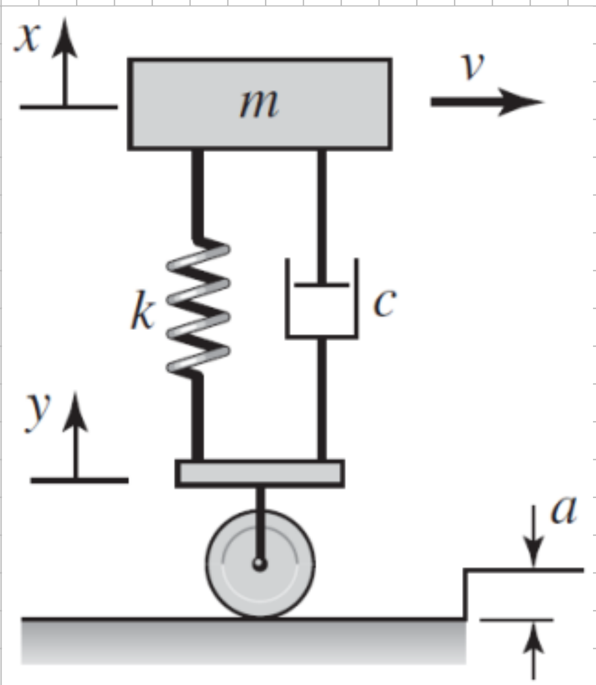
Trabalho 12

Esquema:



Dado que não há ação de forças externas, temos que:

$$y(t) = a\,U(t)$$

e que:

$$m\ddot{x} = -k(x-y) - c(\dot{x}-\dot{y}) \therefore$$

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky \therefore$$

Considerando $\dot{y} = \delta(t)$ temos:

$$m\ddot{x} + c\dot{x} + kx = ac\,\delta(t) + k\,a\,U(t)$$

Aplicando a transformada de Laplace nos dois lados da eq. 1 temos:

$$m[s^2X(s) - x_0 s - \dot{x}_0] + c[s\,X(s) - x_0] + k\,X(s) =$$
$$= ac + a\frac{k}{s}$$

Considerando $x_0 = 0$ e $\dot{x}_0 = 0$ temos:

$$m[s^2X(s) - \cancel{x_0} s - \cancel{\dot{x}_0}] + c[s\,X(s) - \cancel{x_0}] + k\,X(s) =$$
$$= ac + a\frac{k}{s} \therefore$$

$$m[s^2X(s)] + c[s\,X(s)] + k\,X(s) = ac + a\frac{k}{s}$$

$$X(s) = \frac{aC}{ms^2 + Cs + K} + \frac{aK}{s(ms^2 + Cs + K)} \quad \therefore$$

$$X(s) = \frac{aC}{m}\left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right) + a\frac{K}{m}\left[\frac{1}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}\right]$$

$$X(s) = \frac{aC}{K}\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right) + a\left[\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}\right]$$

Aplicando a transformada inversa de Laplace; temos:

$$x(t) = \frac{2\zeta a}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t}\, \mathrm{sen}(\omega_d t) + a\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\, \mathrm{sen}(\omega_d t + \varphi)\right]$$

Onde $\varphi = tg^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$

Passando para o tempo adimensional $\tau = \omega_n t$:

$$\boxed{x(t) = \frac{2\zeta a}{\sqrt{1-\zeta^2}}\, e^{-\zeta\tau}\, \mathrm{sen}\left(\sqrt{1-\zeta^2}\,\tau\right) + a\left[1 - \frac{e^{-\zeta\tau}}{\sqrt{1-\zeta^2}}\, \mathrm{sen}\left(\sqrt{1-\zeta^2}\,\tau + \varphi\right)\right]}$$

Análise Gráfica:

Gráfico da amplitude adimensional em
função do tempo adimensional

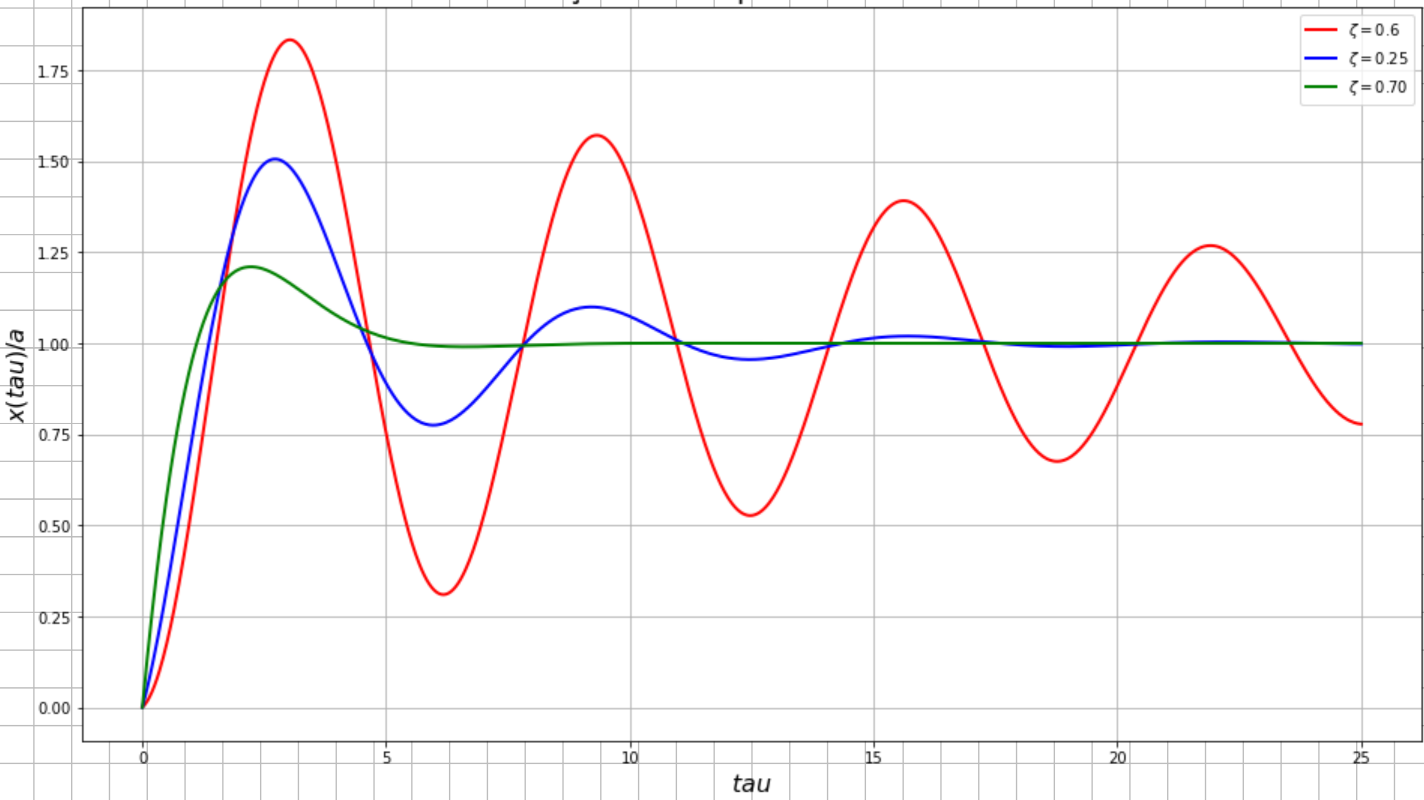


Gráfico da amplitude adimensional em
função do tempo adimensional e do fator de amortecimento