EE 416 Final Project Report

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Task1

Parameter Setup

We set:

A = 1;
k = 1;
s = [1 1 1 1 1 -1 -1 1 1 -1 1 -1 1] (Barker Code 13);
w = randn(13, 1)

Plot Observational Histogram & Theoretical Pdf of V & v

To plot the histogram of distribution of correlator V, we generated 1000 samples of correlator V and obtained the actual mean and variance directly with Matlab commands.

To plot theoretical pdf of V, we calculated theoretical mean and variance of correlator V. Following is the detailed process of derivation.

$$
\begin{aligned}
E[V] &= E[s^2+s*w] \\
&= E[s^2]+E[s*w] \\
&= Var[s]+E[s]^2 \\
&= s^2
\end{aligned}
$$

$$
\begin{aligned}
E[V^2] &= E[(s^2+s*w)^2] \\
&= Var[s^2]+E[s^2]^2+Var[sw]+E[sw]^2+2E[s^3*w] \\
&= E[s^2]^2+Var[sw] \\
&= s^4 + (s^2)*var[w] \\
&= s^4 + s^2
\end{aligned}
$$

$$
\begin{aligned}
var[V] &= E[V^2] - E[V]^2 \\
&= s^4 + s^2 - s^2 \\
&= s^2
\end{aligned}
$$

To plot the histogram of distribution and theoretical pdf of correlator v, which is V/σ_V, We computed the observational mean and variance with Matlab commands. We calculated $E[v] = s^2/\sigma_V$ and $var[v] = s^2/var[V] = 1$ as the theoretical mean and variance of v.

Chi-square goodness-of-fit test

To test the quantify of the match between observation histogram and theoretical pdf of correlator V, we applied chi-square goodness-of-fit test in Matlab. The result is displayed as part of the title on each plot. Due to the very small value of chi-square goodness-of-fit test (the value we got is less than 1), both fits are excellent.

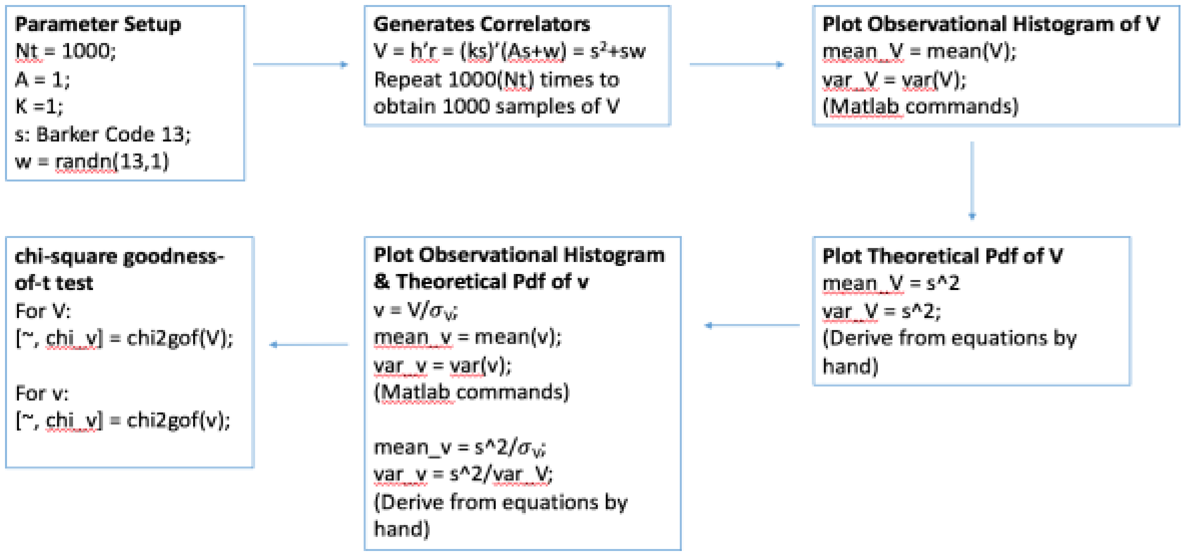


Figure 1 Block Diagram of Task 1

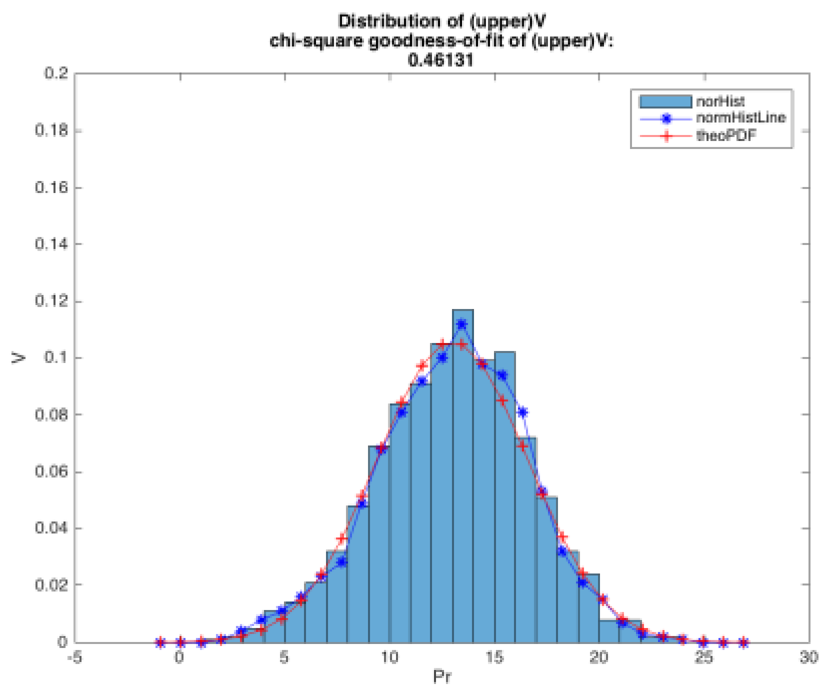


Figure 2 Histogram and theoretical pdf of V

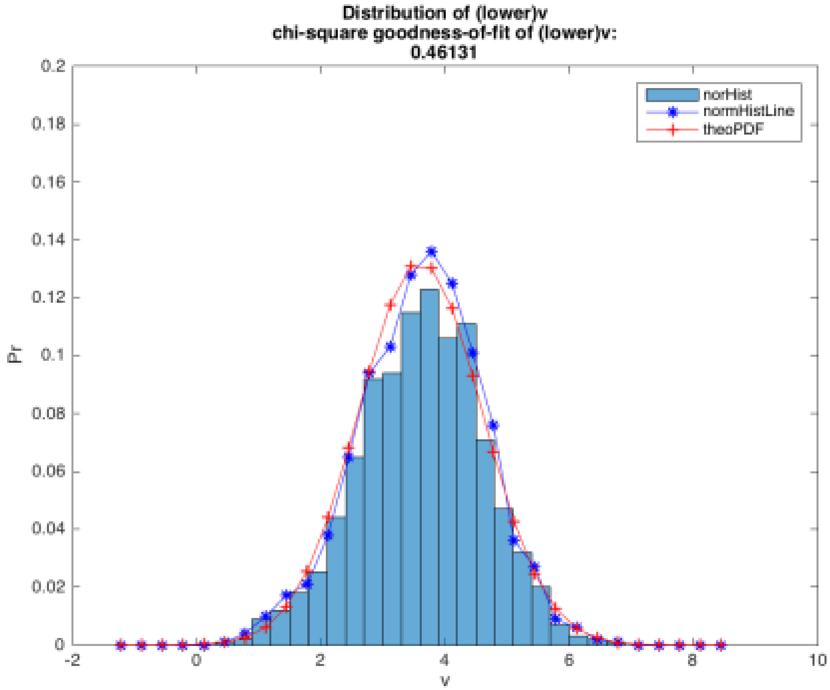


Figure 3 Histogram and theoretical pdf of v

Task2

To plot theoretical pdf and histogram showing probability of detection (Pd) and probability of false alarm (Pfa), we first generated Gaussian distribution of signals under H0 case: V~ N (0, $\sqrt{s's}$) and Gaussian distribution of nosie under H1 case:V~ N(s's, $\sqrt{s's}$). (See Fig 4)

To obtain and display a table with values of Pfa, v0 and Pd, we computed threshold value v0 by $Pfa = Q\left(\frac{v0-0}{\sqrt{s's}}\right)$, with the fixed value of Pfa. Then with known v0, we output Pd by $Pd = Q\left(\frac{v0-s's\sqrt{\frac{SNR}{s's}}}{\sqrt{s's}}\right) = Q\left(\frac{v0-s's}{\sqrt{s's}}\right)$. Here $SNR = \frac{A^2E}{\sigma^2} = \frac{E}{\sigma^2} = 1$, due to my parameter setup initially. (See Fig 5)

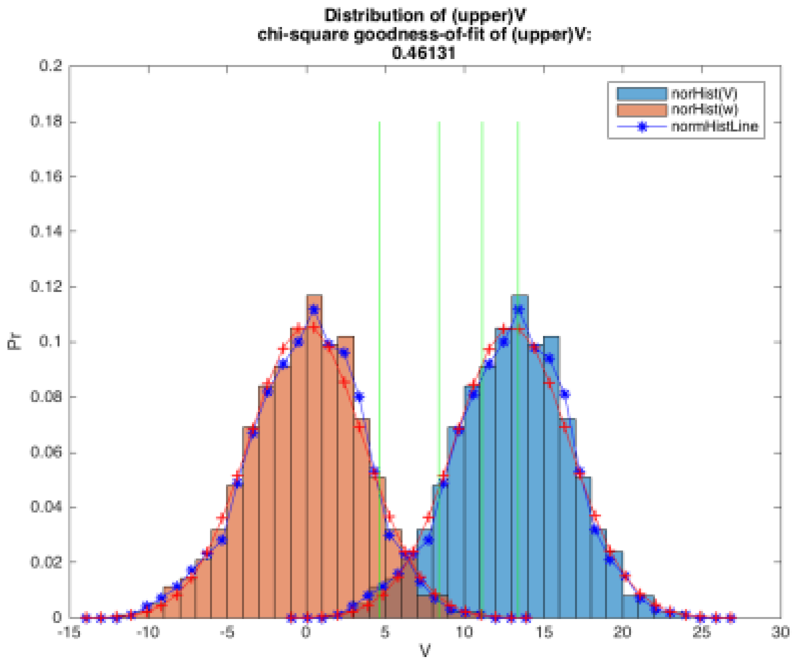


Figure 4 theoretical pdf and histogram showing probability of detection (Pd) and probability of false alarm (Pfa)

----------- Table of Pfa, v0 and Pd -----------				
Pfa:	0.1	0.01	0.001	0.0001
v0:	4.6207	8.38777	11.142	13.4091
Pd:	0.98994	0.89959	0.69683	0.45483

Figure 5 Table of Pfa, v0 and Pd

To plot Pd versus SNR, we derive the equation $Pd = Q\left(\frac{v0 - s's\sqrt{\frac{SNR}{s's}}}{\sqrt{s's}}\right)$, which uses $SNR = \frac{A^2E}{\sigma^2}$ as an input variable to represent Pd. By choosing the range of S dB from 0dB~25dB and with given v0, we finally plot Pd vs SNR.

From the plot, we observed that each curve roughly has a shift from others. To explain it, first, from the previous table, we found that with the increment of threshold v0, Pd will decrease. This is easy to understand because Pd is actually the area of fv|H1(V>v0) with x-axis value from v0 to infinity. Reducing v0 results in the increment of Pd. Thus, high probability of detection can be trade off high probability of false alarm.

When SNR dB = 0 dB, $SNR = \frac{A^2E}{\sigma^2} = 1$, $Pd = Q\left(\frac{v0 - s's\sqrt{\frac{SNR}{s's}}}{\sqrt{s's}}\right) = Q(v0-1)$. So the intersect of Pd is basically determined by the value of v0. Larger value of v0 results in smaller value of Pd as we said before, so that the intersects of Pd are different in this plot. Larger value of v0 results in smaller value of Pd, which is also true for any points on the curve. i.e for the same value of SNR(dB), curve with larger v0 is below the curve with smaller v0.

The shape of four curves are similar mainly because with input variable SNR, Pd calculates the CCDF of normally distributed V.

If SNR is high enough, Pd will be constant as 1 under different threshold v0. In this case, the value of v0 does not affect Pd anymore. This is obvious since when the signal is almost no noises with it, system is able to successfully detect all the signal.

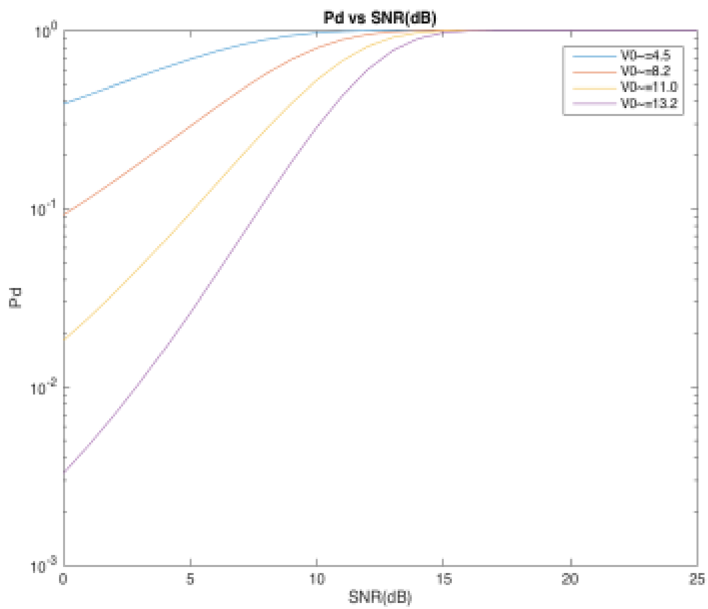


Figure 6 Pd vs SNR(dB)

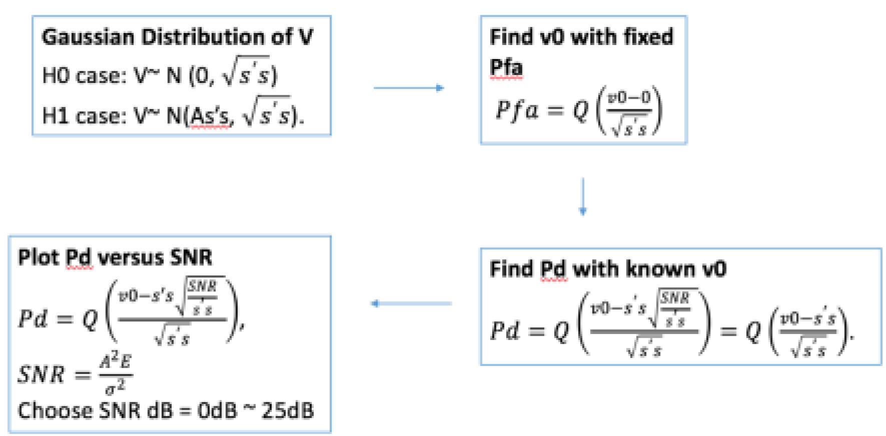


Figure 7 Block Diagram of Task 2

Task3:

Construct a plot of the (auto)-correlation function of the following 3 sequences:

1. The alternating series of length N = 11. This is [01010101010]
2. The N = 11 chip Barker code (used in WiFi 802.11). This is [01001000111]
3. The N = 13 chip Barker Code. This is [1010110011111]

Determine the PSL, the peak sidelobe level.

Generate three codes as required, autocorrelate them using xcorr.m. Plot them and find PSL by getting the largest sidelobe and calculate it in dB.
Plots generated by Task3autocorr.m

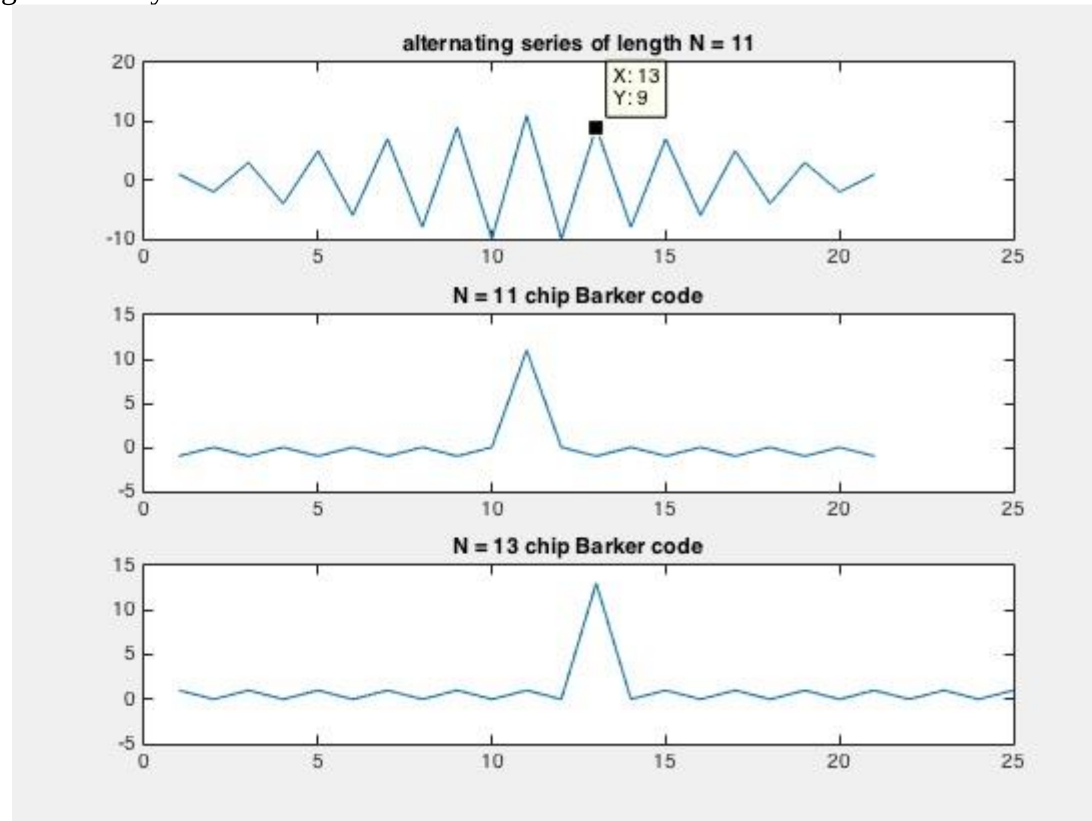


Fig 8 Autocorrelation function of three given sequences

PSL are calculated:

PSL (alternating series)	PSL (11 chip Barker code)	PSL (13 chip Barker code)
19.849 dB	0 dB	0 dB

Table 1. Peak Sidelobe Levels for each autocorrelation function

From the images generated, we can see Barker codes have max autocorrelation sequence which has very low sidelobes and constant envelope, so Barker codes are very good.

Task 3.5:

Using Wikipedia's listing of Barker codes, create longer codes by placing the lengths [2 3 4 5 7 11 13] into a Barker-13 code. Create a table that provides the code Identification, mainlobe maximum in dB, the average sidelobe, and the peak max sidelobe; all in dB.

Ouput generated from Task35longcorr.m:

Code Identification	Barker 2-13 Codes	Barker 3-13 Codes	Barker 4-13 Codes	Barker 5-13 Codes	Barker 7-13 Codes	Barker 11-13 codes	Barker 13-13 codes
Mainlobe Max (Mdb)	28.299466959 dB	31.8212921405 dB	34.3200668727 dB	36.2582671329 dB	39.1808278464dB	43.1067207493 dB	44.5577340923 dB
Average Sidelobe (sidedb)	3.2332308727 dB	0.9601545214 dB	3.1484898138 dB	1.8706054471 dB	2.2307456508 dB	2.5443700581 dB	2.6266988359 dB
Peak Max Sidelobe (13)	22.2789 dB	22.2789 dB	22.2789 dB	22.2789 dB	22.2789 d B	22.2789 dB	22.2789 dB
Sidelobe/Mainlobe Ratio (side2main)	-25.0662360868	-30.861137619	-31.1715770589	-34.3876616857	-36.9500821956	-40.5623506912	-41.9310352564

Table 2. Autocorrelation functions of different combined barker codes with lobe levels
Note: We found different lobe levels by click on figures and find the values of the peaks.

From the table, Barker 3-13 has smallest Average sidelobe, which can be used in antenna engineering, such as optics, and acoustics field such as loudspeaker and sonardesign.

When we look at sidelobe/mainlobe ratio, Barker 13-13 codes have smallest ratio (max mainlobe), meaning this signal exhibits the greatest field strength, which can be used in radar, such as vehicular radar which plays important role in ensuring security of the passengers. For this specific application, wider bandwidth and higher power limitation enable "better resolution and object distinction", which are essential for "pedestrian detection and autonomous emergency braking in urban areas" [1]. So these are the trade-offs from those longer codes and some applications.

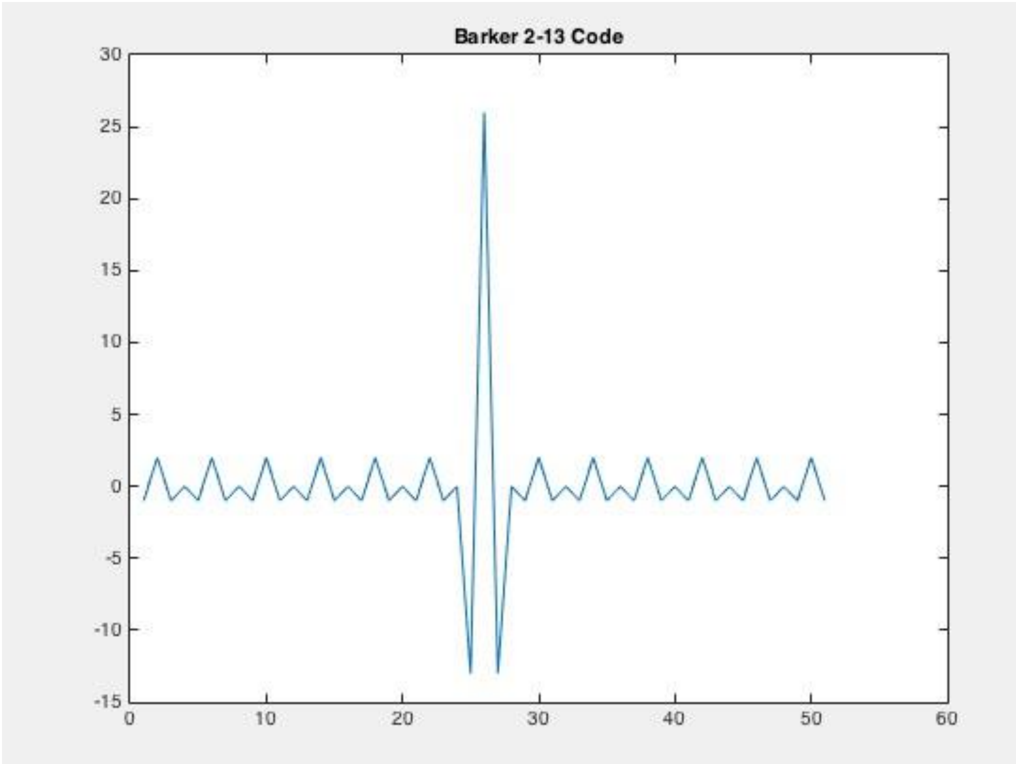


Fig 9 Autocorrelation function of 2-13 combined barker codes

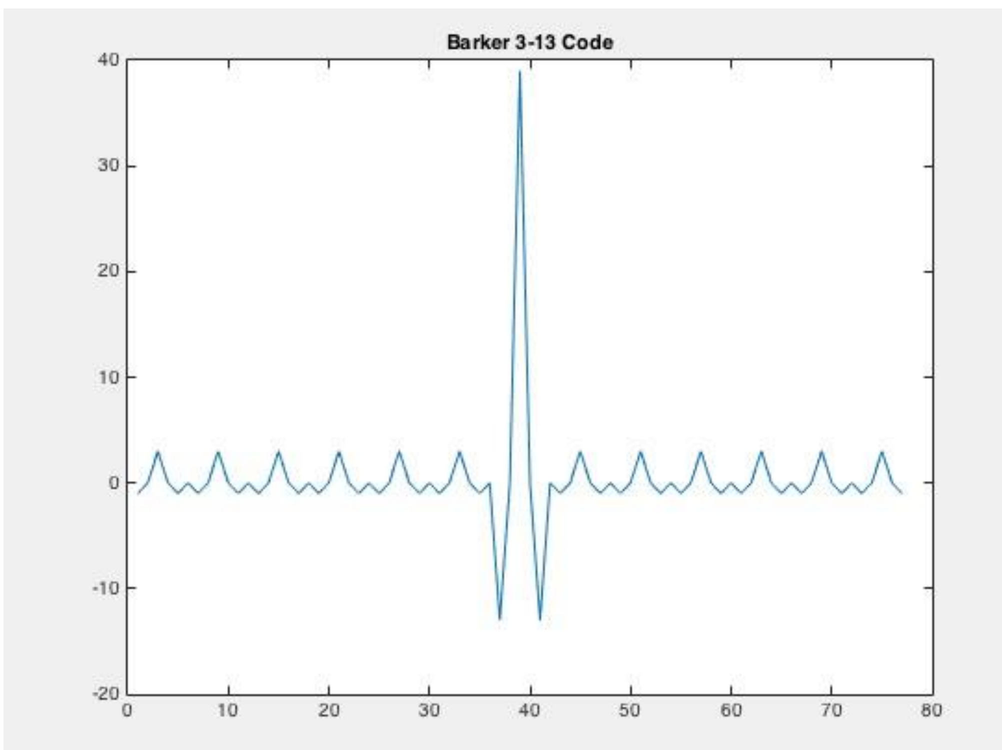


Fig 10 Autocorrelation function of 3-13 combined barker codes

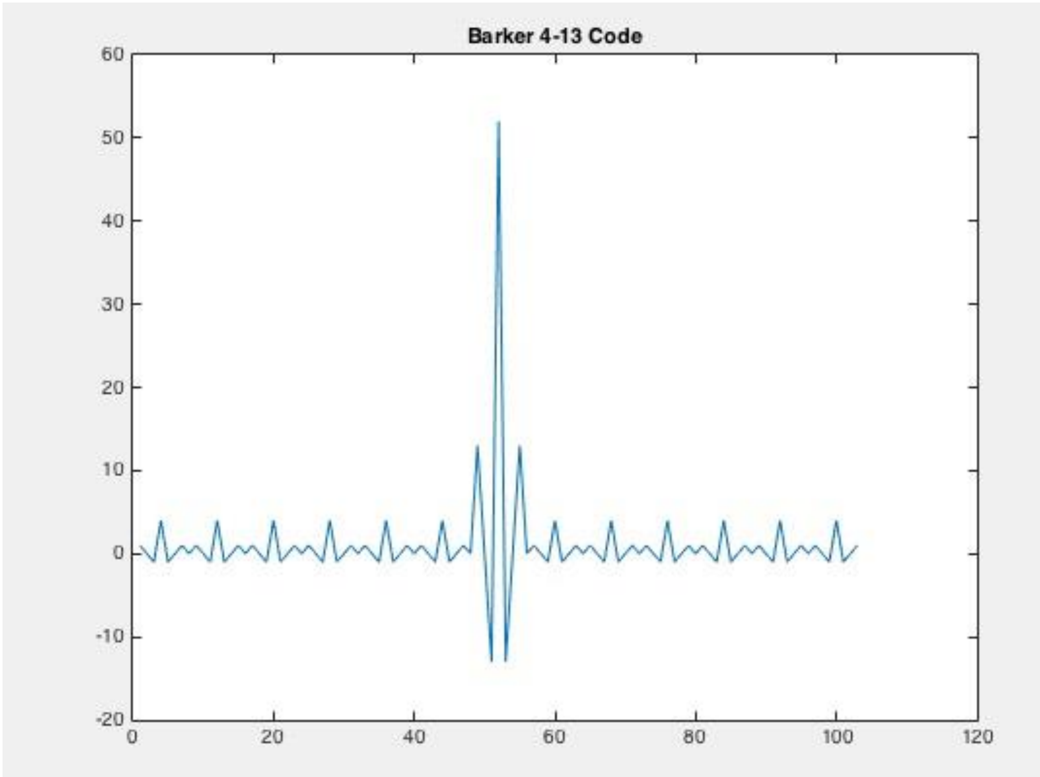


Fig 11 Autocorrelation function of 4-13 combined barker codes

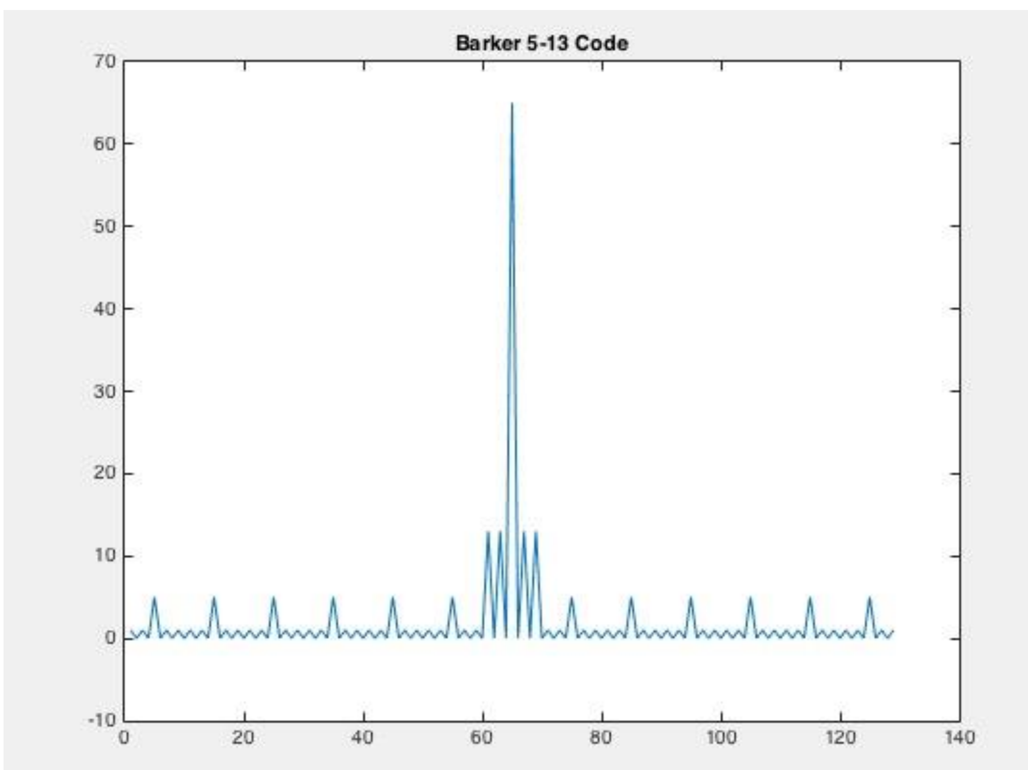


Fig 12 Autocorrelation function of 5-13 combined barker codes

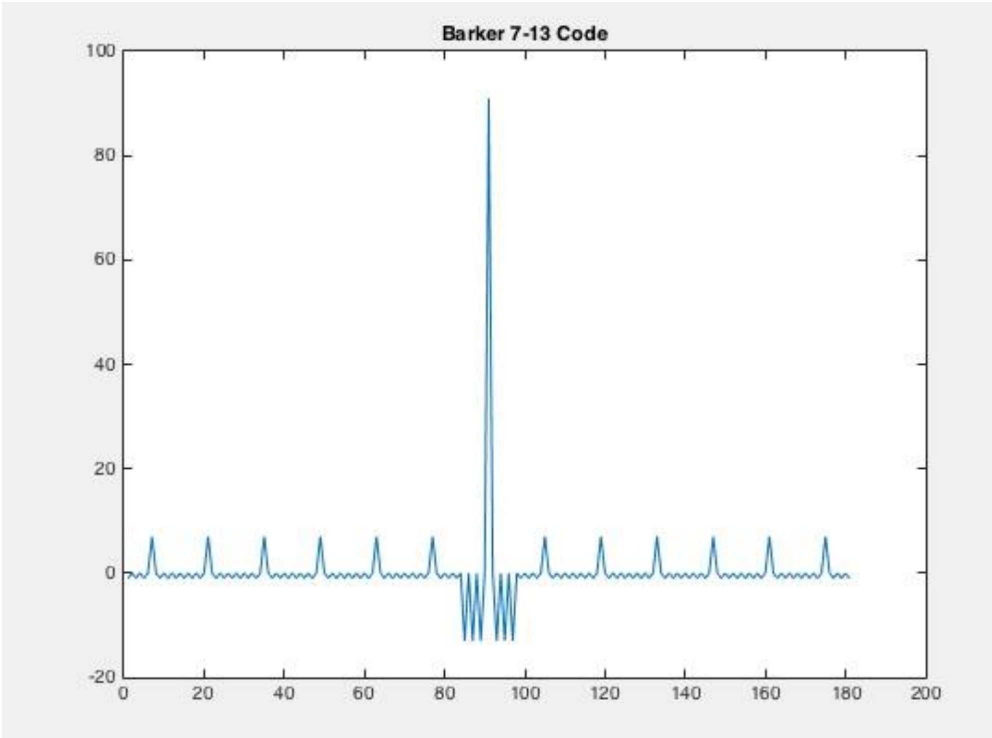


Fig 13 Autocorrelation function of 7-13 combined barker codes

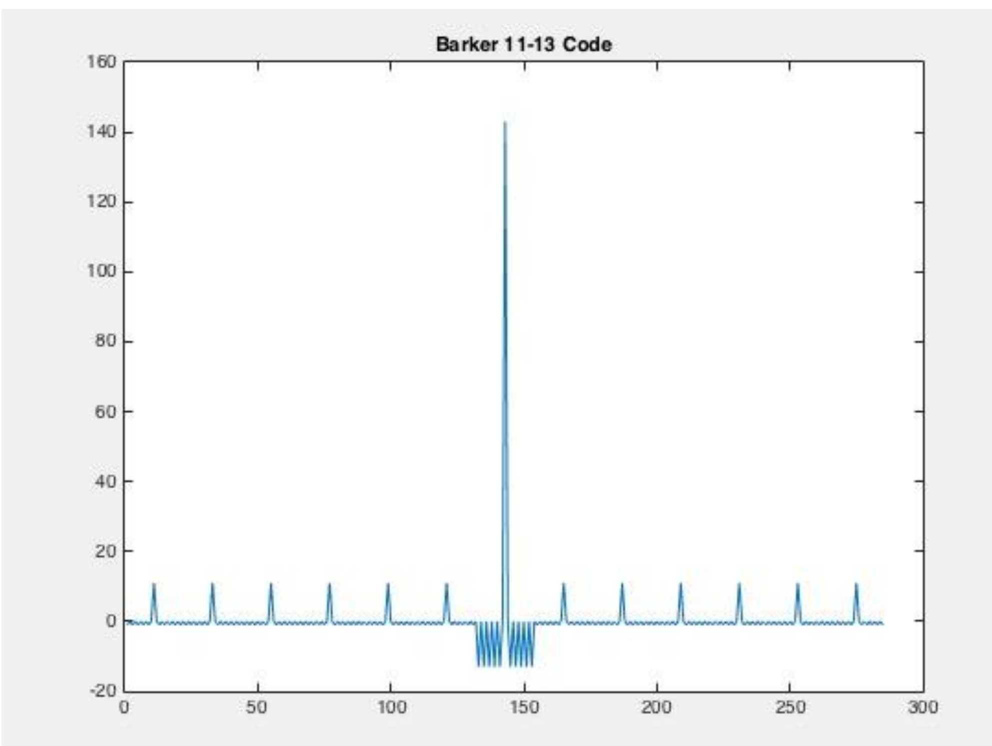


Fig 14 Autocorrelation function of 11-13 combined barker codes

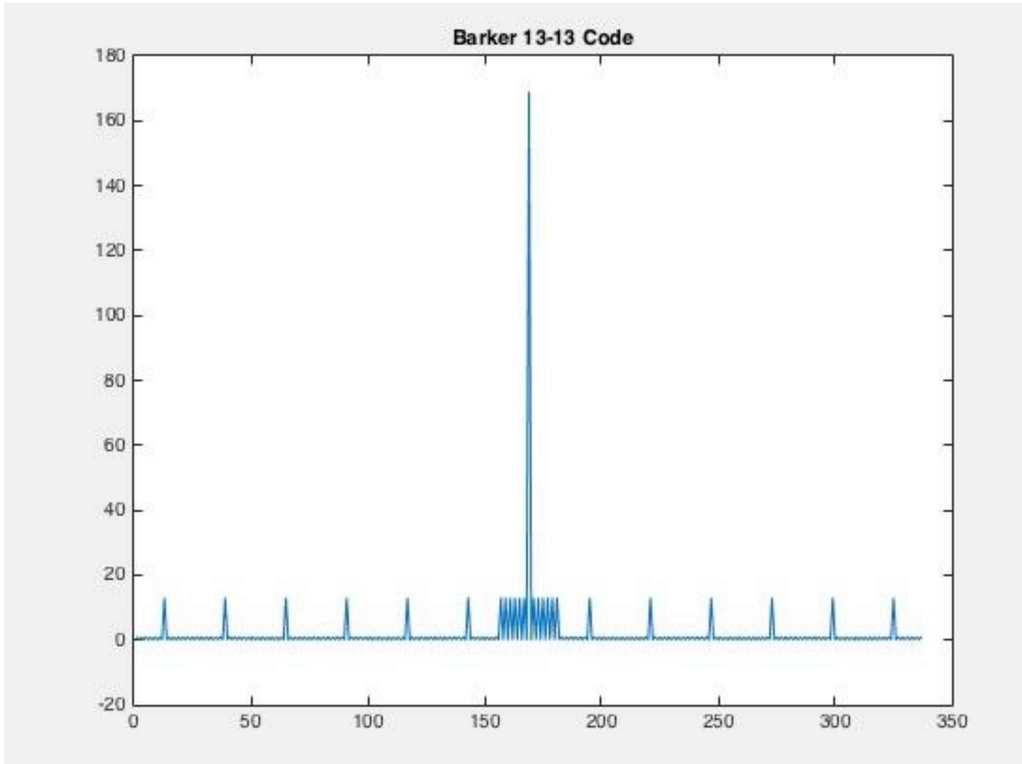


Fig 15 Autocorrelation function of 13-13 combined barker codes

Task 4:

Construct a matched filter decoder that reads in file SGroup4 and determine the starting time of 10 signals (each one is Barker 13, delayed to a random starting time).

Block Diagram and description of our system:

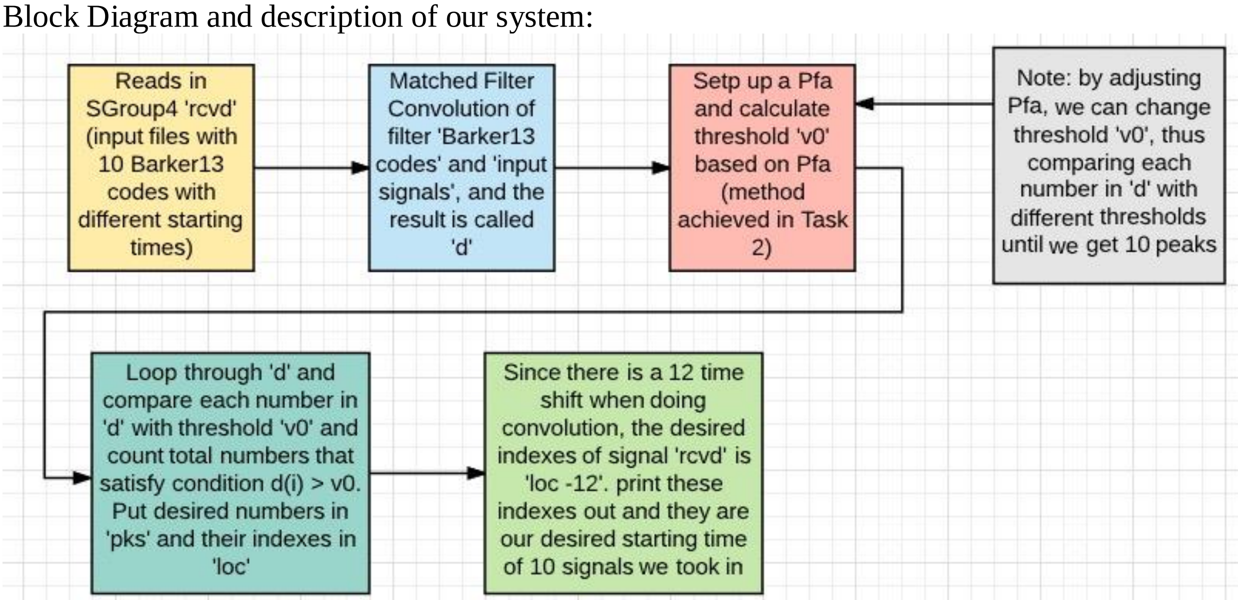


Fig 16 Block Diagram of decoder we used to find starting time of the input signals

Using Task4Decoder.m, the starting time for 10 signals from our data set is:

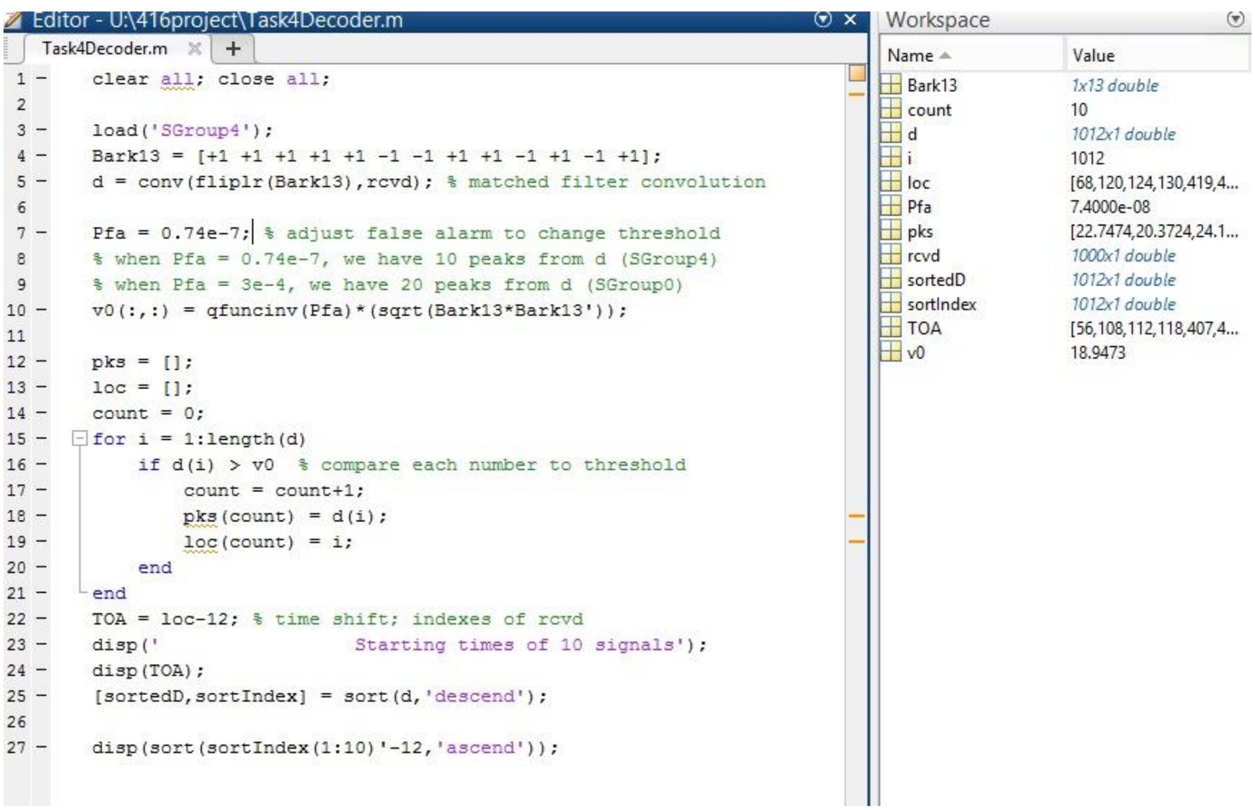
```
>> Task4Decoder
                    Starting times of 10 signals
    56   108   112   118   407   411   528   582   665   766

    56   108   112   118   407   411   528   582   665   766
```

Fig 17 Matlab Output for starting time using two methods

Note: Threshold = 18.9743

As required by the rubric, we also used 10 largest value method to find the starting time, and they match perfectly with our threshold method. Codes are in the screenshot below.

Discussion: When we are designing Pfa to get threshold for matched filter convolution function to compare with, we want to make Pfa small enough so that we can filter out data that's not desired and leave the ones we want. However, there is a trade off between Pfa and Pd that is the probability of detecting the signal: if we decrease Pfa, according to the histogram plotted in Task2, we know the smaller the tail of the H0 hypothesis, the smaller the area of Pd. And that's why when we are testing the data set SGroup0, we cannot locate one starting time (789) and have an extra starting time (648) at which the value of the signal is greater than the threshold value.


Editor - U:\416project\Task4Decoder.m

Task4Decoder.m

```
clear all; close all;

load('SGroup4');
Bark13 = [+1 +1 +1 +1 +1 -1 -1 +1 +1 -1 +1 -1 +1];
d = conv(fliplr(Bark13),rcvd); % matched filter convolution

Pfa = 0.74e-7; % adjust false alarm to change threshold
% when Pfa = 0.74e-7, we have 10 peaks from d (SGroup4)
% when Pfa = 3e-4, we have 20 peaks from d (SGroup0)
v0(:,:) = qfuncinv(Pfa)*(sqrt(Bark13*Bark13'));

pks = [];
loc = [];
count = 0;
for i = 1:length(d)
    if d(i) > v0  % compare each number to threshold
        count = count+1;
        pks(count) = d(i);
        loc(count) = i;
    end
end
TOA = loc-12; % time shift; indexes of rcvd
disp('                    Starting times of 10 signals');
disp(TOA);
[sortedD,sortIndex] = sort(d,'descend');

disp(sort(sortIndex(1:10)'-12,'ascend'));
```

Workspace

Name	Value
Bark13	1x13 double
count	10
d	1012x1 double
i	1012
loc	[68,120,124,130,419,4...
Pfa	7.4000e-08
pks	[22.7474,20.3724,24.1...
rcvd	1000x1 double
sortedD	1012x1 double
sortIndex	1012x1 double
TOA	[56,108,112,118,407,4...
v0	18.9473

Fig 18 Matlab Code for Task 4 using threshold and 10 largest methods

Task 5:

Find another application of the matched filter concept.

Matched filter in Gravitational-Wave Astronomy

Matched filters play a central role in gravitational-wave astronomy [2]. Astronomers directly detected the gravitational waves for the first time on September 2015 14th using larger-scale filtering of each detector's output for signals that resembles the expected shape of the waves [2][3]. As mentioned in *Gravitational-wave Astronomy: Aspect of the Theory of Binary Sources and Interferometric Detectors*, "the detection and measurement of gravitational waves is an arduous task" [4]. One source of gravitational wave is compact binaries, from which the equations of motion can be derived, and the gravitational waveforms that binary systems emit can be calculated [4]. Using data from LIGO instruments (detectors) and other observatories, astronomers can gain a larger bank of matched filters with with the data can be cross-correlated [4]. There are different waveforms emitted, and the inspiral and ringdown are well-understood because their sinusoidal waveform, but the merger need to be analyzed using "fully nonlinear mathematical machinery of general relativity" [4]. So Matched filter technique plays very important role in detecting and analyzing data received because they are designed based on the information scientists know and get correlated and matched with waves so that the gravitational waves can be detected with larger SNR. Then "false-alarm rates, and the statistical significance of the detection" can be computed using "resampling methods" [2]. Detecting and analyzing waveforms and match with gravitational waves is arduous, but matched filtering is critical the first observation of gravitational field and will be more promising in other observations in the future.

Reference

[1] (2016, Dec 14). D. Brizzolara [Online]. Future trends for automotive radars: Towards the 79 GHz band. Availabe: http://itunews.itu.int/en/3935-Future-trends-for-automotive-radars-Towards-the-79GHz-band.note.aspx
[2] (2016, Dec 13). Matched Filter [Wikipedia]. Available: https://en.wikipedia.org/wiki/Matched_filter
[3] (2016, Dec 13). First Observation of Gravitational Waves [Wikipedia]. Available: https://en.wikipedia.org/wiki/First_observation_of_gravitational_waves
[4] S. A. Hughes, "Gravitational-wave Astronomy: Aspect of the Theory of Binary Sources and Interferometric Detectors", California Institute of Technology: 1998, pp 13-14. Available: http://web.mit.edu/sahughes/www/thesis.pdf