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MF850: Advanced Computational Methods

Problem Set 2

If you run into problems, try to verify each component of your code on simple cases to make sure they work independently.

Due date: See Blackboard.

Problem 2.1 For business and regulatory reasons, it is necessary to quantify risk in simpler terms than the complete risk distribution. One common measure is the *Value-at-Risk*, which for the profit-and-loss distribution X and at the *level* α is the quantity $\mathrm{VaR}_\alpha(X)$ for which $X + \mathrm{VaR}_\alpha(X) \geq 0$ with probability $1 - \alpha$. More precisely, it is the minimum amount that can be added to X to keep the sum non-negative with probability $1 - \alpha$. Mathematically, it is the $(1 - \alpha)$-quantile of the negative of the distribution:[1]

$$\mathrm{VaR}_\alpha(X) = \inf\{x \in \mathbb{R} : \mathbb{P}[X + x \geq 0] \geq 1 - \alpha\} = F_{-X}^{-1}(1 - \alpha).$$

The accompanying csv file contains returns $(r_i)_{i=1,\dots,N}$ data of an asset. Consider an investor who invests $W = 100$ in this asset.

Remark: The hints provided are for Julia, but similar functions are available in other languages/libraries.

(a) Compute an estimate $\widehat{\mathrm{VaR}}_\alpha$ for the Value-at-Risk at a level $\alpha = 0.05$ of the portfolio returns by empirically finding the quantile of the distribution.

Hint: Use `quantile` from the `Statistics` library.

(b) Compute an estimate $\widehat{\mathrm{VaR}}_\alpha$ for the Value-at-Risk at a level $\alpha = 0.05$ of the portfolio returns by assuming a normal distribution, finding its parameters $\hat{\mu}$ and $\hat{\sigma}$, and finally using an expression for the quantiles of a normal distribution.

Hint: Use `quantile` and `Normal` from the `Distributions` library.

(c) The measure calculated above gives a one-period value-at-risk estimate. Hence, it does not reflect compounding effects. Therefore, in practice the value-at-risk is often estimated as

$$(e^q - 1)W,$$

where q is the value-at-risk quantile of the returns data.

Estimate this value.

(d) You want to asses how accurate the estimates in (a) and (b) are. Using bootstapping, estimate the standard deviation of the estimates in (c), i.e., estimate $\mathrm{SE}(\widehat{\mathrm{VaR}}_\alpha)$.

Hint: Use `sample` from the `Distributions` library and `std` from the `Statistics` library. In general, it might be preferable to use a library such as `Bootstrap` instead of doing it manually.

[1] In risk management, negative outcomes are commonly denoted by positive values because they require (positive amounts of) capital to mitigate risk. Therefore analysis focuses on the loss distribution $-X$ instead of X.

