

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n \frac{dx}{dt} + \omega_n^2 x = 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y$$

$$\frac{d^2x}{d\tau^2} + 2\zeta \frac{dx}{d\tau} + x = 2\zeta \frac{dy}{d\tau} + y \quad (5.76)$$

$$y(\tau) = y_o \sin(\Omega\tau) \quad (5.77)$$

$$\frac{d^2x}{d\tau^2} + 2\zeta \frac{dx}{d\tau} + x = 2\zeta y_o \Omega \cos(\Omega\tau) + y_o \sin(\Omega\tau) \quad (5.78)$$

$$x(\tau) = y_o H(\Omega) \sqrt{1 + (2\zeta\Omega)^2} \sin(\Omega\tau - \theta(\Omega)) + \varphi \quad (5.79)$$

$$\varphi = \tan^{-1} 2\zeta\Omega$$

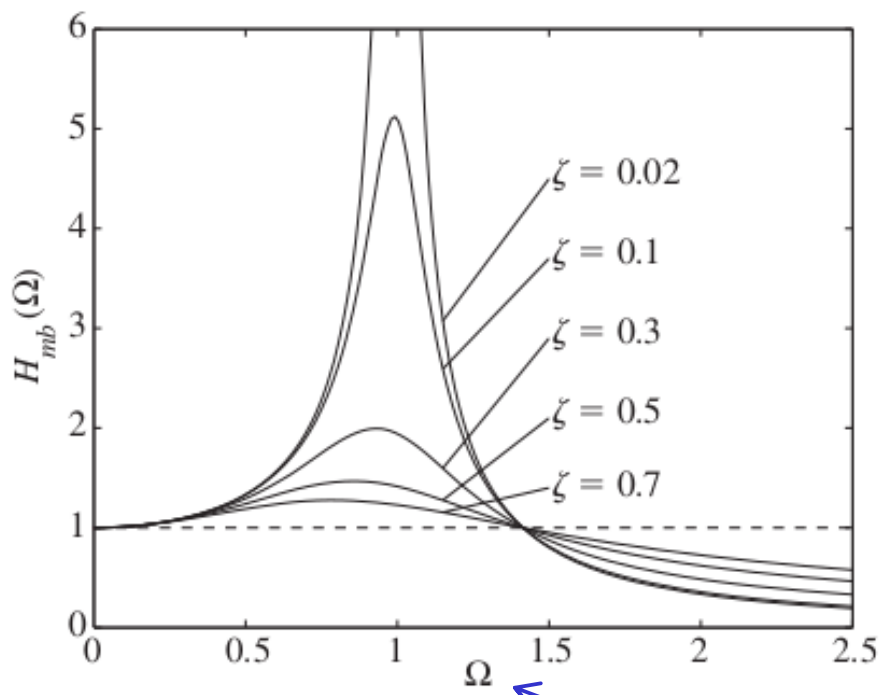
$$x(\tau) = y_o \underbrace{H_{mb}(\Omega)}_{\text{Displacement magnitude}} \sin(\Omega\tau - \underbrace{\psi(\Omega)}_{\text{Phase}})$$

$$H_{mb}(\Omega) = \frac{\sqrt{1 + (2\zeta\Omega)^2}}{\sqrt{(1 - \Omega^2)^2 + (2\zeta\Omega)^2}}$$

$$\psi(\Omega) = \tan^{-1} \frac{2\zeta\Omega^3}{1 + \Omega^2(4\zeta^2 - 1)}$$

$$v(\tau) = y_o \omega_n \Omega H_{mb}(\Omega) \cos(\Omega\tau - \psi(\Omega))$$

$$a(\tau) = -\Omega^2 \omega_n^2 x(\tau) \quad (5.83)$$



(a)

frequency ratio Ω

$$\Omega = \omega/\omega_n$$

$$\omega = \frac{2\pi f}{\gamma}$$

var essa
frequência
no plot

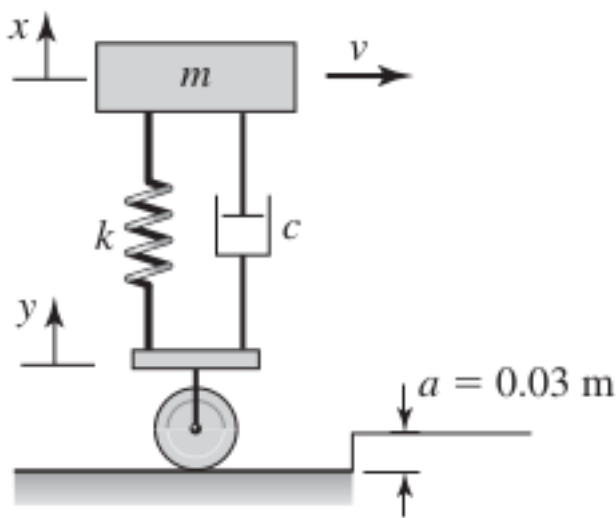


FIGURE 6.13

Car suspension encountering a step change in road conditions.

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n \frac{dx}{dt} + \omega_n^2 x = 2\zeta\omega_n \frac{dy}{dt} + \omega_n^2 y \quad (\text{a})$$

$$y(t) = au(t - t_o) \quad (\text{b})$$

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n \frac{dx}{dt} + \omega_n^2 x = 2\zeta\omega_n a \delta(t - t_o) + a\omega_n^2 u(t - t_o) \quad (\text{c})$$

$$\frac{du(t - t_o)}{dt} = \delta(t - t_o)$$

$$x(t) = \frac{2\zeta a}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n(t-t_o)} \sin(\omega_d(t - t_o)) u(t - t_o) + a \left[1 - \frac{e^{-\zeta\omega(t-t_o)}}{\sqrt{1 - \zeta^2}} \sin(\omega_d(t - t_o) + \varphi) \right] u(t - t_o)$$

Também há frequências
 ↑
 Ugar no plot?

$$\omega_n = \sqrt{\frac{4 \times 10^5 \text{ N/m}}{1100 \text{ kg}}} = 19.07 \text{ rad/s}$$

$$\omega_d = 19.07 \sqrt{1 - 0.358^2} = 17.81 \text{ rad/s}$$

$$\varphi = \tan^{-1} \frac{\sqrt{1 - 0.358^2}}{0.358} = 1.21 \text{ rad}$$

$$\zeta = \frac{15 \times 10^3}{2 \times \sqrt{400 \times 10^3 \times 1100}} = 0.358$$

Oo seja:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$\varphi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta}$$