1a) a1) $0 \leq P(A \cup B) = P(A) + P(B) - \underbrace{P(A \cap B)}_{\geq 0} \leq P(A) + P(B)$

$\therefore P(A) = P(B) = 0 \Rightarrow 0 \leq P(A \cup B) \leq 0 \Rightarrow P(A \cup B) = 0$

a2) $1 \geq P(A \cap B) = 1 - P(A^c \cup B^c) = 1 - P(A^c) - P(B^c) + \underbrace{P(A^c \cap B^c)}_{\geq 0} \geq 1 - P(A^c) - P(B^c)$

$\therefore P(A) = P(B) = 1 \Rightarrow P(A^c) = P(B^c) = 0 \Rightarrow 1 \geq P(A \cap B) \geq 1 \Rightarrow P(A \cap B) = 1$

1b) $P(A) = p$, $P(B) = q$ y $A \perp\!\!\!\perp B$

b1) $P(A \triangle B) = P((A \cap B^c) \cup (A^c \cap B)) = P(AB^c) + P(A^c B)$ $(AB^c \cap A^c B = \phi)$

$= P(A) P(B^c) + P(A^c) P(B)$ $(A \perp\!\!\!\perp B \Rightarrow A \perp\!\!\!\perp B^c$ y $A^c \perp\!\!\!\perp B)$

$= p(1-q) + (1-p) q$

b2) $P(A^c \cup B^c) = P((A \cap B)^c)$

$= 1 - P(A \cap B)$

$= 1 - P(A) P(B)$

$= 1 - pq$.

②

2a) $P(X=-1) = F_X(-1) - F_X(-1^-) = 1-p-0 = 1-p \Rightarrow F_X$ "salta" en $x=-1$

$$\left.\begin{aligned} P(X=0) &= F_X(0) - F_X(0^-) = 1-p+\tfrac{1}{2}p\times 0-(1-p) = 0 \\ P(-1<X\leq 0) &= F_X(0) - F_X(-1) = 1-p+\tfrac{1}{2}p\times 0-(1-p) = 0 \end{aligned}\right\} \Rightarrow X \text{ no asume valores en } (-1,0)$$

$$\left.\begin{aligned} P(0<X\leq 1) &= F_X(1) - F_X(0) = 1-p+\tfrac{1}{2}p\times 1-(1-p) = p/2 \\ P(X\geq 1) &= 1-P(X<1) = 1-F_X(1^-) = 1-(1-p+\tfrac{1}{2}p\times 1) = p/2 \end{aligned}\right\} \begin{aligned} &X \text{ es cont.} \\ &\text{en } [0,2) \\ &\text{con f.d.p.} \\ &f_X(x) = p/2 \end{aligned}$$

b) Como X es mixta, con recorrido $\mathcal{X} = \{-1\} \cup [0,2) \Rightarrow$:

$$E(X) = -1\times(1-p) + \int_0^2 \underbrace{\frac{d F_X(x)}{dx}}_{=p/2} dx \quad \leftarrow \frac{d F_X(x)}{dx} = f_X(x) = \begin{cases} p/2 & \text{si } 0\leq x<2 \\ 0 & \sim \end{cases}$$

$$= -1+p+\frac{p}{2}x\Big|_0^2 = -1+p+p = \underline{2p-1}$$

Alternativamente, de la fórmula general de esperanza,

$$\Rightarrow E(X) = \int_0^2 \underbrace{\{1-F_X(x)\}}_{\text{en } [0,2)} dx - \int_{-1}^0 \underbrace{F_X(x)}_{\text{en } [-1,0)} dx$$

$$= \int_0^2 \{\underbrace{1-(1-p+\tfrac{1}{2}px)}_{p-\frac{1}{2}px}\} dx - \int_{-1}^0 (1-p)\, dx = px - \frac{1}{4}px^2\Big|_0^2 - (1-p)x\Big|_{-1}^0$$

$$= 2p-p-(1-p) = 2p-1$$

La mediana m depende de p! ③

i) $p < 1/2 \Rightarrow 1-p > 1/2 \Rightarrow P(X \le m) = 1-p > 1/2 \quad \forall m \in [-1,0)$
$P(X \ge m) = 1-p > 1/2 \quad \forall m \in [-1,0)$

$\Rightarrow$ Cualquier $m \in [-1,0)$ es una mediana

ii) $p = 1/2 \Rightarrow P(X \le 0) = 1-p = 1/2$
$P(X \ge 0) = 1 - P(X<0) = 1-p = 1/2$ $\Big\} \Rightarrow m=0$ es mediana

iii) $p > 1/2 \Rightarrow F_X(m) = 1-p + \frac{1}{2} p m = 1/2 \Leftrightarrow m = 2\left(\frac{1-p}{p}\right)$
$\Rightarrow 1-p < 1/2$ p.e. si $p = 3/4 \Rightarrow m = 2 \frac{1/4}{3/4} = 2/3$

P3) a) $P(X>s \mid X>t) = \frac{P(X>s \wedge X>t)}{P(X>t)} = \frac{P(X>s)}{P(X>t)}, \; s>t$

$= \frac{1-P(X\le s)}{1-P(X\le t)} = \frac{1-\int_0^s e^{-x}dx}{1-\int_0^t e^{-x}dx} = \frac{1-(1-e^{-s})}{1-(1-e^{-t})}$

$= e^{-(s-t)} = 1-(1-e^{-(s-t)}) = 1-F_X(s-t) = \overline{P(X>s-t)}$

b) $E(cX+1) = c\,E(X)+1$, donde $E(X) = \int_0^\infty x e^{-x}dx$ $\quad u = x \Rightarrow du = dx$
$= -xe^{-x}\Big|_0^\infty + \int_0^\infty e^{-x}dx$ $\quad dv = e^{-x}dx \Rightarrow v = -e^{-x}$
$= 0 - 0 + 1 = 1$

$= c \times 1 + 1$
$= c + 1$