Tarefa Básica - Regra de Cramer

① a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$$

$$Dx = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$$

$$Dy = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 + 2 = -4$$

$x = \frac{3}{5}$

$y = \frac{-4}{5}$

$V = \left\{\left(\frac{3}{5}, \frac{-4}{5}\right)\right\}$

b) $\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$

$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix}$ (0 + 9 + 4 = 13; 0 − 12 + 2 = −10)

$= -10 - 13 = -23$

$D_x = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix}$ (0 + 3 − 2 = 1; 0 − 21 − 1 = −22)

$= -22 - 1 = -23$

$x = \frac{-23}{-23} = 1$

$D_y = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{vmatrix}$ (6 + 12 + 14 = 32; 0 − 3 − 14 = −17)

$= 32 - 55 = -23$

$y = \frac{-23}{-23} = 1$

$D_z = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{vmatrix}$ (0 + 4 + 2 = 6)

$= 6 + 17 = 23$

$z = \frac{23}{-23} = -1$

$V = \{(1, 1, -1)\}$

② $\begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$

$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix}$ (−5 − 12 + 48 = 31; 45 + 8 + 8 = 61)

$= 61 - 31 = 30$

$D_y = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix}$ (−12 + 48 + 12 = 48; 108 + 2 − 32 = 78)

$= 78 - 48 = 30$

$y = \frac{30}{30} = 1$

Alternativa (A)

③ $\begin{cases} x+2y+z=1 \\ 3x+y-11z=-2 \\ 2x+3y-z=1 \end{cases}$

$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{matrix}$ (2 − 33 − 6 = −37; −1 − 44 + 9 = −36)

$= -36 + 37 = 1$

$D_x = \begin{vmatrix} 1 & 2 & 1 \\ -2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix} \begin{matrix} 1 & 2 \\ -2 & 1 \\ 1 & 3 \end{matrix}$ (1 − 33 + 4 = −28; −1 − 22 − 6 = −29; −4 − 11 − 3 = −18)

$= -29 + 28 = -1$ $\quad x = \frac{-1}{1} = -1$

$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & -11 \\ 2 & 1 & -1 \end{vmatrix} \begin{matrix} 1 & 1 \\ 3 & -2 \\ 2 & 1 \end{matrix}$ (2 − 22 + 3 = −17; 2 − 6 + 6 = 2)

$= -17 + 18 = 1$ $\quad y = \frac{1}{1} = 1$

$D_z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{matrix}$ (1 − 8 + 9 = 2)

$= 2 - 2 = 0$ $\quad z = \frac{0}{1} = 0$

$a + b + c = -1 + 1 + 0 = 0$ Alternativa ©

④ $\begin{cases} x+2y-3z=29 \\ x+3y+2z=4 \\ x-y-2z=8 \end{cases}$

$D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{matrix}$ (−9 − 2 − 4 = −15; −6 + 4 + 3 = 1)

$= 1 + 15 = 16$

$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} \begin{matrix} 29 & 2 \\ 4 & 3 \\ 8 & -1 \end{matrix}$ (−72 − 58 − 16 = −146; −174 + 32 + 12 = −130)

$= -130 + 146 = 16$ $\quad x = \frac{16}{16} = 1$

$D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} \begin{matrix} 1 & 29 \\ 1 & 4 \\ 1 & 8 \end{matrix}$ (−12 + 16 − 58 = −54; −8 + 58 − 24 = 26)

$= 26 + 54 = 80$ $\quad y = \frac{80}{16} = 5$

87 - 4 + 16 = 99

$$D_z = \begin{vmatrix} 1 & 2 & 29 & 1 & 2 \\ 1 & 3 & 4 & 1 & 3 \\ 1 & -1 & 8 & 1 & -1 \end{vmatrix}$$

24 + 3 - 29 = 3

$3 - 99 = -96$ $\qquad z = \frac{-96}{16} = -6$

$x + y + z = 1 + 5 - 6 = 0$ $\qquad$ Alternativa (A)

⑤
$$\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$$

0 + 4 + 0 = 4

$$D = \begin{vmatrix} 2 & 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & 0 & 2 \\ 3 & 2 & 1 & 3 & 2 \end{vmatrix} = 7 - 4 = 3$$

4 + 3 + 0 = 7

0 + 10 + 3 = 13

$$D_x = \begin{vmatrix} 5 & 1 & 0 & 5 & 1 \\ 3 & 2 & 1 & 3 & 2 \\ 7 & 2 & 1 & 7 & 2 \end{vmatrix} = 17 - 13 = 4$$

10 + 7 + 0 = 17

$x = \frac{4}{3}$

$2 \cdot \frac{4}{3} + y = 5$ $\qquad 2 \cdot \frac{7}{3} + z = 3$ $\qquad$ Alternativa (D)

$\frac{8}{3} + y = 5$ $\qquad \frac{14}{3} + z = 3$

$y = \frac{15 - 8}{3} = \frac{7}{3}$ $\qquad z = \frac{9 - 14}{3} = \frac{-5}{3}$

⑥
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ 2x + y \\ -x + 2y + 2z \end{bmatrix}$$

Alternativa (E)

$x = 3$ $\qquad 2x + y = 7$ $\qquad -3 + 2 \cdot 1 + 2z = -1$

$2 \cdot 3 + y = 7$ $\qquad -1 + 2z = -1$

$y = 1$ $\qquad 2z = 0$

$z = 0$

Tarefa Básica - Escalonamento (Gauss)

① $\begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$ $\quad V = \{(-2, 4, -1)\}$

$$\begin{pmatrix} 2 & -1 & -3 & | & -5 \\ 1 & 3 & -1 & | & 11 \\ 1 & 0 & -5 & | & 3 \end{pmatrix} \xrightarrow{\substack{3 \to \\ \oplus \\ \oplus \\ -2 \quad -1}} \begin{pmatrix} 0 & -1 & 7 & | & -11 \\ 0 & 3 & 4 & | & 8 \\ & & & | & \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 25 & | & -25 \\ & & & | & \end{pmatrix}$$

$x + 5 = 3$
$x = -2$

$3y - 4 = 8$
$3y = 12$
$y = 4$

$25z = -25$
$z = -1$

② $\begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases}$

$x = 2y = 3z$

$2y + y + \frac{2y}{3} = 11$

$\frac{6y + 3y + 2y}{3} = \frac{33}{3}$

$11y = 33$
$y = 3$

$x = 2.3$
$x = 6$

$2.3 = 3z$
$z = \frac{6}{3} = 2$

$6 + 2.3 + 3.2 = 6 + 6 + 6 = 18$

Alternativa (B)

③ $\begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{pmatrix} \xrightarrow{\substack{-2 \\ +}} \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{pmatrix} \xrightarrow{2 \to} \sim \begin{pmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & -5 & -10 \end{pmatrix}$$

$-5z = -10$

$z = 2$

Alternativa (D)

④ $x + y + z = 68$

$y + \frac{20}{100}z = x \Rightarrow y + \frac{z}{5} = x \Rightarrow \frac{5y + z}{5} = \frac{5x}{5} \Rightarrow 5y + z = 5x$

$\frac{20}{100}x + z = 3y \Rightarrow \frac{x}{5} + z = 3y \Rightarrow \frac{x + 5z}{5} = \frac{15y}{5} \Rightarrow x + 5z = 15y$

$$\begin{cases} x + y + z = 68 \\ -5x + 5y + z = 0 \\ x - 15y + 5z = 0 \end{cases}$$

Ali $\Rightarrow x =$ R$ 20,00

Bia $\Rightarrow y =$ R$ 13,00

Caio $\Rightarrow z =$ R$ 35,00

$$\begin{pmatrix} 1 & 1 & 1 & 68 \\ -5 & 5 & 1 & 0 \\ 1 & -15 & 5 & 0 \end{pmatrix} \xrightarrow{\substack{-1 \\ 5 \\ +}} \begin{pmatrix} 1 & 1 & 1 & 68 \\ 0 & 10 & 6 & 340 \\ 0 & -16 & 4 & -68 \end{pmatrix} \xrightarrow{\substack{4 \to \\ -6 \to}} \sim \begin{pmatrix} \cdots & \cdots & \cdots & \cdots \\ 0 & 136 & 0 & 1768 \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

$x + 13 + 35 = 68$

$x = 20$

$10 \cdot 13 + 6z = 340$

$130 + 6z = 340$

$6z = 210$

$z = 35$

$136y = 1768$

$y = 13$

Alternativa (A). Porque Caio tem 35 Reais e Ali 20 Reais, então Ali tem 15 Reais a menos que Caio.

⑤ $A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix}$ $\cdot$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $AX = \begin{bmatrix} 3y + 4z \\ x + 5z \\ 2x + y \end{bmatrix}$

$$\begin{cases} 3y + 4z = 134 & \longrightarrow \text{Alfeu} \\ x + 5z = 115 & \longrightarrow \text{Bento} \\ 2x + y = 48 & \longrightarrow \text{Cíntia} \end{cases}$$

$$\begin{pmatrix} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 2 & 1 & 0 & 48 \end{pmatrix} \xrightarrow{\cdot(-2)\ \oplus} \sim \begin{pmatrix} 0 & 3 & 4 & 134 \\ & & & \\ 0 & 1 & -10 & -182 \end{pmatrix} \xrightarrow{\cdot(-3)} \sim \begin{pmatrix} & & & \\ & & & \\ 0 & 0 & 34 & 680 \end{pmatrix}$$

$x + 5.20 = 115$

$x + 100 = 115$

$x = 15$

$3y + 4.20 = 134$

$3y + 80 = 134$

$3y = 54$

$y = 18$

$34z = 680$

$z = 20$

Alternativa (A)

$x + y + z = 15 + 18 + 20 = 53 \Rightarrow R\$\,53{,}00$