$

$-2(x-2)=9$ $\qquad$ $x=-\frac{9}{2}+2=-\frac{5}{2}$

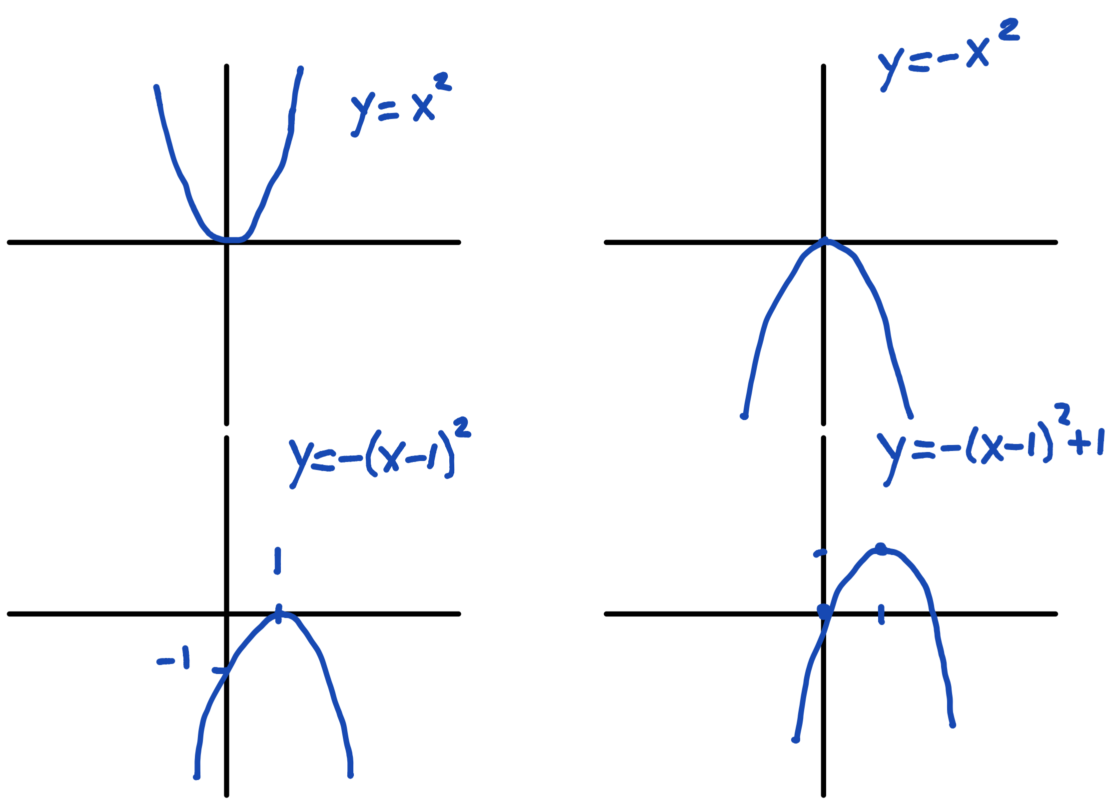
• Ej: Trazar la gráfica de

$$p(x) = 2x - x^2$$

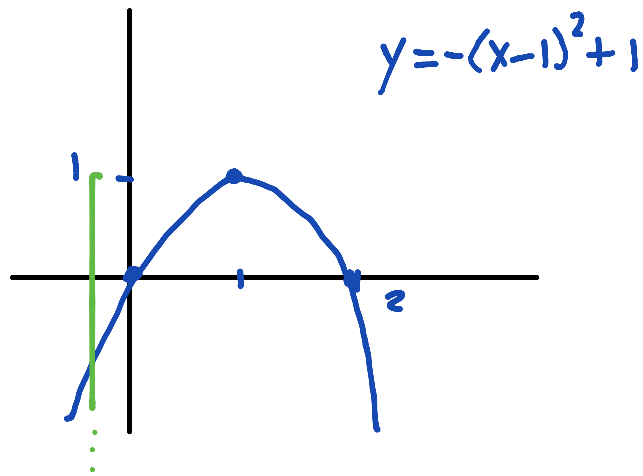
Sol:

$$p(x) = Af(w(x-h)) + C$$

$$\begin{aligned} p(x) &= 2x - x^2 \\ &= -(x^2 - 2x) \\ &= -(x^2 - 2x + 1) + 1 \\ &= -(x-1)^2 + 1 \end{aligned}$$



• <u>Obs</u>:



$$\text{Rec } p = (-\infty, 1]$$

<u>DEM</u>: • Sea $y \leq 1$

$$y=-(x-1)^2+1 \iff (x-1)^2 = 1-y \geq 0$$

$$\iff |x-1| = \sqrt{1-y}$$

$$\iff x = 1 \pm \sqrt{1-y}$$

$$\implies (-\infty, 1] \subseteq \text{Rec } f$$

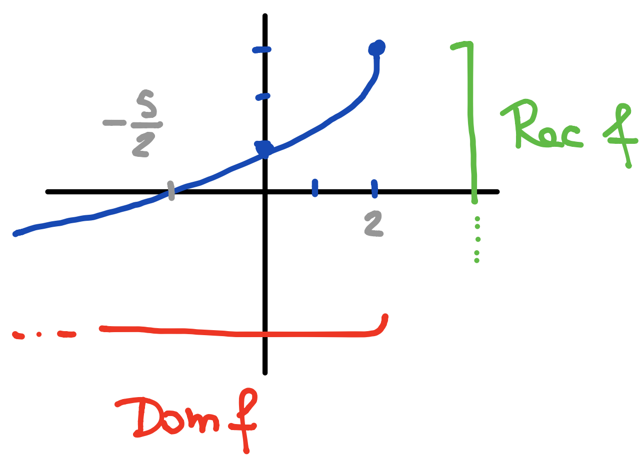
• $\text{Rec} f \subseteq (-\infty, 1]$

$$\iff (1, \infty) \subseteq (\text{Rec} f)^c$$

$$\left.\begin{array}{l} \text{Si } y>1 \\ y \in \text{Rec} f \end{array}\right\} 0 \leq (x-1)^2 = 1-y < 0 \quad \rightarrow\!\!\!\!\times$$

$\square$

• Ej: $f(x) = 3 - \sqrt{-2(x-2)}$



Demostrar que: i) $\text{Dom} f = (-\infty, 2]$

ii) $\text{Rec} f = (-\infty, 3]$

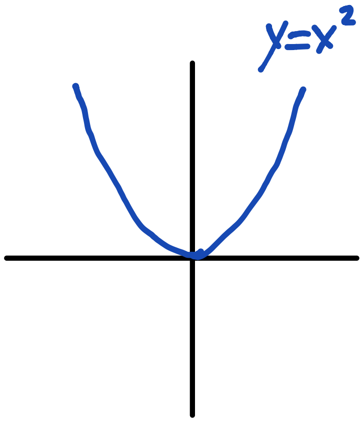
• Ej: Esbozar la gráfica de

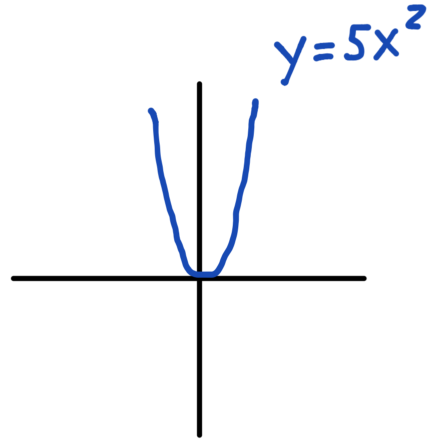
$$p(x) = 5x^2 + 30x + 49$$

Sol: $p(x) = 5(x^2 + 6x) + 49$

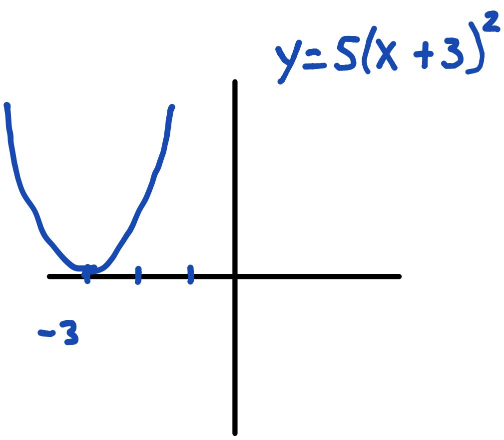
$= 5(x^2 + 6x + 9) + 49 - 45$

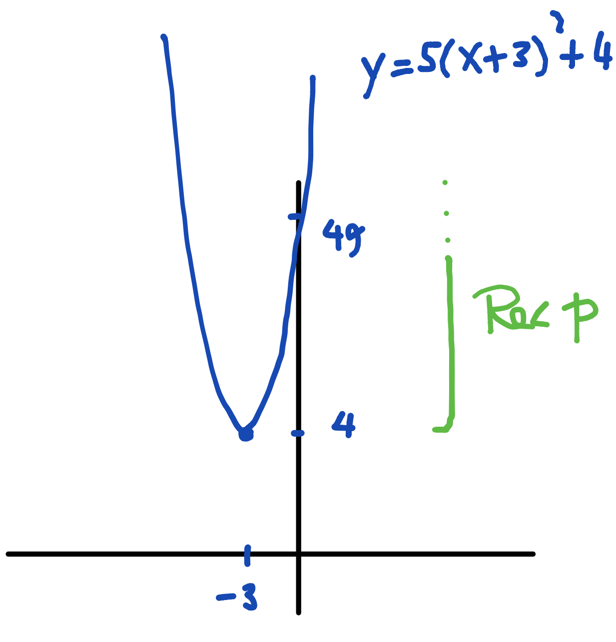
$= 5(x+3)^2 + 4$





$x+3 = x-(-3)$





- <u>Completación de cuadrado:</u>

$$ax^2+bx+c = a\left(x^2+\frac{b}{a}x\right)+c$$

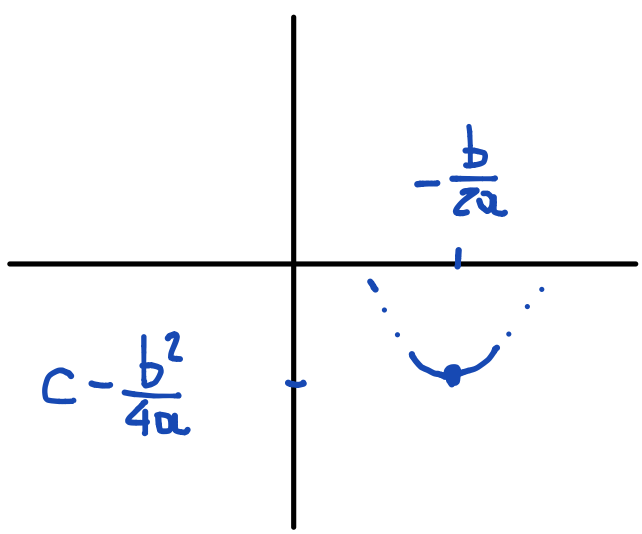
$$= a\left(x^2+2\cdot\frac{b}{2a}x\right)+c$$

$$= a\left(x^2+2\cdot\frac{b}{2a}x+\frac{b^2}{4a^2}\right)+c-\frac{b^2}{4a}$$

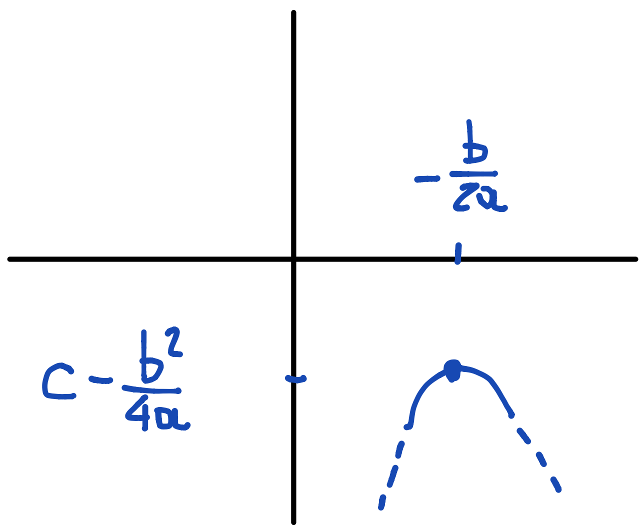
$$= a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}$$

$$p(x)=ax^2+bx+c = a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}$$

- <u>$a>0$</u>:



• <u>$a<0$</u>:



• <u>Lema</u>: Sea $p(x)=ax^2+bx+c$, $a\neq 0$.

1.- Si $a>0$, entonces:

i) p alcanza su valor mínimo en $x=-\frac{b}{2a}$

y $p\left(-\frac{b}{2a}\right)=c-\frac{b^2}{4a}$

ii) $\operatorname{Rec} p=\left[c-\frac{b^2}{4a},\infty\right)$

2.- Si $a<0$, entonces:

i) p alcanza su valor máximo en $x=-\frac{b}{2a}$

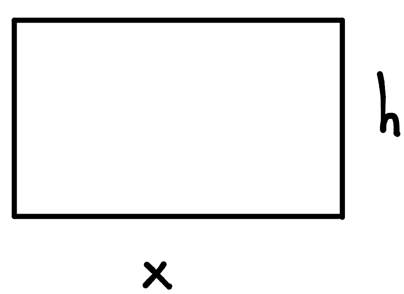
y $p\left(-\frac{b}{2a}\right)=c-\frac{b^2}{4a}$

ii) $\operatorname{Rec} p = \left(-\infty, c - \frac{b^2}{4a}\right]$

DEM: Ejercicio $\square$

- **Ej**: Encuentre el rectángulo de perímetro 20 que maximize el área.

Sol:



x: largo de la base

h: altura

Dato:

$$2x + 2h = 20$$

$$\rightarrow h = 10 - x$$

Área: $xh = x(10-x) = A(x)$

$$A(x) = 10x - x^2$$

$$= -(x^2 - 10x)$$

$$= -(x^2 - 10x + 25) + 25$$

$$= -(x-5)^2 + 25$$

Luego, A alcanza su valor máximo en $x = 5$

con $A(5) = 25$

<u>Conclusión</u>: el rectángulo buscado es un cuadrado de lado 5.

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• <u>Obs</u>: $ax^2 + bx + c = 0$, $a \neq 0$

$$\Leftrightarrow \quad a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} = 0$$

$$\Leftrightarrow \quad a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c = \frac{b^2 - 4ac}{4a}$$

$$\Leftrightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\Leftrightarrow \quad \left| x + \frac{b}{2a} \right| = \sqrt{\frac{b^2-4ac}{4a^2}} \qquad (b^2-4ac > 0)$$

$$\Leftrightarrow \quad x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$\Leftrightarrow \quad x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\Leftrightarrow \quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

• <u>Obs</u>: outra forma (Po-Shen Loh)

$$ax^2 + bx + c = 0 \qquad (a \neq 0)$$

$$\Leftrightarrow \quad x^2 + \tilde{b}x + \tilde{c} = 0, \quad \tilde{b} = \frac{b}{a}, \quad \tilde{c} = \frac{c}{a}$$

Buscamos m, n tq

$$x^2 + \tilde{b}x + \tilde{c} = (x-m)(x-n)$$

$$= x^2 - (m+n)x + mn$$

Luego, $m+n=-\tilde{b}$

$$\Rightarrow \frac{m+n}{2}=-\frac{\tilde{b}}{2}$$

Por lo tanto, buscamos z tq

$$m=-\frac{\tilde{b}}{2}+z, \quad n=-\frac{\tilde{b}}{2}-z$$

Ahora, como $mn=\tilde{c}$

$$\left(-\frac{\tilde{b}}{2}+z\right)\left(-\frac{\tilde{b}}{2}-z\right)=\tilde{c}$$

$$\Rightarrow \frac{\tilde{b}^2}{4}-z^2=\tilde{c}$$

$$\Rightarrow z^2=\frac{\tilde{b}^2}{4}-\tilde{c}=\frac{b^2}{4a^2}-\frac{c}{a}=\frac{b^2-4ac}{4a^2}$$

$$\Rightarrow z=\pm\sqrt{\frac{b^2-4ac}{4a^2}} \quad (b^2-4ac>0)$$

$$\Rightarrow m=-\frac{b}{2a}+\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$n=-\frac{b}{2a}-\sqrt{\frac{b^2-4ac}{4a^2}}$$