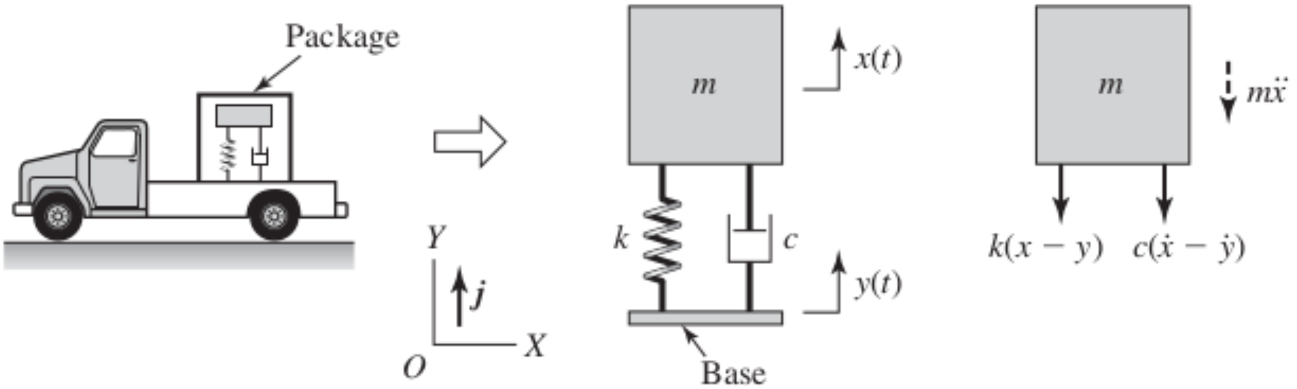


$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n^2 x = 2\zeta\omega_n\frac{dy}{dt} + \omega_n^2 y$$

$$\frac{d^2x}{d\tau^2} + 2\zeta\frac{dx}{d\tau} + x = 2\zeta\frac{dy}{d\tau} + y \tag{5.76}$$

$$y(\tau) = y_o \sin(\Omega\tau) \tag{5.77}$$

$$\frac{d^2x}{d\tau^2} + 2\zeta\frac{dx}{d\tau} + x = 2\zeta\, y_o\Omega\cos(\Omega\tau) + y_o\sin(\Omega\tau) \tag{5.78}$$

$$x(\tau) = y_o H(\Omega)\sqrt{1 + (2\zeta\Omega)^2}\,\sin(\Omega\tau - \theta(\Omega) + \varphi) \tag{5.79}$$

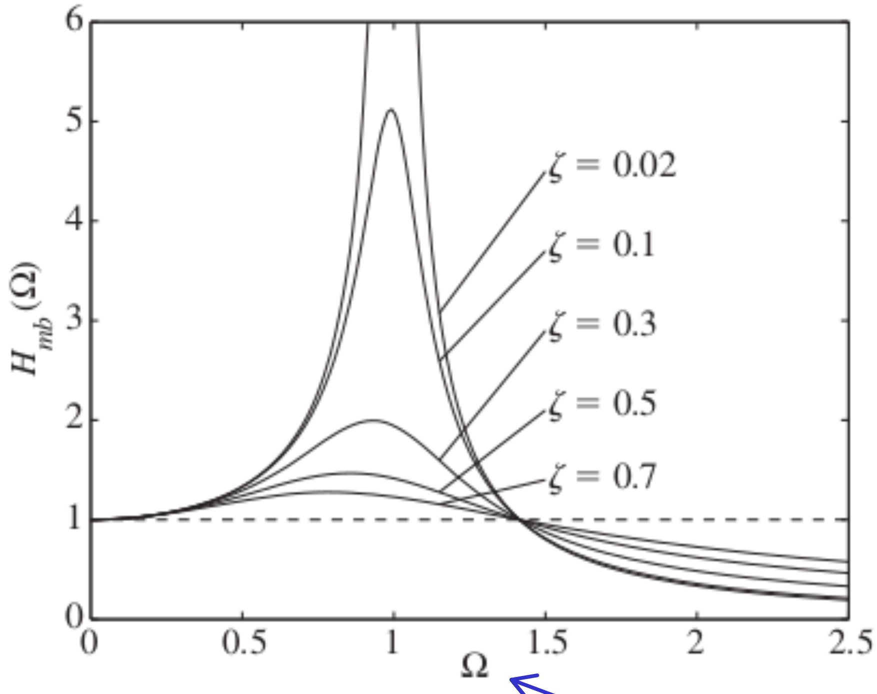
$$\varphi = \tan^{-1} 2\zeta\Omega$$

$$x(\tau) = \underbrace{y_o H_{mb}(\Omega)}_{\text{Displacement magnitude}}\sin(\Omega\tau - \underbrace{\psi(\Omega)}_{\text{Phase}}))$$

$$H_{mb}(\Omega) = \frac{\sqrt{1 + (2\zeta\Omega)^2}}{\sqrt{(1 - \Omega^2)^2 + (2\zeta\Omega)^2}}$$

$$\psi(\Omega) = \tan^{-1}\frac{2\zeta\Omega^3}{1 + \Omega^2(4\zeta^2 - 1)}$$

$$\begin{aligned} v(\tau) &= y_o\omega_n\Omega H_{mb}(\Omega)\cos(\Omega\tau - \psi(\Omega)) \\ a(\tau) &= -\Omega^2\omega_n^2 x(\tau) \end{aligned} \tag{5.83}$$



(a)

$\boxed{\text{frequency ratio } \Omega}$

$\boxed{\Omega = \omega/\omega_n}$

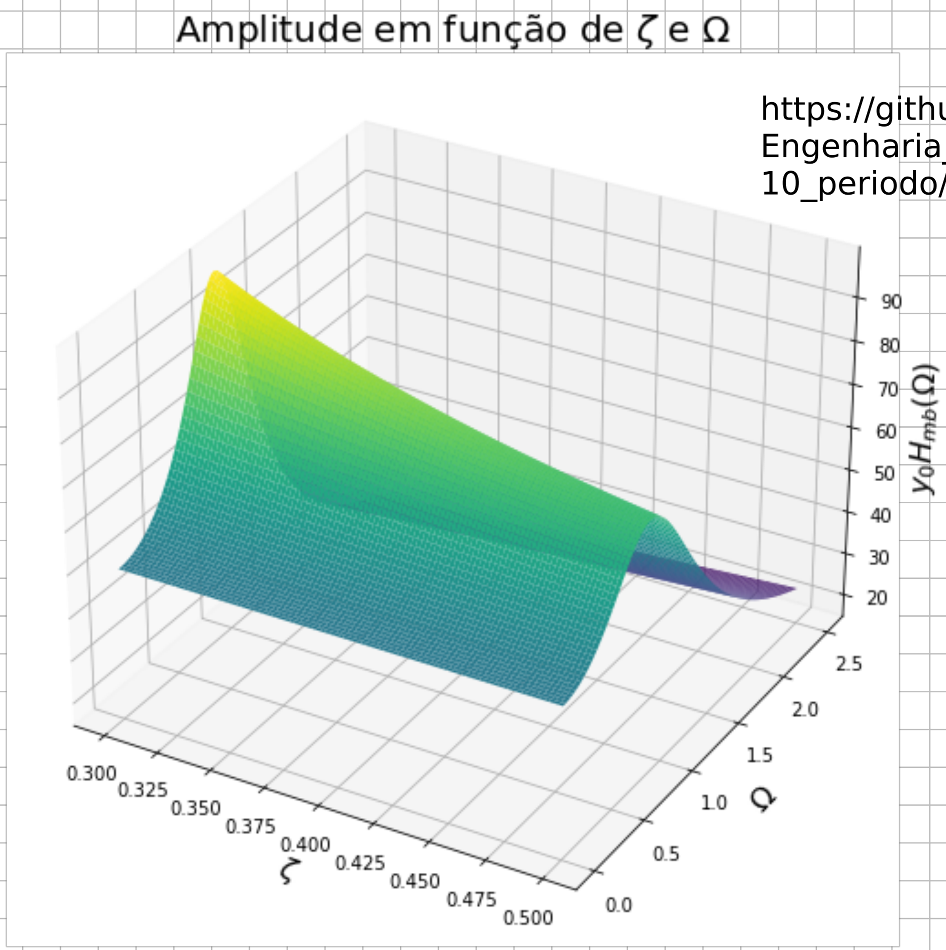
$\omega = \frac{2\pi v}{\lambda}$

Usar essa frequência no plot

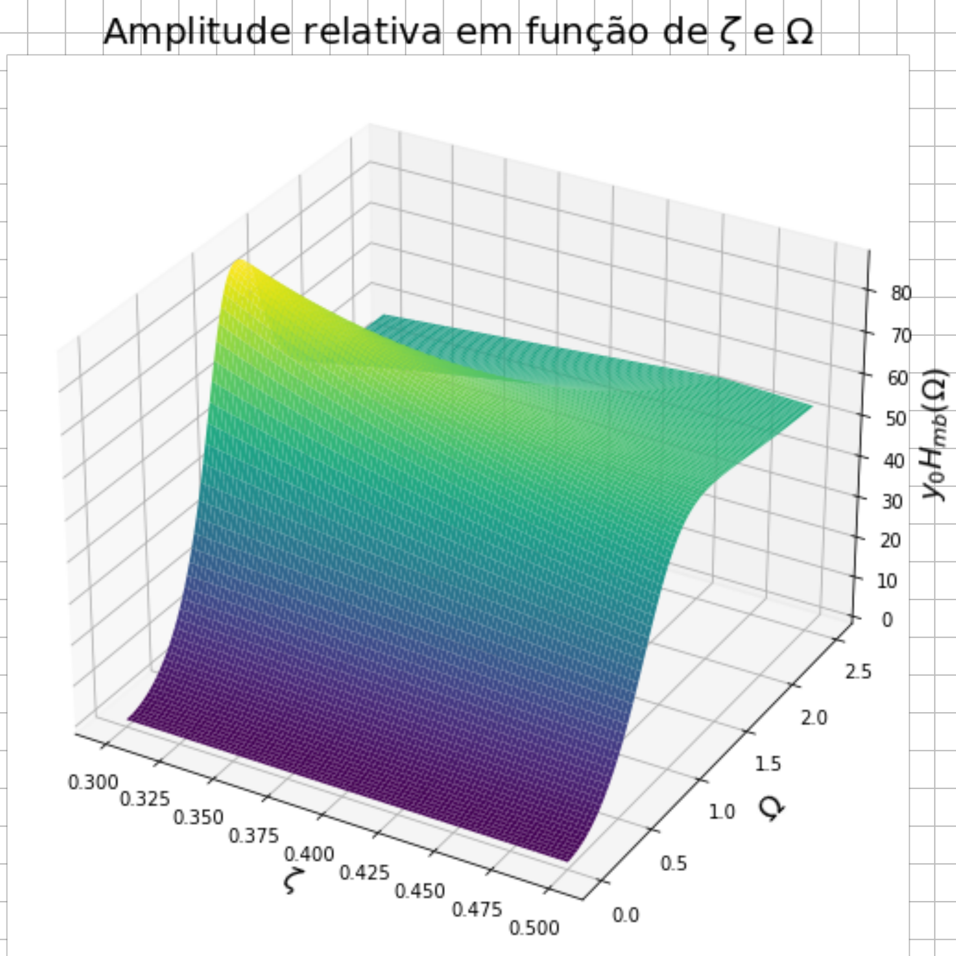
$$
\begin{aligned}
z = x(\tau) - y(\tau) &= y_o[H_{mb}(\Omega)\sin(\Omega\tau - \psi(\Omega)) - \sin(\Omega\tau)] \\
&= y_o[\{H_{mb}(\Omega)\cos(\psi(\Omega)) - 1\}\sin(\Omega\tau) - H_{mb}(\Omega)\sin(\psi(\Omega))\cos(\Omega\tau)] \\
&= y_o X_y(\Omega)\sin(\Omega\tau - \varphi_b(\Omega))
\end{aligned} \tag{5.84}
$$

$$
\begin{aligned}
X_y(\Omega) &= \sqrt{H_{mb}^2 - 2H_{mb}\cos(\psi(\Omega)) + 1} \\
\varphi_b(\Omega) &= \tan^{-1}\frac{H_{mb}(\Omega)\sin(\psi(\Omega))}{H_{mb}(\Omega)\cos(\psi(\Omega)) - 1}
\end{aligned} \tag{5.85}
$$

Gráficos encontrados



Código Disponível em https://github.com/Esterci/Engenharia_Mecanica_UFJF/blob/master/10_periodo/vibracoes/Trabalho_final.ipynb



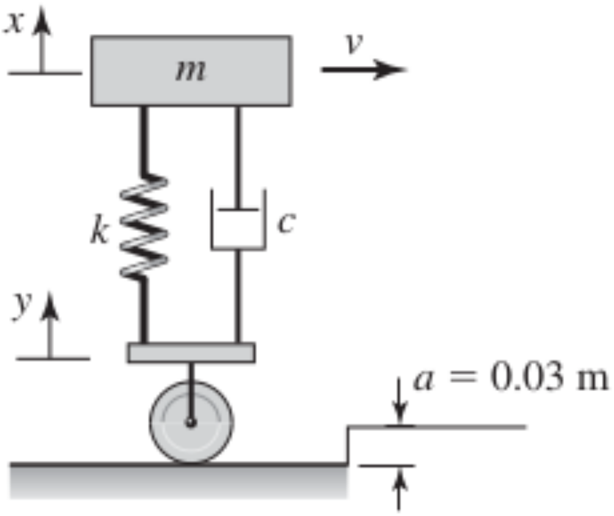


FIGURE 6.13
Car suspension encountering a step change in road conditions.

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n^2 x = 2\zeta\omega_n\frac{dy}{dt} + \omega_n^2 y \quad \text{(a)}$$

$$y(t) = au(t - t_o) \quad \text{(b)}$$

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n\frac{dx}{dt} + \omega_n^2 x = 2\zeta\omega_n a\delta(t - t_o) + a\omega_n^2 u(t - t_o) \quad \text{(c)}$$

$$\frac{du(t - t_o)}{dt} = \delta(t - t_o)$$

$$x(t) = \frac{2\zeta a}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n(t-t_o)} \sin(\omega_d(t - t_o))u(t - t_o)$$

$$+ a\left[1 - \frac{e^{-\zeta\omega(t-t_o)}}{\sqrt{1-\zeta^2}} \sin(\omega_d(t - t_o) + \varphi)\right]u(t - t_o)$$

Também são frequências

Usar no Plot?

$$\omega_n = \sqrt{\frac{4 \times 10^5 \text{ N/m}}{1100 \text{ kg}}} = 19.07 \text{ rad/s}$$

$$\omega_d = 19.07\sqrt{1 - 0.358^2} = 17.81 \text{ rad/s}$$

$$\varphi = \tan^{-1} \frac{\sqrt{1 - 0.358^2}}{0.358} = 1.21 \text{ rad}$$

$$\zeta = \frac{15 \times 10^3}{2 \times \sqrt{400 \times 10^3 \times 1100}} = 0.358$$

Ocsejа:

- $\omega_d = \omega_n\sqrt{1-\zeta^2}$
- $\varphi = \mathrm{tg}^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$