Trabalho 15:

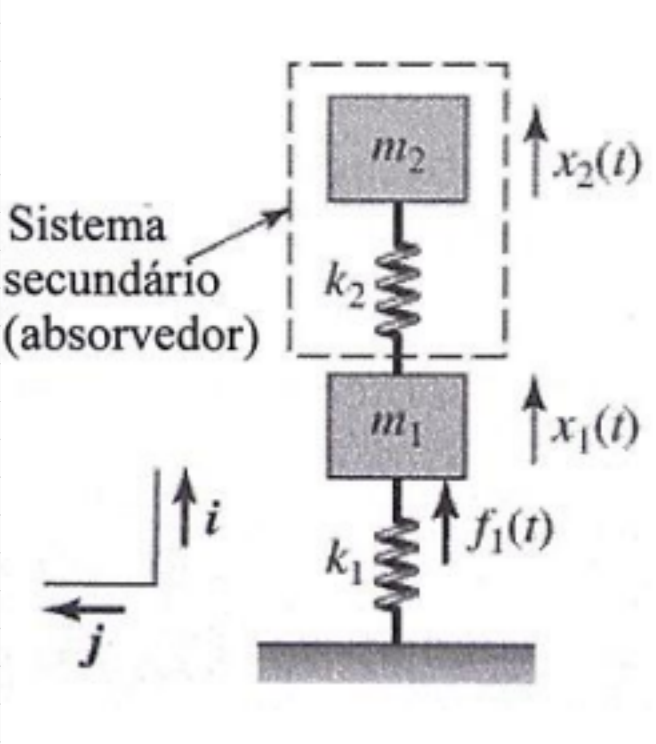
Dados:

$\omega_{min} = 670 \times 2\pi / 60$ rad/s $m_1 = 9{,}8N = m_2$

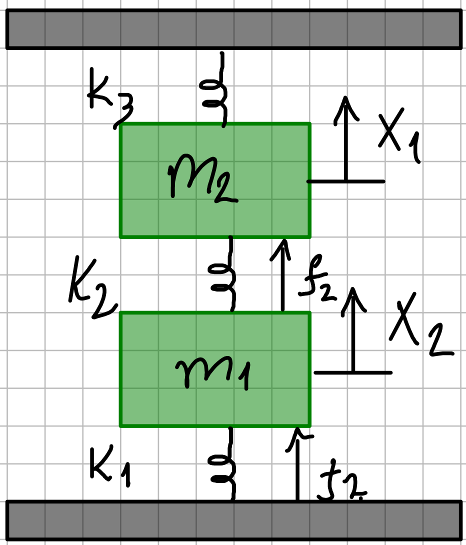
$\omega_{max} = 1350 \times 2\pi / 60$ rad/s $F_1 = 100 N$

$K_1 = 10 KN/m$

Esquema:



(I)



(II)

Análise:

Considerando o sistema II com duas massa excitado por uma força cossenoidal em cada massa, temos que:

$f_1(t) = F_1 \cos \omega t$

$f_2(t) = F_2 \cos \omega t$

O sistema pode ser representado por:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2+K_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \cos\omega t \quad (1)$$

A solução deste sistema é dado por:

$$\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \cos \omega t \quad (2)$$

Substituindo a eq. 2 e sua 2ª derivada na eq. 1, temos:

$$\begin{bmatrix} K_1+K_2-\omega^2 m_1 & -K_2 \\ -K_2 & K_2+K_3-\omega^2 m \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (3)$$

Considerando os seguintes parâmetros adimensionais

$\omega_{n_j}^2 = \dfrac{K_j}{m_j}$ p $j = 1,2$; $m_r = \dfrac{m_2}{m_1}$; $\Omega = \dfrac{\omega}{\omega_{n1}}$;

$\omega_r = \dfrac{\omega_{n2}}{\omega_{n_1}} = \dfrac{1}{\sqrt{m_r}} \sqrt{\dfrac{K_2}{K_1}}$; $K_{32} = \dfrac{K_3}{K_2}$

A solução da eq. 3 pode ser reescrita como:

$$\begin{cases} [1+\omega_r^2 m_r - \Omega^2] X_1 - m_r \omega_r^2 X_2 = F_1/K_1 \\ -\omega_r^2 X_1 + [\omega_r^2 (1+K_{32}) - \Omega^2] X_2 = \omega_r^2 F_2/K_2 \end{cases}$$

Neste caso:

$$X_1 = \frac{1}{D_0}\left[F_1/K_1\left[\omega_r^2(1+K_{32}) - \Omega^2\right] + (F_2/K_2)\, m_r \omega_r^4\right] \quad (4)$$

$$X_2 = \frac{\omega_r^2}{D_0}\left[(F_1/K_1) + (F_2/K_2)\left[1 + \omega_r m_r - \Omega^2\right]\right] \quad (5)$$

Onde:

$$D_0 = \Omega^4 - a_1 \Omega^2 + a_2 \quad (6) \quad \begin{cases} a_1 = 1 + \omega_r^2(1 + m_r + K_{32}) \\ a_2 = \omega_r^2\left[1 + K_{32}(1 + \omega_r^2 m_r)\right] \end{cases}$$

Para o esquema I, temos que:

$f_2(t) = 0$; $K_3 = 0$; $m_1 = m_2 = m$;

Neste caso:

$\omega_1 = \sqrt{\frac{K_1}{m}}$; $\omega_2 = \sqrt{\frac{K_2}{m}}$; $\omega_3^2 = 0$; $m_r = 1$; $\Omega = \omega\sqrt{\frac{m}{K_1}}$

$\omega_r = \frac{\sqrt{K_2/m}}{\sqrt{K_1/m}} = \sqrt{\frac{K_2}{K_1}}$; $K_{32} = 0$;

Portanto temos que as eq. 4 e 5 são simplificadas para:

$$X_1 = \frac{1}{D_0}\left[F_1/K_1\left[\omega_r^2(1+\cancelto{0}{K_{32}}) - \Omega^2\right] + (F_2/K_2)\, \cancelto{1}{m_r} \omega_r^4\right] \therefore$$

$$X_1 = \frac{1}{D_0}\left[F_1/K_1\left[\omega_r^2 - \Omega^2\right] + (F_2/K_2)\, \omega_r^4\right] \quad (7)$$

$$X_2 = \frac{\omega_r^2}{D_0}\left[(F_1/k_1) + (F_2/k_2)\left[1 + \omega_r \cancel{m_r}^{1} - \Omega^2\right]\right] \therefore$$

$$X_2 = \frac{\omega_r^2}{D_0}\left[(F_1/k_1) + (F_2/k_2)\left[1 + \omega_r - \Omega^2\right]\right] \quad (8)$$

A eq. 6 fica:

$$D_0 = \Omega^4 - a_1\Omega^2 + a_2 \begin{cases} a_1 = 1 + \omega_r^2\left(1 + \cancel{m_r}^{1} + \cancel{k_{32}}^{0}\right) \therefore \\ a_2 = \omega_r^2\left[1 + \cancel{k_{32}}_{0}\left(1 + \omega_r^2 \cancel{m_r}_{1}\right)\right] \end{cases}$$

$$D_0 = \Omega^4 - a_1\Omega^2 + a_2 \begin{cases} a_1 = 1 + 2\omega_r^2 \\ a_2 = \omega_r^2 \end{cases} \quad (9)$$

Portanto, temos as seguintes amplitudes de deslocamento das massas:

$$\frac{X_1}{F_1/K_1} = H_{11}(\Omega) = \frac{\omega_r^2 - \Omega^2}{D_0} \quad (10)$$

$$\frac{X_2}{F_2/K_2} = H_{21}(\Omega) = \frac{\omega_r^2}{D_0} \quad (11)$$

Para o caso no qual $H_{11} = 0$:

$$\Omega^2 = \omega_r^2 \therefore \omega\sqrt{\frac{m}{K_1}} = \sqrt{\frac{K_2}{K_1}} \therefore \omega^2\frac{m}{\cancel{K_1}} = \frac{k_2}{\cancel{K_1}} \therefore$$

$$K_2 = \omega^2 m \quad (12)$$

$$\boxed{K_2 = 98\,014{,}44\ N/m} \rightsquigarrow \text{Letra A}$$

Segundo as eq. 10 e 11 temos:

$$\left.\begin{aligned} H_{11_{max}} &= 0,97 \\ H_{21_{max}} &= -0,97 \\ H_{11_{min}} &= -0,8 \\ H_{21_{min}} &= -0,15 \end{aligned}\right\} \text{Letra B}$$

Para o sistema sem absorvedor, temos que:

$k_2 = 0 \rightsquigarrow \omega_r = 0 \therefore$

$$D_0 = \Omega^4 - \overset{1}{\cancel{a_1}}\,\Omega^2 + \overset{0}{\cancel{a_2}} \quad \left|\begin{aligned} a_1 &= 1 + \overset{0}{\cancel{2\omega_r^2}} \therefore \\ a_2 &= \cancel{\omega_r^2}\ 0 \end{aligned}\right.$$

$$D_0 = \Omega^4 - \Omega^2 \therefore$$

$$H_{11}(\Omega) = \frac{\overset{0}{\cancel{\omega_r^2}} - \Omega^2}{D_0} \therefore$$

$$H_{11}(\Omega) = \frac{-\Omega^2}{\Omega^4 - \Omega^2} = \frac{1}{-\Omega^4/\Omega^2 + 1} = \frac{1}{1 + \Omega^2}$$

Análise gráfica: