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6.2.36.

(1). $z = a_1x_1+\cdots+a_px_p$, $|x_i|=1$. $x_i x_j = \begin{cases}1, & i=j \\ 0, & i\neq j\end{cases}$.

$z^TAz = (a_1x_1+\cdots+a_px_p)^T A(a_1x_1+\cdots+a_px_p)$.

$= (a_1x_1+\cdots+a_px_p)^T(a_1\lambda_1x_1+\cdots+a_p\lambda_px_p)$

$= \lambda_1a_1^2+\cdots+\lambda_p a_p^2 \geq 0$

同理, $z^TAz = \mu_1b_1^2+\cdots+\mu_q b_q^2 \leq 0$.

(2) $z^TAz=0$.

而 $x^TAx >0$. $(\forall x\neq 0)$.

$\therefore z=0 \Rightarrow \alpha_1\cdots\alpha_p=0$.

$b_1=\cdots=b_q=0$.

$\therefore x_1\cdots x_p, b_1\cdots b_q$ 线性无关.

$\therefore p+q \leq n$.

(3) 同理 $n-p+n-q \leq n$

$\Rightarrow p+q \geq n$.

37. $\mathrm{rank}(C^TAC) \leq \mathrm{rank}(A)$.

$\mathrm{rank}(C^TAC) \geq \mathrm{rank}((C^T)^{-1}C^TACC^{-1})$

$= \mathrm{rank}(A)$.

$\therefore \mathrm{rank}(C^TAC)=\mathrm{rank}(A)$

38. 有 $\frac{n}{2}$ 个正特征值和 $\frac{n}{2}$ 个负特征值

6.3.

1. $A^TA = \begin{bmatrix}1 & 2\\4 & 8\end{bmatrix}\begin{bmatrix}1 & 4\\2 & 8\end{bmatrix} = \begin{bmatrix}5 & 20\\20 & 80\end{bmatrix}$

$\sigma_1^2 = 85$. $v_1 = \begin{bmatrix}\frac{1}{\sqrt{17}}\\ \frac{4}{\sqrt{17}}\end{bmatrix}$ $V = \begin{bmatrix}\frac{1}{\sqrt{17}} & \frac{4}{\sqrt{17}}\\ \frac{4}{\sqrt{17}} & -\frac{1}{\sqrt{17}}\end{bmatrix}$

$\sigma_2 = 0$. $v_2 = \begin{bmatrix}\frac{4}{\sqrt{17}}\\ -\frac{1}{\sqrt{17}}\end{bmatrix}$.

2. ~~(a) $v_1 = \frac{Au_1}{\sigma_1} =$~~

(a) $AA^T = \begin{bmatrix}1 & 4\\2 & 8\end{bmatrix}\begin{bmatrix}1 & 2\\4 & 8\end{bmatrix}$

$= \begin{bmatrix}17 & 34\\34 & 68\end{bmatrix}$.

$\sigma_1^2 = 85$, $u_1 = \begin{bmatrix}\frac{1}{\sqrt5}\\ \frac{2}{\sqrt5}\end{bmatrix}$. $U = \begin{bmatrix}\frac{1}{\sqrt5} & \frac{2}{\sqrt5}\\ \frac{2}{\sqrt5} & -\frac{1}{\sqrt5}\end{bmatrix}$.

$\sigma_2^2 = 0$. $u_2 = \begin{bmatrix}\frac{2}{\sqrt5}\\ -\frac{1}{\sqrt5}\end{bmatrix}$.

$U\Sigma V^T = \begin{bmatrix}\frac{1}{\sqrt5} & \frac{2}{\sqrt5}\\ \frac{2}{\sqrt5} & -\frac{1}{\sqrt5}\end{bmatrix}\begin{bmatrix}\sqrt{85} & \\ & 0\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{17}} & \frac{4}{\sqrt{17}}\\ \frac{4}{\sqrt{17}} & -\frac{1}{\sqrt{17}}\end{bmatrix}$.

$= \begin{bmatrix}\sqrt{17} & 0\\ 2\sqrt{17} & 0\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{17}} & \frac{4}{\sqrt{17}}\\ \frac{4}{\sqrt{17}} & -\frac{1}{\sqrt{17}}\end{bmatrix}$.

$= \begin{bmatrix}1 & 4\\ 2 & 8\end{bmatrix} = A$.

(b) $C(A) = \mathrm{Span}\{u_1\}$. $N(A)=\mathrm{Span}\{u_2\}$.

$C(A^T)=\mathrm{Span}\{v_1\}$. $N(A^T)=\mathrm{Span}\{v_2\}$.

5. $A^TA = \begin{bmatrix}1 & 1 & 0\\1 & 2 & 1\\0 & 1 & 1\end{bmatrix}$. $AA^T = \begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$.

$\sigma_1^2 = 1$. $\sigma_2^2 = 3$.

$v_1 = \begin{bmatrix}\frac{1}{\sqrt2}\\ 0\\ -\frac{1}{\sqrt2}\end{bmatrix}$ $v_2 = \begin{bmatrix}\frac{1}{\sqrt6}\\ \frac{2}{\sqrt6}\\ \frac{1}{\sqrt6}\end{bmatrix}$ $v_3 = \begin{bmatrix}\frac{1}{\sqrt3}\\ -\frac{1}{\sqrt3}\\ \frac{1}{\sqrt3}\end{bmatrix}$.

$u_1 = \begin{bmatrix}\frac{1}{\sqrt2}\\ -\frac{1}{\sqrt2}\end{bmatrix}$. $u_2 = \begin{bmatrix}\frac{1}{\sqrt2}\\ \frac{1}{\sqrt2}\end{bmatrix}$.

$A = \begin{bmatrix}\frac{1}{\sqrt2} & \frac{1}{\sqrt2}\\ -\frac{1}{\sqrt2} & \frac{1}{\sqrt2}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & \sqrt3\\ 0 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt2} & 0 & -\frac{1}{\sqrt2}\\ \frac{1}{\sqrt6} & \frac{2}{\sqrt6} & \frac{1}{\sqrt6}\\ \frac{1}{\sqrt3} & -\frac{1}{\sqrt3} & \frac{1}{\sqrt3}\end{bmatrix}$.

6. $A = UV^T$.

10.

$U = [u_1 \; u_2]$

$\Lambda = \begin{bmatrix} 3 & \\ & -2 \end{bmatrix}$

$V^T = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}$.

12(a) V ~~[illegible]~~ Σ [illegible] 2Σ

(b) $A^T = V \cancel{\Lambda} U^T$. Σ^T

$A^{-1} = V\Sigma^{-1}U^T$.

13. 因为 $UV^T \neq I$.

17. $A^TA = \frac{1}{10}\begin{bmatrix} 10 & 0 \\ 6 & 8 \end{bmatrix}\begin{bmatrix} 10 & 6 \\ 0 & 8 \end{bmatrix}$

$= \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$.

$\lambda_1 = 4 \quad v_1 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$\lambda_2 = 16. \quad v_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$S = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 2 & \\ & 4 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$.

$= \begin{bmatrix} \sqrt{2} & 2\sqrt{2} \\ -\sqrt{2} & 2\sqrt{2} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

$= \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$.

$Q = AS^{-1} = \frac{1}{\sqrt{10}}\begin{bmatrix} 10 & 6 \\ 0 & 8 \end{bmatrix}\begin{bmatrix} \frac{3}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{3}{8} \end{bmatrix}$

$= \frac{1}{\sqrt{10}}\begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}$.

$A = QS$.