Martes 3/Mayo/2016.

Intro a Estadística.
Ayudantía.

1 X = # de huevos que pone un insecto. $x \in \mathbb{N}^0$

a. $X \sim \text{Poisson}(\lambda)$

Y = # de huevos que sobreviven.

$Y|X=x$: "# de huevos que sobreviven de un total x con prob. p de ~~sobrev~~ sobrevivir." $y \in \{0, \dots, x\}$

$$Y|X=x \sim \text{Binomial}(x,p)$$

$$\mathbb{P}(Y=x) = \sum_{x=y}^{\infty} \mathbb{P}(Y=y|X=x)\cdot\mathbb{P}(X=x)$$

$$= \sum_{x=y}^{\infty} \binom{x}{y} \cdot p^y (1-p)^{x-y} \cdot \frac{\lambda^x \cdot e^{-\lambda}}{x!}$$

$$= \sum_{x=y}^{\infty} \frac{\cancel{x!}}{(x-y)!\cdot y!} \cdot \left(\frac{p}{1-p}\right)^y \cdot \frac{((1-p)\lambda)^x \cdot e^{-\lambda}}{\cancel{x!}}$$

$$= \left(\frac{p}{1-p}\right)^y \cdot \frac{1}{y!} \cdot e^{-\lambda} \cdot \sum_{x=y}^{\infty} \frac{((1-p)\lambda)^x}{(x-y)!} \qquad \boxed{u = x-y}$$

$$= \frac{1}{y!} \cdot \left(\frac{p}{1-p}\right)^y e^{-\lambda} \sum_{u=0}^{\infty} \frac{((1-p)\lambda)^{u+y}}{u!}$$

$$= \frac{1}{y!} \cdot \frac{p^y}{\cancel{(1-p)^y}} \cdot e^{-\lambda} \cdot \cancel{(1-p)^y} \cdot \lambda^y \cdot \underbrace{\sum_{u=0}^{\infty} \frac{((1-p)\lambda)^u}{u!}}_{e^{\lambda(1-p)}}$$

$$= \frac{1}{y!} \cdot e^{-\lambda} \cdot (\lambda p)^y \cdot e^{\lambda(1-p)}$$

$$= \frac{e^{-\lambda p} \cdot (\lambda p)^y}{y!}$$

Luego $Y \sim \text{Poisson}(\lambda p)$.

b. $\mathbb{E}(Y) = \sum_{y} x \cdot \mathbb{P}(Y = x)$.

$\star\ \mathbb{E}(\mathbb{E}(Y|X)) = \mathbb{E}(Xp) = p \cdot \mathbb{E}(X) = p \cdot \lambda$. ✓✓

$\star\ \mathbb{E}(Y) = \sum_{y=0}^{\infty} y \cdot \frac{e^{-\lambda p} \cdot (\lambda p)^y}{y!}$

$= e^{-\lambda p} \cdot \left(\sum_{y=1}^{\infty} \frac{(\lambda p)^y}{(y-1)!} + 0 \cdot \frac{1}{1} \right)$ ✓ $\quad y=0 \quad u = y-1$

$= e^{-\lambda p} \cdot \sum_{u=0}^{\infty} \frac{(\lambda p)^{u+1}}{u!}$

$= e^{-\lambda p} \cdot (\lambda p) \cdot \sum_{u=0}^{\infty} \frac{(\lambda p)^u}{u!}$

$= \cancel{e^{-\lambda p}} \cdot (\lambda p) \cdot \cancel{e^{\lambda p}} = \lambda p$. ✓✓

c. $Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X) \cdot \mathbb{E}(Y)$

$= \mathbb{E}(XY) - \lambda \cdot \lambda p$

$\mathbb{E}(XY) = \mathbb{E}(\cancel{\mathbb{E}(XY|Y)})$ $\quad = \mathbb{E}(\mathbb{E}(Y \cdot X | X))$

$= \cancel{\mathbb{E}(Y \cdot \mathbb{E}(X|Y))}$ $\quad = \mathbb{E}(X \cdot \mathbb{E}(Y|X))$

$= \mathbb{E}(X \cdot Xp)$

$= p \cdot \mathbb{E}(X^2)$

$= p \cdot [Var(X) + \mathbb{E}^2(X)]$

$= p[\lambda + \lambda^2]$

$\therefore Cov(X, Y) = \lambda p + \cancel{\lambda^2 p} - \cancel{\lambda^2 p} = \lambda p$.

②

$$\begin{array}{cc} & Y \\ & \begin{array}{ccc} -1 & 0 & 1 \end{array} \\ X & \left[\begin{array}{c|ccc} 1 & 0.3 & 0.05 & 0.05 \\ 2 & 0.05 & 0.2 & 0.05 \\ 3 & 0.1 & 0.1 & 0.1 \end{array}\right. \end{array}$$

a) $\cdot\ \mathbb{P}(X=1, Y=0) \overset{?}{=} \mathbb{P}(X=1)\cdot\mathbb{P}(Y=0)$

$0.05 \overset{?}{=} 0.4 \cdot 0.35 \overset{!!}{=} 0$

$\cdot\ \mathbb{P}(X=1 \mid Y=0) \overset{?}{=} \mathbb{P}(X=1 \mid Y=-1) = \mathbb{P}(X=1, Y=-1)$

$\frac{0.05}{0.35} = \frac{1}{7} \neq \frac{0.05}{0.20} = \frac{1}{4} \neq \frac{1}{20} = \mathbb{P}(X=1)$

$\therefore$ no son independientes.

a) $Cov(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\cdot\mathbb{E}(Y)$

$\star\ \mathbb{E}(XY) \sum_{Y=-1}^{1}\sum_{X=1}^{3} x\cdot y\, \mathbb{P}(X=x, Y=y)$

$= -1\cdot 1\cdot 0.3 + 1\cdot 0.05 \cancel{-2\cdot 0.05 + 2\cdot 0.05} \cancel{-0.3 + 0.3}$

$= -0.3 + 0.05$

$= 0.25$

$\mathbb{P}(X=x) = \sum_{Y=-1} \mathbb{P}(X=x, Y=y) \Rightarrow \mathbb{P}(X=x) \begin{cases} 0.4 & x=1 \\ 0.3 & x=2 \\ 0.3 & x=3 \\ 0 & e.o.c \end{cases}$

$\mathbb{P}(Y=y) = \sum_{X=1}^{3} \mathbb{P}(X=x, Y=y) \Rightarrow \mathbb{P}(y=y) = \begin{cases} 0.45 & y=-1 \\ 0.35 & y=0 \\ 0.2 & y=1 \\ 0 & e.o.c \end{cases}$

$\mathbb{E}(X) = 0.4 \cdot 1 + 0.3 \cdot 2 + 0.3 \cdot 3 = 0.4 + 0.6 + 0.9 = 1.9$

$\mathbb{E}(Y) = -0.45 + 0 \cdot 0.35 + 1 \cdot 0.2 = (-0.25)$

$$\begin{aligned} Cov(X,Y) &= -0.25 - 1.9(-0.25) \\ &= -0.25(1 - 1.9) \\ &= -0.25 \cdot -0.9 = 0.225 \end{aligned}$$

- $Cor(X,Y) = \dfrac{Cov(X,Y)}{\sqrt{Var(X) \cdot Var(Y)}}$

$Var(X) = \sum_x (x - \underset{1.9}{\underset{\shortparallel}{\mathbb{E}(X)}})^2 \, \mathbb{P}(X=x)$

$= (-0.9)^2 \cdot 0.4 + (0.1)^2 \cdot 0.3 + (1.1)^2 \cdot 0.3 = 0.69$

$Var(Y) = 0.25401$

$\Rightarrow Cor(X,Y) = \dfrac{0.225}{\sqrt{0.69 \cdot 0.25401}} = 0.53734$

- $Var(X+Y) = Var(X) + Var(Y) - Cov(X,Y)$

c) $X+Y$:

	-1	0	1
1	0	1	2
2	1	2	3
3	2	3	4

$\mathbb{P}(X+Y=1) = \mathbb{P}(X=1, Y=0) + \mathbb{P}(X=2, Y=-1)$

d) • $\mathbb{P}(Y \le X) = 1$

(3) X = ingresos por familia en u.m.

X ~ Exponencial(1/2)

$\hookrightarrow E(X) = \frac{1}{\lambda} = 2 \Rightarrow \lambda = \frac{1}{2}$

p. $P(X > 6) = 1 - P(X \leq 6)$

$= 1 - (1 - e^{-\lambda 6}))$

$= e^{-\lambda 6}$

$= e^{-3} = 0.049$

Y: "# de familias entrevistadas hasta encontrar a la 3°
con ingreso superior a 6 u.m."

$Y \sim BinNeg(r = 3, p = 0.049) \quad y \in \{3, \ldots\}$

$P(Y > 5) = 1 - P(Y \leq 5)$

$= 1 - P(Y = 3) - P(Y = 4) - P(Y = 5)$

$= 1 - 3$

$= 0,9988$

(4) Sean M piezas defectuosas

X = cant. de piezas defectuosas de un total de K piezas
extraidas de un total de 100 piezas con n defectuosas.

X ~ Hipergeométrica (N = 100, n = K, r = M)

$P(X = 0) \leq 0.1$

A: Aceptar lote defectuoso

$P(A) = P(X = 0)$

Para $M = 6$, la prob. de aceptar un lote defectuoso se hace más grande.

$$\mathbb{P}(X=0) = \frac{\binom{6}{0}\binom{94}{k}}{\binom{100}{k}} = \frac{\frac{94!}{(94-k)!\cdot \cancel{k!}}}{\frac{100!}{(100-k)!\cdot\cancel{k!}}} = \frac{94!}{100!}\cdot\frac{(100-k)!}{(94-k)!}$$

$$= \frac{(100-k)(99-k)(98-k)\cdots(95-k)}{100\cdot 99\cdots 95} \leq 0.1$$

$k = 31$ $\quad \mathbb{P}(X=0) = 0.10056$

$k = 32$ $\quad \mathbb{P}(X=0) = 0.9182.$