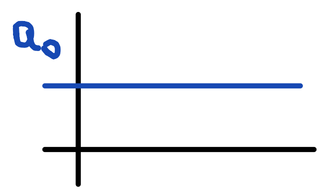
CLASE 12: GRÁFICAS DE POLINOMIOS

- Reservaciones: hasta hoy 12:00 am
- Temario I3: hasta transf. de fns (incluidas)

- $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

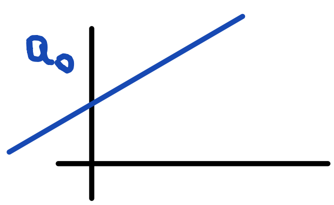
  $n \geq 0,\ a_n \neq 0$

- $\underline{n=0}$: $p(x) = a_0$

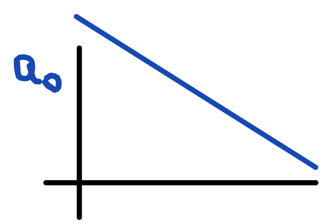


- $\underline{n=1}$: $p(x) = a_1 x + a_0$

$a_1 > 0$

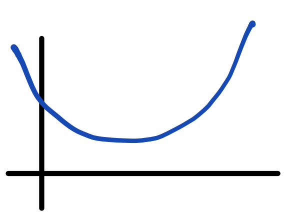


$a_1 < 0$

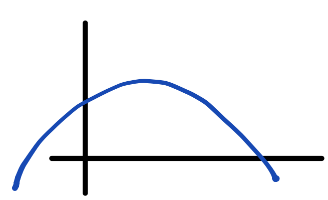


- $\underline{n=2}$: $p(x) = a_2 x^2 + a_1 x + a_0$

$a_2 > 0$

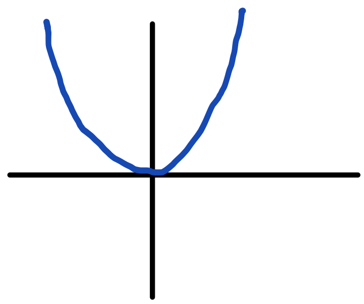
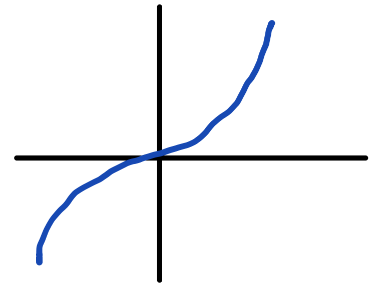
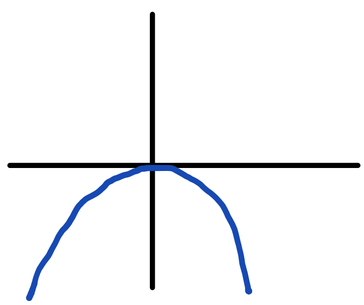
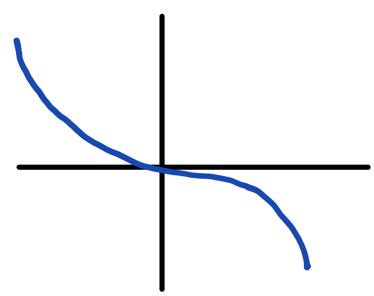


$a_2 < 0$

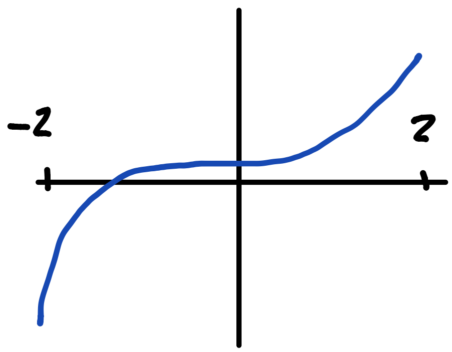


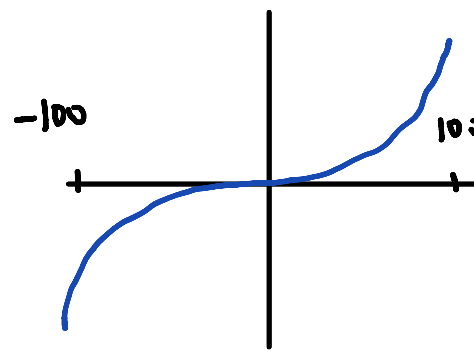
- $n \geq 3$: Caso por caso

- **Caso simples**: $p(x) = a x^n, \ n \geq 2$

	n par	n impar
$a>0$		
$a<0$		

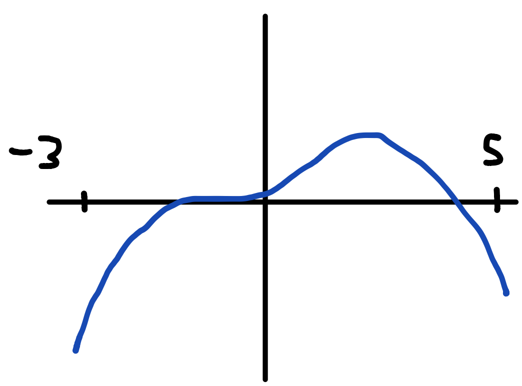
• Ej: $p(x) = 3x^5 + 7x^3 - x^2 + 14$

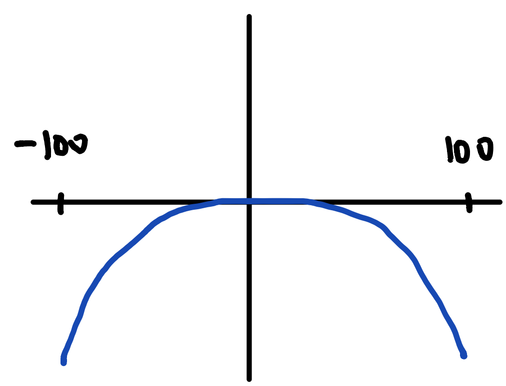




$p(x) \approx 3x^5$

• Ej: $p(x) = -x^4 + 4x^3 + 1$





$p(x) \approx -x^4$

• **Pregunta**: ¿Qué significa $p(x) \approx a_m x^m$?

$$p(x) - a_m x^m = a_{m-1} x^{m+1} + \dots + a_1 x + a_0$$

• **Obs**:

A vs B	$A-B$	$\frac{A-B}{A}$
20 vs 10	10	$\frac{1}{2}$
50 vs 40	10	$\frac{1}{5}$
100 vs 90	10	$\frac{1}{10}$
1000 vs 990	10	$\frac{1}{100}$

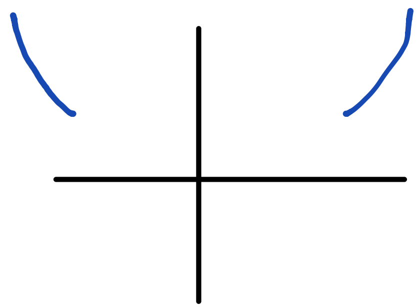
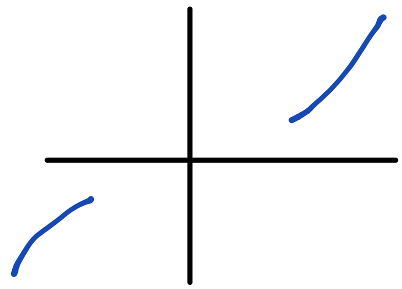
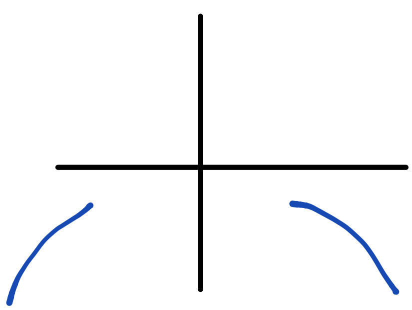
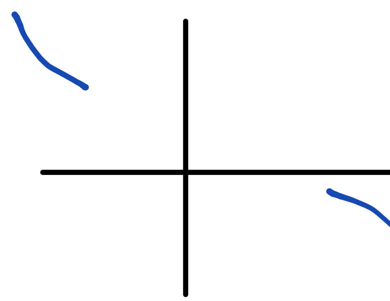
En el caso de los polinomios:

$$\frac{p(x) - a_m x^m}{a_m x^m} = \underbrace{\frac{a_{m-1}}{a_m}\frac{1}{x} + \frac{a_{m-2}}{a_m}\frac{1}{x^2} + \dots + \frac{a_0}{x^m}}_{\text{pequeño si } |x| \text{ es grande}}$$

$\longrightarrow$ $p(x) \approx a_m x^m$: el error relativo es pequeño cuando $|x|$ es grande

• <u>Cuatro casos:</u> $p(x) = a_m x^m + \cdots + a_1 x + a_0$

$$\simeq a_m x^m$$

	m par	m impar
$a_m > 0$	 $p(x) \to \infty$ si $x \to \infty$ $p(x) \to \infty$ si $x \to -\infty$	 $p(x) \to \infty$ si $x \to \infty$ $p(x) \to -\infty$ si $x \to -\infty$
$a_m < 0$	 $p(x) \to -\infty$ si $x \to \infty$ $p(x) \to -\infty$ si $x \to -\infty$	 $p(x) \to -\infty$ si $x \to \infty$ $p(x) \to \infty$ si $x \to -\infty$

Esto se conoce como "Comportamiento en infinito"

- Ej: $p(x) = 3x^5 + 7x^3 - x^2 + 14$

  $p(x) \longrightarrow \infty$ si $x \longrightarrow \infty$

  $p(x) \longrightarrow -\infty$ si $x \longrightarrow -\infty$

- Ej: $p(x) = -x^4 + 4x^3 + 1$

  $p(x) \longrightarrow -\infty$ si $x \longrightarrow \infty$

  $p(x) \longrightarrow -\infty$ si $x \longrightarrow -\infty$

- Falta averiguar el comportamiento de $p(x)$ para x "pequeño", es decir, cerca de las raíces del polinomio.

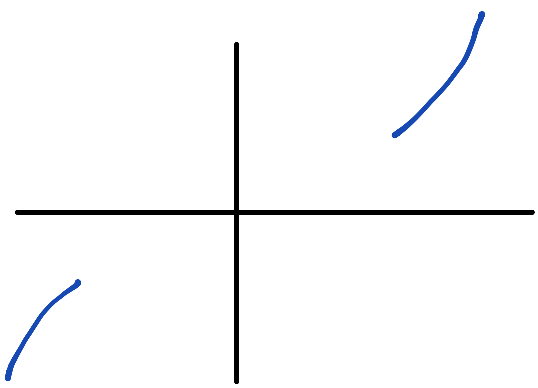
  $\hookrightarrow p(c) = 0$

- Ej: $p(x) = x^3 - 7x + 6$

i) Comportamiento en infinito

$$p(x) \longrightarrow \infty \text{ si } x \longrightarrow \infty$$

$$p(x) \longrightarrow -\infty \text{ si } x \longrightarrow -\infty$$



ii) Raíces:

$$p(x) = x^3 - 7x + 6$$

$$p(1) = 0 \Longrightarrow p(x) = (x-1)\,q(x)$$

$$p(x) = x^3 - 7x + 6$$

$$= x^3 - x^2 + x^2 - 7x + 6$$

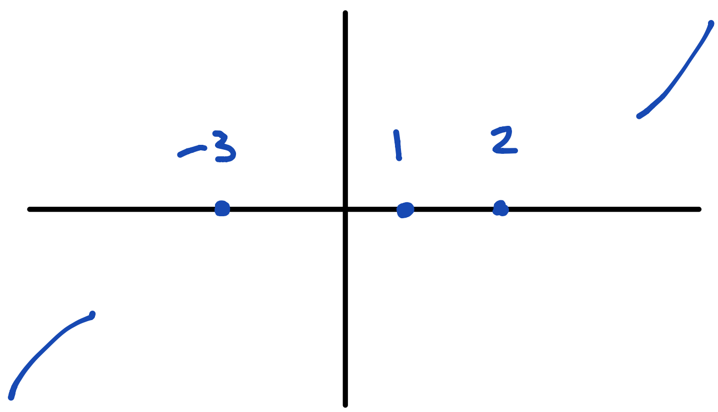
$$= x^3 - x^2 + x^2 - x - 6x + 6$$

$$= x^2(x-1) + x(x-1) - 6(x-1)$$

$$= (x^2+x-6)(x-1)$$

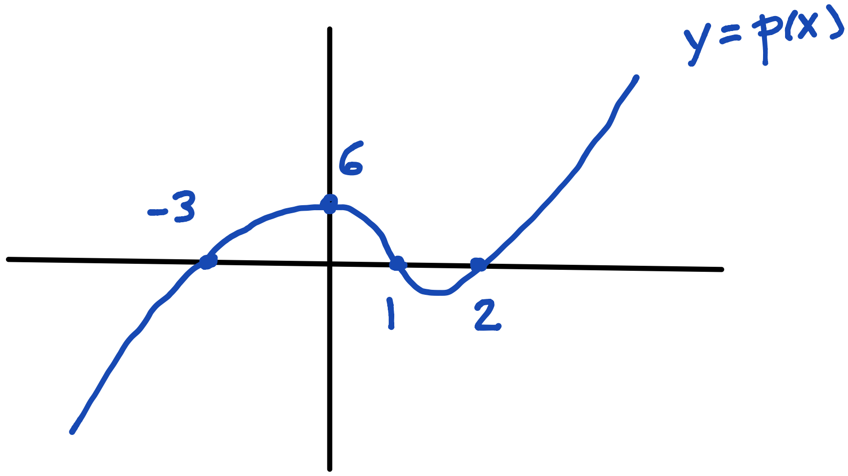
$$= (x+3)(x-2)(x-1)$$

<u>Raíces</u>: $-3, 1, 2$



- Signos:

	-3		1		2	
$x+3$	$-$	0	$+$		$+$	$+$
$x-1$	$-$		$-$	0	$+$	$+$
$x-2$	$-$		$-$		$-$ 0	$+$
$p(x)$	$-$	0	$+$	0	$-$ 0	$+$



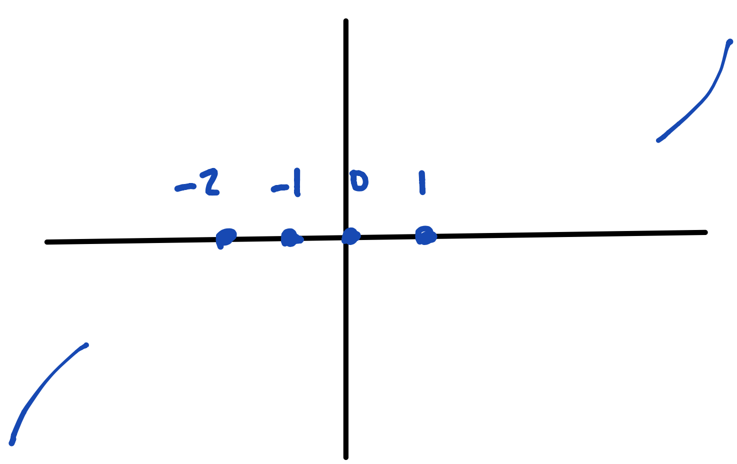
- Ej: $p(x) = (x+2)(x+1)^2 x^3 (x-1)$

  - $n = 7,\ a_7 = 1$

    $p(x) \longrightarrow \infty$ si $x \longrightarrow \infty$

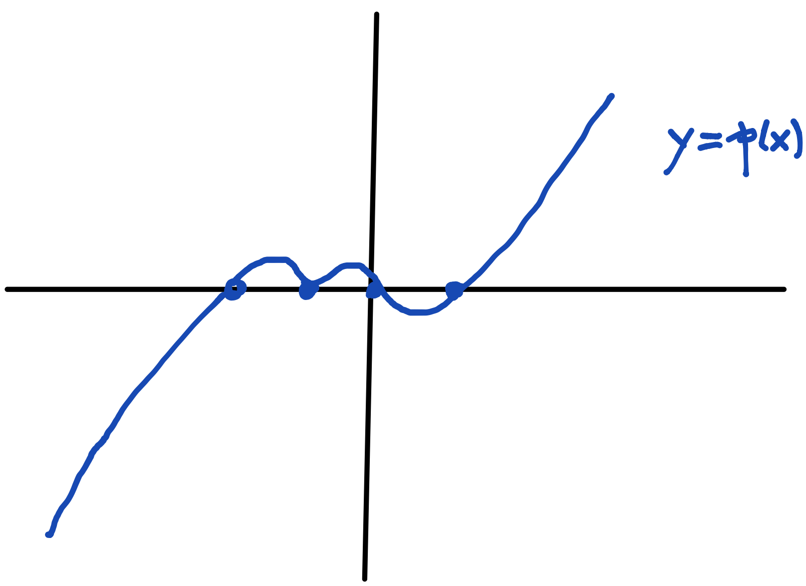
    $p(x) \longrightarrow -\infty$ si $x \longrightarrow -\infty$

  - Raíces: $-2, -1, 0, 1$

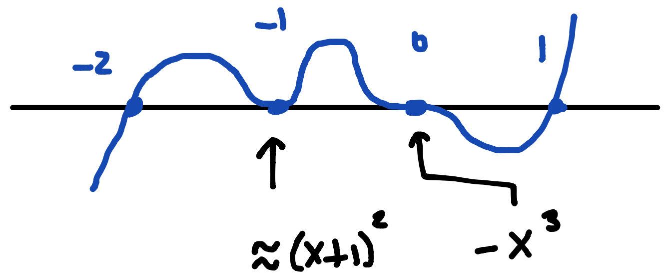


• Signo: $p(x) = (x+2)(x+1)^2 x^3 (x-1)$

		-2		-1		0		1	
$x+2$	$-$	0	$+$		$+$		$+$		$+$
$(x+1)^2$	$+$		$+$	0	$+$		$+$		$+$
x^3	$-$		$-$		$-$	0	$+$		$+$
$x-1$	$-$		$-$		$-$		$-$	0	$+$
$p(x)$	$-$	0	$+$	0	$+$	0	$-$	0	$+$



• Obs: podríamos ser más precisos



$$p(x) = A(x-c_1)^{m_1}(x-c_2)^{m_2} \cdots (x-c_k)^{m_k}$$

$$A \neq 0, \quad m_1 + \cdots + m_k = m$$

Cerca de $x = c_j$:

$$p(x) \approx A(c_j-c_1)^{m_1}(c_j-c_2)^{m_2} \cdots (x-c_j)^{m_j} \cdots (c_j-c_k)^{m_k}$$

$$\approx B(x-c_j)^{m_j}$$

m_j par

m_j impar

$B>0$

$B<0$