Sustentación= 4,3
Informe = 4,4
Códigos = 4,0
Nota2 (70%) = 4,2

NOTA FINAL:
Andres
Esteban } 3,8
Juan David = 4,1

Digital control laboratory report

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Abstract: In this report a mathematical analysis is made on the temperature plant called TCLab, approximations, optimizations of mathematical models, stability analysis are performed of the system by the locus of the poles, locus of the roots and bode plot for then with the help of control theory and different tunings for controllers arrive to control that plant.

Keywords: Temperature, control, TC-LAB, analysis , energy balance and heat transfer, optimization

1. INTRODUCTION

This report contains the studies carried out in the temperature plant called TCLab in order to implement all the control algorithms carried out for Control the temperature of said plant and thus be able to resemble the previous studies that must be done to a system before wanting to control it. This educational model is characterized by having three temperature sensors (LM35) and two heaters, in the Arduino Uno embedded system.

This study arises as a laboratory practice of the Digital Control Systems subject with the objective of applying the theoretical concepts to a physical plant such as: the selection of the sampling period, the identification of the characteristic equation of the plant (transfer function) , optimization of the found model, stability analysis and design of different PI and PID controls.

2. DIDACTIC PLANTS

To build our TC-LAB, see 1. we use an Arduino UNO as an embedded system, where two digital outputs were sent to send pulse width modulation (PWM), all 3 analog inputs to detect the temperature with the help of the LM35. The PWM is used to saturate the transistors that they are used as heaters, these heaters have a 12V supply and for the Arduino we use the USB from the computer to harge the code.

When we allow our heaters to enter a "cut" state, a current is supplied from the source flowing from the collector to emitter and with the help of a shunt resistor the electric current flowing through the transistors. Said current measurements are treated by an operational amplifier in order to match impedance and low-pass filter, and thus read the current flowing through the transistors, see Fig. 2.The same happens for the temperature reading, the sensors deliver a proportional voltage for each centigrade (10 mV/°C) and the Arduino has the facility to do the A/D conversion. We must also make it clear that our controlled variable is PWM and the manipulated variable is temperature, see Fig. 3.

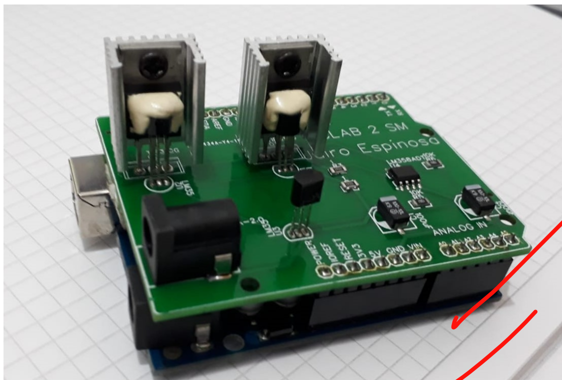

Fig. 1. TCLAB 2 SM, Jairo Espinosa

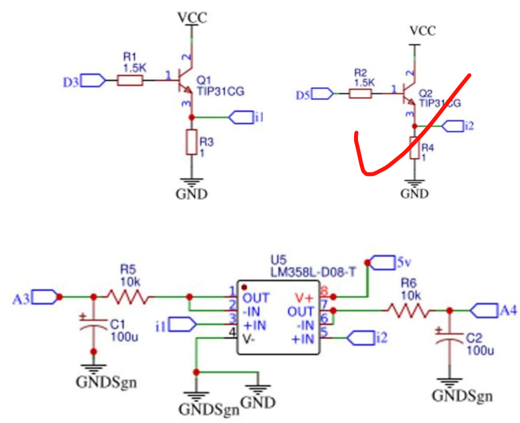

Fig. 2. Treatment of current measurement.

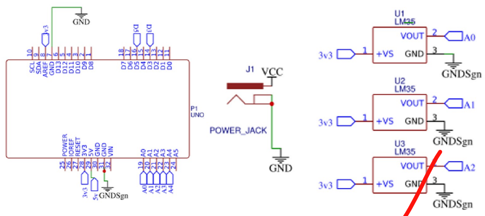

Fig. 3. Arduino y temperature sensor.

3. FIT PHYSICS-BASED MODEL

For this section we seek to understand the phenomenological modeling of TC-LAB and develop a script in python with the phenomenological modeling of TC-LAB.

In our temperature plant there is heat transfer by all three types. Radiation heat transfer comes from the movement of particles and is emitted as electromagnetic photons .Convection transfer occurs through surrounding physical contact , such as air. Heat transfer by conduction is through physical contact of two materials where energy transfer occurs, but, unlike convection, the contacting material is stationary, fig. 4.

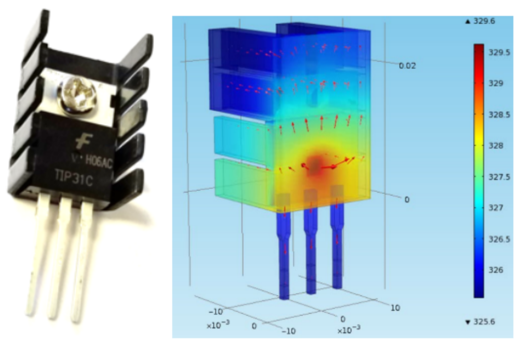

Fig. 4. Model transistor, input temperature: electric current, output temperature: convection and radiation.

In our system, the 3 types of heat transfer interact, but the convection given by the surrounding air and the radiation of the transistor when receiving electric current act to a greater extent. We now seek to mathematically define how the different forms of energy in our system interact, so we make use of an energy balance of our system defined as (1).

3.1 Energy balance

Searching for full steady state.

$$\frac{\Delta Q}{\Delta t} = Q_{in} - Q_{Out} \tag{1}$$

- Q: System heat.
- Q_{in}: Heat input.
- Q_{Out}: Head output.

For the mathematical modeling of convection we use Newton's law of cooling, this equation describes the behavior of a thermal system with the surrounding medium, this model is described in (2). And to describe the phenomenon of transfer through radiation we use the Stefan-Boltzmann law that is exposed in (3).

$$A \cdot U \cdot (T - T_{amb}). \tag{2}$$

Quantity	Value
Ambient temperature	296.15 K (23°C)
Temperature og the BJT	296.15 K (23°C)
Heat input	0 to 1 W
Parameter that relates the heater output	0.014 W/ % heater
Heat capacity	500 J / Kg-K
Area of the BJT	$1.2 \cdot 10^{-3}$
Mass	0.004 Kg (4 gm)
Heat transfer coefficient	$5W/m^2 - K$
Emissivity	0.9
Stefan - Boltzmann constant	$5.67 \cdot 10^{-8} W/m^2 - K^4$

Table 1. phenomenological model constants

A : Area of the BJT, U: Heat transfer coefficient , $Tamb$: ambient temperature and T: Temperature of the BJT.

$$\varepsilon \cdot \sigma \cdot A \cdot (T^4 - T_{amb}^4) \tag{3}$$

σ : Stefan -boltzman constant and ε: Emissivity .

We now use the derivative to relate the variation of heat with time as defined in (1).

$$\frac{\partial Q}{\partial t} = \alpha \cdot Q_{in} - A \cdot U(T - T_{amb}) - \varepsilon \cdot \sigma \cdot A \cdot (T^4 - T_{amb}^4)(4)$$

With the help of Thermodynamics we can define heat in terms of temperature and some constants depending on the materials which can be germanium, gallium arsenic or silicon in our case, see table 1.

Specific heat equation.

$$Q = m \cdot C_p \cdot (T - T_{ref}) \tag{5}$$

- m: Mass.
- C_p: Specific heat.
- T: BJT Temperature.
- T_{ref}: Reference. temperature.

Also for the phenomenological model we define a hypothesis where e is the temperature of the environment. hypothesis where e the temperature of the environment is constant. After substituting Q to obtain a differential equation as a function of the transistor temperature as shown in the equation (6).

$$mC_p \frac{\partial T}{\partial t} = \alpha \cdot Q_i + A \cdot U \cdot (T_{amb} - T) + \in \cdot \sigma \cdot A(T_{amb}^4 - T^4)(6)$$

$K_T(T - T_{amb})$: Newton's law of cooling (Convection).

$\in \sigma A(T^4 - T_{amb}^4)$: Stefan & Boltzmann's law (Radiation).

- K_T : constant loss to the environment.
- T : BJT Temperature.
- T_{amb} : Environment temperature.
- $\in$: Emissivity factor $(0 - 1)$.
- σ : Stefan & Boltzmann's constant.
- A : BJT area.

by using the phenomenological model applying the thermodynamic equations of TCLAB, shown in Fig (5).

4. CONNECTION OF THE TC-LAB IN PYTHON

In this part of the experimental work we initially connect the TC-lab and the computer through the Arduino IDE in order to start a communication, later through Python using the Spyder IDE you connect to Arduino with the

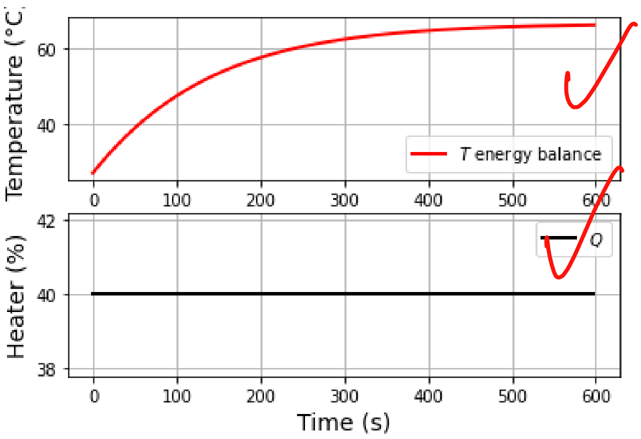

Fig. 5. Phenomenological model

tclab_cae.tclab_cae library, from it, the TClab operation is presented, inserting a step with an amplitude of 40 in the second 10 for 10 minutes,The data is then saved in a .txt file for analysis to obtain the first and second order models with delay, see fig (6).

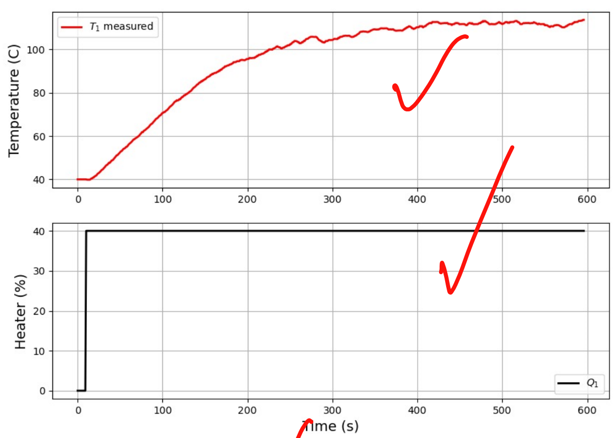

Fig. 6. 40% step response

5. TC-LAB MODEL

First-order systems by definition are those that have a single pole and are represented by first-order ordinary differential equations. It means that the maximum order of the derivative is order 1. Considering the case of first-order linear differential equations, with constant coefficients and zero initial condition as shown in equation (7) (Castaño, 2022d).

$$a_1 \cdot \frac{\mathrm{d}y(t)}{\mathrm{d}t} + a_0 \cdot y(t) = b_0 \cdot y(t) \tag{7}$$

whose transfer function in the Laplace transformed domain is (8)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{Ke^{-\theta s}}{\tau s + 1} \tag{8}$$

5.1 First order model

In the curves obtained as a response, two representative points are chosen. In general, these points are those for which the response reaches 28.3% and 63.2% of its final value; these points occur when the times elapsed from the moment of application of the step.(Castaño, 2022b)

see Fig 7.

$$Gp(s) = \frac{1.84e^{-25.09s}}{115.6s + 1} \tag{9}$$

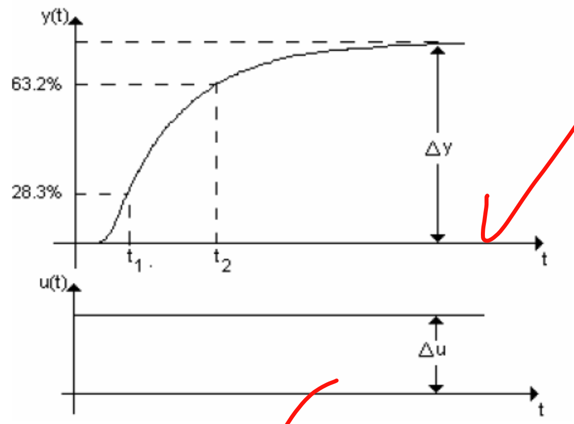

Fig. 7. First order model

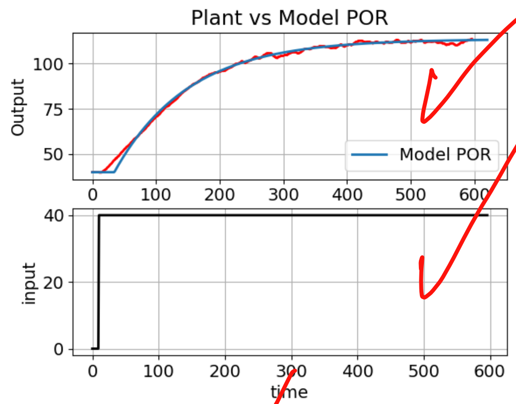

Fig. 8. First order model

With the data obtained in the previous graph, the equations are formulated:

$$\theta + \frac{\tau}{3} = t_1 \tag{10}$$

$$\theta + \tau = t_2 \tag{11}$$

t1 and t2 are read directly from the cards given by the graphs or are estimated from the database. Solving equations (10) and (11) simultaneously, we estimate the values of THETA and TAU.(Castaño, 2022b)

For the first order model, we can observe the settling time Tss, the steady state output Yee and the delay after applying the 40 % step as shown in equation (9) and figure (2).

5.2 Second order model

Second order systems like figure 9 are all those that have two poles and are typically represented by second order

Table 2. Thermal model of the BJT transistor

Variable	Definition	Values	Unit
T	Temperature	113	°C
τ	Time constant	115.6	s
k	Gain	1.84	°C/%
H	Heater	40	%
θ	Delay	25.09	%

ordinary differential equations. Considering the case of second-order linear differential equations, with constant coefficients and zero initial condition as shown in equation (12) (Castaño, 2022c).

$$a_2 \cdot \frac{\mathrm{d}y^2(t)}{\mathrm{d}t} + a_1 \cdot \frac{\mathrm{d}y(t)}{\mathrm{d}t} + a_0 \cdot y(t) = b_0 \cdot y(t) \quad (12)$$

whose transfer function in the Laplace transformed domain is:

$$G(s) = \frac{Kw_n e^{-\theta s}}{s^2 + 2\zeta w_n s + w_n^2}, \zeta < 1 \quad (13)$$

$$G(s) = \frac{ke^{-\theta s}}{(r_1 s + 1)(r_2 s + 1)}, \zeta \geq 1 \quad (14)$$

Where:

$$r_{1,2} = \frac{\zeta \pm \sqrt{\zeta^2 - 1}}{w_n} \quad (15)$$

The second order equation after finding the three times in the reaction curve and applying the equations (16), (17), (18), (19), (20) and (21) .

$$x = \frac{t_2 - t_1}{t_3 - t_1} \quad (16)$$

$$\zeta = \frac{0.0805 - 5.547(0.475 - x)^2}{x - 0.356} \quad (17)$$

$$F_2(\zeta) = \begin{cases} 2.6\zeta - 0.6; \zeta \geq 1 \\ 0.708(2.811)^{\zeta} < 1 \end{cases} \quad (18)$$

$$w_n = \frac{F_2(\zeta)}{t_3 - t_1} \quad (19)$$

$$F_3(\zeta) = 0.922(1.66)^{\zeta} \quad (20)$$

$$\theta = t_2 - \frac{F_3(\zeta)}{w_n} \quad (21)$$

If when applying equation (21) a negative value is obtained, it is assumed that the plant model has no delay. The values of the parameters estimated with equations (17), (19) and (21) are replaced in equation (13) or in equation (14) and this the experimental model of the plant is obtained (22).

$$Gp(s) = \frac{1.84}{4927s^2 + 149.1s + 1} \quad (22)$$

k: plant gain, w_n: natural frequency, ζ: coefficient of damping, θ: plant dead time, r_1, r_2: time constants.

For plant identification, two models were approximated, a first order (9) and a second order (22) model. they were represented graphically in fig (8) and (9), respectively.

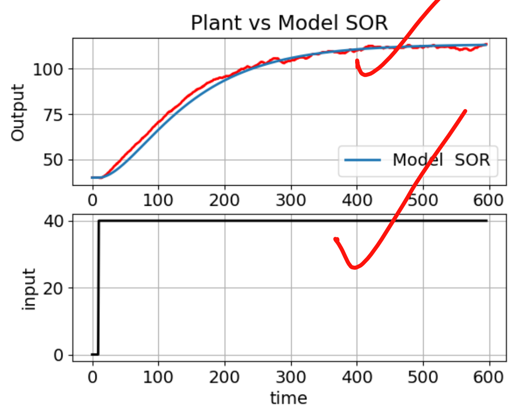

Fig. 9. Second order model

6. SAMPLING PERIOD SELECTION AND DISCRETIZATION

When sampling signals, special care must be taken when selecting the period demonstrated, since with said selection it is defined how close the analog and digital conversion is, and therefore the loss of information is less or greater. The smaller the sampling time, the more faithful the discrete signal is to the signal in the continuous domain, but to reduce this period, a large computing capacity must be available to process said information.

A basic concept is aliasing, aliasing is the effect that causes distinct continuous signals to be indistinguishable like in the fig (10) tomada . In this case, we can't recover the original signal uniquely from the digital signal. In other words, we have different signals, but they coincide in the samples taken, so the recovery of our signal isn't possible (Fernández, 2018).

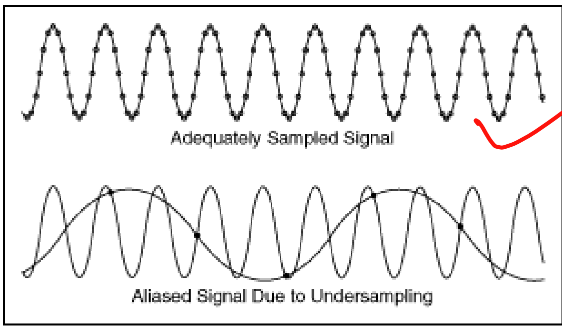

Fig. 10. Aliasing, extracted from Ambitiously (2022)

The conversion of the analog signal into Digital (A/D Conversion) is performed, among other reasons because digital signals have great advantages when being transmitted and/or processed: greater immunity to noise,easier processing and ease of multiplexing. (Jimmy. C, 2008)

To obtain the sampling period we use the bandwidth criterion. Let W [rad/s] be the band of the closed-loop system, Let G(jw) be the sinusoidal transfer function of the system, Let Gw be the closed-loop transfer function. The sampling frequency is defined as Ws SENSORICX (2022) and is calculated with equation (23).

$$\frac{\pi}{6 \cdot W_c} \leq T \leq \frac{\pi}{4 \cdot W_c} \quad (23)$$

As recommendation, For the next time, try to select a nearest T_s, such as $T_s = 26$ or $T_s = 27$

After finding the cutoff frequency, the sampling period is obtained, which for our temperature plant is 26.7004 seconds With the sampling time, the pulse transfer function is obtained (24).

$$HGp(z) = \frac{0.02428 \cdot z + 0.3552}{z - 0.7938} \cdot z^{-1} \quad (24)$$

$T_s = 26.7004$

7. MATHEMATICAL MODEL OPTIMIZATION

For Peña (2019) the deterministic models are those in which we have a mathematical model previously designed, that is to say we have a function and constraints to optimize. As specified above, the TCLAB system works as a deterministic model, adopting some physical constraints observed in table (1) and its behavior simulated and optimized in fig (11).

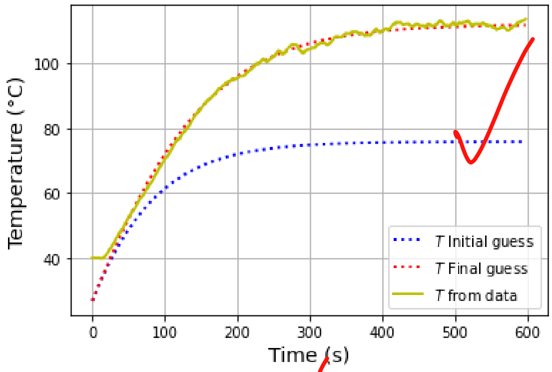

Fig. 11. Optimization model

8. PSEUDO RANDOM SIGNALS

When the systems are very linear and it is desired to have a valuable database of information, a PRBS signal is applied that allows to see the behavior of the system at high and low frequencies. In the manipulated variable, a pseudo-random signal is injected to vary the behavior of the plant, this signal covers several frequency ranges of the machine so that the model represents the dynamic behavior in a good way.

When there are positive steps the TC-LAB dynamics is fast and for negative steps the deportment is very slow, since the temperature variable has a lot of inertia, see figure (12).

These are signals that assume only two values. A PRBS signal is a periodic, deterministic signal that has properties similar to white noise. It can be generated from the equation in differences. (Gómez, 2022)

The result yielded by the optimization algorithm can be visualized in Equation (25) and its response in the compared to the approximation fig (13).

$$G1(s) = \frac{1.80909e^{-11.8264s}}{205.90337s + 1} \quad (25)$$

9. STABILITY ANALYSIS

When it is required to determine the stability of a plant, it is necessary that we start by identifying the poles and

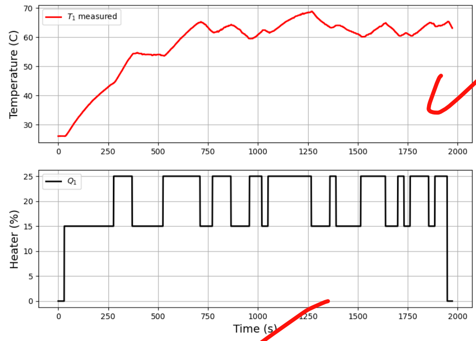

Fig. 12. Signal PBRS

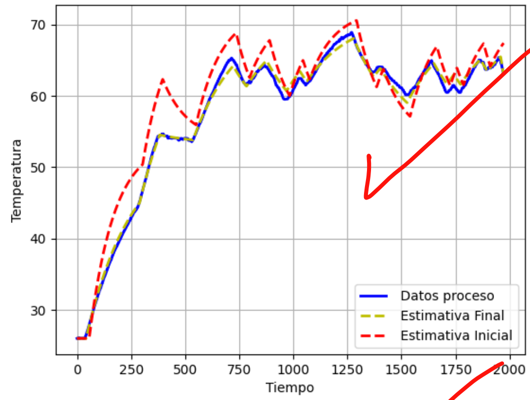

Fig. 13. Optimization model of PBRS

zeros, mainly those since they are the ones that dominate the behavior of the system.

Starting from the above, we extracted the poles in continuous time which is located at: -0.0086+0j without having zeros, therefore we did the same in discrete time obtaining a zero at: -14.6293+0j and 2 poles that describe the dynamics in : 0.7938+0j and 0+0j, the latter on the origin of the Z plane.

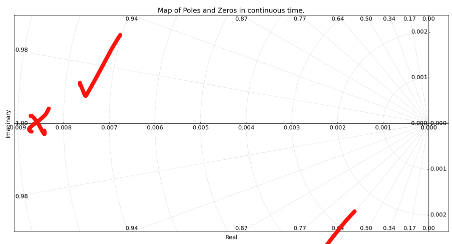

Fig. 14. Poles in continuous time

It is evident that the only pole of the system in continuous time (see table 3) located at sigma = -0.00865051 + 0j describes its stabilization time, that is, (-1/-0.00865051) * 4 = 462.40 seconds, by carefully analyzing the response graph of the plant (see figure 6) we can confirm this time and see that it is effectively stable around this value; on the other hand the system is stable since the pole is in the negative half plane of the S plane, in addition to a slow

Table 3. Poles and zeros of the system

Continuous time		
Poles:	1°	-0.00865+0j
Zeros:	Does not have	
Discrete time		
Poles:	1°	0.7938+0j
	2°	0+0j
Zeros:	1°	-14.6293+0j

response since it is close to the origin, we also see that the single pole has no imaginary part, which describes a dynamic without oscillations or overshoot.

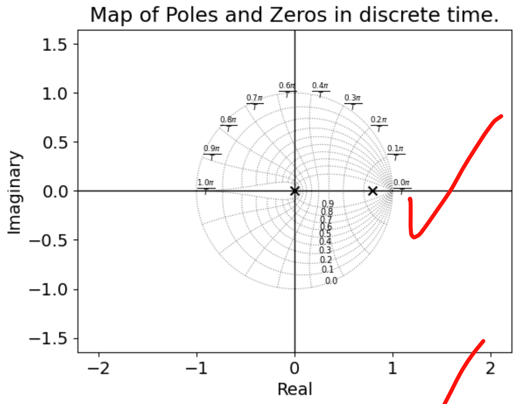

Fig. 15. Poles and zeros in discrete time

Likewise, of the poles and zeros in discrete time (see table 3), a zero is evident in the negative half plane which refers to the gain of the system in addition to being shown as a discrete delay (d = 1), on the other hand there is a pole at the origin which does not contribute anything, another pole that dominates the dynamics of the system located at 0.7938+0j, both poles being within the unitary circumference giving rise to the fact that the system is stable and will not have strange dynamics.

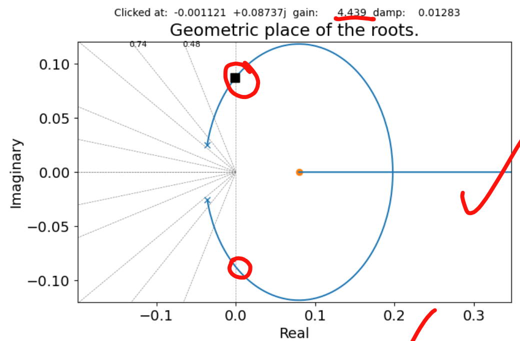

Fig. 16. Geometric place of the roots

By adding the delay to the transfer function via Python's Pade() method, What is obtained is the inclusion of a pole to the system, which results in a pair of complex conjugated poles, as well as an unstable zero, which shows that the feedback system becomes unstable when the gain is greater than 4.5 (see figure 16), that is. it becomes oscillatory; It must also be clarified that it is neither possible nor coherent to have a very high gain because that would mean having an infinite energy source, which is not possible to have.

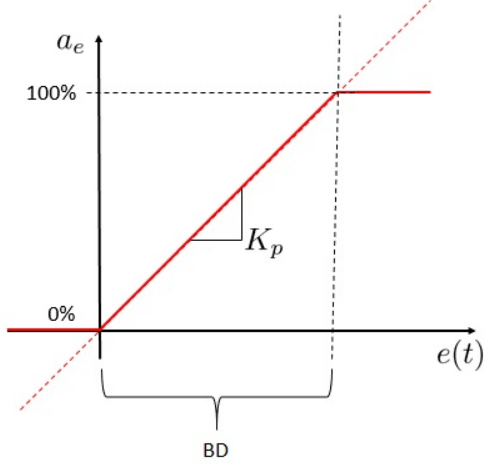

Fig. 17. image extracted from Controlautomaticoeducacion.com

9.1 Bode diagram

A Bode graphics consists of two plots: one is a diagram of the logarithmic modulus of the sinusoidal transfer function and the other is a diagram of the phase angle. Both plots are graphed as a function of frequency on a logarithmic scale. the essence of Bode is to take a system and inject a sinusoidal with a given frequency.The Bode diagram shows the frequency variations of the input signal and the dynamic behavior of the system at different frequency changes.Where commonly the control systems are low pass filters that have a very good response to low oscillations.

The gain margin is the quantity of gain in dB that can be added to the loop before the closed-loop system becomes unstable.(Kuo, 1996)

The phase margin indicates the effect on system stability due to changes in system parameters.Phase margin is the amount of pure delay that can be added to the system before the closed-loop system becomes unstable.(Kuo, 1996)

where is the bode plot?

10. PROPORTIONAL CONTROL

The proportional control is divided into different variables observed in the fig 17, as expressed Castaño (2022a) the proportional band (BD) is expressed in the total output divided between the gain (Kp), meaning that for an error is very large, the heater opens much more and when the error decreases the opening of the heater is closing.

- BD: Proportional band.
- Kp: Proportional gain.

10.1 Heater Opening Calculations

$$\alpha_e = Kp(Tr - T) \tag{26}$$

- α_e: Opening of Heater.
- $Tref$: Reference Temperature.
- T: BJT Temperature.

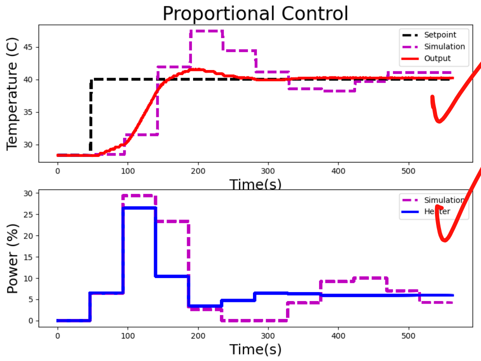

Fig. 18. Proportional control response

10.2 Calculations for the proportional band

$$BD = \frac{100\%}{Kp} \tag{27}$$

10.3 Stabilization Time Calculation

$$Tss = \frac{KpK}{1 + KpK} \tag{28}$$

- Tss: steady-state weather.
- K: System gain.
- $1 + KpK$: System feedback.

the proportional control is calculated by means of an algorithm with the equations (26) (27)(28).

The proportional controller was applied to the TCLAB which results in a reduction of the stabilization time although the steady state error increases as shown in fig (18). for a gain of 1.840349.

11. CONCLUSION

- The sampling period is of vital importance, since it eliminates or attenuates the loss of information when the A/D conversion is done. On the other hand, choosing the wrong sampling time is equivalent to having greater processing capacity.
- Analyzing and modeling the stability of a system is essential since generally the processes where one or more variables are controlled are financially and administratively expensive, so setting up a controller without having previously analyzed its behavior in the plant could render our final control element useless, delaying the project in progress as well as harming the company and our name.
- Based on all the analysis performed with the approximate models, discrete and frequency domain stability analysis, there should be a high correlation coefficient when the respective silver implementations are made.
- With the bode diagram it is possible to obtain a parameter that indicates the gain margin, which refers to the limiting gain that the system allows to enter an unstable state.
- The pseudo-random signal as input in a control system allows to obtain data that faithfully represent the real plant, this is due to the different input steps and their different frequencies, since with these samples a much more approximate first order model is obtained compared to the reaction curve method.
- proportional control performs an action so that the stabilization time decreases although it has a penalty based on the error, because if the error is very large proportional action also applies or is reflected in the response of the system.

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PID Controllers are missing

I Tested all your PIDs Controllers scripts in Python and all of them work very well. I don't know why you don't put that results.