Diagonalisation and applications questions 26/09/2025

1) $T=\begin{bmatrix}6&-1\\2&3\end{bmatrix}$ $C=\begin{bmatrix}1&1\\1&2\end{bmatrix}$ Calculate $D=C^{-1}TC$.

$$C^{-1}=\begin{bmatrix}1&1\\1&2\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}\to R_2-R_1$$

$$\begin{bmatrix}1&1\\0&1\end{bmatrix}\begin{bmatrix}1&0\\-1&1\end{bmatrix}\to R_1-R_2$$

$$\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}2&-1\\-1&1\end{bmatrix}=C^{-1}\quad\begin{bmatrix}2&-1\\-1&1\end{bmatrix}$$

$$\cancel{T^2=CD^2C^{-1}=\begin{bmatrix}1&1\\1&2\end{bmatrix}\begin{bmatrix}6&0\\0&3\end{bmatrix}^2\begin{bmatrix}2&-1\\0&1\end{bmatrix}\begin{bmatrix}0&0\\-1&0\end{bmatrix}}$$

$$\cancel{\begin{bmatrix}1&1\\1&2\end{bmatrix}\begin{bmatrix}36&0\\0&9\end{bmatrix}\begin{bmatrix}0&0\\-1&0\end{bmatrix}=\begin{bmatrix}1&1\\1&2\end{bmatrix}\begin{bmatrix}0&0\\-9&0\end{bmatrix}}$$

$$\cancel{=\begin{bmatrix}-9&0\\-18&\end{bmatrix}}$$

$$D=C^{-1}TC$$

$$\cancel{\begin{bmatrix}0&0\\-1&0\end{bmatrix}\begin{bmatrix}6&-1\\2&3\end{bmatrix}\begin{bmatrix}1&1\\1&2\end{bmatrix}=\begin{bmatrix}0&0\\-6&1\end{bmatrix}\begin{bmatrix}1&1\\1&2\end{bmatrix}}$$

$$\cancel{=\begin{bmatrix}0&0\\-5&-4\end{bmatrix}}$$ ~~NOT an option~~ :(

$$\begin{bmatrix}2&-1\\-1&1\end{bmatrix}\begin{bmatrix}6&-1\\2&3\end{bmatrix}\begin{bmatrix}1&1\\1&2\end{bmatrix}=\begin{bmatrix}2&-1\\-1&1\end{bmatrix}\begin{bmatrix}5&4\\5&8\end{bmatrix}$$

$$=\begin{bmatrix}5&0\\0&4\end{bmatrix}\qquad D=\begin{bmatrix}5&0\\0&4\end{bmatrix}$$

2).

$$\begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$R_2 + \frac{1}{7} R_1$

$$\begin{bmatrix} 7 & 1 \\ 0 & \frac{3}{7} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{3}{7} & 1 \end{bmatrix}$$

$\frac{7}{3} R_2$

$$\begin{bmatrix} 7 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & \frac{7}{3} \end{bmatrix}$$

$R_1 - R_2$

$$\begin{bmatrix} 7 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -\frac{7}{3} \\ 1 & \frac{7}{3} \end{bmatrix}$$

$R_1 - \frac{1}{7} R_1$ Scale.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -\frac{1}{3} \\ 1 & \frac{2}{3} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & -\frac{1}{3} \\ 1 & \frac{7}{3} \end{bmatrix}$$

$$\begin{bmatrix} 0 & -\frac{1}{3} \\ 1 & \frac{7}{3} \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix} = \cancel{\begin{bmatrix} 0 & -\frac{1}{2} \\ 1 & 2/3 \end{bmatrix}}$$

$$\begin{bmatrix} 0 & -\frac{1}{3} \\ 1 & \frac{7}{3} \end{bmatrix} \begin{bmatrix} -7 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

3) $= T = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$C - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ ✓ ithink lest

$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $R_2 - R_1$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ ✓

$D = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$

one method? $D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

4) $D = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ inverse $C^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

$T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & -2a \\ 0 & a \end{bmatrix}$

$= \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

5) $T^3 = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}^3 \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

$T^3 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 125 & 0 \\ 0 & 64 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 250 & -125 \\ -64 & 64 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 250 & -125 \\ -64 & 64 \end{bmatrix} =$

$= \begin{bmatrix} 186 & -61 \\ 122 & 3 \end{bmatrix}$

6) $T^3 = \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} 0 & -1/3 \\ 1 & 7/5 \end{bmatrix}$

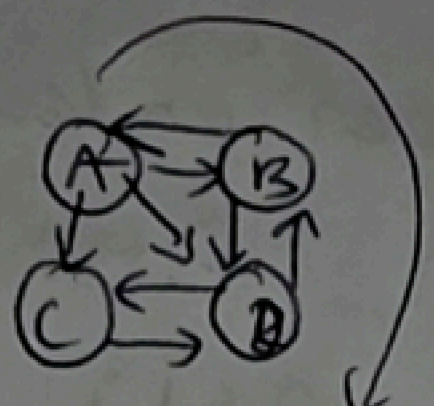
$T^3 = \begin{bmatrix} -7 & 8 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1/3 \\ 1 & 7/5 \end{bmatrix} = \begin{bmatrix} 8 & 21 \\ 0 & -1 \end{bmatrix}$

7) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} =$

$= \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

Eigen problems

Page Rank.



because A has 3 options

$L_A = (0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ (A, B, C, D)

$L_B = (\frac{1}{2}, 0, 0, \frac{1}{2})$

$L_C = (0, 0, 0, 1)$

$L_D = (0, \frac{1}{2}, \frac{1}{2}, 0)$.

Now can build link matrix L.

$$L = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 1 & 0 \end{pmatrix}$$

r_A equals sum from $j=1$ to n where n is all web pages to link matrix relevant to whatever's multiplied by rank of j.

$$r_A = \sum_{j=1}^{n} L_{A,j} r_j$$

$$r = \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix}$$

$r^i = Lr$. $r^{i+1} = Lr^{i+1}$

$d = \cancel{0.1}$. $0 < d < 1$

ie $r^{i+1} = d(Lr^i) + \frac{1-d}{n}$

Page Rank Lab

$F = (0, 0, \frac{1}{2}, \frac{1}{2}, 0, 0)$

Question Sheet

1) $\begin{bmatrix} -2/\sqrt{9} \\ -2/\sqrt{9} \\ \cdot 1/\sqrt{9} \end{bmatrix}$

$\begin{bmatrix} -3 \\ -3 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 1/2 \\ -1/2 \\ \cdot -1 \end{bmatrix}$

2)
0001
1000
0100
0010

eigen values
$-0.+0.j$
$0.+1.j$
$0.-1.j$
$1.+0.j$

- Because of the loop
- Some of the eigenvectors complex
- ~~not small component~~!
- system to small.

3)

5) $A = \begin{bmatrix} 3/2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$

Calc. characteristic polynomial

$\det A - \lambda y$

$I \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\lambda I \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

4) L =
0100
1000
0001
0010

1 -1 1 -1

$\det(A - \lambda I) = \begin{matrix} 3/2 - \lambda & -1 \\ -1/2 & 1/2 - \lambda \end{matrix} \quad \begin{bmatrix} 3/2 - \lambda & -1 \\ -1/2 & 1/2 - \lambda \end{bmatrix}$

$\lambda^2 - (3/2 + 1/2)\lambda + (3/4 - \frac{1}{2})$

$\lambda^2 - 2\lambda + \frac{1}{4}$