Ayudantía 3 MAT 1107

① a)

	Ase.	Ens.
M :	3	1
S :	2	2

x_1: Tiempo Aserrando mesas
x_2: Tiempo Aserrando sillas
y_1: Tiempo Ensambaje mesas
y_2: Tiempo Ensambaje sillas

Máximo: $3x_1 + 2x_1 < 12$
$y_1 + 2y_2 < 8$

O Sea: { Ensamblan máx entre 8 mesas y 0 sillas hasta 0 mesas y 4 sillas
Casos límites no entran { Aserran máx entre 4 mesas y 0 sillas hasta 0 mesas y 6 sillas

∴ No pueden producir más de 4 mesas completas al día ni tampoco más de 4 sillas. Quedan las siguientes combin.

(mesas, sillas) : (0, 0) (1, 0) (2, 0) (3, 0) ~~(4, 0)~~ (0, 1) (0, 2) (0, 3) ~~(0, 4)~~ (1, 1) (1, 2) (1, 3) (2, 1) (3, 1) (2, 2)

b)

$$\sqrt{x^2+1} > 2 \Rightarrow x^2+1 > 4 \Rightarrow x^2 > 3 \quad / \sqrt{\ }$$

$$\Rightarrow |x| > 3 \quad \therefore \quad x > \sqrt{3} \vee x < -\sqrt{3}$$

c) $|x+1| \leq \sqrt{x^2-4x+4} + 3$

Ambos lados positivo: $(x+1)^2 \leq x^2 - 4x + 4 + 6 \cdot \sqrt{x^2-4x+4} + 9$

$$\Rightarrow \cancel{x^2} + 2x + 1 \leq \cancel{x^2} - 4x + 13 + 6 \cdot (x-2) \quad / +4x - 13$$

$$6x - \cancel{12} \leq 6x - \cancel{12}$$

$x \leq x$ Se cumple para todos los reales ($\mathbb{R}$)

$\therefore x \in \mathbb{R}$

② a) $|5x+5|-8 \leq 17$

$|5x+5| \leq 25 \Rightarrow -25 \leq 5x+5 \leq 25$

$-30 \leq 5x \leq 20 \quad \therefore \quad -6 \leq x \leq 4 \quad \Rightarrow x \in [-6;4]$

b) $\left|\frac{x^2+5x+4}{x^2-4x-5}\right| > 2 \Rightarrow \left|\frac{(x+4)\cdot\cancel{(x+1)}}{(x-5)\cdot\cancel{(x+1)}}\right| > 2 \Rightarrow \left|\frac{x+4}{x-5}\right| > 2$

C1: $\frac{x+4}{x-5} > 2 \Rightarrow x+4 > 2x-10 \Rightarrow 14 > x \qquad S_1 =]-\infty, 14[$

C2: $\frac{x+4}{x-5} < -2 \quad /\cdot(x-5)$

a) $(x-5)<0 \Rightarrow x+4 > -2x+10 \Rightarrow 3x > 6 \Rightarrow x > 2 \qquad S_2 =]2;\infty[$

b) $(x-5)>0 \Rightarrow x+4 < -2x+10 \Rightarrow 3x < 6 \Rightarrow x < 2 \Rightarrow (x-5)<0$ ⚠

$\therefore \quad x \in S_1 \cap S_2 =]2, 14[$

c) $|3x+2| \geq |x+1| + |2x+1| \Rightarrow |3x+2| - |x+1| - |2x+1| \geq 0$

$f(x)$	$-\infty$ C1 -1	C2 $-2/3$	C3 $-1/2$	C4 $+\infty$
$3x+2$	$-$	$-$	$+$	$+$
$x+1$	$-$	$+$	$+$	$+$
$2x+1$	$-$	$-$	$-$	$+$

C1: $x \in]-\infty, -1]$

$\Rightarrow -(3x+2)+(x+1)+(2x+1) \geq 0$

$\Rightarrow 0 \geq 0 \quad \therefore \quad S_1 =]-\infty, -1]$

C2: $x \in]-1, -2/3] \Rightarrow -(3x+2)-(x+1)+(2x+1) \geq 0$

$\Rightarrow -2x-2 \geq 0 \Rightarrow x \leq -1 \quad \therefore S_2 =]-\infty,-1] \cap]-1,-2/3] = \emptyset$

C3: $x \in]-2/3; -1/2] \Rightarrow (3x+2)-(x+1)+(2x+1) \geq 0$

$\Rightarrow 4x+2 \geq 0 \Rightarrow x \geq -1/2 \quad \therefore S_3 =]-2/3;-1/2] \cup [-1/2, \infty[= [-1/2]$

C4: $x \in]-1/2; \infty[\Rightarrow (3x+2)-(x+1)-(2x+1) \geq 0$

$\Rightarrow 0 \geq 0 \quad \therefore S_4 =]-1/2; \infty[$

$S_T = S_1 \cup S_2 \cup S_3 \cup S_4 =]-\infty; -1] \cup [-1/2; \infty[$

d) $|x^2-2x| + x\cdot|x-3| \geq 3 \Rightarrow |x|\cdot|x-2| + x\cdot|x-3| \geq 3$

	$-\infty$	0	2	3 $+\infty$
x	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$		$+$
$x-3$	$-$	$-$	$-$	$+$

C1: $x \in (-\infty, 0)$

$\Rightarrow -x\cdot(-(x-2)) + x\cdot(-(x-3)) \geq 3$

$\Rightarrow \cancel{x^2} - 2x - \cancel{x^2} + 3x \geq 3$

$\Rightarrow x \geq 3 \quad \therefore S_1 =]-\infty, 0[\cap [3, \infty[$
$= \emptyset$

C2: $x \in [0, 2[$

$\Rightarrow x\cdot(-(x-2)) + x\,(-(x-3)) \geq 3 \Rightarrow -x^2 + 2x - x^2 + 3x \geq 3$

$\Rightarrow -2x^2 + 5x - 3 \geq 0 \Rightarrow 2x^2 - 5x + 3 \leq 0 \Rightarrow (2x-3)\cdot(x-1) \leq 0$

a) $2x-3 \leq 0 \wedge x-1 \geq 0$ b) $2x-3 \geq 0 \wedge x-1 \leq 0$

$\Rightarrow x \leq 3/2 \wedge x \geq 1$ $x \geq 3/2 \wedge x \leq 1$

$\Rightarrow x \in [1, 3/2]$ $x \in]-\infty; 1] \cap [3/2, \infty[= \emptyset$

$S_2 = [1; 3/2] \cap [0; 2[\Rightarrow [1; 3/2]$

C3: $x \in [2, 3[$

$\Rightarrow x\cdot(x-2) + x\cdot(-(x-3)) \geq 3 \Rightarrow \cancel{x^2} - 2x - \cancel{x^2} + 3x \geq 3$

$\Rightarrow x \geq 3 \quad \therefore S_3 = [2, 3[\cap [3, \infty[= \emptyset$

C4: $x \in [3, \infty[$

$\Rightarrow x\cdot(x-2) + x\cdot(x-3) \geq 3 \Rightarrow x^2 - 2x + x^2 - 3x \geq 3$

$\Rightarrow 2x^2 - 5x - 3 \geq 0 \Rightarrow (2x+1)(x-3) \geq 0$

$\Rightarrow$ a) $2x+1 \geq 0 \wedge x-3 \geq 0$ b) $2x+1 \leq 0 \wedge x-3 \leq 0$

$x \geq -1/2 \wedge x \geq 3$ $x \leq -1/2 \wedge x \leq 3$

$x \in [3, \infty[$ $x \in]-\infty, -1/2] \cap [3, \infty[= \emptyset$

$S_4 = [3, \infty[$

$\therefore S_T = S_1 \cup S_2 \cup S_3 \cup S_4 = [1; 3/2] \cup [3; \infty[$

⑤ a) $a^2+d^2=1=c^2+d^2$

$$ac+bd\le 1$$

$(a-c)^2\ge 0 \Rightarrow a^2+c^2-2ac\ge 0 \Rightarrow a^2+c^2\ge 2ae$

$(b-d)^2\ge 0 \Rightarrow b^2+d^2\ge 2\cdot bd$

$\therefore (a-c)^2+(b-d)^2\ge 0 \iff a^2+b^2+c^2+d^2\ge 2(ac+bd)$

$\therefore a^2+b^2+c^2+d^2=2 \Rightarrow 2/2\ge ac+bd$

$\Rightarrow ac+bd\le 1$ //

b) $x\in\mathbb{R}^+$

$x^3+\frac{1}{x^3}\ge x+\frac{1}{x} \quad \Rightarrow \quad x^3+\frac{1}{x^3}-x-\frac{1}{x}\ge 0$

$\Rightarrow (x^3-x)+\left(\frac{1}{x^3}-\frac{1}{x}\right) = x(x^2-1)-\left(\frac{x^2-1}{x^3}\right)$

$\Rightarrow (x^2-1)\cdot(x-1/x^3) = \frac{(x^2-1)\cdot(x^4-1)}{x^3} = \frac{(x^2-1)\cdot(x^2-1)\cdot(x^2+1)}{x^3}$

$= \frac{(x^2-1)^2\cdot(x^2+1)}{x^3}$

Luego: $(x^2-1)^2\ge 0 \ \forall x$; $(x^2+1)>0$: $x^2>0$

y $x^3>0$ porque $x>0$

$\therefore \frac{(x^2-1)^2\cdot(x^2+1)}{x^3}\ge 0$

$\Rightarrow x^3+\frac{1}{x^3}\ge x+\frac{1}{x}$ //