线性代数 0231班 12112627 李乐平

Week 10.

4.3.

5.

$$F_n=\begin{vmatrix} 1 & -1 & 0 & & & 0 \\ 1 & 1 & -1 & 0 & & \\ 0 & 1 & 1 & -1 & 0 & \\ & 0 & \ddots & \ddots & \ddots & 0 \\ & & & \ddots & \ddots & -1 \\ & & & & 1 & 1 \end{vmatrix}_{n\times n}$$

$$=1\times\begin{vmatrix} 1 & -1 & & \\ 1 & 1 & -1 & \\ & \ddots & \ddots & -1 \\ & & 1 & 1 \end{vmatrix}_{(n-1)\times(n-1)} - (-1)\times\begin{vmatrix} 1 & -1 & & \\ 0 & 1 & -1 & \\ & \ddots & \ddots & -1 \\ & & 1 & 1 \end{vmatrix}_{(n-1)\times(n-1)}$$

$$=F_{n-1}+\begin{vmatrix} 1 & -1 & & \\ 1 & \ddots & \ddots & \\ & \ddots & \ddots & -1 \\ & & 1 & 1 \end{vmatrix}_{(n-2)\times(n-2)}+(-1)\times 0$$

$$=F_{n-1}+F_{n-2}.$$

11. ∵由提示.

① $m<n$. 如 $m=2$. $n=3$.

$$A=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},\ B=\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix}=\left|\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}\right|$$

$$=1\times\begin{vmatrix} 0 & 0 & 1 & 1 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{vmatrix}$$

$$=1\times(-1)\times\begin{vmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$=1\times(-1)\times(-1)\times\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}=-1.$$

$$\det\{AB\}=\left|\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right|=-1.=\det\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix}.$$

② $n<m$: 如 $n=1$. $m=2$.

$$A=\begin{bmatrix} 1 \\ 2 \end{bmatrix}\quad B=\begin{bmatrix} 3 & 1 \end{bmatrix}$$

$$\det\begin{bmatrix} 0 & A \\ -B & I \end{bmatrix}=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ -3 & -1 & 1 \end{bmatrix}=0.$$

$$\det AB=\begin{vmatrix} 3 & 1 \\ 6 & 2 \end{vmatrix}=0.$$

在②中 $\operatorname{rank}(AB)_{m\times m}\le n<m$.

∴ $AB$ 不可逆.

∴ $\det AB=0$.

16. (a) 考虑矩阵的第3.4.5行. 因其每行均至多有2个非零元. ∴ $\dim(\operatorname{Span}\{R_3,R_4,R_5\})\le 2$.

∴ $R_3,R_4,R_5$ 必线性相关.

∴ $|A|=0$.

(b) 不妨从第5行展开.

$$|A|=-a_{54}\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{25} \\ 0 & 0 & 0 & a_{35} \\ 0 & 0 & 0 & a_{45} \end{vmatrix}+a_{55}\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & a_{34} \\ 0 & 0 & 0 & a_{44} \end{vmatrix}$$

$$=-a_{54}\left(-a_{35}\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{vmatrix}\right)+a_{55}\left(-a_{34}\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{vmatrix}\right)$$

$=0+0$

$=0.$

23. 偶置换数 ~~[illegible]~~ 共有12个.

分别为 (1.2.3.4) (1.3.4.2) (1.4.2.3)

(2.1.4.3) (2.3.1.4) (2.4.3.1)

(3.1.2.4) (3.2.4.1) (3.4.1.2)

(4.1.3.2) (4.2.3.1) (4.3.2.1)

按顺序. 其对应的 $\det(I+P_{even})$ 分别为

16, 4, 4, 0, 4, 4, 4, 4, 0, 4, 4, 0.

故可能取值为 0. 4. 16.

28. (a) $C_1=0$. $C_2=-1$. $C_3=0$. $C_4=1$

(b) 观察可知 $C_n=-C_{n-2}$ ∴ $C_{10}=C_6=C_2=-1$.

29. 当 $j_i=i\pm 1$ 时 ($j_i>0$, $j_i\le n$)

且 $j_i\ne j_k$ ($i\ne k$). 可保证.

当 $n$ 为偶数时. 可视为第 $2i$ 列与第 $2i-1$ 列置换

∴ 当 $n=4.8\cdots$ 时 $\frac{n}{2}$ 为偶数. 置换 $\frac{n}{2}$ 次. 为偶置换.

$n=2.6\cdots$ 时. 为奇置换.

$n=1.3.\cdots$ 时. 无法得到满足条件的序列.

故为0.

于是结论显而易见。

34.(a)

$$\begin{vmatrix} A & B \\ O & D \end{vmatrix} = \left| \begin{bmatrix} A & B \\ O & D \end{bmatrix} \begin{bmatrix} I & -A^{-1}B \\ O & I \end{bmatrix} \right|$$

$$= \left| \begin{vmatrix} A & O \\ O & D \end{vmatrix} \right| = |A||D|.$$

(b)

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \left| \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} I & -A^{-1}B \\ O & I \end{bmatrix} \right|$$

$$= \begin{vmatrix} A & O \\ C & D-CA^{-1}B \end{vmatrix}$$

$$= |A||D-CA^{-1}B| = |AD-ACA^{-1}B| \neq |AD|-|CB|$$

如 $\begin{vmatrix} 1 & & 1 & \\ & 1 & & 1 \\ & 1 & 1 & \\ 1 & & & 1 \end{vmatrix} = 0 \neq 1\times1 - 1\times(-1) = 2.$

(c). 如 $\begin{vmatrix} 0&0&0&1 \\ 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \end{vmatrix} = -1 \neq |AD-CB| = 0.$

36. 无需证明.

37. $\begin{vmatrix} 1&2&3 \\ 4&5&6 \\ 7&8&9 \end{vmatrix}$

$= 45+84+96-105-48-72$

$= 0.$

$\because R_3 = 2R_2 - R_1.$

$\therefore A$ 不可逆.

4.4.

1. $|A| = 4\begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 20.$

$C_{11}=20.\ C_{12}=0\ C_{13}=0$

$C_{21}=-10\ C_{22}=5\ C_{23}=0$

$C_{31}=-12\ C_{32}=0\ C_{33}=4.$

$$AC^T = \begin{bmatrix} 1 & 3 & 3 \\ & 4 & \\ & & 5 \end{bmatrix} \begin{bmatrix} 20 & -10 & -12 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix} = |A|I.$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{3}{5} \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$$

2. A: $C_{11}=3.\ C_{12}=2.\ C_{13}=1.$

$C_{21}=2.\ C_{22}=4\ C_{23}=2.$

$C_{31}=1\ C_{32}=2\ C_{33}=3.$

$$\therefore A^{-1} = \frac{1}{|A|}C^T = \frac{1}{4}\begin{bmatrix} 3&2&1 \\ 2&4&2 \\ 1&2&3 \end{bmatrix}$$

4. (a) $\det M = x_j$

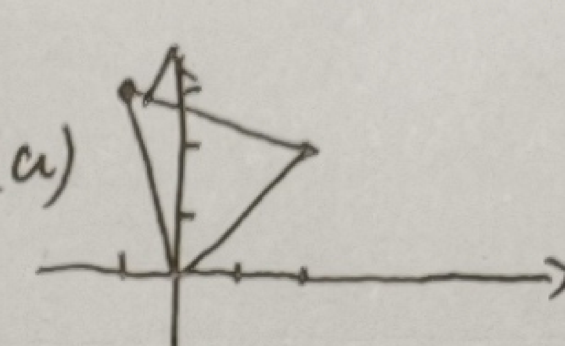
(b) $AM = [a_1 \cdots a_n]M$

$= [a_1 \cdots a_{j-1}\ Ax\ a_{j+1} \cdots a_n]$

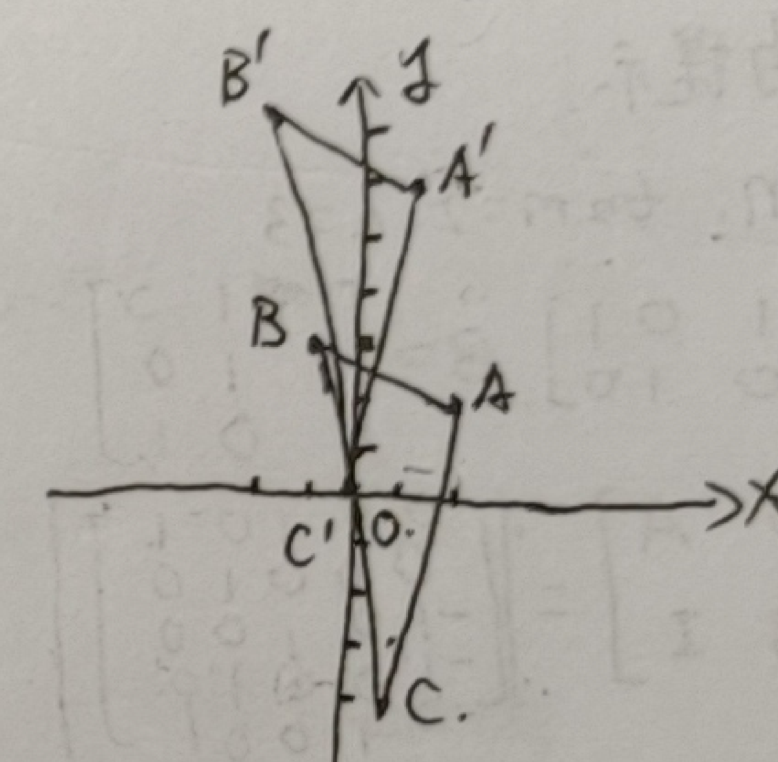
$= [a_1 \cdots a_{j-1}\ b\ a_{j+1} \cdots a_n] = B_j$

(c) $|A||M| = |B_j|.\ \therefore x_j = \frac{|B_j|}{|A|}.$

5. (a)



$A = \frac{1}{2}|(2,2)\times(1,3)| = 4.$




(b). $A = \begin{vmatrix} 1 & 6 \\ -2 & 7 \end{vmatrix} = 19.$

A'B'C'为ABC将C平移至原点得到的新三角形.

9. $P = \begin{bmatrix} 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&1&0&0&0 \\ 1&0&0&0&0 \end{bmatrix}$

(a) $P^2(1,2,3,4,5) = P(5,4,1,2,3) = (3,2,5,4,1)$

(b) $P^{-1} = \begin{bmatrix} 0&0&0&0&1 \\ 0&0&0&1&0 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \end{bmatrix}$

$P^{-1}(1,2,3,4,5) = (3,4,5,2,1).$

13.(b) $|A| = 4$

$X = \frac{1}{4}[D_1\ D_2\ D_3]^T$

$= \frac{1}{4}[-137\ 278\ -139]^T$

18(b) $|A| = 4$

$C_{11}=3\ C_{12}=2.\ C_{13}=1$

$C_{21}=2\ C_{22}=4\ C_{23}=2$

$C_{31}=1\ C_{32}=2\ C_{33}=3$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3&2&1 \\ 2&4&2 \\ 1&2&3 \end{bmatrix}.$$

(a) $|A| = 3.$

$C_{11}=3\ C_{12}=0\ C_{13}=0$

$C_{21}=-2\ C_{22}=1\ C_{23}=-4$

$C_{31}=0\ C_{32}=0\ C_{33}=3.$

$$A^{-1} = \begin{bmatrix} 1 & -\frac{2}{3} & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & -\frac{4}{3} & 1 \end{bmatrix}.$$