STA201 运筹与优化 Homework 5

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1.

	x1	x2	x3	x4	x5	x6	x7	x8	b
x6	1	-1	-2	3	-1	1	0	0	10
x7	1	0	-1	1	-2	0	1	0	17
x8	2*	-1	-6	7	-6	0	0	1	15
	-1	1	1	-1	0	M	M	M	0
	-1-4M	1+2M	1+9M	-1-11M	9M	0	0	0	-42M

	x1	x2	x3	x4	x5	x6	x7	x8	b
x6	0	-1/2	1*	-1/2	2	1	0	-1/2	5/2
x7	0	1/2	2	-5/2	1	0	1	-1/2	19/2
x1	1	-1/2	-3	7/2	-3	0	0	1/2	15/2
	0	1/2	-3M-2	3M+5/2	-3M-3	0	0	2M+1/2	-12M+15/2

	x1	x2	x3	x4	x5	x6	x7		b
x3	0	-1/2	1	-1/2	2	1	0		5/2
x7	0	3/2*	0	-3/2	-3	-2	1		9/2
x1	1	-2	0	2	3	3	0		15
	0	-1/2-3M/2	0	3M/2+3/2	3M+1	3M+2	0		-9M/2+25/2

	x1	x2	x3	x4	x5		x7		b
x3	0	0	1	-1	1		1/3		4
x2	0	1	0	-1	-2		2/3		3
x1	1	0	0	0	-1		4/3		21
	0	0	0	1	0		M+1/3		14

Feasible region unbounded because the elements in 4th column are all non-positive.

Optimal BFS: $[21\ 3\ 4\ 0\ 0]^T$, not unique

Maximum: 14

Generic Optimal Feasible Solution:

$$(21+t,3+2t,4-t,0,t),t\in[0,4]$$

Solution non-basic if $t\neq 0$ and $t\neq 4$.

2.

	x1	x2	x3	x4	x5	x6		b	b./x1
x4	1	2	0	1	0	0		5	5
x5	1*	1	-1	0	1	0		2	2*
x6	7	3	-5	0	0	1		20	20/7
	-3	-1	0	0	0	0		0	

	x1	x2	x3	x4	x5	x6		b	b./x3
x4	0	1	1*	1	-1	0		3	3*
x1	1	1	-1	0	1	0		2	-2(Ignored)
x6	0	-4	2	0	-7	1		6	3
	0	2	-3	0	3	0		6	

	x1	x2	x3	x4	x5	x6		b	
x3	0	1	1	1	-1	0		3	
x1	1	2	0	1	0	0		5	
x6	0	-6	0	-2	-5	1		0	
	0	5	0	3	0	0		15	

Optimal BFS: $[5\ 0\ 3\ 0\ 0\ 0]^T$, degenerate because basic variable $x_6 = 0$.

Maximum: 15

Generic Optimal Feasible Solution:

$$(5,0,3+t,0,t,5t), t \geq 0$$

Solution non-basic if $t \neq 0$.

3.

Phase I:

	x1	x2	x3	x4	x5	x6	x7		b
x5	1	1	1	0	0	1	0		10
x6	1*	-1	0	-1	0	0	1		1
x7	2	3	1	0	1	0	0		20
	0	0	0	0	1	1	1		0
	-4	-3	-2	1	0	0	0		-31

	x1	x2	x3	x4	x5	x6	x7		b
x5	0	2	1	1	0	1	-1		9
x1	1	-1	0	-1	0	0	1		1
x7	0	5*	1	2	1	0	-2		18
	0	-7	-2	-3	0	0	4		-27

	x1	x2	x3	x4	x5	x6	x7		b
x5	0	0	3/5*	1/5	-2/5	1	-1/5		9/5
x1	1	0	1/5	-3/5	1/5	0	3/5		23/5
x2	0	1	1/5	2/5	1/5	0	-2/5		18/5
	0	0	-3/5	-1/5	7/5	0	6/5		-9/5

	x1	x2	x3	x4	x5	x6	x7		b
x3	0	0	1	1/3	-2/3	5/3	-1/3		3
x1	1	0	0	-2/3	1/3	-1/3	2/3		4
x2	0	1	0	1/3	1/3	0	-1/3		3
	0	0	0	0	1	1	1		0

Phase II:

	x1	x2	x3	x4	x5				b
x3	0	0	1	1/3*	-2/3				3
x1	1	0	0	-2/3	1/3				4
x2	0	1	0	1/3	1/3				3
	-4	-5	3	0	0				0
	0	0	0	-2	5				22

	x1	x2	x3	x4	x5				b
x4	0	0	3	1	-2				9
x1	1	0	2	0	-1				10
x2	0	1	-1	0	1				0
	0	0	6	0	1				40

Optimal BFS: $[10\ 0\ 0\ 9\ 0]^T$

Maximum: 40

4.

Phase I:

	x1	x2	x3	x4		b
	-1	2	-4	1		1
	1	6	-5	2		19
	1	-4	3	-1		0

↓ Paper operation

	x1	x2	x3	x4		b
	-1	2	-4	1		1
	3	2	3	0		17
	0	-2	-1	-1		1

Initial tableau for phase I:

	x1	x2	x3		y1	b
y1	3	2*	3		1	17
	0	0	0		1	0
	-3	-2	-3		0	-17

Final tableau for phase I:

	x1	x2	x3		y1	b
x2	3/2	1	3/2		1/2	17/2
	0	0	0		1	0

Phase II:

	x1	x2	x3			b
?	3/2	1*	3/2			17/2
	0	-2	-1			1

	x1	x2	x3			b
x2	3/2	1	3/2			17/2
	3	0	2			18

Maximum: 18

Optimal Feasible Solution: $[0\ 17/2\ 0\ -16]^T$

5.

(a) At direction $\overrightarrow{x_6}$, the feasible region is unbounded because the elements y in 6^{th} column are all non-positive.

(b) The optimal solution is not unique because the number of 0 in the last row is greater than 3. By giving out the generic optimal feasible solution

$$(t,t,6-2t,0,0,0,0,2-3t), t \in [0,\frac{2}{3}],$$

we can easily get another optimal feasible solution

$$(\frac{1}{2},\frac{1}{2},5,0,0,0,0,\frac{1}{2}).$$