线性代数 12112627 李乐轩.

Week 15 补充题.

1. $C(A) = \text{Span}\left\{\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\\ \frac{1}{2}\\ \frac{1}{2}\end{bmatrix}, \begin{bmatrix}\frac{1}{2}\\ -\frac{1}{2}\\ -\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}\right\}.$

$N(A) = \text{Span}\left\{\begin{bmatrix}\frac{1}{3}\\ -\frac{2}{3}\\ \frac{2}{3}\end{bmatrix}\right\}.$

$C(A^T) = \text{Span}\left\{\begin{bmatrix}\frac{2}{3}\\ \frac{2}{3}\\ \frac{1}{3}\end{bmatrix}, \begin{bmatrix}-\frac{2}{3}\\ \frac{1}{3}\\ \frac{2}{3}\end{bmatrix}\right\}.$

$N(A^T) = \text{Span}\left\{\begin{bmatrix}\frac{1}{2}\\ -\frac{1}{2}\\ \frac{1}{2}\\ -\frac{1}{2}\end{bmatrix}, \begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\\ -\frac{1}{2}\\ -\frac{1}{2}\end{bmatrix}\right\}.$

2. (a). $\because A^TAv = \lambda v.$

但 $\text{rank} A = r.$

$\therefore v \subset C(A^T).$

又 v 正交. $\therefore \{v_1, \cdots v_r\}$ 是 $C(A^T)$ 的标准正交基.

(b). $A = U\Sigma V^T.$

$A^T = V\Sigma^T U^T.$

$AA^T = U\Sigma\Sigma^T U^T.$

$A^TA = V\Sigma^T\Sigma V^T.$

所以 A^TA 与 AA^T 特征值相等.

$A^TA \sim AA^T.$

3. $P^2 = AA^+AA^+$

$= (AA^+A)A^+$

$= AA^+ = P.$

$A^+ = \cancel{A^T(AA^T)^{-1}}\ V\Sigma^+U^T.$

$P = AA^+ = U\Sigma\Sigma^+U^T.$

($\cancel{AA^+}$) $P^T = U(\Sigma\Sigma^+)^TU^T$

$= U\Sigma\Sigma^+U^T = P.$