Sol. Ayudantía I2 hasta clase 12

[redacted]

Integra 1

Ej: $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2+4}\, dx = \frac{\pi}{2}$

$$\int_{-\infty}^{\infty} \frac{2}{x^2+4} + \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{x^2}{2}\right\} dx$$

$$= 2\int_{-\infty}^{\infty} \frac{1}{x^2+4}\, dx + \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{x^2}{2}\right\} dx$$

$$= \pi + 1$$

1. a) $\pi/4$ b) $1/2$ c) 1

2. a) $\displaystyle\int_0^1 \int_0^2 C(x+2y)\, dx\, dy$

$$= \int_0^1 \left(C\frac{x^2}{2} + 2Cxy\right)\Big|_0^2 dy = \int_0^1 2C + 4yC\, dy$$

$$= 2Cy + 4\frac{y^2}{2}C\Big|_0^1$$

$$= 2C + 2C - 0 = 4C$$

$$\Rightarrow \boxed{C = \frac{1}{4}}$$

b) $\displaystyle\int_0^1 f(x,y)\, dy = C\int_0^1 x+2y\, dy = C\cdot(xy + y)\Big|_0^1$

$$= C\cdot(x+1)$$

$$\Rightarrow f(x) = \frac{(x+1)}{4} \mathbb{1}_{(0,2)}$$

$$\frac{1}{4}\int_0^y\int_0^x u+2v\,du\,dv$$

$$=\frac{1}{4}\int_0^y \frac{u^2}{2}+2vu\Big|_0^x\,dv$$

$$=\frac{1}{4}\int_0^y\left(\frac{x^2}{2}+2xv-0\right)dv=\frac{1}{4}\left(\frac{x^2v}{2}+2x\frac{v^2}{2}\right)\Big|_0^y$$

$$=\frac{1}{4}\left(\frac{x^2y}{2}+xy^2\right)$$

$$=\frac{x^2y}{8}+\frac{xy^2}{4}$$

$$\Rightarrow F_{XY}(x,y)=\begin{cases}0 & \text{si } x\le 0,\ y\le 0\\ \frac{x^2y}{8}+\frac{y^2x}{4} & \text{si } 0<x<2 \text{ y } 0<y<1\\ \frac{y}{2}+\frac{y^2}{2} & \text{si } 2\le x \text{ y } 0<y<y\\ \frac{x^2}{8}+\frac{x}{4} & \text{si } 0<x<2 \text{ y } 1\le y\\ 1 & \text{si } 2\le x \text{ y } 1\le y\end{cases}$$

Si $x\geq 2$
$x=2$
$\frac{4y}{8}$
$\frac{y}{2}$

$1\le y$
$0<x<2$
$\frac{1}{4}\int_0^1\int_0^x$

d) $z=\frac{9}{(x+1)^2}=g(x)$ es monótona en $0<x<2$

$\Rightarrow z=\frac{9}{(x+1)^2}\Rightarrow x=\left(\frac{9}{z}\right)^{\frac{1}{2}}-1$

$$\frac{\partial x}{\partial z}=\frac{1}{2}\left(\frac{9}{z}\right)^{-\frac{1}{2}}\cdot 9\cdot(-1)z^{-2}$$

$$f_Z(z)=\frac{1}{4}\left(\sqrt{9/z}\right)\cdot\frac{9}{2}\left(\sqrt{9/z}\right)\cdot\frac{1}{z}$$

$$=\frac{9}{8}\cdot\frac{9}{z}\cdot\frac{1}{z}=\frac{9}{8}\cdot z^{-2}\,\mathbb{1}_{(1,9)}$$

$g(0)=9$
$g(2)=\frac{9}{9}=1$

3. El soporte (U,V) es $\left\{ (u,v) : \begin{array}{l} u = 1, 2, \ldots \\ v = u+1, u+2 \ldots \end{array} \right\}$

No es un prod. cruz.

Equivalentemente $\underbrace{U = u}\ \Rightarrow\ \underbrace{V > u}$ $\therefore$ no son independientes

4. i) $f(x,y) \geq 0$ pues el $\frac{\text{num.}}{\text{denom.}}$ es ≥ 0

ii) $x = r\cos\theta$
$y = r \operatorname{sen}\theta$

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial x}{\partial \theta} \end{vmatrix}$$

$f \times c \quad \frac{\partial f}{\partial c}$

$$= \begin{vmatrix} \cos\theta & -r\operatorname{sen}\theta \\ \operatorname{sen}\theta & r\cos(\theta) \end{vmatrix} = r\cos^2\theta + r\operatorname{sen}^2\theta = r$$

$$\int_0^{\frac{\pi}{2}} \int_0^{\infty} \frac{2g(r)}{\pi \cdot r} \cdot r \, dr \, d\theta$$

$$= \frac{2}{\pi} \int_0^{\frac{\pi}{2}} 1 \, d\theta = \frac{2}{\pi} \cdot \frac{\pi}{2} = 1$$

5. a) Sop. de (U,V) es $\{ (u,v) : u \in \mathbb{Z}_+ , v \in \mathbb{Z} \}$
$1, 2, \ldots$

$U = \min(X,Y)$
$V = X - Y$

Si $V > 0 \Rightarrow X > Y$ $\quad v = 1, 2, \ldots$

$$f_{UV}(u,v) = P(U = u, V = v) = P(Y = u, X = u+v)$$
$$= p(1-p)^{u+v-1} \cdot p(1-p)^{u}$$

Si $V < 0 \Rightarrow X < Y$ $\quad v = -1, -2, \ldots$

$$f_{UV}(u,v) = P(X = u, Y = u - v)$$

Si $V = 0$ $\quad f_{UV}(u,v) = P(X = u, Y = u)$

Lo ant. se resume en

$$f_{U,V}(u,v) = p^2 (1-p)^{2u+|v|-2}$$

$$= \underbrace{(p^2(1-p)^{2u})}_{\text{Solo dep. de } u} \cdot \underbrace{(1-p)^{|v|-2}}_{\text{Solo dep. de } V}, \quad u = 1,2,\dots \quad v = \pm 1, \pm 2, \dots$$

$\therefore$ Son indep.

b) Z todas las frac. $\frac{r}{s}$ $r,s \in \mathbb{Z}_+$

y $r < s$ sin factores en común.

$\Rightarrow \frac{X}{X+Y} = \frac{r}{s} \Rightarrow (ir, i(s-r)) \quad i = 1,2,\dots$

$$P\left(Z = \frac{r}{s}\right) = \sum_{i=1}^{\infty} P(X = ir, Y = i(s-r))$$

$$= \sum_{i=1}^{\infty} p(1-p)^{ir-1}\, p\,(1-p)^{i(s-r)-1}$$

$$= \frac{p^2}{(1-p)^2} \sum_{i=1}^{\infty} ((1-p)^s)^i = \frac{p^2}{(1-p)^2} \frac{(1-p)^s}{1-(1-p)^s}$$

$$= \frac{p^2(1-p)^{s-2}}{1-(1-p)^s}$$

c) $P(X = x, \overbrace{X+Y = t}^{Z}) = P(X = x, Y = t-x) = P(X = x)P(Y = t-x)$

Convolución $= p^2(1-p)^{t-2} \Rightarrow \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\,dx = f_Z(z)$