STA5103 Selected Topics in Frontiers of Statistics

Homework 3

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Question 1

Answer:

Now we are going to solve the $\boldsymbol{z}_1$-subproblem:

$$\boldsymbol{z}_1 = L_\rho^{(\text{ADMM})}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{u}) = \arg\min_{\boldsymbol{z}} \left(\langle \mathbf{1}, \boldsymbol{z} - \boldsymbol{b} \circ \ln \boldsymbol{z} \rangle + \frac{\rho}{2} \| K\boldsymbol{x} - \boldsymbol{z} + \boldsymbol{u} \|_2^2 \right)$$

Differentiate the objective function with respect to $\boldsymbol{z}$ at $\boldsymbol{z}_1$ and we get

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{z}} L_\rho^{(\text{ADMM})}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{u}) \Big|_{z=z_1} &= \frac{\partial}{\partial \boldsymbol{z}} \left(\langle \mathbf{1}, \boldsymbol{z} - \boldsymbol{b} \circ \ln \boldsymbol{z} \rangle + \frac{\rho}{2} \| K\boldsymbol{x} - \boldsymbol{z} + \boldsymbol{u} \|_2^2 \right) \Big|_{z=z_1} \\ &= \frac{\partial}{\partial \boldsymbol{z}} \left(-\ln \boldsymbol{z} \circ \boldsymbol{b} + \boldsymbol{z}^T \mathbf{1} + \frac{\rho}{2} \| K\boldsymbol{x} - \boldsymbol{z} + \boldsymbol{u} \|_2^2 \right) \Big|_{z=z_1} \\ &= -[\text{diag}(\boldsymbol{z}_1)]^{-1} \boldsymbol{b} + \mathbf{1} + \rho(\boldsymbol{z}_1 - K\boldsymbol{x} + \boldsymbol{u}) \\ &= \mathbf{0} \end{aligned}$$

With reference to each elements of $\boldsymbol{z}$, we have

$$\begin{aligned} \frac{\partial}{\partial z_j} L_\rho^{(\text{ADMM})}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{u}) \Big|_{z=z_1} &= -\frac{b_j}{z_{1j}} + 1 + \rho(z_{1j} - (K\boldsymbol{x} + \boldsymbol{u})_j) \\ &= 0 \end{aligned},$$

i.e.,

$$\rho z_{1j}^2 + (1 - \rho(K\boldsymbol{x} + \boldsymbol{u})_j) z_{1j} - b_j = 0,$$

Solving the equations and we hereby derive the element-wise closed-form solution as follows.

$$z_{1j} = \frac{\rho(K\boldsymbol{x} + \boldsymbol{u})_j - 1 + \sqrt{(\rho(K\boldsymbol{x} + \boldsymbol{u})_j - 1)^2 - 4\rho b_j}}{2\rho}, \quad j = 1, \ldots, n$$

(Ignoring the negative solution.)

Question 2

Answer:

Please refer to `script.py` to see the corresponding python script.

The origin video is of 8 frames/second with frame shape 121×72. Though the new video should have a frame shape of 241×143, the new video is resized to 240×142. This is might due to the internal alignment requirement of MJPG codec.