运筹与优化 Homework3

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1. Denote $A=[\vec{a_1}\ \vec{a_2}\cdots\vec{a_m}]^T$ $b=\begin{bmatrix} b_1\\ \vdots \\ b_m\end{bmatrix}$, $x=\begin{bmatrix} x_1\\ \vdots \\ x_n\end{bmatrix}$.

$a_i=[a_{i1}\cdots a_{in}]^T$. $i=1,2,\cdots,m$.

Then.

(1). $\min \sum_{k=1}^{m}\left[\cancel{(-b_k+\sum_{i=1}^{n}a_{ki}x_i)+(-b_k+\sum_{i=1}^{n}a_{ki}x_i)}\ y_k^+ + y_k^-\right]$

s.t. $-1\le x_1\le 1$, $y_1^+-y_1^-=-b_1+\sum_{i=1}^{n}a_{1i}x_i$

$\vdots$

$-1\le x_n\le 1$, $y_m^+-y_m^-=-b_m+\sum_{i=1}^{n}a_{mi}x_i$

$y_1^+,\cdots,y_m^-\ge 0$.

(2). $\min \sum_{k=1}^{n}(x_k^+ + x_k^-)$

s.t. $-1\le -b_1+\sum_{i=1}^{n}a_{1i}(x_i^+ + x_i^-)\le 1$

$\vdots$

$-1\le -b_m+\sum_{i=1}^{n}a_{mi}(x_i^+ + x_i^-)\le 1$

$x_1^+,x_1^-,\cdots,x_n^+,x_n^-\ge 0$.

(3). $\min \sum_{k=1}^{m}\left[\cancel{(-b_k+\sum_{i=1}^{n}a_{ki}x_i)+(-b_k+\sum_{i=1}^{n}a_{ki}x_i)}\ y_k^+ + y_k^-\right]+w$

s.t. $w\ge x_1^+ + x_1^-$ $\quad y_1^+-y_1^-=-b_1+\sum_{i=1}^{n}a_{1i}x_i$

$w\ge x_2^+ + x_2^-$

$\vdots$

$w\ge x_n^+ + x_n^-$ $\quad y_m^+-y_m^-=-b_m+\sum_{i=1}^{n}a_{mi}x_i$

$x_1=x_1^+ + x_1^-$

$\vdots$

$x_n=x_n^+ + x_n^-$

$x_1^+,\cdots,x_n^-\ge 0$, $y_1^+,\cdots,y_m^-\ge 0$.

(4). $\min \sum_{k=1}^{m} w_k$

s.t. $w_1\ge 0$

$w_1\ge(\sum_{i=1}^{n}a_{1i}x_i)+b_1$

$\vdots$

$w_m\ge 0$

$w_m\ge(\sum_{i=1}^{n}a_{mi}x_i)+b_m$.

2. Canonical form:

maximize $3x+2y$

subject to $-2x+y\le -6$

$-2x-y\le -10$

$x\ge 0, y\ge 0$

Standard form:

maximize $3x+2y$

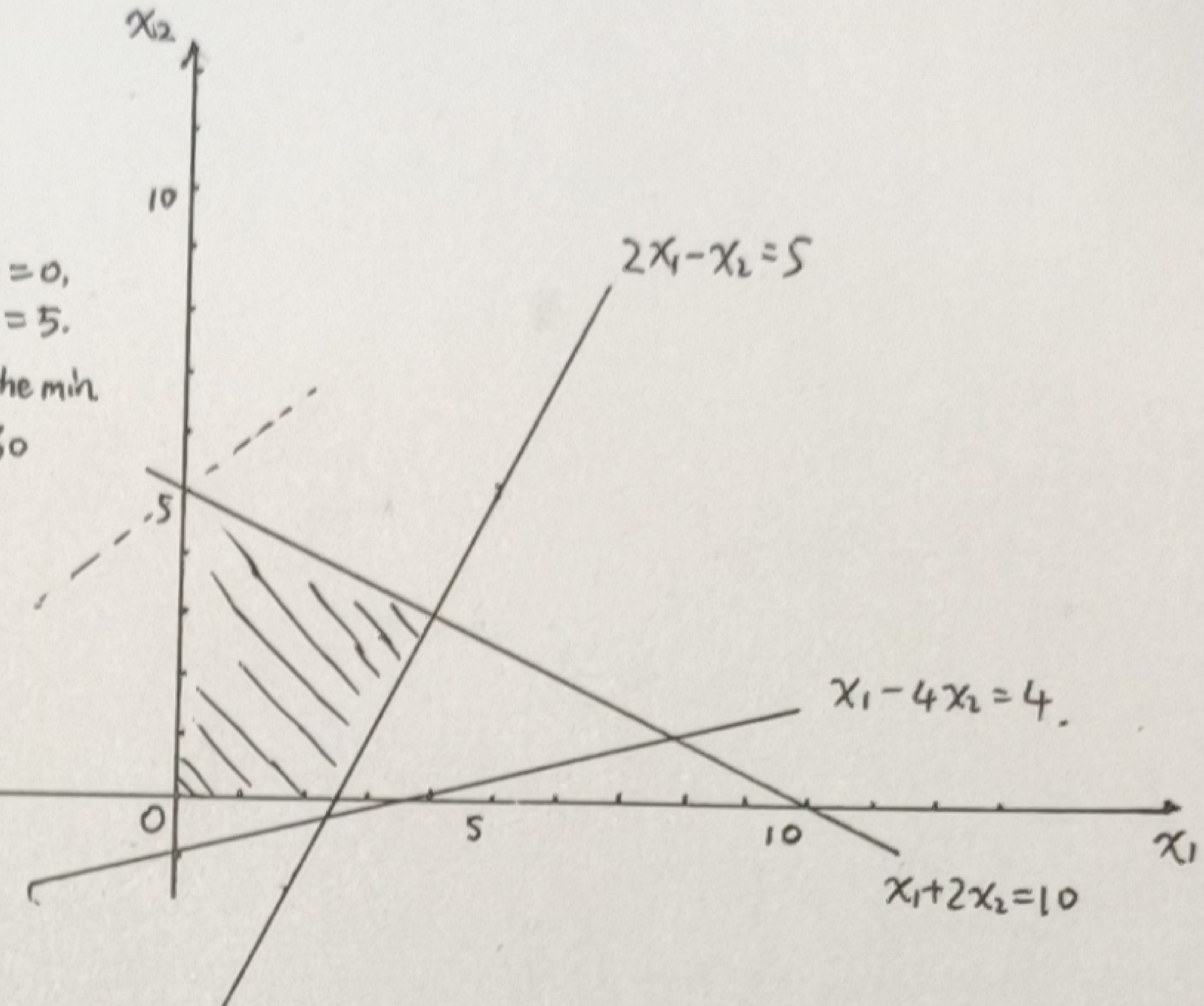
subject to $2x-y-m=6$

$2x+y-n=10$

$x\ge 0, y\ge 0, m\ge 0, n\ge 0$

3. (a)

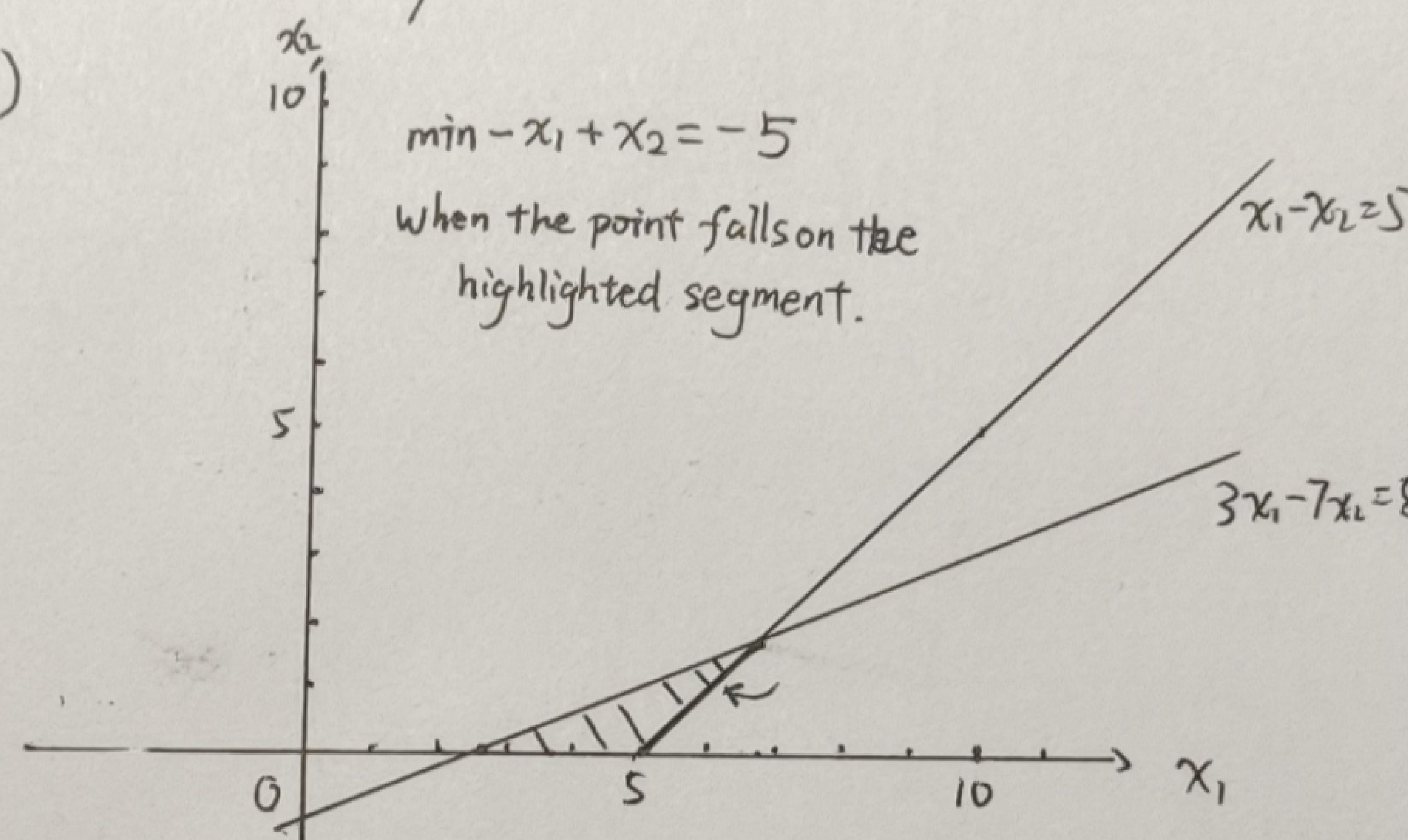
When $x_1=0$, $x_2=5$, $5x_1-6x_2$ achieves the min value -30



(b)

$\min -x_1+x_2=-5$

when the point falls on the highlighted segment.



4. (a) $\because 2-1+3+0=4$. $\quad 2\ge 0$

$-2+1+2\times 1+0=1$, $\quad 1\ge 0$

$-3\times 2+1+1+0=-4$ $\quad 3\ge 0$

$0\ge 0$

$\cdots$

$\therefore (2,1,1,3,0,0,0)^T$ ~~follows~~ satisfies all the constraint and is a feasible solution.

(b) Since there are only 3 equations and $(2,1,1,3,0,0,0)^T$ has 4 non-zero entries.

$\therefore$ It is not a BFS.

~~Yet we can easily construct a BFS: $(0,0,0,4,0,1,-4)$ where $\{x_4,x_6,x_7\}$ is the basis.~~

To construct a BFS, we must vanish some column of x_1,x_2,x_3,x_4 to 0. It is easy to find that x_1,x_3,x_4 are linear independent.

$\therefore$ By solving $\begin{bmatrix}1&0&1\\-1&2&0\\3&1&0\end{bmatrix}\begin{bmatrix}x_1\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}4\\1\\-4\end{bmatrix}\Rightarrow\begin{bmatrix}x_1\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}0&\frac{1}{5}&-\frac{2}{5}\\0&\frac{3}{5}&-\frac{1}{5}\\1&-\frac{1}{5}&\frac{2}{5}\end{bmatrix}\begin{bmatrix}4\\1\\-4\end{bmatrix}=\begin{bmatrix}\frac{9}{5}\\\frac{7}{5}\\\frac{11}{5}\end{bmatrix}$

$\Rightarrow[\frac{9}{5},0,\frac{7}{5},\frac{11}{5},0,0,0]^T$ is a BFS. while the basis is $\{x_1,x_3,x_4\}$