Project 5) Diffusion-limited aggregation - simplified version on 2nd page

Consider a system of N particles.

One of the particles, here the blue particle, is very heavy compared to the remaining particles $m_{blue} \gg m_{orange}$.

In addition, the blue particle is initialised with a very small velocity $|\vec{v}_{blue}| \ll |\vec{v}_{orange}|$ and its initial position is the centre of the box $\vec{r}_{blue}(t=0) = (x_{center}, y_{center})$.

When an orange particle collides with a blue particle, the following things happen:

1) a dimer bond is formed (see Fig 2)
2) the orange particle transforms into a blue particle by changing its mass ($m_{orange} \rightarrow m_{blue}$). This change of mass should preserve kinetic energy, i.e. the speed must be updated $|\vec{v}| * = \sqrt{m_{orange}/m_{blue}}$ (see Fig 3)
3) the velocities of the particles are updated as normal

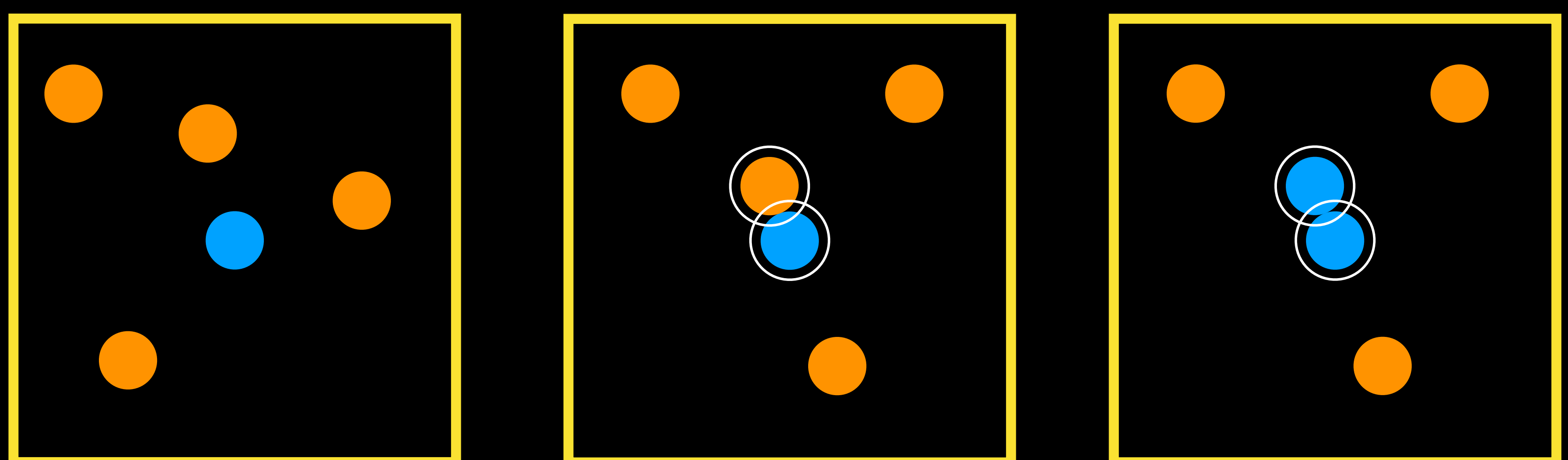

Fig. 1: Initial state Fig. 2: seed collision Fig. 3: transformation

The blue aggregate will grow until it contains all orange particles. We want to stop the simulation once all orange particles disappeared. Because $m_{blue} \gg m_{orange}$ and because the initial state satisfied $|\vec{v}_{blue}| \ll |\vec{v}_{orange}|$, the orange particles move much faster. We can say that the time scale of the orange particles is much faster than for the blue particles.

We are interested in how the final structure of the blue aggregate looks like. The question is also how the ratio of the masses influence the final structure.

If $m_{blue} \gg m_{orange}$, then the final aggregate should be a fractal. (See also wikipedia)

If $m_{blue} \sim m_{orange}$, then the final structure should not be a fractal.

Project 5) simplified version

The simplified version is to consider that the initial blue particle has infinite mass and zero speed. Every time an orange particle collides with the blue particle, the orange particle freezes (i.e. $v_x = v_y = 0$) and the particle becomes also infinitely heavy.

In this simplified version, one could:

1) finalise the aggregation process until there is only one orange particle left. This last survivor never joins the aggregate, i.e. it moves freely and interacts via elastic collisions with the infinitely heavy particles of the aggregate.
2) study how this survivor particle explores the space. You can trace the position as a function of time and also record the histogram of the position of the particle. What do you observe?

velocity update with white particles

in general

$$\vec{v_1}' = \vec{v_1} - \frac{2m_2}{m_1+m_2}\left(\hat{\vec{\delta}} \cdot \vec{\Delta v}\right)\hat{\vec{\delta}} \text{ and}$$

$$\vec{v_2}' = \vec{v_2} + \frac{2m_1}{m_1+m_2}\left(\hat{\vec{\delta}} \cdot \vec{\Delta v}\right)\hat{\vec{\delta}}.$$

consider $m_1 \to \infty$ & m_2 finite

$$\Rightarrow \lim_{m_1 \to \infty} \frac{2m_2}{m_1+m_2} = 0 \,, \quad \lim_{m_1 \to \infty} \frac{m_1}{m_1+m_2} = 1$$

in addition $\vec{V}_1 = (0,0)$

therefore

$$\boxed{\vec{V}_1{}' = \vec{V}_1 = (0,0) \;\&\; \vec{V}_2{}' = \vec{V}_2 + 2\left(\hat{\vec{\delta}} \cdot \vec{\Delta V}\right)\hat{\vec{\delta}}}$$