1 Triangular pdf

For each item we can calculate the average or any other percentile. The average cost of an item is $\frac{a+b+c}{3}$. The expected cost x with likelihood p is defined by the quantiles for each item, given by:

$$x_p = \begin{cases} a + \sqrt{(b-a)(c-a)p} & \text{for } 0 \leq p \leq F(c) \\ b - \sqrt{(b-a)(b-c)(1-p)} & \text{for } F(c) \leq p \leq 1 \end{cases}$$

Where $F(c)$ is the result of the cumulative distribution: $F(c) = \frac{(x-a)^2}{(b-a)(c-a)}$. Note that these formulas only work when: $a \leq c$ and $b \geq c$.

2 MIP Formulation

Decision Variables

- $x_i \in \{0, 1\}$: Binary variable indicating whether finance option i is selected.

Parameters

- C_i: Total cost of option i (excluding balloon payment)
- B_i: Balloon payment associated with option i
- M_i: Monthly payment for option i
- O_i: Ownership indicator for option i (1 if user will own the car at end, 0 otherwise)
- E_i: Excess mileage penalty indicator for option i (1 if excess mileage applies, 0 otherwise)
- $M_{\max}$: Maximum allowable monthly payment (user-defined budget)

Objective Function

$$\min \sum_i (C_i + B_i \cdot O_i) \cdot x_i \tag{1}$$

Constraints

$$\sum_i x_i = 1 \qquad \text{(Select exactly one finance option)} \tag{2}$$

$$\sum_i M_i \cdot x_i \leq M_{\max} \qquad \text{(Monthly budget constraint)} \tag{3}$$

$$\sum_i O_i \cdot x_i = 1 \qquad \text{(Must own the car at the end)} \tag{4}$$

$$\sum_i E_i \cdot x_i \leq 0 \qquad \text{(Avoid options with mileage penalties)} \tag{5}$$

$$x_i \in \{0, 1\} \quad \forall i \qquad \text{(Binary decision variables)} \tag{6}$$