

① número binomial  $\binom{8}{3}$

$$\binom{8}{3} = \frac{8!}{3!(8-3)!}$$

$$\frac{8 \cdot 7 \cdot 6 \cdot 5!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$$

② número binomial  $\binom{200}{198}$

$$\frac{200!}{198!(200-198)!} = \frac{200 \cdot 199 \cdot 198!}{198! \cdot 2!} = \frac{200 \cdot 199}{2} = 19900$$

③ Resolver a equação

$$\binom{n-1}{2} = \binom{n+1}{4} \Rightarrow \frac{(n-1)!}{2!(n-3)!} = \frac{(n+1)!}{4!(n-3)!}$$

$$\Rightarrow \frac{(n-1)!}{2} = \frac{(n+1)!}{24} \Rightarrow 12(n-1) = (n+1) \Rightarrow 12n - 12 = n + 1 \Rightarrow 11n = 13 \Rightarrow n = \frac{13}{11}$$

Como  $n$  é inteiro, não há solução.

④ O valor de  $\binom{20}{13} + \binom{20}{14}$

$$\binom{20}{13} + \binom{20}{14} = \binom{21}{14} = \frac{21!}{14!(21-14)!} = \frac{21!}{14!7!}$$

⑤ Quanto vale  $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$ ?

$$1 + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n} = 2^n$$

⑥

$$\sum_{p=0}^{10} \binom{10}{p} = \binom{10}{0} + \binom{10}{1} + \dots + \binom{10}{10} = 2^{10} = 1024$$

$$\sum_{p=0}^9 \left( \binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \dots + \binom{10}{9} \right) = 1024 - 1$$

$$\sum_{p=2}^9 \left( \binom{9}{2} + \binom{9}{3} + \dots + \binom{9}{9} \right) = \binom{9-9}{1 \ 0} = 512 - 9 - 1$$

$$\sum_{p=4}^{10} \left( \binom{4}{4} + \binom{5}{4} + \dots + \binom{10}{4} \right) = \binom{11}{5}$$

$$11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6! = 924 = 462$$

$$\sum_{p=5}^{10} \left( \binom{5}{5} + \binom{6}{5} + \dots + \binom{10}{5} \right) = \binom{11}{6}$$

$$11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6! = 924 = 462$$

⑦

$$\sum_{k=0}^m \binom{m}{k} = 2^m = 512$$

$$\binom{m}{k} = \binom{m}{0} + \dots + \binom{m}{m} = 512$$

$$2^m = 512 = m = 9$$

$$2^9 = 512$$

