Optimizing Hospital Bed Utilization

Linear Programming Final Project

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Background and Motivation

The problem

- The hospital provides inpatient services across four main clinical departments: Emergency, ICU, General Medicine, and Surgery.
- Each service competes for access to a shared pool of hospital beds.
- Goal: formulate and apply a mathematical optimization model designed to improve hospital bed allocation under constrained capacity for a baseline scenario and a demand surge scenario.

Problem Statement

Determine the allocation of hospital beds to departments that minimizes unmet patient demand while ensuring that critical-care units retain sufficient capacity to handle longer and more complex treatments for a baseline scenario and a demand surge scenario.

Datasets Description

Dataset	Key Variables and Use
`patients.csv`	One row per patient. Variables: arrival_date, departure_date, service, age, satisfaction. Length of stay computed; dates converted to `datetime`.
`services_weekly.csv`	Weekly bed usage per service. Variables: available_beds, patients_request, patients_admitted, patients_refused. Derived: occupancy rate and unmet demand.
`staff_schedule.csv`	Weekly staff presence and service assignment. Used to assess workforce stability and staff-to-service distribution.
`staff.csv`	Staff roster with roles and service assignments. Used to summarize skill mix; mostly redundant with `staff_schedule.csv`.

Table 1: Summary of datasets used in the exploratory analysis. Data sets taken from kaggle.

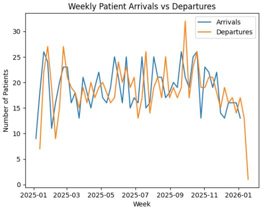
Data Cleaning

A critical inconsistency was identified in the staff data. There is a mismatch between the `staff_id` in the master list versus the schedule:

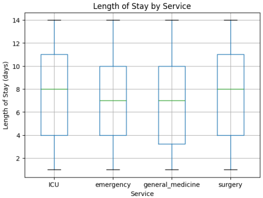
- **Master List:** 110 unique staff members.
- **Schedule:** 126 unique staff members.
- **Issue:** Employees appear with different IDs in different files (e.g., "Allison Hill" has ID `STF-5ca26577` in the master list but `STF-b77cdc60` in the schedule).

To resolve this, data merging was performed based on `staff_name` rather than ID to ensure accurate capacity calculation.

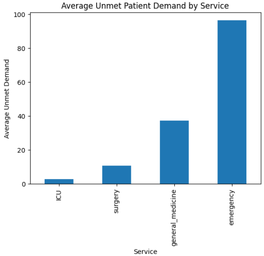
Data Analysis



(a) Patients arrivals vs departures



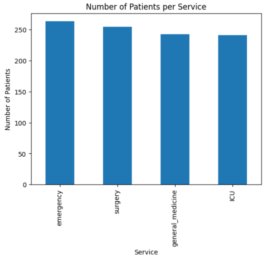
(b) Lenght of stay per service



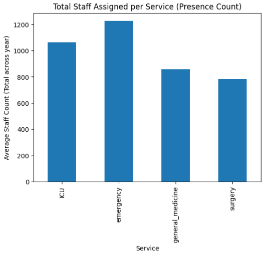
(c) Unmet patients by service

Figure 1: Exploratory Data Analysis

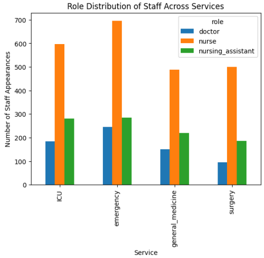
Data Analysis



(a) Patients per service



(b) Staff per service



(c) Role distribution across service

Figure 2: Exploratory Data Analysis

1. Introduction
2. Data Analysis & Cleaning
3. **Methodology: The LP Model**
4. Methodology: The LP Model
5. Results

Mathematical Formulation

We selected **Option A: Minimize Unmet Demand** as our objective. The problem is formulated as a weighted minimization problem to reflect the medical urgency of different departments.

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Mathematical Formulation

Sets

- T: Set of weeks $\{1, \ldots, 52\}$.
- I: Set of services $\{ICU, Emergency, Surgery, General\_Medicine\}$.

Parameters:

- $D_{t,i}$: Patient demand for service i in week t.
- C_t: Total available hospital beds in week t.
- P_i: Penalty weight for refusing a patient in service i.

Mathematical Formulation

Decision Variables:

- $x_{t,i} \geq 0$: Number of beds allocated to service i in week t.
- $y_{t,i} \geq 0$: Unmet demand (patients refused) for service i in week t.

Objective Function:

Minimize the weighted sum of refused patients:

$$\min Z = \sum_{t \in T} \sum_{i \in I} P_i \cdot y_{t,i} \tag{1}$$

Mathematical Formulation

Constraints:

- **Capacity Constraint:** The sum of allocated beds cannot exceed weekly capacity.

$$\sum_{i \in I} x_{t,i} \leq C_t \quad \forall t \in T \tag{2}$$

- **Demand Balance:** Demand must be split between allocated and unmet.

$$x_{t,i} + y_{t,i} = D_{t,i} \quad \forall t, i \tag{3}$$

- **ICU Safety:** ICU demand must be met fully (Hard Constraint).

$$y_{t,ICU} = 0 \quad \forall t \tag{4}$$

Mathematical Formulation

Priority Weights

To prioritize critical care, the following weights (P_i) were assigned:

- **ICU:** 10,000 (Highest Priority - Life threatening).
- **Emergency:** 100 (High Priority).
- **Surgery:** 10 (Medium Priority).
- **General Medicine:** 1 (Lowest Priority - Deferrable).

The full implementation can be found at this GitHub Repository.

Scenario 1: Baseline

The first scenario consists of assuming that no change in the trend of the demand though the year occurs. The model was implemented in ZIMPL and solved using SCIP and yielded the following results:

Performance

- **Historical Refusals:** 7,642 patients.
- **Optimized Refusals:** 7,181 patients.
- **Improvement:** 461 fewer refusals (6% reduction).

Allocation Strategy

The complete allocation plan per week can be found at this link.

Pandemic Scenario

A "War Time" scenario was simulated starting in Week 12 to test system resilience.

Scenario Definition

- **Demand Shock:** ICU demand increases by **600%** starting Week 12.
- **Operational Change:** Average Length of Stay (LOS) for ICU drops from 7.6 days to ~2 days.
- **Modeling Implication:** The bed consumption factor for ICU drops to 0.285 (1 bed can treat 3.5 patients/week).

Pandemic Scenario

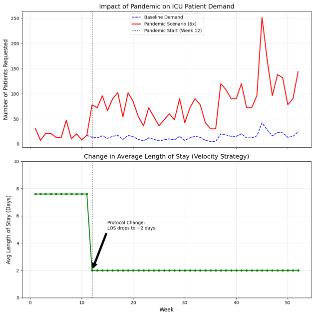

Figure 3: Scenario Parameters: 600% Demand Increase vs. Reduced Length of Stay

Impact on Services:

Impact on other services

- **ICU:** 0 Refusals (Fully Served).
- **Emergency:** Severe service reduction in peak weeks (e.g., Week 45 saw 142 refusals).
- **General Medicine:** Effectively shut down during peak pandemic waves to free up beds.

Conclusions

This project demonstrates that mathematical optimization can significantly improve hospital efficiency. The baseline model found "hidden capacity" worth 461 patients/year simply by better allocation. The pandemic analysis highlighted a critical operational insight: when physical capacity is fixed, **velocity (Length of Stay)** is the only variable that can absorb a massive surge. The recommended strategy for future crises is to prioritize rapid-turnover protocols over physical expansion.

Thank you for listening !

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