1

a) $\frac{5x+5}{2} - 8 \leq 17$

$\frac{5x+5}{2} \leq 25 \quad \Rightarrow \quad \leq 5x+5 \leq 50 \Rightarrow x \leq 9$

$x \in (-\infty, 9)$

b) $\frac{(x+2)\cdot(x-7)}{(x-3)^2} \geq 0$

$(x-3)^2 \geq 0 \ \forall x$

$\therefore (x+2)\cdot(x-7) \overset{x=?}{\geq} 0$

	$-\infty$		-2		3		7		∞
$x+2$		$-$		$+$		$+$		$+$	
$x-7$		$-$		$-$		$-$		$+$	
$x-3$		$-$		$-$	0	$+$		$+$	
$\frac{(x+2)(x-7)}{(x+3)^2}$		$+$	0	$-$	$\Vert$	$-$	0	$+$	

$S_T = (-\infty, -2] \cup [7, \infty)$

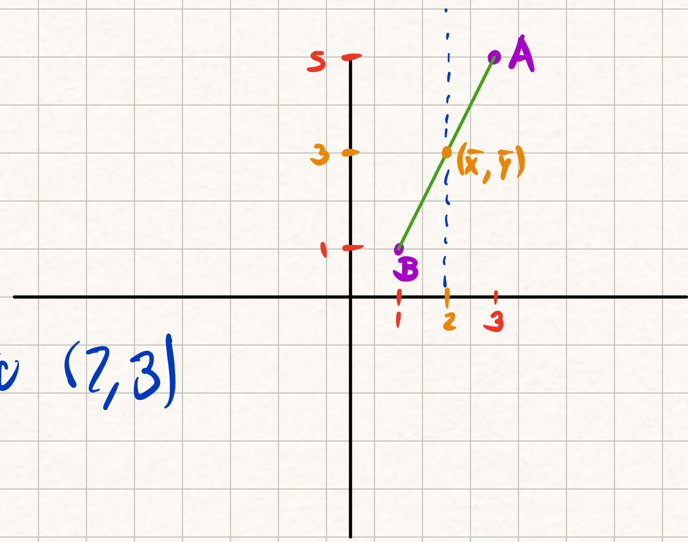
c) $\frac{(x+2)\cdot(x-7)}{(x+3)^2} \geq 0$

$(x+3)^2 \geq 0 \ \forall x$

	$-\infty$		-3		-2		7		∞
$x+2$		$-$		$-$		$+$		$+$	
$x-7$		$-$		$-$		$-$		$+$	
$x+3$		$-$	0	$+$		$+$		$+$	
$\frac{(x+2)(x-7)}{(x+3)^2}$		$+$	$\Vert$	$+$	0	$-$	0	$+$	

$\therefore S_T = (-\infty, -3) \cup (-3, -2] \cup [7, \infty)$

② $A=(3,5) \qquad B=(1,1)$



a) $\bar{x} = \dfrac{x_1+x_2}{2} \; ; \; \bar{y} = \dfrac{y_1+y_2}{2}$

$$\left.\begin{array}{l} \Rightarrow \bar{x} = \dfrac{3+1}{2} = \dfrac{4}{2} = 2 \\[2ex] \Rightarrow \bar{y} = \dfrac{5+1}{2} = \dfrac{6}{2} = 3 \end{array}\right\} \text{El punto } (2,3)$$

b) $d = \sqrt{(A_x - B_x)^2 + (A_y - B_y)^2}$

c) $d = \sqrt{(3-1)^2 + (5-1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

d) $d_{B,(2,a)} = \sqrt{(2-1)^2 + (a-1)^2} \leq \sqrt{5}$

$\Rightarrow 1 + (a-1)^2 \leq 5$

$\Rightarrow 1 + (a-1)^2 \leq 5$

$(a-1)^2 \leq 4 \quad \Rightarrow \left.\begin{array}{l} a-1 \leq 2 \\ a-1 \geq -2 \end{array}\right\} \begin{array}{l} a \leq 3 \\ a \geq -1 \end{array}$

$a \in [-1,3]$

(3) $\frac{x+1}{x} \leq \frac{x+1}{x-1} - \frac{3}{x} \Rightarrow 0 \leq \frac{x+1}{x-1} - \frac{x+1}{x} - \frac{3}{x}$

$x \neq 1 \wedge x \neq 0 \Rightarrow 0 \leq \frac{(x+1)\cdot x - (x+1)\cdot(x-1) - 3\cdot(x-1)}{(x-1)\cdot x}$

$0 \leq \frac{(x+1)\cdot(x-x+1) - 3(x-1)}{(x-1)\cdot x} = \frac{-2x+4}{(x-1)\cdot x} \Rightarrow 0 \leq \frac{-2(x-2)}{(x-1)\cdot x}$

	$-\infty$ — 0	0 — 1	1 — 2	2 — $+\infty$
x	$-$	$+$	$+$	$+$
$x-1$	$-$	$-$	$+$	$+$
$x-2$	$-$	$-$	$-$	$+$
-2	$-$	$-$	$-$	$-$
$\frac{-2(x-2)}{(x-1)\cdot x}$	$+$	$-$	$+$	$-$

$\therefore\ x \in (-\infty, 0) \cup (1, 2]$

$: \ 0 \leq \frac{-2(x-2)}{(x-1)\cdot x}$

(4) $|a| \neq |c|$

$|ax+b| = |cx+d|$

$|ax+b| - |cx+d| = 0$

Caso 1: $ax+b-cx-d=0 \Rightarrow (a-c)\cdot x = -(b-d) \Rightarrow x = \frac{-(b-d)}{a-c} = \frac{d-b}{a-c}$

Caso 2: $ax+b+cx+d=0 \Rightarrow x\cdot(a+c) = -(b+d) \Rightarrow x = \frac{-(b+d)}{a+c}$

Caso 3: $-ax-b+cx+d=0 \Rightarrow x(c-a) = b-d \Rightarrow \frac{b-d}{c-a} = \frac{d-b}{a-c}$

Caso 4: $-ax-b-cx-d=0 \Rightarrow -x\cdot(a+c) = b+d \Rightarrow x = -\frac{(b+d)}{a+c}$

$\therefore\ C1 = C3\ ;\ C2 = C4\ :\ C1 \neq C2$

$\Rightarrow$ Hay 2 soluciones y no más. (Podría ser 1 si $d-\cancel{b} = \cancel{b}-d$ $\Rightarrow d = 0$)

5 • $x^2 - 20x + 91 > 0 \Rightarrow (x-7)\cdot(x-13) > 0$

$\therefore (x-7>0 \wedge x-13>0) \vee (x-7<0 \wedge x-13<0)$

$x>7 \wedge x>13 \quad \vee \quad x<7 \wedge x<13$

$x \in (13,\infty) \quad \vee \quad x \in (-\infty, 7)$

$\therefore x \in (-\infty,7) \cup (13,\infty)$

• $x^3 - 20x + 91x > 0 \Rightarrow x\cdot(x-7)\cdot(x-13) > 0$

ptos. criticos: $0;\ 7;\ 13$

	$-\infty$		0		7		13		∞
x		$-$	0	$+$		$+$		$+$	
$x-7$		$-$		$-$	0	$+$		$+$	
$x-13$		$-$		$-$		$-$	0	$+$	
$x^3-20x+91x$		$-$	0	$+$	0	$-$	0	$+$	

$\therefore x \in (0,7) \cup (13,\infty)$

• $x^4 - 20x^2 + 91 > 0$

$\Rightarrow (x^2-13)\cdot(x^2-7) > 0$

$\therefore (x^2-13>0 \wedge x^2-7>0) \quad \vee \quad (x^2-13<0 \wedge x^2-7<0)$

$x^2>13 \quad x^2>7 \qquad x^2<13 \wedge x^2<7$

$x>\sqrt{13};\ x<-\sqrt{13} \wedge x>\sqrt{7};\ x<-\sqrt{7} \quad \vee \quad -\sqrt{13}<x<\sqrt{13} \wedge -\sqrt{7}<x<\sqrt{7}$

$\Rightarrow \left.\begin{matrix} x<-\sqrt{13} \\ x>\sqrt{13}\end{matrix}\right\} x\in(-\infty,-\sqrt{13}) \cup (\sqrt{13},\infty) \quad \vee \quad -\sqrt{7}<x<\sqrt{7}$

$\Rightarrow x \in (-\sqrt{7},\sqrt{7})$

$\therefore x \in (-\infty;-\sqrt{13}) \cup (-\sqrt{7};\sqrt{7}) \cup (\sqrt{13},\infty)$

Pese a que se ven similares, su conjunto solución varía bastante entre cada caso.

⑥ $z > 0 : z \in \mathbb{R}$; $A_z \Rightarrow z : \dfrac{|x^2 + z\cdot x + z^2|}{z\cdot x + 2z^2} \leq 1$

$0 < z_1 < z_2$; $x > -2z \Rightarrow A_{z_1} \overset{?}{>} A_{z_2}$

Como $x > -2z \Rightarrow x + 2z > 0$ $/\cdot z\,(>0)$

$\therefore\ x\cdot z + 2z^2 > 0$

$\dfrac{|x^2 + z\cdot x + z^2|}{z\cdot x + 2z^2} \leq 1 \Rightarrow |x^2 + z\cdot x + z^2| \leq z\cdot x + 2z^2$

$\Rightarrow -zx - 2z^2 \leq x^2 + zx + z^2 \leq z\cdot x + 2z^2$

$\Rightarrow -2z^2 \leq x^2 + 2zx + z^2 \quad \wedge \quad x^2 + z^2 \leq 2z^2$

$-2z^2 \leq (x+z)^2$

Sabemos que $z > 0 \Rightarrow -2z^2 < 0$

Ademas $(x+z)^2 > 0\ \forall x$

$\therefore$ La igualdad se cumplira siempre

$x \in \mathbb{R}$

$x^2 - z^2 \leq 0 \Rightarrow (x+z)\cdot(x-z) \leq 0$

Lo cual se cumple si:

(1) $x + z \geq 0 \quad \wedge \quad x - z \leq 0$

$x \geq -z \quad \wedge \quad x \leq z \Rightarrow x \in [-z, z]$

(2) $x + z \leq 0 \quad \wedge \quad x - z \geq 0$

$x \leq -z \quad \wedge \quad x \geq z \Rightarrow x \in [-z, z]$

Luego, el conjunto solución es $(-\infty, \infty) \cap [-z, z] = [-z, z]$

Por ende, si $z_1 < z_2$, Es evidente que A_{z_2} es más grande que A_{z_1}.