Teorema do Binômio

① Desenvolvimento $(1+2x^2)^6$ de x^8

$$(1+2x^2)^6 = \binom{6}{k} \cdot 1^{6-k} \cdot (2x^2)^k : 6$$

$$2^k \cdot x^{2k} \rightarrow \text{valor de } k \quad k = \frac{8}{2} = 4$$

Substituir → $\binom{6}{4} 1^{6-4} \cdot 2^4 \cdot x^{2 \cdot 4}$

$$\binom{6}{4} 1^2 \cdot 16 \cdot x^8 = \frac{6!}{4!\,2 \cdot 1} \cdot 1^2 \cdot 16 \cdot x^8$$

$$\frac{6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}\,2 \cdot 1} = 15 \cdot 16x^8 = \underline{240x^8}$$

C

② A soma de todos os coeficientes do desenvimento $(14\overset{1}{x} - 13\overset{1}{y})^{237}$

$$(14x - 13y)^{237} \quad (14-13)^{237}$$

$$1^{237} = 1$$

③ Termos do desenvolvimento de $(x+a)^{11}$ igual a $1386x^5$

$$11 \div \binom{11}{k} \cdot x^{11-k} \cdot a^k \leftrightarrow k = 11-5 = 6$$

$$11 : \binom{11}{6} \cdot x^{11-6} \cdot a^6$$

$$\frac{11}{6!\,\underset{5}{(11-6)!}} \cdot x^5 \cdot a^6 = 1386x^5$$

$$\rightarrow \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}\,5 \cdot 4 \cdot 3 \cdot 2} \cdot a^6 = 1386$$

$$462a^6 = 1386$$

$$a^6 = \frac{1386}{462} = \sqrt[6]{3}$$

④ Desenvolvimento do binômio $\left(x + \frac{1}{x^2}\right)^9$

$$\left(x + \frac{1}{x^2}\right)^9 \rightarrow (x + x^{-2})^9$$

$$\binom{9}{0} x^{9-0} \cdot (x^{-2})^0 + \binom{9}{1} x^{9-1} \cdot (x^{-2})^1 + \binom{9}{2} x^{9-2} \cdot (x^{-2})^2 + \binom{9}{3} x^{9-3} \cdot (x^{-2})^3$$

$$\binom{9}{0} x^9 \cdot x^0 + \binom{9}{1} x^8 \cdot x^{-2} \binom{9}{2} x^7 \cdot x^{-4} + \binom{9}{3} x^6 \cdot x^{-6} +$$

$$\binom{9}{0} x^9 + \binom{9}{1} x^6 + \binom{9}{2} x^3 + \binom{9}{3} x^0$$

$$\binom{9}{3}$$

⑤ Desenvolvimento de $\left(x + \frac{1}{x^2}\right)^n$ termo independente

$n \quad (x + x^{-2})^n \quad \binom{n}{k} x^{n-k} \cdot x^{-2k}$

$x = n - k - 2k$

$n - 3k = 0$

$k = \frac{n}{3} = C$

⑥ $\left(3x^3 + \frac{2}{x^2}\right)^5 \cdot \left(243x^{15} + 810x^{10} + 1080x^5 + \frac{240}{x^5} + \frac{32}{x^{10}}\right)$

$(3x^3 + 2x^{-2})^5 = \binom{5}{0} \cdot (3x^3)^5 (2x^{-2})^0 \ldots \binom{5}{5} (3x^3)^0 \cdot (2x^{-2})^5$

$243x^{15} + 810x^{10} + 1080x^5 + 720 + \frac{240}{x^5} + \frac{32}{x^{10}}$

$243x^{15} + 810x^{10} + 1080x^5 + 720 + \frac{240}{x^5} + \frac{32}{x^{10}}$

$-243x^{15} - 810x^{10} - 1080x^5 - \frac{240}{x^5} - \frac{32}{x^{10}}$

$- K = 720$

⑦ A soma dos coeficientes do desenvolvimento de $(2\cancel{x}+\cancel{y})$

Soma $= 1$ $\cancel{xy}$ $(2+1)^5 = 3^5 = 242$