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Week 8 补充题

1. (a) 这是因为 $b \notin C(A)$

(b) $\hat{x}=(A^TA)^{-1}A^Tb$

$$=\begin{bmatrix}4&-5&1\\-5&7&-2\\1&-2&9\end{bmatrix}^{-1}\begin{bmatrix}-1&1&1&1\\1&-2&-1&-1\\0&1&2&-2\end{bmatrix}\begin{bmatrix}2\\-2\\-1\\3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{59}{24}&\frac{43}{24}&\frac{1}{8}\\\frac{43}{24}&\frac{35}{24}&\frac{1}{8}\\\frac{1}{8}&\frac{1}{8}&\frac{1}{8}\end{bmatrix}\begin{bmatrix}-2\\4\\-10\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$$

(c) $b_c=A\hat{x}=\begin{bmatrix}0\\-2\\-2\\2\end{bmatrix}$

$b_l=b-b_c=\begin{bmatrix}2\\0\\1\\1\end{bmatrix}$.

2. 证明：对于矩阵 A 和任意 $b_{m\times1}$.

若 $Ax=b$ 有解. 则 $A^TAx=A^Tb$.

若 $Ax=b$ 无解. 则 $\exists\hat{x}$. s.t $b-A\hat{x}\perp C(A)$.

$\therefore A^T(b-A\hat{x})=0 \Rightarrow A^TA\hat{x}=A^Tb$.

$\because A^TAx \subset C(A^TA)$. $A^Tb\subset C(A^T)$.

$\forall b$, $A^TAx=A^Tb$ 均有解.

$\therefore C(A^T)\subset C(A^TA)$.

而 $C(A^TA)\subset C(A^T)$

$\therefore C(A^TA)=C(A^T)$.

3. 即求两个正交单位向量属于

$R(\mathrm{Span}\{\alpha_1,\alpha_2\})^{\perp}$.

容易给出两个正交向量.

$(2,2,3,1)$, $(0,1,-1,1)$

化为单位向量 $(\frac{\sqrt2}{3},\frac{\sqrt2}{3},\frac{\sqrt2}{2},\frac{\sqrt2}{6})$

$(0,\frac{\sqrt3}{3},-\frac{\sqrt3}{3},\frac{\sqrt3}{3})$.

$\therefore$ 所求 Q 可为 $\begin{bmatrix}\frac{1}{3}&-\frac{2}{\sqrt6}&\frac{\sqrt2}{3}&0\\-\frac{2}{3}&0&\frac{\sqrt2}{3}&\frac{\sqrt3}{3}\\0&\frac{1}{\sqrt6}&\frac{\sqrt2}{3}&-\frac{\sqrt3}{3}\\\frac{2}{3}&\frac{1}{\sqrt6}&\frac{\sqrt2}{6}&\frac{\sqrt3}{3}\end{bmatrix}$.

4. $q_1=[1\ 1\ 2\ 2]^T$

$q_2=v_2-\frac{v_2^Tq_1}{\|q_1\|^2}q_1=[\frac{1}{10}\ \frac{11}{10}\ -\frac{8}{10}\ \frac{2}{10}]^T$.

给出 $q_3=[-2,0,0,1]^T$.

则 $q_4=[1,-\frac{5}{3},-\frac{5}{3},2]^T$.

$\therefore Q=\begin{bmatrix}1&\frac{1}{10}&-2&1\\1&\frac{11}{10}&0&-\frac{5}{3}\\2&-\frac{4}{5}&0&-\frac{5}{3}\\2&\frac{1}{5}&1&2\end{bmatrix}$.