(1) $\lambda^T = \lambda$ ...

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week14 补充题

1. (1) f对应矩阵为

$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ 特征值为 5, -1, -1.

为双叶双曲面.

(2). 因示为双叶双曲面

故有 1个正特征值.

2. 曲面方程对应矩阵

$[x\ y\ z]\begin{bmatrix} 1 & b & 1 \\ b & a & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

因可化为椭圆柱面方程.

故其有 2正特征值, 1零特征值. 特征值为 1, 4, 0.

$\begin{bmatrix} 1 & b & 1 \\ b & a & 1 \\ 1 & 1 & 1 \end{bmatrix}$ 的特征多项式为

$-\lambda^3 + (2+a)\lambda^2 + (1-2a+b^2)\lambda - (b-1)^2 = 0.$

$\because$ 0为一解 $\therefore b = 1.$

$-\lambda^3 + (2+a)\lambda^2 + (2-2a)\lambda = 0.$

$= -\lambda(\lambda-1)(\lambda-4)$

$a = 3.$

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. $\lambda = 0 \Rightarrow v_3 = \begin{bmatrix} \frac{1}{\sqrt2} \\ 0 \\ -\frac{1}{\sqrt2} \end{bmatrix}$. $\lambda = 1 \Rightarrow v_2 = \begin{bmatrix} \frac{1}{\sqrt3} \\ -\frac{1}{\sqrt3} \\ \frac{1}{\sqrt3} \end{bmatrix}$.

$\lambda = 4 \Rightarrow v_1 = \begin{bmatrix} \frac{1}{\sqrt6} \\ \frac{2}{\sqrt6} \\ \frac{1}{\sqrt6} \end{bmatrix}$.

$\therefore P = \begin{bmatrix} \frac{1}{\sqrt6} & \frac{1}{\sqrt3} & \frac{1}{\sqrt2} \\ \frac{2}{\sqrt6} & -\frac{1}{\sqrt3} & 0 \\ \frac{1}{\sqrt6} & \frac{1}{\sqrt3} & -\frac{1}{\sqrt2} \end{bmatrix}$.

3. 由题意知.

(1). A的三个特征值为 1, 1, 0.

且 $\lambda=0$ 时对应特征向量 $\begin{bmatrix} \frac{1}{\sqrt2} \\ 0 \\ \frac{1}{\sqrt2} \end{bmatrix}$.

设 $Q = \begin{bmatrix} a & b & \frac{1}{\sqrt2} \\ c & d & 0 \\ e & f & \frac{1}{\sqrt2} \end{bmatrix}$.

由 $A = Q\begin{bmatrix} 1 & & \\ & 1 & \\ & & 0 \end{bmatrix}Q^T$.

$= \begin{bmatrix} a^2+b^2 & ac+bd & ae+bf \\ ac+bd & c^2+d^2 & ce+df \\ ae+bf & ce+df & e^2+f^2 \end{bmatrix}$.

$QQ^T = Q^TQ = I.$

$\begin{bmatrix} \frac{1}{\sqrt2} \\ 0 \\ \frac{1}{\sqrt2} \end{bmatrix} \in N(A).$

$\Rightarrow a = -e \quad b = -f$

$a^2+c^2+e^2 = 1 \quad b^2+d^2+f^2 = 1$

$ac+bd = 0$

$ae+bf = -\frac{1}{2}$

$2a^2+c^2 = 1 \quad 2b^2+d^2 = 1$

$c^2+d^2 = 1 \quad a^2+b^2 = \frac{1}{2}.$

$\therefore A = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$

(2) $A+I = \begin{bmatrix} \frac{3}{2} & & -\frac{1}{2} \\ & 2 & \\ -\frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$.

因其特征值 2, 2, 1 均为正.

所以A+I 正定.

4(1) 二次型对应矩阵 $A = \begin{bmatrix} 1-a & 1+a & \\ 1+a & 1-a & \\ & & 2 \end{bmatrix}$

$\because \operatorname{rank} A = 2.$ $\therefore a = 0.$

(2) $A = \begin{bmatrix} 1 & 1 & \\ 1 & 1 & \\ & & 2 \end{bmatrix}$ 特征值为 2, 2, 0

$\lambda = 0 \Rightarrow v_3 = \begin{bmatrix} \frac{1}{\sqrt2} \\ -\frac{1}{\sqrt2} \\ 0 \end{bmatrix}$

$\lambda = 2 \Rightarrow v_1 = \begin{bmatrix} \frac{1}{\sqrt2} \\ \frac{1}{\sqrt2} \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

$\therefore Q = \begin{bmatrix} \frac{1}{\sqrt2} & 0 & \frac{1}{\sqrt2} \\ \frac{1}{\sqrt2} & 0 & -\frac{1}{\sqrt2} \\ 0 & 1 & 0 \end{bmatrix}$ $y^TQ^TAQy = 2y_1^2 + 2y_2^2.$

(3) ~~A+I 的特征值为~~

$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + 2x_3^2 + 2x_1x_2 = 0$

$= (x_1+x_2)^2 + 2x_3^2 = 0.$

$\therefore$ 解为 $\begin{cases} x_1 = -x_2. \\ x_3 = 0 \end{cases} \Rightarrow \lambda\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

5. (1). $A = \begin{bmatrix} a & 0 & 1 \\ 0 & a & -1 \\ 1 & -1 & a-1 \end{bmatrix}$.

$\lambda_1 = a$

$\lambda_2 = a-2.$

$\lambda_3 = a+1.$

(2) 因A的规范形为 $y_1^2 + y_2^2$.

即特征值两正一零.

$\therefore a = 2.$