Lab 3 STAT 151A

Data Table

X	Y	X_iY_i	X_i^2	$\hat{Y}_i$	$E_i = Y_i - \hat{Y}_i$	E_i^2
5	3.5	17.5	25	3.539048	−0.03904762	0.0015247166
6	3.8	22.8	36	3.750476	0.04952381	0.0024526077
3	3.1	9.3	9	3.116190	−0.01619048	0.0002621315
7	4	28	49	3.961905	0.03809524	0.0014512472
4	3.2	12.8	16	3.327619	−0.12761905	0.0162866213
2	3	6	4	2.904762	0.09523810	0.0090702948

Lab 4 STAT 151A

Step ①: We have the data table and numerical result from Lab 3 in the first page

$A = 2.1481905$, $B = 0.1211 4286$

$SSR = 0.03104762$, $RMS = 0.007761905$

$t = 10.03919$, $\bar{Y} = 3.433$

step ②: ANOVA Table

Source of Variation	Sum of squares	Degree of Freedom	Mean square	F
Regression	RegSS = 0.7822857	1	RegMS = 0.7822857	F = 100.7853
Residual	RSS = 0.03104762	4	RMS = 0.007761905	
Total	TSS = 0.8133	5		

step ③:

$RegSS = \sum_{i=1}^{6}[\hat{Y}_i - \bar{Y}]^2 = \sum_{i=1}^{6}[\hat{Y}_i - 3.433]^2 = 0.7822857$

$RSS = \sum_{i=1}^{6}[Y_i - \hat{Y}_i]^2 = 0.03104762$

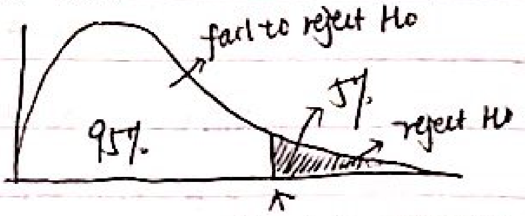
$TSS = \sum_{i=1}^{6}[Y_i - \bar{Y}]^2 = 0.8133$

$RegMS = \frac{RegSS}{1} = RegSS = 0.7822857$

$RMS = \frac{RSS}{6-2} = \frac{RSS}{4} = \frac{0.03104762}{4} = 0.007761905$

$F = \frac{RegMS}{RMS} = \frac{0.7822857}{0.007761905} = 100.7853$

step ④ $F = \frac{RegMS_{df=1}}{RMS_{df=4}} \sim F_{1,4}$



By R. $F\text{-critical}_{\alpha=0.05} = 7.708647$

f_critical = 7.708647

step ⑤: Since $F = 100.7853$, $\Rightarrow F > F\_critical$

$\Rightarrow$ We reject the null hypothesis that there is no linear relationship and conclude that there is a linear relationship between # of hours of study in (explanatory variable)

(response variable)
a student life and his/her GPA.

step ⑥: Yes, there is a linear relationship

$$R^2 = \frac{RegSS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{0.0310476^2}{0.813333} = 0.9618267$$

Since R^2 (0.96) explains the proportion of variability in the response that can be explained by the Model, 96% of the variation in the response variable can be attributed to the explanatory variable. 4% of the variation in the response variable can be attributed to other variables.

step ⑦: Yes, did this with t-test and obtained the same conclusion

step ⑧: Proof: $F = t^2$ where $t = \dfrac{B}{\sqrt{\frac{RMS}{S_{xx}}}}$

$$F = \frac{RegMS}{RMS} = \frac{RegSS/1}{RSS/n-2} = \frac{RegSS}{RSS/n-2} = \frac{RegSS}{RSS} \cdot (n-2)$$

$$= \frac{RegSS}{RMS}.$$

Since $RegSS = \sum_{i=1}^{n}[(\hat{Y} - \bar{Y})^2] = \sum_{i=1}^{n}[(A+BX-(A+B\bar{X}))^2]$ $\left(\begin{matrix}\hat{Y}=A+BX \\ \bar{Y}=A+B\bar{X}\end{matrix}\right)$

$$= \sum_{i=1}^{n}[A+BX-A-B\bar{X}]^2 = \sum_{i=1}^{n}[BX-B\bar{X}]^2$$

$$= \sum_{i=1}^{n} B^2(X-\bar{X})^2 = B^2 \cdot \sum_{i=1}^{n}(X-\bar{X})^2 \quad (B \text{ is constant})$$

$$= B^2 \cdot S_{xx} \quad \left(S_{xx} = \sum_{i=1}^{n}(X-\bar{X})^2\right)$$

$$\therefore F = \frac{B^2 S_{xx}}{RMS} = \left(\frac{B}{\sqrt{RMS}}\sqrt{S_{xx}}\right)^2 = \left(\frac{B}{\sqrt{\frac{RMS}{S_{xx}}}}\right)^2 = t^2$$

Thus prove $F = t^2$ □

STAT151A-Lab4

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```
x = c(5,6,3,7,4,2)
y = c(3.5,3.8,3.1,4,3.2,3)
x_bar = mean(x)
x_bar
```

```
## [1] 4.5
```

```
y_bar = mean(y)
y_bar
```

```
## [1] 3.433333
```

```
X_iY_i = x * y
X_iY_i
```

```
## [1] 17.5 22.8  9.3 28.0 12.8  6.0
```

```
X_sqr = x ^ 2
X_sqr
```

```
## [1] 25 36  9 49 16  4
```

```
n = 6
sum(X_iY_i)
```

```
## [1] 96.4
```

```
sum(x^2)
```

```
## [1] 139
```

```
B = (sum(X_iY_i) - n*x_bar*y_bar) / (sum(x^2) - n*x_bar^2)
B
```

```
## [1] 0.2114286
```

```
A = y_bar - B*x_bar
A
```

```
## [1] 2.481905
```

```
y_hat <- A + B*x
y_hat
```

```
## [1] 3.539048 3.750476 3.116190 3.961905 3.327619 2.904762
```

```
E <- y - y_hat
E
```

```
## [1] -0.03904762  0.04952381 -0.01619048  0.03809524 -0.12761905  0.09523810
```

```
E_sqr = E^2
E_sqr
```

```
## [1] 0.0015247166 0.0024526077 0.0002621315 0.0014512472 0.0162866213
## [6] 0.0090702948
```

```r
SSR = sum(E_sqr)
SSR
```

```
## [1] 0.03104762
```

```r
RMS = SSR/4
RMS
```

```
## [1] 0.007761905
```

```r
S_xx = sum(x^2) - 6*x_bar^2
S_xx
```

```
## [1] 17.5
```

```r
t = B/(sqrt(RMS/S_xx))
t
```

```
## [1] 10.03919
```

```r
TSS = sum((y - y_bar)^2)
TSS
```

```
## [1] 0.8133333
```

```r
RSS = sum((y - y_hat)^2)
RSS
```

```
## [1] 0.03104762
```

```r
RegSS = sum((y_hat - y_bar)^2)
RegSS
```

```
## [1] 0.7822857
```

```r
RegMS = RegSS/1
RegMS
```

```
## [1] 0.7822857
```

```r
RMS = RSS/4
RMS
```

```
## [1] 0.007761905
```

```r
F_value = RegMS/RMS
F_value
```

```
## [1] 100.7853
```

```r
F_critical = qf(0.95,1,4)
F_critical
```

```
## [1] 7.708647
```

```r
R_sqr = 1- RSS/TSS
R_sqr
```

```
## [1] 0.9618267
```