Matriz Inversa

01 - $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$ é inversa de $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$ c

Soma $x+y$ Sinal

$\begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} \cdot \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$ $x+y = 2 + 5 = -3$

$3x - 5 = 1$ $x \cdot y + 10 = 0$

$3x = 1 + 5$ $2y = \frac{-10}{2} = -5$

$3x = 6$

$x = \frac{6}{3} = 2$

02 - Os valores de k para que a matriz $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$ não admita inversa são:

det

detA $\begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix}$

$\det A = \left[\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ k & 1 & 3 & k & 1 \\ 1 & k & 3 & 1 & k \end{array}\right]$

diagonal

$3k + 1$

$3 + 0 + k^0 = k^2 + 3$

$k^2 - 3k + 2 = 0$

$(3)^2 - 4 \cdot 1 \cdot 2$

$\Delta : 9 - 8 = 1$ →

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$\pm \quad I = \frac{3 + \sqrt{1}}{2} = \frac{4}{2} = 2$ $\qquad$ 1 e 2

$II = \frac{3 - 1}{2} = \frac{2}{2} = 1$

03- Se B é a matriz inversa de $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$

$B? = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$ dividido por 2

$\begin{bmatrix} 2 & -\frac{5}{2} \\ -1 & -\frac{3}{2} \end{bmatrix}$

04- $\begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix}$ é inversível

$\left[\begin{array}{ccc|cc} x & 1 & 2 & x & 1 \\ 3 & 1 & 2 & 3 & 1 \\ 10 & 1 & x & 10 & 1 \end{array}\right]$ diagonal

$x^2 + 26 - (20 + 5x)$ ← $20x + 2x + 3x = 20 + 5x$

$x^2 - 5x + 6 \neq 0$ $\quad x^2 + 20 + 6 = x^2 + 26$

$(5)^2 - 4 \cdot 1 \cdot 6$

$\Delta = 25 - 24 = 1$ $\quad \mathrm{I} \neq \frac{5+1}{2} = 3$ $\quad \{x \neq 3, x \neq 2\}$

$\mathrm{II} \neq \frac{5-1}{2} = 2$

A/.

05 -

$$A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \quad \text{Seja } A^{-1}$$

$$\det A = \left[\begin{array}{ccc|cc} -1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & 2 & 1 \\ 1 & 1 & -1 & 1 & 1 \end{array}\right]$$

diagonal

$$2+2+2=6 \qquad 7-6=1$$

$$1+2+4=7$$

$$A' = \begin{bmatrix} (-1-(-2)) & (-2-(-2)) & (2-1) \\ (1-2) & (1-2) & (-1-(-1)) \\ (2-2) & (2-4) & (-1-(-2)) \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \qquad \frac{\bar{A}}{1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

Soma

$$A + A^{-1} = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

06 - A e B matrizes de mesma ordem

$(X \cdot A)^t = B$ — matriz normal

$(X \cdot A)^T = B^T$

$X \cdot A = B^T$

$X \cdot A \cdot A^{-1}$

$X = B^T \cdot A^{-1}$

07 -

$B = \begin{bmatrix} x \\ y \end{bmatrix}$ $C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$ inversa da matriz A

AB = C

A^{-1}

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$ $\begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix}$ $24 - 25 = -1$

inverso $\begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} 1/_{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}$

08 - $A = \begin{pmatrix} 2 & K \\ -2 & 1 \end{pmatrix}$ Soma dos valores de K - det A =

$\rightarrow$ det A · det A^{-1} | det A^{-1}

$\det A = 2 + 2K$

$(2 + 2K) \cdot (2 + 2K) = 1$

$4 + 4K + 4K + 4K^2 = 1$

$4 + 8K + 4K^2 = 1$

$4K^2 + 8K + 4 - 1 = 0$

$4K^2 8K + 3 = 0$

$\rightarrow (8)^2 - 4 \cdot 4 \cdot 3$

$\Delta = 64 - 48 = 16$

$I = \frac{-8 + 4}{8} = \frac{-4}{8}$

$II \quad \frac{-8 - 4}{8} = \frac{-12}{8}$

$-\frac{4}{8} - \frac{12}{8} = \frac{-16}{8} = -2 \qquad -2$

09- A e B são matrizes quadradas de ordem 2

$(\det(A) \neq 0$ e $\det(B) \neq 0$

a- $(A+B) \cdot (A-B)$

$A^2 - AB + BA - B^2$

$\downarrow$

$AB \neq BA$

b- $(A+B)^2 = A^2 + 2 \cdot A \cdot B + B^2$

A e B $= 2 \cdot A \cdot B = AB = BA$

c-

$\underline{\det(A)}$

$\det(-A) = (-\underline{1})^2 \cdot \det A = \underline{1}$

d B inversa de A $\quad B = A^{-1}$

$\det A \cdot \det B = \underline{1}$

$\det B = \dfrac{\underline{1}}{\det A}$