

运筹与优化 Homework 4

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| | x_1 | x_2 | x_3 | x_4 | b | $1/x_1$ |
|-------|-------|-------|-------|-------|-----|--------------|
| x_3 | 1 | 1* | 1 | 0 | 2 | 2 |
| x_4 | 1 | -1 | 0 | 1 | 1 | -1 (Ignored) |
| | 1 | -2 | 0 | 0 | 0 | |

| | | | | | | |
|-------|---|---|---|---|---|--------------------|
| x_2 | 1 | 1 | 1 | 0 | 2 | |
| x_4 | 2 | 0 | 1 | 1 | 3 | \Rightarrow End. |
| | 3 | 0 | 2 | 0 | 4 | |

Optimal BFS: $[0 \ 2 \ 0 \ 3]^T$.
Maximum: 4.

| | x_1 | x_2 | x_3 | x_4 | x_5 | b | b/x_1 |
|-------|-------|-------|-------|-------|-------|-----|---------|
| x_3 | 1 | 1 | 1 | 0 | 0 | 2 | 2 |
| x_4 | 0 | 1 | 0 | 1 | 0 | 1 | Inf |
| x_5 | 1* | -1 | 0 | 0 | 1 | 1 | 1 |
| | -2 | 0 | 0 | 0 | 0 | 0 | |

| | x_1 | x_2 | x_3 | x_4 | x_5 | b | b/x_1 |
|-------|-------|-------|-------|-------|-------|-----|--------------|
| x_3 | 0 | 2* | 1 | 0 | -1 | 1 | $1/2$ |
| x_4 | 0 | 1* | 0 | 1 | 0 | 1 | 1 |
| x_1 | 1 | -1 | 0 | 0 | 1 | 1 | -1 (Ignored) |
| | 0 | -1 | 0 | 0 | 0 | 2 | 2 |

| | | | | | | |
|-------|---|---|---|----|----|----|
| x_3 | 0 | 0 | 1 | -2 | -1 | -1 |
| x_2 | 0 | 1 | 0 | 1 | 0 | 1 |
| x_1 | 1 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | |
|-------|---|---|--------|---|---------|-------|
| x_2 | 0 | 1 | $1/2$ | 0 | $-1/2$ | $1/2$ |
| x_4 | 0 | 0 | $-1/2$ | 1 | $1/2$ * | $1/2$ |
| x_1 | 1 | 0 | $1/2$ | 0 | $1/2$ | $3/2$ |
| | 0 | 0 | $1/2$ | 0 | $3/2$ | $5/2$ |

| | | | | | | |
|-------|---|---|----|----|---|---|
| x_2 | 0 | 1 | 0 | 1 | 0 | 1 |
| x_5 | 0 | 0 | -1 | 2 | 1 | 1 |
| x_1 | 1 | 0 | 1 | -1 | 0 | 1 |
| | 0 | 0 | 0 | 1 | 0 | 3 |

\Rightarrow End.
Optimal BFS: $[3/2 \ 1/2 \ 0 \ 1/2 \ 0]^T$
Maximum: $5/2$.

3. \Rightarrow maximize $z = -4x_1 - x_2$
subject to $3x_1 + x_2 = 3$
 $4x_1 + 3x_2 - x_3 = 6$
 $x_1 + 2x_2 + x_4 = 4$
 $x_1, x_2, x_3, x_4 \geq 0$.

| | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | x_7 | b | b/x_1 |
|-------|-------|-------|-------|-------|-------|-------|-------|-----|---------|
| x_5 | 3 | 1 | 0 | 0 | 1 | 0 | 0 | 3 | Inf |
| x_6 | 4 | 3 | -1 | 0 | 0 | 1 | 0 | 6 | Inf |
| x_7 | 1 | 2 | 0 | 1* | 0 | 0 | 1 | 4 | 4. |
| | 4 | 1 | 0 | 0 | M | M | M | 0 | |

$4-8M \ 1-6M \ M-M \ 0 \ 0 \ 0 \ -13M$

| | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | x_7 | b | b/x_1 |
|-------|-------|-------|-------|-------|-------|-------|-------|-----|---------|
| x_5 | 3 | 1 | 0 | 0 | 1 | 0 | 0 | 3 | 3 |
| x_6 | 4 | 3 | -1 | 0 | 0 | 1 | 0 | 6 | 2 |
| x_4 | 1 | 2* | 0 | 1 | 0 | 0 | 1 | 4 | 2 |

$4-7M \ 1-4M \ M \ 0 \ 0 \ 0 \ -9M$

| | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | b | b/x_1 |
|-------|---------|-------|-------|-------|-------|-------|-----|---------|
| x_5 | $5/2$ | 0 | 0 | 0 | 1 | 0 | 1 | $2/5$ |
| x_6 | $5/2$ * | 0 | -1 | 0 | 0 | 1 | 0 | 0 |
| x_2 | $1/2$ | 1 | 0 | $1/2$ | 0 | 0 | 2 | 4 |

$7/2 \ -5M \ 0 \ 0 \ 0 \ 0 \ -M-2$

| | x_1 | x_2 | x_3 | x_5 | x_6 | b | b/x_1 |
|-------|-------|-------|---------|-------|----------|--------|--------------|
| x_5 | 0 | 0 | 1* | 1 | -1 | 1 | 1 |
| x_1 | 1 | 0 | $-2/5$ | 0 | $2/5$ | 0 | -0 (Ignored) |
| x_2 | 0 | 1 | $1/5$ | 0 | $-1/5$ | 2 | 10 |
| | 0 | 0 | $7/5-M$ | 0 | $2M-7/5$ | $-M-2$ | |

| | x_1 | x_2 | x_3 | x_5 | b |
|-------|-------|-------|-------|---------|---------|
| x_3 | 0 | 0 | 1 | 1 | 1 |
| x_1 | 1 | 0 | 0 | $2/5$ | $2/5$ |
| x_2 | 0 | 1 | 0 | $-1/5$ | $9/5$ |
| | 0 | 0 | 0 | $M-1/5$ | $-17/5$ |

\Rightarrow End.
Optimal BFS: $[2/5 \ 9/5]^T$.
Maximum: $-17/5$
Minimum: $17/5$

| | x_1 | x_2 | x_3 | x_4 | x_5 | y_1 | y_2 | y_3 | b | b/x_4 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|---------|
| y_1 | 2 | -1 | 3 | 2 | 1 | 1 | 0 | 0 | 10 | 5 |
| y_2 | 1 | -3 | 2 | 2* | 1 | 0 | 1 | 0 | 8 | 4 |
| y_3 | 1 | -2 | 5 | 2 | 2 | 0 | 0 | 1 | 18 | 9 |
| | -2 | 0 | -1 | -1 | 0 | M | M | M | 0 | |

$$-2-4M \quad 6M-1-10M \quad -1-6M \quad -4M \quad 0 \quad 0 \quad 0 \quad -36M$$

| | x_1 | x_2 | x_3 | x_4 | x_5 | y_1 | y_2 | y_3 | b | b/x_3 |
|-------|------------------|-------------------|-------|-------|------------------|-------|--------|-------|----------|----------------|
| y_1 | 1 | 2 | 1* | 0 | 0 | 1 | -1 | 0 | 2 | 2 |
| x_4 | $\frac{1}{2}$ | $-\frac{3}{2}$ | 1 | 1 | $\frac{1}{2}$ | 0 | 1 | 0 | 4 | 4 |
| y_3 | 0 | 1 | 3 | 0 | 1 | 0 | -1 | 1 | 10 | $\frac{10}{3}$ |
| | $-\frac{3}{2}-M$ | $-3M-\frac{3}{2}$ | $-4M$ | 0 | $-M+\frac{1}{2}$ | 0 | $6M+1$ | 0 | $-12M+4$ | |

| | x_1 | x_2 | x_3 | x_4 | x_5 | y_1 | y_3 | b | b/x_5 |
|-------|----------------|----------------|-------|-------|---------------|-------|-------|-----|---------|
| x_3 | 1 | 2 | 1 | 0 | 0 | 1 | 0 | 2 | Inf |
| x_4 | $-\frac{1}{2}$ | $-\frac{7}{2}$ | 0 | 1 | $\frac{1}{2}$ | -1 | 0 | 2 | 4 |
| y_3 | -3 | -5 | 0 | 0 | 1* | -3 | 1 | 4 | 4 |

$$-\frac{3}{2}+3M \quad 5M-\frac{3}{2} \quad 0 \quad 0 \quad -M+\frac{1}{2} \quad 0 \quad 0 \quad -4M+4$$

| | x_1 | x_2 | x_3 | x_4 | x_5 | y_3 | b |
|-------|-------|-------|-------|-------|-------|-----------------|-----|
| x_3 | 1 | 2 | 1 | 0 | 0 | 0 | 2 |
| x_4 | 1 | -1 | 0 | 1 | 0 | $-\frac{1}{2}$ | 0 |
| x_5 | -3 | -5 | 0 | 0 | 1 | 1 | 4 |
| | 0 | 1 | 0 | 0 | 0 | $M-\frac{1}{2}$ | 2 |

\Rightarrow End.

~~Optimal BFS:~~

Optimal BFS: $[0 \ 0 \ 2 \ 0 \ 4]^T$

Maximum: 2.