运筹与优化 Homework6.

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1. (a)

Minimize $50u_1+10u_2+23u_3$

Subject to: $3u_1+2u_2+u_3\ge 40$

$-3u_1 - u_3 \ge 20$

$5u_1+u_2+4u_3=1$

$u_1,u_2,u_3\ge 0$

(b)

Maximize $5u_1+10u_2+6u_3$.

s.t. $2u_1+u_2+u_3=1$

$u_1+u_2-4u_3=1$

$u_1,u_2\le 0$ $u_3\ge 0$.

(c).

Minimize $10u_1+5u_2+8u_3+15u_4+20u_5$

s.t. $4u_1+4u_2+3u_3+3u_4+u_5\ge 2$.

$2u_1+2u_2+5u_3+5u_4+u_5\ge 1$

$5u_1+5u_2+4u_3+4u_4+u_5\ge 3$.

$5u_1+5u_2+u_3+u_4+5u_5\ge 4$.

$u_1,u_4\ge 0$, $u_2,u_3\le 0$, u_5 free.

2. (a) ~~Dual problem:~~

Rewrite Primal problem:

maximize: $x_0=\vec{c}^T\vec{x}$

s.t. $A\vec{x}=\vec{b}$

$\vec{x}\ge\vec{l}$ $\Rightarrow$ $\begin{bmatrix}A\\I\end{bmatrix}\vec{x}\ge\begin{bmatrix}\vec{b}\\\vec{l}\end{bmatrix}$

$\vec{x}\le\vec{u}$. $-\begin{bmatrix}A\\I\end{bmatrix}\vec{x}\ge\begin{bmatrix}-\vec{b}\\-\vec{u}\end{bmatrix}$

$\vec{x}$ free.

Dual problem:

minimize $[\vec{b}^T\ \vec{l}^T\ -\vec{b}^T\ -\vec{u}^T]\,\vec{v}$

s.t. $\begin{bmatrix}A\\I\\-A\\-I\end{bmatrix}^T\vec{v}=\vec{c}$

$\vec{v}\le\vec{0}$.

(b). Easy to obtain that

$$\begin{bmatrix}\vec{0}_m\\ \min(0,c_1)\\ \vdots\\ \min(0,c_n)\\ \vec{0}_m\\ \min(0,-c_1)\\ \vdots\\ \min(0,-c_n)\end{bmatrix}$$ is a feasible solution.

Q.E.D.

(c). The ~~dual~~ primal problem is bounded <or infeasible> because the dual problem always has a feasible solution. then the primal cannot be unbounded. (Weak Duality's corollary).

3. Dual Problem:

minimize $6u_1+12u_2+5u_3$

s.t. $2u_1+5u_2=9$

$u_1+4u_2+2u_3=14$

$3u_1+u_2=7$.

$u_1,u_2,u_3\ge 0$.

Easy to solve out the only feasible solution to the dual is $\begin{cases}u_1=2\\u_2=1\\u_3=4.\end{cases}$ and the objective value is 44.

∴ For the Primal. there holds $\vec{c}^T\vec{x}\le 44$.

yet feasible solution $\vec{x}=\begin{bmatrix}\frac{5}{26}\\\frac{5}{2}\\\frac{27}{26}\end{bmatrix}$

satisfies $\vec{c}^T\vec{x}=44$.

∴ $\vec{x}$ is an optimal solution to the primal LPP. (Strong duality).

Q.E.D.

4.

	x_1	x_2	s_1	s_2	s_3	b
s_1	1	2	1			10
s_2	3	5		1		26
s_3	1	1			1	8
	3	4	0	0	0	0

$\Rightarrow$ End.

By the Complementary Slackness condition, $(1\times w_1-0)\cdot s_1=0$ yet $s_1=10\ne 0$.

∴ In the optimal solution of dual problem, there must has $w_1=0$.