Tarefa Básica

① $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} = B^{-1}$ $\quad B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$ $\quad x + y$

$2 - 5 = -3$

$\underbrace{x = 2 \qquad y = -5}$
pela regra das matrizes de ordem 2

Alternativa ⓒ

② $A = \begin{bmatrix} 1 & 0 & 1 \\ K & 1 & 3 \\ 1 & K & 3 \end{bmatrix}$ $\quad \det A = 0$

Alternativa ⓒ

$1 + 3K + 0 = 3K + 1$

$$\left| \begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ K & 1 & 3 & K & 1 \\ 1 & K & 3 & 1 & K \end{array} \right| = K^2 + 3 - 3K - 1 = K^2 - 3K + 2 = 0$$

$3 + 0 + K^2 = K^2 + 3$

$\Delta = 9 - 4.1.2$

$\Delta = 1$

$K = \dfrac{3 \pm 1}{2}$ $\begin{cases} K' = 2 \\ K'' = 1 \end{cases}$

③ $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ $\quad B = A^{-1} = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \div 2$

$\det A = 12 - 10$

$\det A = 2$

$B = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$

Alternativa ⓒ

④ $20 + 2x + 3x = 5x + 20$

$$\begin{vmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{vmatrix} \begin{matrix} x & 1 \\ 3 & 1 \\ 10 & 1 \end{matrix} = x^2 + 26 - x - 20 = x^2 - 5x + 6 \neq 0$$

$x^2 + 20 + 6 = x^2 + 26$

$\Delta = 25 - 4 . 1 . 6$

$\Delta = 1$

$x = \frac{5 \pm 1}{2} \rightarrow x' = 3$, $x'' = 2$

Alternativa Ⓐ.

⑤ $2 + 2 + 2 = 6$

$$A = \begin{vmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix} \begin{matrix} -1 & -1 \\ 2 & 1 \\ 1 & 1 \end{matrix} = 7 - 6 = 1$$

$1 + 2 + 4 = 7$

$$A' = \begin{bmatrix} 1 & 0 & 1 \\ +1 & -1 & 0 \\ 0 & +2 & 1 \end{bmatrix} \qquad \bar{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \div 1 = A^{-1}$$

$$S = A + A^{-1} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

Alternativa Ⓑ.

⑥ $(X . A)^t = B$

$(X^t . A^t)^t = B^t$

$X . A = B^t$

$X . A . A^{-1} = B^t . A^{-1}$

$X = B^t . A^{-1}$

Alternativa Ⓑ

⑦ $B = \begin{bmatrix} x \\ y \end{bmatrix} \qquad C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$

$B = \begin{bmatrix} x \\ y \end{bmatrix}$

$A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix}$ $\qquad AB = C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$

$\det A = 24 - 25 = -1$

$A^{'} = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \div (-1) = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}$ Alternativa D

8) $A = \begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix}$ $\qquad \det A = 2 + 2k \qquad \det A^{-1} = \frac{1}{2+2k}$

$2 + 2k = \frac{1}{2+2k}$

$(2+2k)^2 = 1$

$4 + 8k + 4k^2 = 1$

$4k^2 + 8k + 3 = 0$

$\Delta = 64 - 4.4.3$

$\Delta = 16$

$k = \frac{-8 \pm 4}{8}$ $\begin{cases} k' = \frac{-4}{8} = \frac{-1}{2} \\ k'' = \frac{-12}{8} = \frac{-3}{2} \end{cases}$

$\frac{-3}{2} - \frac{1}{2} = \frac{-4}{2} = -2$ Alternativa B

9) a) $(A+B).(A-B) = A^2 - BA + BA - B^2$

b) $(A+B).(A+B) = A^2 + AB + BA + B^2$ $\qquad (A+B)^2 = A^2 + 2AB + B^2$

Só podemos representar com 2AB, se AB e BA forem iguais.

c) $\frac{\det A}{\det(-A)}$ $\qquad \det(-A) = (-1)^2 . \det A$

$\det(-A) = \det A.$

$\dfrac{\det A}{\det A} = 1$ usando a propriedade do fator comum

d) $\det B = \dfrac{1}{\det A}$ $B = \dfrac{1}{A}$