线性代数 02班 12112627 李乐平 Week9.

3.4.

15.

$q_1 = \frac{1}{\sqrt{9}}\begin{bmatrix}1\\2\\-2\end{bmatrix} = \begin{bmatrix}\frac{1}{3}\\\frac{2}{3}\\-\frac{2}{3}\end{bmatrix}$.

$q_2' = \begin{bmatrix}-1\\-1\\4\end{bmatrix} - \frac{-3}{1}\begin{bmatrix}\frac{1}{3}\\\frac{2}{3}\\-\frac{2}{3}\end{bmatrix}$

$= \begin{bmatrix}2\\1\\2\end{bmatrix} \quad q_2 = \begin{bmatrix}\frac{2}{3}\\\frac{1}{3}\\\frac{2}{3}\end{bmatrix}$

$q_3 = q_1 \times q_2 = \begin{bmatrix}\frac{2}{3}\\-\frac{2}{3}\\-\frac{1}{3}\end{bmatrix} \subset N(A^T)$

$\hat{x} = (A^TA)^{-1}A^Tb = \begin{bmatrix}1\\2\end{bmatrix}$.

17. $q_1 = \begin{bmatrix}1\\2\\2\end{bmatrix}$ $q_2 = \begin{bmatrix}0\\1\\-1\end{bmatrix}$

$e_1 = \begin{bmatrix}\frac{1}{3}\\\frac{2}{3}\\\frac{2}{3}\end{bmatrix}$ $e_2 = \begin{bmatrix}0\\\frac{\sqrt{2}}{2}\\-\frac{\sqrt{2}}{2}\end{bmatrix}$

$\therefore Q = \begin{bmatrix}\frac{1}{3} & 0\\\frac{2}{3} & \frac{\sqrt{2}}{2}\\\frac{2}{3} & -\frac{\sqrt{2}}{2}\end{bmatrix}$

$R = \begin{bmatrix}3 & 3\\0 & \sqrt{2}\end{bmatrix}$

$R\hat{x} = Q^Tb = \begin{bmatrix}\frac{5}{3}\\0\end{bmatrix}$.

$\hat{x} = \frac{1}{3\sqrt{2}}\begin{bmatrix}\sqrt{2} & -3\\0 & 3\end{bmatrix}\begin{bmatrix}\frac{5}{3}\\0\end{bmatrix}$

$= \begin{bmatrix}\frac{5}{9}\\0\end{bmatrix}$.

27. $q_1 = \begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$

$q_2 = a_2 - \frac{q_1^Ta_2}{\|q_1\|^2}q_1 = a_2 + \frac{1}{2}q_1 = \begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\\-1\\0\end{bmatrix}$

$q_3 = a_3 + \frac{2}{3}q_2 = \begin{bmatrix}\frac{1}{3}\\\frac{1}{3}\\\frac{1}{3}\\-1\end{bmatrix}$

$e_1 = \begin{bmatrix}\frac{\sqrt{2}}{2}\\-\frac{\sqrt{2}}{2}\\0\\0\end{bmatrix}$ $e_2 = \begin{bmatrix}\frac{\sqrt{6}}{6}\\\frac{\sqrt{6}}{6}\\-\frac{\sqrt{6}}{3}\\0\end{bmatrix}$

$e_3 = \begin{bmatrix}\frac{\sqrt{3}}{6}\\\frac{\sqrt{3}}{6}\\\frac{\sqrt{3}}{6}\\-\frac{\sqrt{3}}{2}\end{bmatrix}$ $\{e_1, e_2, e_3\}$ 即为所求.

30.

~~Q^TQ~~

$A^TA = R^TR$ = 下三角乘上三角

$\alpha = [1\ -1\ 0\ 0]^T$.

$\beta = [\frac{1}{2}\ \frac{1}{2}\ -1\ 0]^T$

$\gamma = [\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}\ -1]^T$.

显然 α, β, γ 正交. 且

$\|\alpha\|^2 = 2, \|\beta\|^2 = \frac{3}{2}, \|\gamma\|^2 = \frac{4}{3}$

4.2.

2. $\det(\frac{1}{2}A) = -\frac{1}{8}$

$\det(-A) = 1$

$\det(A^2) = 1$

$\det(A^{-1}) = -1$

4. $\begin{vmatrix}1&2&-2&0\\2&3&-4&1\\-1&-2&0&2\\0&2&5&3\end{vmatrix} = \begin{vmatrix}1&2&-2&0\\0&-1&0&1\\0&0&-2&2\\0&2&5&3\end{vmatrix}$

$= \begin{vmatrix}-1&0&1\\0&-2&2\\2&5&3\end{vmatrix} = \begin{vmatrix}-1&0&1\\0&-2&2\\0&5&5\end{vmatrix}$

$= -\begin{vmatrix}-2&2\\5&5\end{vmatrix} = 20$.

$\begin{vmatrix}2&-1&0&0\\-1&2&-1&0\\0&-1&2&-1\\0&0&-1&-2\end{vmatrix} = \begin{vmatrix}2&-1&0&0\\0&\frac{3}{2}&-1&0\\0&-1&2&-1\\0&0&-1&-2\end{vmatrix}$

$= \begin{vmatrix}2&-1&0&0\\0&\frac{3}{2}&-1&0\\0&0&\frac{4}{3}&-1\\0&0&0&-\frac{11}{4}\end{vmatrix} = -11$

交换3,4行得

$\begin{vmatrix}2&-1&0&0\\0&\frac{3}{2}&-1&0\\0&0&-1&-2\\0&0&\frac{4}{3}&-1\end{vmatrix} = \begin{vmatrix}2&-1&0&0\\0&\frac{3}{2}&-1&0\\0&0&-1&-2\\0&0&0&-\frac{11}{3}\end{vmatrix}$

$= 11$.

6. 需要 $\lfloor \frac{n}{2} \rfloor$ 次.

$\det P_n = (-1)^{n+1}$.

7. (a). $|A| = 0$.

(b) $|U| = 16$

(c) $|U^T| = 16$

(d) $|U^{-1}| = -16$

(e) $|M| = 16$.

10. $\because |Q^TQ| = |Q^T||Q| = |Q|^2 = |I| = 1$

$\therefore |Q| = \pm 1$.

是一个正方体

12. $\begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix} = \begin{vmatrix}1&a&a^2\\0&b-a&b^2-a^2\\0&c-a&c^2-a^2\end{vmatrix}$

$= \begin{vmatrix}1&a&a^2\\0&b-a&b^2-a^2\\0&0&(c-b)(c-a)\end{vmatrix} = (b-a)(c-a)(c-b)$

13(a) 这是因为每一行均乘以了-1

$\because \det K = \det K^T = \det(-K) = -\det K$

$\therefore \det K = 0$.

(b) 如 $\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{bmatrix}$.

14. (a) 错误. 如 $A = \begin{bmatrix}1&1\\1&1\end{bmatrix}$ $B = \begin{bmatrix}2&1\\1&1\end{bmatrix}$

$\det B = 1$ $\det A = 0$.

(b) 错误. 如 $A = \begin{bmatrix}0&1\\1&0\end{bmatrix}$ $|A| = -1 \neq 1$

(c) 错误. 如 $A = \begin{bmatrix}1&0\\0&1\end{bmatrix}$ $B = -\begin{bmatrix}1&0\\0&0\end{bmatrix}$.

(d) 正确. $\because |AB| = |A||B| = 0$.

(e) 错误. 如 $A = \begin{bmatrix}1&0\\0&1\end{bmatrix}$ $B = \begin{bmatrix}1&0\\1&1\end{bmatrix}$.

$\det(AB - BA) = \left|\begin{bmatrix}0&1\\-1&0\end{bmatrix}\right| = 1 \neq 0$

15.

∵A的行向量之和为0.

∴A的行向量线性相关.

∴A不可逆.

∴$\det A = 0$.

若行向量求和各分量为1.

则 $A-I$的行向量之和为0.

∴ $A-I$不可逆.

$\det(A-I)=0$

此时 $\det A$ 不一定为1.

如 $A=\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. $\det A=0$

$\det(A-I)=0$.

19. 论证过程中,行列式求错了.

∵$CD = -DC$.

∴ $|CD| = |-DC|$.

∴ $|C||D| = |-D||C| = (-1)^n|C||D|$

若 n为偶数,则论证错误

C, D可能均可逆.

23.
$$\begin{vmatrix} 1&2&3&0 \\ 2&6&6&1 \\ -1&0&0&3 \\ 0&2&0&7 \end{vmatrix} = \begin{vmatrix} 1&2&3&0 \\ 0&2&0&1 \\ 0&2&3&3 \\ 0&2&0&7 \end{vmatrix}$$
$$= \begin{vmatrix} 1&2&3&0 \\ 0&2&0&1 \\ 0&0&3&2 \\ 0&0&0&6 \end{vmatrix} = 36.$$

$$\begin{vmatrix} 2&1&1&1 \\ 1&2&1&1 \\ 1&1&2&1 \\ 1&1&1&2 \end{vmatrix} = \begin{vmatrix} 1&1&1&2 \\ 1&1&2&1 \\ 1&2&1&1 \\ 2&1&1&1 \end{vmatrix}$$
$$= \begin{vmatrix} 1&1&1&2 \\ 0&-1&1&0 \\ 0&1&0&-1 \\ 0&-1&-1&-3 \end{vmatrix} = \begin{vmatrix} 1&1&1&2 \\ 0&-1&1&0 \\ 0&0&1&-1 \\ 0&0&-2&-3 \end{vmatrix}$$
$$= \begin{vmatrix} 1&1&1&2 \\ 0&-1&1&0 \\ 0&0&1&-1 \\ 0&0&0&-5 \end{vmatrix} = 5$$

29.

若A, A^T非方阵.

则无$|A|$和$|A^T|$的定义.

35.
$$\begin{vmatrix} 1+a&b&c&d \\ a&1+b&c&d \\ a&b&1+c&d \\ a&b&c&1+d \end{vmatrix}$$
$$= \begin{vmatrix} 1+a&b&c&d \\ a&1+b&c&d \\ 0&-1&1&0 \\ 0&0&-1&1 \end{vmatrix}$$
$$= \begin{vmatrix} 1+a&b&c&d \\ -1&1&0&0 \\ 0&-1&1&0 \\ 0&0&-1&1 \end{vmatrix}$$
$$= \begin{vmatrix} 1+a&b&c+d&d \\ -1&1&0&0 \\ 0&-1&1&0 \\ 0&0&0&1 \end{vmatrix} = \begin{vmatrix} 1+a&b+c+d&c+d&d \\ -1&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{vmatrix}$$
$$= \begin{vmatrix} 1+a+b+c+d&b+c+d&c+d&d \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{vmatrix}$$
$$= 1+a+b+c+d$$