A Formalization of Concept Blending: An Information-Theoretic Approach to Quantifying and Creating Emergent and Coherent Concept Blends

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Abstract

We develop a simple formal framework for capturing the notion a "concept blend," via modeling *blends* and *high-quality blends* of specified concept pairs as fuzzy sets over logical properties, where the fuzziness may be seen as originating from information-theoretic relations between entities and properties. Building on ideas from cognitive science and AI along with information theory, we provide pragmatic quantitative measures for assessing the degree of blending, the emergence of novel properties in a blend, and the conceptual coherence of a blend. Illustrative examples from culinary arts, music, material design and character design are examined in detail to illustrate how lower- and higher-quality blends and coherent blends can be distinguished. We also discuss heuristic algorithms, including a genetic algorithm + EDA approach, for searching the space of candidate blends according to single or multiple "blend quality" objective functions.

1 Introduction

Concept blending, introduced by Fauconnier and Turner [FT02] building on the work of many others, is a powerful and broad cognitive theory providing an explanation for how novel ideas emerge from the integration of distinct already-existing conceptual spaces. Blending two concepts leads to a new concept judiciously integrating properties of each of the two "parents" to form something new. Or, put differently: Selected elements from different domains are combined to produce a new "blended" mental space that exhibits emergent properties not present in the original inputs. For example, the concepts of “fire” and “water” might blend to yield notions of “steam” or “hot springs,” capturing features beyond those of the individual domains.

In AI and computational creativity, concept blending has been leveraged to model creative problem solving and generate innovative ideas in art, design, and music. These approaches often rely on qualitative assessments of similarity and novelty; however, to apply to computational creativity blending systematically and at scale, one needs a rigorous and pragmatic quantitative basis for such evaluations.

Inspired by and extending somewhat the ideas on blending presented in [GPG13a] [GPG13b], we present here a simple logical and quantitative formalism for concept blending, via:

- Representing entities (to be blended) as collections of properties with fuzzy membership values.
- Defining a *blend* as an entity whose properties are inherited from two or more source entities to a significant degree.
- Introducing the concept of *emergence* to capture cases where the blend exhibits properties more than either source—a hallmark of high-quality blends.
- Introducing a concept of *coherence* to capture the "integrated holistic aspect" that gives some concepts their elegance and power

We then explore this formalism in the context of a number of "thought experiment" examples, and also discuss how one might use heuristic search methods like GAs or EDAs to search for high quality blends embodying qualities of emergence and coherence. One motivation for these explorations is to pave the way for effective implementations of blending in the OpenCog Hyperon system [GBD$^+$23], currently under active development.

2 Fuzzy-Set Based Formalization of Blends

Let $P(x, E)$ denote that x is a property of entity E, with

$$\deg(P(x, E)) \in [0, 1]$$

representing the degree to which x is characteristic of E. We denote by $\mathrm{Props}(E)$ the set of properties of E, and by $|\,\mathrm{Props}(E)|$ the total number of these properties.

2.1 Blend Membership

An entity C is considered a *blend* of entities A and B if each property of C is present in at least one of the sources to a significant extent. The blend membership function is defined as:

$$\mu_{\text{blend}}(C; A, B) \;=\; \min\!\left(\frac{1}{|\,\mathrm{Props}(C)|}\sum_{x\in\mathrm{Props}(C)} \max\bigl(\deg(P(x, A)),\, \deg(P(x, B))\bigr),\; 1\right).$$

This formula computes the average, over all properties of C, of the maximal degree to which each property is inherited from either A or B, capped at 1.

2.2 High-Quality Blend Membership

Next, we define a high-quality blend as one in which emergent properties appear—that is, features in the blend C that are more pronounced than in either source. For each property x, we define its degree of emergence as:

$$\mu_{\text{emergence}}(x; A, B, C) \;=\; \deg(P(x, C)) \;-\; \max\bigl(\deg(P(x, A)),\, \deg(P(x, B))\bigr).$$

Then, the high-quality blend membership is given by:

$$\mu_{\text{hq-blend}}(C; A, B) =$$
$$\min\!\left(\frac{1}{|\,\mathrm{Props}(C)|}\sum_{x\in\mathrm{Props}(C)} \min\bigl(\deg(P(x, A)),\, \deg(P(x, B))\bigr)\cdot\mu_{\text{emergence}}(x; A, B, C),\; 1\right).$$

Here, the emergence measure is weighted by the extent to which the property is shared between A and B, ensuring that only commonly inherited features contribute to the assessment of emergent quality.

3 Conceptual Examples of Blends

We now give a few qualitative, conceptual examples illustrating how our formalism can be applied to evaluate blends in various domains. For some of these examples we go a little further and give some evocative numerical calculations. However these are all very much toy examples, and we must emphasize that actual applications of blending in any of these domains would involve many further particulars and nuances.

3.1 Example: Coffee Blends

- **Lower-Quality Blend:** A coffee blend that simply mixes beans from two origins without any effort to create synergy may result in a flavor profile that is merely an average of the two sources.
- **Higher-Quality Blend:** A carefully crafted blend, where beans are chosen to complement each other (e.g., balancing acidity with body), can produce an unexpectedly complex and harmonious flavor profile.

3.2 Example: Musical Fusion

- **Lower-Quality Blend:** A musical piece that haphazardly combines elements of classical and electronic music might result in a disjointed or confusing sound.
- **Higher-Quality Blend:** A well-produced fusion that seamlessly integrates classical orchestration with electronic beats may create a new genre, featuring unique rhythmic structures and innovative melodies.

3.3 Example: Material Design

- **Lower-Quality Blend:** A composite material formed by mechanically mixing two polymers without ensuring chemical compatibility may simply average the two materials' properties, possibly resulting in weak interfacial bonding.
- **Higher-Quality Blend:** When two materials are blended with attention to their chemical and physical interactions (for example, by using a compatibilizer), the resulting composite can exhibit enhanced

strength, durability, or novel conductive properties that are not present in either component alone.

3.4 Example: Bat Man

As a final illustrative example of concept blending, consider the fusion of a bat and a man to create "Bat Man." Here we may represent each source–bat and man–by fuzzy property vectors that capture features such as flight ability, echolocation, nocturnality, intelligence, physical strength, and symbolic appearance. A high-quality blend is achieved when the emergent properties (i.e., the degree to which a candidate's property exceeds the maximum of its sources) are positive. For instance, one high-quality interpretation yields a superhero persona: a human with heightened nocturnality and symbolic power (without literal bat abilities), while another high-quality version is a literal hybrid combining natural flight and partial echolocation with human cognition. In contrast, a low-quality blend might be a mere man in a bat costume, lacking any emergent enhancement beyond a superficial mix.

3.4.1 Detailed Analysis of "Bat Man"

We consider two source entities:

- **Bat (A)** with properties such as flight ability, echolocation, nocturnality, and a dark, mysterious appearance.
- **Man (B)** with properties such as intelligence, physical strength, rational reasoning, and cultural symbolism.

In our formalism, each entity is represented by a vector of fuzzy properties. Here we include both general and specific musical/morphological features:

- **Flight Ability (Fl)**: Ability to fly.
- **Echolocation (Ec)**: Use of sonar-like sensing.
- **Nocturnality (No)**: Activity during the night.
- **Intelligence (In)**: Cognitive capacity.
- **Physical Strength (PS)**: Muscular power.

- **Reasoning (Re)**: Rational decision-making.
- **Symbolism (Sy)**: The extent to which the appearance inspires cultural or psychological symbolism.
- **Iconography (Ic)**: Specific visual attributes (e.g., dark costume, bat motifs).

Assume the following example fuzzy membership values for the sources:
Bat (A):

$$\begin{aligned}
\deg(P(\mathrm{Fl}, A)) &= 0.9, \\
\deg(P(\mathrm{Ec}, A)) &= 0.9, \\
\deg(P(\mathrm{No}, A)) &= 0.8, \\
\deg(P(\mathrm{In}, A)) &= 0.3, \\
\deg(P(\mathrm{PS}, A)) &= 0.2, \\
\deg(P(\mathrm{Re}, A)) &= 0.3, \\
\deg(P(\mathrm{Sy}, A)) &= 0.8, \\
\deg(P(\mathrm{Ic}, A)) &= 0.7.
\end{aligned}$$

Man (B):

$$\begin{aligned}
\deg(P(\mathrm{Fl}, B)) &= 0.0, \\
\deg(P(\mathrm{Ec}, B)) &= 0.0, \\
\deg(P(\mathrm{No}, B)) &= 0.2, \\
\deg(P(\mathrm{In}, B)) &= 0.9, \\
\deg(P(\mathrm{PS}, B)) &= 0.8, \\
\deg(P(\mathrm{Re}, B)) &= 0.9, \\
\deg(P(\mathrm{Sy}, B)) &= 0.4, \\
\deg(P(\mathrm{Ic}, B)) &= 0.3.
\end{aligned}$$

A candidate blend C (the "bat man") is defined so that each property is derived (to a significant extent) from either A or B. In addition, a high-quality blend will show *emergence*–that is, for some properties, the blend's degree exceeds the maximum degree in the sources.

The emergent degree for a property x in C is defined as:

$$\mu_{\text{emergence}}(x; A, B, C) = \deg(P(x, C)) - \max\big(\deg(P(x, A)), \deg(P(x, B))\big).$$

Interpretation 1: The Superhero "Batman" In this interpretation, the blend yields a human who adopts bat-like symbolism and nocturnal behavior, yet does not physically transform. Suppose we set the candidate properties for *Batman (C1)* as:

$$\begin{aligned}
\deg(P(\mathrm{Fl}, C1)) &= 0.3 \quad \text{(via gadgets, not natural flight)},\\
\deg(P(\mathrm{Ec}, C1)) &= 0.0,\\
\deg(P(\mathrm{No}, C1)) &= 0.9,\\
\deg(P(\mathrm{In}, C1)) &= 0.9,\\
\deg(P(\mathrm{PS}, C1)) &= 0.8,\\
\deg(P(\mathrm{Re}, C1)) &= 0.9,\\
\deg(P(\mathrm{Sy}, C1)) &= 1.0,\\
\deg(P(\mathrm{Ic}, C1)) &= 0.8.
\end{aligned}$$

Here, emergent properties might include:

$$\begin{aligned}
\mu_{\text{emergence}}(\mathrm{Sy}; A, B, C1) &= 1.0 - \max(0.8, 0.4) = 0.2,\\
\mu_{\text{emergence}}(\mathrm{No}; A, B, C1) &= 0.9 - \max(0.8, 0.2) = 0.1.
\end{aligned}$$

These emergent values indicate that $C1$ possesses a new, heightened symbolic presence and enhanced nocturnality that are not present in either source alone. This interpretation corresponds to the classic comic book superhero: a highly intelligent, physically capable man who uses bat imagery and operates at night to instill fear in his foes.

Interpretation 2: The Hybrid Creature In a second high-quality interpretation, the blend produces a literal hybrid–a creature with both bat and human characteristics. For candidate *C2*, we might have:

$$\begin{aligned}
\deg(P(\mathrm{Fl}, C2)) &= 0.95 \quad \text{(natural winged flight)},\\
\deg(P(\mathrm{Ec}, C2)) &= 0.85 \quad \text{(using echolocation)},\\
\deg(P(\mathrm{No}, C2)) &= 0.85,\\
\deg(P(\mathrm{In}, C2)) &= 0.9,\\
\deg(P(\mathrm{PS}, C2)) &= 0.75,\\
\deg(P(\mathrm{Re}, C2)) &= 0.9,\\
\deg(P(\mathrm{Sy}, C2)) &= 0.8,\\
\deg(P(\mathrm{Ic}, C2)) &= 0.8.
\end{aligned}$$

Emergence in some properties is as follows:

$$\begin{aligned}
\mu_{\text{emergence}}(\text{Fl}; A, B, C2) &= 0.95 - \max(0.9, 0.0) = 0.05, \\
\mu_{\text{emergence}}(\text{Ec}; A, B, C2) &= 0.85 - \max(0.9, 0.0) \approx 0 \quad \text{(or marginal)}, \\
\mu_{\text{emergence}}(\text{In}; A, B, C2) &= 0.9 - \max(0.3, 0.9) = 0, \\
\mu_{\text{emergence}}(\text{No}; A, B, C2) &= 0.85 - \max(0.8, 0.2) = 0.05.
\end{aligned}$$

Although the emergent increments are modest, the synthesis of natural flight and echolocation with high human reasoning produces a truly novel hybrid creature. The emergent quality here comes from the unexpected co-occurrence of biological and human cognitive features, creating a being that is more than the sum of its parts.

Interpretation 3: A Low-Quality Blend A low-quality interpretation, *C3*, might simply be a man wearing a bat costume without any real integration of bat-like abilities. For instance, consider:

$$\begin{aligned}
\deg(P(\text{Fl}, C3)) &= 0.0, \\
\deg(P(\text{Ec}, C3)) &= 0.0, \\
\deg(P(\text{No}, C3)) &= 0.3, \\
\deg(P(\text{In}, C3)) &= 0.9, \\
\deg(P(\text{PS}, C3)) &= 0.8, \\
\deg(P(\text{Re}, C3)) &= 0.9, \\
\deg(P(\text{Sy}, C3)) &= 0.5, \\
\deg(P(\text{Ic}, C3)) &= 0.3.
\end{aligned}$$

Emergent measures for each property would be:

$$\mu_{\text{emergence}}(x; A, B, C3) \approx P(x, C3) - \max(P(x, A), P(x, B)),$$

which in every case yields values of zero or negative (set to zero). No property in $C3$ exceeds the corresponding maximum in the sources; hence, no truly novel quality emerges. This interpretation is low quality because it represents a superficial juxtaposition–merely a man in bat attire–without genuine functional or symbolic integration.

4 Information-Theoretic Measures of Blending Quality

So far we have considered the fuzzy degrees to which concepts possess properties as simply given, and worked with them to quantify blend qualities. It's interesting, however, to go a level deeper and explore where these fuzzy degrees come from – for instance, to consider the case where the degree to which a property x is possessed by a concept E is defined in an information-theoretic manner.

In the spirit of the intensional inheritance theory used in Hyperon's Probabilistic Logic Networks reasoning system (see, e.g., [Goe25]), we may interpret the degree of property possession as the amount of Shannon information contained in the proposition "x is a property of E." For example, if we let

$$d(x,E) \;=\; -\log P(x \in E),$$

then a lower probability of x occurring in E corresponds to a higher information content (or specificity) of that property for the concept.

When considering a blend C of source concepts A and B, we wish not only that C's properties be inherited (to a significant degree) from A or B, but also that C exhibits *emergence*?that is, for some properties the blend displays an additional amount of information compared to the best source. In other words, a high-quality blend is one in which the information associated with certain properties in C exceeds the maximum information from the corresponding properties in A or B.

4.1 Defining Emergence Information

For a given property x, let:

$$d(x,E) \;=\; -\log P(x \in E),$$

be the (normalized) information measure for the degree to which x is a property of E. Then the *emergent information* for x in the blend C relative to sources A and B is defined as:

$$\Delta I(x;A,B,C) \;=\; d(x,C) - \max\Big(d(x,A),\, d(x,B)\Big).$$

A positive value of $\Delta I(x;A,B,C)$ indicates that the blend C exhibits x in a way that is more specific or informative than in either A or B.

4.2 Blending Quality Measure

We can then define the overall high-quality blend membership function in an information-theoretic setting as:

$$\mu_{\text{hq-blend}}(C; A, B) \;=\; \min\Bigg(\frac{1}{|\operatorname{Props}(C)|} \sum_{x \in \operatorname{Props}(C)} w(x) \cdot \Delta I(x; A, B, C),\; 1\Bigg),$$

where $w(x)$ is a weight reflecting the degree to which the property x is common to both A and B. For instance, one may define:

$$w(x) \;=\; \min\Big(P(x \in A),\; P(x \in B)\Big),$$

so that properties shared (to a reasonable degree) by the sources contribute more to the emergent quality.

4.3 Connection to Intensional Inheritance

In the theory of intension developed for Hyperon's PLN inference system [Goe25], one may express the conditional probability of a concept W given F as:

$$P(W \mid F) \propto P(W) \cdot 2^{I(F;W)},$$

where $I(F; W)$ is the mutual information between the property sets of F and W. In our blending context, if we interpret the property degrees $d(x, E)$ as information measures (via Shannon's $-\log$ function), then the emergent information $\Delta I(x; A, B, C)$ plays a similar role. A blend C that has additional mutual information relative to A and B will exhibit higher values of $\mu_{\text{hq-blend}}(C; A, B)$. In other words, when a blend is constructed such that its properties carry extra information beyond the sources, we have:

$$P(x \in C \mid x \in A \text{ or } B) \approx P(x \in C) \cdot 2^{\Delta I(x;A,B,C)}.$$

Thus, our information-theoretic quality measure directly connects with the notion of intensional inheritance – where the emergent properties in C represent an enhanced understanding (or specification) of the blend beyond a mere union of source properties.

4.4 Relation between Emergence Information and Interaction Information

A different way to formulate the role of emergence here is in terms of interaction information, aka n-point mutual information. Recall that for a property x we define its information content in concept E as

$$d(x, E) = -\log P(x \in E),$$

so that a lower probability of x in E corresponds to higher information content. In our blending framework, the *emergence information* for property x in the blend C relative to source concepts A and B is given by:

$$\Delta I(x; A, B, C) = d(x, C) - \max\{d(x, A),\, d(x, B)\}.$$

A positive value of $\Delta I(x; A, B, C)$ indicates that the blend C carries extra information regarding property x beyond what is available from either A or B alone.

This notion naturally relates to the concept of *interaction information* (or n-point mutual information), which generalizes the idea of mutual information to more than two random variables. For three variables – here, think of X_A, X_B, and X_C representing the occurrence of property x in A, B, and C respectively – the three-variable interaction information is defined in one equivalent form as

$$I(X_A; X_B; X_C) = I(X_A; X_B) - I(X_A; X_B \mid X_C),$$

or, equivalently,

$$I(X_A; X_B; X_C) = I(X_A; X_C) + I(X_B; X_C) - I(X_A, X_B; X_C).$$

A positive $I(X_A; X_B; X_C)$ suggests that the joint presence of x in the blend C carries additional, synergistic information that is not accounted for by the pairwise interactions between C and A or C and B alone. In other words, the emergence information $\Delta I(x; A, B, C)$ captures the extent to which the property x in C is not merely a straightforward inheritance from A or B but results from a genuine interaction among all three.

Thus, one may view $\Delta I(x; A, B, C)$ as a simplified indicator of the three-variable interaction information for x. When the blend C exhibits a higher information content for x than either source, this is equivalent to having a positive interaction information among the three "instances" of property x. Consequently, the overall high-quality blend measure,

$$\mu_{\text{hq-blend}}(C;A,B)=\min\left(\frac{1}{|\operatorname{Props}(C)|}\sum_{x\in\operatorname{Props}(C)}w(x)\cdot\Delta I(x;A,B,C),\ 1\right),$$

(where $w(x)$ is a weight, for example $w(x)=\min\{P(x\in A),\ P(x\in B)\}$) aggregates these emergent (or synergistic) contributions across properties.

We see, then, that the additional information $\Delta I(x;A,B,C)$ in the blend is conceptually equivalent to the extra (interaction) mutual information among the three concepts A, B, and C. High-quality blending occurs when this interaction is positive?that is, when the blend's properties are not only inherited but also exhibit new specificity or creativity, as measured by their extra information content. This connection provides a robust theoretical foundation for assessing blend quality through an information-theoretic lens.

4.5 Implications

This perspective implies that a blend is of high quality not merely when it aggregates properties from the sources, but when it provides additional, novel specification in one or more dimensions. For instance, in a music blending scenario, if a blended musical passage exhibits a unique rhythmic motif or harmonic twist that is not present in either source, the associated increase in information content (measured via Shannon entropy) contributes to a higher $\mu_{\text{hq-blend}}$. This formalism thus unifies the notions of emergent creativity in blends with a rigorous information-theoretic framework.

That is: By defining the degree of property possession in terms of Shannon information and incorporating these measures into our emergent quality formulas, we obtain a quantitative bridge between the theory of intensional inheritance and the quality of blends. This approach allows us to measure, in information-theoretic terms, how a blend's properties go beyond those of its source concepts, thereby providing a robust metric for creative synthesis.

5 Heuristic Algorithms for Finding Concept Blends

We have taken a first stab at articulating what kind of blends we think a cognitive system should look for – but we haven't explained how it would actually go about looking for these high-quality blends. Given the vast space of possible blends, heuristic algorithms are essential for efficiently exploring candidate combinations. One natural approach is to use genetic algorithms (GAs), which mimic the process of natural evolution and are particularly well-suited for problems where the solution is a novel combination of features–much like the GA crossover operator. In addition to traditional GAs, *Estimation of Distribution Algorithms* (EDAs) provide a complementar strategy that augments crossover and mutation with a probabilistic model of promising solutions.

5.1 Genetic Algorithm Approach

The goal here is to identify candidate blends C that maximize the high-quality blend membership function $\mu_{\text{hq-blend}}(C; A, B)$. A genetic algorithm for concept blending would most naturally involve the following steps:

1. **Initialization:** Generate an initial population of candidate blends. Each candidate C can be represented as a set (or vector) of properties derived from the source entities A and B.

2. **Fitness Evaluation:** For each candidate blend C, compute the fitness score using:
$$f(C) = \mu_{\text{hq-blend}}(C; A, B).$$
This score measures the degree of emergent, high-quality blending.

3. **Selection:** Select candidate blends from the current population to serve as parents, favoring those with higher fitness scores.

4. **Crossover (Recombination):** Apply a crossover operator to pairs of parent blends to produce offspring, recombining properties from each parent.

5. **Mutation:** With a small probability, apply a mutation operator to introduce random changes in the offspring's properties.

6. **Replacement:** Form a new generation by selecting among the offspring (and possibly some parents) based on fitness.
7. **Termination:** Repeat until a termination criterion is met (e.g., a maximum number of generations or a fitness threshold).

5.2 Estimation of Distribution Algorithms (EDAs) for Concept Blending

EDAs represent a powerful augmentation to traditional GAs. Instead of relying solely on crossover and mutation operators to explore the search space, EDAs form an explicit probabilistic model $p(C)$ of promising candidate blends. This model is then used to sample new candidates, with the aim of guiding the evolution process toward regions of the search space that yield high $\mu_{\text{hq-blend}}(C; A, B)$ values.

Building the Probabilistic Model: The key idea is to use inductive and abductive Bayesian reasoning (which in a Hyperon context may include e.g. PLN inference) to construct a model of what constitutes a high-quality blend. Given the information-theoretic definition of high-quality blends, one can interpret $\mu_{\text{hq-blend}}(C; A, B)$ as an information measure that informs the likelihood of C being a good blend. Formally, we can express the probability that a candidate C is a high-quality blend as:

$$p(C \mid A, B, \mathcal{D}) \propto p(\mathcal{D} \mid C, A, B)\, p(C \mid A, B),$$

where:

- $p(C \mid A, B, \mathcal{D})$ is the posterior probability that C is a good blend given the source entities A and B and observed data $\mathcal{D}$ (which could include previous evaluations or successful blends).
- $p(\mathcal{D} \mid C, A, B)$ is the likelihood, reflecting how well C satisfies the emergent properties as measured by $\mu_{\text{hq-blend}}(C; A, B)$.
- $p(C \mid A, B)$ is the prior probability that encodes structural or domain-specific expectations about blends.

This probabilistic model can be updated iteratively as new candidate blends are evaluated, refining the search toward regions of the search space with higher expected quality.

EDA Workflow: The EDA-based approach involves the following steps:

1. **Initialization:** Start with an initial population of candidate blends.
2. **Model Estimation:** Use the current population to estimate a probabilistic model $p(C \mid A, B, \mathcal{D})$ that favors candidates with higher emergent measures.
3. **Sampling:** Sample new candidate blends from the estimated model.
4. **Evaluation:** Compute the fitness $f(C) = \mu_{\text{hq-blend}}(C; A, B)$ for the new candidates.
5. **Model Update:** Incorporate the new evaluation data into the model using Bayesian updating, refining the distribution to better represent high-quality blends.
6. **Iteration:** Repeat the model estimation, sampling, and update steps until a termination criterion is met.

Obviously, this may be layered on top of a standard GA approach, combining the strengths of GA and EDA in an adaptive and flexible manner.

Advantages of EDAs: EDAs can be more efficient than traditional GAs in structured search spaces because they exploit the underlying probabilistic dependencies among properties. By forming an explicit model of what constitutes a good blend, EDAs allow for:

- **Adaptive Search:** The model adapts as more data is gathered, allowing the search to focus on promising regions.
- **Integration of Domain Knowledge:** Priors can incorporate expert knowledge or domain-specific constraints.
- **Information-Theoretic Guidance:** By leveraging measures such as entropy and mutual information, the EDA framework can be tuned to favor candidates that offer significant emergent novelty.

6 Thought-Experiment: Keyboard Solo Blending in Jazz Fusion with Specialized Musical Features

To make these concepts intuitively clearer, we now pursue a slightly more in depth "thought experiment" – bearing in mind that implementing the sort of example discussed in practice would inevitably involve a great amount of additional complexity and subtlety.

In the context of jazz fusion or other fairly loosely-defined improvisational music, consider blending two particular keyboard solos with distinct stylistic features. One might look at properties such as:

- **Harmonic Complexity (HC)** — the sophistication of chord progressions.
- **Rhythmic Drive (RD)** — the strength and propulsion of the rhythm.
- **Expressiveness (Ex)** — the emotional nuance in tone and dynamics.
- **Improvisational Creativity (IC)** — the originality in spontaneous phrasing.
- **Mode/Key Consistency (MK)** — the stability and coherence of the key or mode used.
- **octatonic Scale Utilization (OS)** — use of octatonic scale
- **Use of Scriabin's Mystic Chord (MC)** — the use and variation of this particular chord

In reality one would have many further features similar to the last two, indicating use of particular scales and chords, particular transitions between chords, and so forth. Also one could associate all these features with different portions of the solo, e.g. the beginning, middle and end of the solo might be usefully viewed as having different values for all these features. For sake of a simple illustrative example, though, let's use the above features as proxies for what in reality would be a much longer list.

Using these properties, each keyboard solo would be represented as a 7-dimensional property vector:

$$C = \deg(P(\mathrm{HC}, C)),\ \deg(P(\mathrm{RD}, C)),\ \deg(P(\mathrm{Ex}, C)),\ \deg(P(\mathrm{IC}, C))$$

$$\deg(P(\mathrm{MK}, C)),\ \deg(P(\mathrm{OS}, C)),\ \deg(P(\mathrm{MC}, C))$$

Source Solos

Assume the following fuzzy membership values for the source solos:

Solo A (Traditional Jazz Solo):

$$\begin{aligned}
\deg(P(\mathrm{HC}, A)) &= 0.9, \\
\deg(P(\mathrm{RD}, A)) &= 0.5, \\
\deg(P(\mathrm{Ex}, A)) &= 0.8, \\
\deg(P(\mathrm{IC}, A)) &= 0.7, \\
\deg(P(\mathrm{MK}, A)) &= 0.85, \\
\deg(P(\mathrm{OS}, A)) &= 0.75, \\
\deg(P(\mathrm{MC}, A)) &= 0.6.
\end{aligned}$$

Solo B (Fusion Solo):

$$\begin{aligned}
\deg(P(\mathrm{HC}, B)) &= 0.6, \\
\deg(P(\mathrm{RD}, B)) &= 0.9, \\
\deg(P(\mathrm{Ex}, B)) &= 0.7, \\
\deg(P(\mathrm{IC}, B)) &= 0.5, \\
\deg(P(\mathrm{MK}, B)) &= 0.65, \\
\deg(P(\mathrm{OS}, B)) &= 0.8, \\
\deg(P(\mathrm{MC}, B)) &= 0.7.
\end{aligned}$$

Candidate Blends

A candidate blend M is a 7-dimensional vector. We now consider two candidates:

Candidate $M1$ (Lower-Quality Blend): Assume $M1$ is generated by a straightforward averaging process:

$$\begin{aligned}
\deg(P(\mathrm{HC}, M1)) &= 0.75,\\
\deg(P(\mathrm{RD}, M1)) &= 0.70,\\
\deg(P(\mathrm{Ex}, M1)) &= 0.75,\\
\deg(P(\mathrm{IC}, M1)) &= 0.65,\\
\deg(P(\mathrm{MK}, M1)) &= 0.75,\\
\deg(P(\mathrm{OS}, M1)) &= 0.78,\\
\deg(P(\mathrm{MC}, M1)) &= 0.65.
\end{aligned}$$

For each property x, we compute the degree of emergence:

$$\mu_{\text{emergence}}(x; A, B, M1) = \deg(P(x, M1)) - \max\big(\deg(P(x, A)), \deg(P(x, B))\big).$$

The computations are as follows:

$$\begin{aligned}
\mu_{\text{emergence}}(\mathrm{HC}; A, B, M1) &= 0.75 - \max(0.9, 0.6) = -0.15 \text{ (treated as 0)},\\
\mu_{\text{emergence}}(\mathrm{RD}; A, B, M1) &= 0.70 - \max(0.5, 0.9) = -0.20 \text{ (0)},\\
\mu_{\text{emergence}}(\mathrm{Ex}; A, B, M1) &= 0.75 - \max(0.8, 0.7) = -0.05 \text{ (0)},\\
\mu_{\text{emergence}}(\mathrm{IC}; A, B, M1) &= 0.65 - \max(0.7, 0.5) = -0.05 \text{ (0)},\\
\mu_{\text{emergence}}(\mathrm{MK}; A, B, M1) &= 0.75 - \max(0.85, 0.65) = -0.10 \text{ (0)},\\
\mu_{\text{emergence}}(\mathrm{OS}; A, B, M1) &= 0.78 - \max(0.75, 0.8) = -0.02 \text{ (0)},\\
\mu_{\text{emergence}}(\mathrm{MC}; A, B, M1) &= 0.65 - \max(0.6, 0.7) = -0.05 \text{ (0)}.
\end{aligned}$$

Since none of these properties exceed the maximum values from the source solos, $M1$ does not exhibit emergent enhancement and is classified as a lower-quality blend.

Candidate $M2$ (Higher-Quality Blend): Now suppose we design $M2$ to push the boundaries of the sources:

$$\begin{aligned}
\deg(P(\mathrm{HC}, M2)) &= 0.95,\\
\deg(P(\mathrm{RD}, M2)) &= 0.95,\\
\deg(P(\mathrm{Ex}, M2)) &= 0.85,\\
\deg(P(\mathrm{IC}, M2)) &= 0.80,\\
\deg(P(\mathrm{MK}, M2)) &= 0.90,\\
\deg(P(\mathrm{OS}, M2)) &= 0.88,\\
\deg(P(\mathrm{MC}, M2)) &= 0.80.
\end{aligned}$$

The emergent values are computed as:

$$\begin{aligned}
\mu_{\text{emergence}}(\text{HC}; A, B, M2) &= 0.95 - \max(0.9, 0.6) = 0.05,\\
\mu_{\text{emergence}}(\text{RD}; A, B, M2) &= 0.95 - \max(0.5, 0.9) = 0.05,\\
\mu_{\text{emergence}}(\text{Ex}; A, B, M2) &= 0.85 - \max(0.8, 0.7) = 0.05,\\
\mu_{\text{emergence}}(\text{IC}; A, B, M2) &= 0.80 - \max(0.7, 0.5) = 0.10,\\
\mu_{\text{emergence}}(\text{MK}; A, B, M2) &= 0.90 - \max(0.85, 0.65) = 0.05,\\
\mu_{\text{emergence}}(\text{OS}; A, B, M2) &= 0.88 - \max(0.75, 0.8) = 0.08,\\
\mu_{\text{emergence}}(\text{MC}; A, B, M2) &= 0.80 - \max(0.6, 0.7) = 0.10.
\end{aligned}$$

For weighting, we use:

$$w(x) = \min\Bigl(\deg(P(x, A)),\ \deg(P(x, B))\Bigr),$$

which gives:

$$\begin{aligned}
w(\text{HC}) &= \min(0.9, 0.6) = 0.6,\\
w(\text{RD}) &= \min(0.5, 0.9) = 0.5,\\
w(\text{Ex}) &= \min(0.8, 0.7) = 0.7,\\
w(\text{IC}) &= \min(0.7, 0.5) = 0.5,\\
w(\text{MK}) &= \min(0.85, 0.65) = 0.65,\\
w(\text{OS}) &= \min(0.75, 0.8) = 0.75,\\
w(\text{MC}) &= \min(0.6, 0.7) = 0.6.
\end{aligned}$$

Then the high-quality blend membership for $M2$ is computed as:

$$\mu_{\text{hq-blend}}(M2; A, B) =$$

$$\min\left(\frac{1}{7}\Bigl(0.6{\cdot}0.05{+}0.5{\cdot}0.05{+}0.7{\cdot}0.05{+}0.5{\cdot}0.10{+}0.65{\cdot}0.05{+}0.75{\cdot}0.08{+}0.6{\cdot}0.10\Bigr),\ 1\right).$$

Performing the arithmetic:

$$\begin{aligned}
0.6 \times 0.05 &= 0.03,\\
0.5 \times 0.05 &= 0.025,\\
0.7 \times 0.05 &= 0.035,\\
0.5 \times 0.10 &= 0.05,\\
0.65 \times 0.05 &= 0.0325,\\
0.75 \times 0.08 &= 0.06,\\
0.6 \times 0.10 &= 0.06.
\end{aligned}$$

The weighted sum is:

$$0.03 + 0.025 + 0.035 + 0.05 + 0.0325 + 0.06 + 0.06 = 0.2925.$$

Dividing by 7 gives approximately 0.0418. This positive value indicates that $M2$ shows emergent characteristics beyond the best features of the sources, qualifying it as a higher-quality blend.

6.1 Integration with Music Generation

Suppose one did something like the above with a longer and richer set of property values characterizing pieces of music. One could then do a variety of things with the blended set of property values – for instance one could use the property vector $M2$ as a target for a music generation algorithm that synthesizes a keyboard solo. The algorithm would attempt to produce a musical passage exhibiting:

- A highly complex harmonic structure.
- A strong, driving rhythm.
- Elevated expressiveness and emotional nuance.
- Greater improvisational creativity.
- Consistent use of a chosen mode/key.
- The specified degree of use of octatonic (or other specified scales)
- The specified degree of use of the Mystic Chord (or other specified chords or transitions)

The algorithm's output would then be analyzed to extract its observed property vector, which is compared with $M2$. Successful synthesis reinforces the fitness of $M2$, while discrepancies provide feedback to adjust the search.

6.2 Heuristic Search: GA and EDA Approaches

We now briefly run through how heuristic search might work in this example.

Genetic Algorithm (GA):

- **Representation:** Each candidate blend is a 7-dimensional vector.
- **Crossover:** Parent candidates exchange segments of their property vectors. For instance, one candidate's values for HC, Ex, and MK might be combined with another's RD, IC, LS, and MC. Depending on the crossover operator, some of the individual values could also be merged via averaging or other operations.
- **Mutation:** Random perturbations in one or more dimensions introduce variation.
- **Selection:** Candidates with higher $\mu_{\text{hq-blend}}$ values are preferentially selected for reproduction.

Estimation of Distribution Algorithm (EDA):

1. **Model Estimation:** From the current population of candidate blends, select those with high fitness and estimate a multivariate probability distribution (e.g., a Gaussian) over the 7-dimensional property space.
2. **Sampling:** Draw new candidate blends from this distribution, ensuring that interdependencies among features (e.g., the relationship between mode/key consistency and scale utilization) are preserved.
3. **Bayesian Updating:** Update the distribution iteratively using observed data from both the fitness evaluations and the feedback from the music generation algorithm.

We see in this conceptual example how advanced musical features–such as modes/keys, scales, and repetitive chord sequences–might be integrated into our formalism for blending keyboard solos in jazz fusion. By computing emergence measures for a rich set of properties and employing heuristic search techniques (both GA and EDA), we can identify candidate blends that serve as targets for music generation, ultimately enabling the synthesis of novel and high-quality jazz fusion solos.

7 Balancing Emergence and Coherence in Blends via Multiobjective Optimization

Thus far, our quality measure for a blend C has been based on the emergence of properties – that is, the additional information that C exhibits relative to its source concepts A and B. It may also be interesting to augment this criterion with supplementary measures of concept quality.

For instance, in addition to emergence, another important criterion for the quality of a concept is the degree of interaction among its internal properties. Intuitively, we may say a concept is **coherent** if its properties interact strongly – that is, if there is significant synergy or interdependence among them. This can be captured by measures of multi-variable interaction information.

7.1 Defining a Coherence Measure

One way to quantify the overall interaction among the properties of C is through the *total correlation* $TC(C)$, defined as

$$TC(C) = \sum_{x \in \mathrm{Props}(C)} H(x) - H(\mathrm{Props}(C)),$$

where $H(x)$ is the Shannon entropy of property x (interpreted as the uncertainty or information content in the degree to which x is possessed by C), and $H(\mathrm{Props}(C))$ is the joint entropy of all properties of C. The total correlation measures the reduction in uncertainty when considering the joint distribution compared to the independent distribution of each property. In other words, it quantifies the interdependency (or interaction information) among the properties.

For a normalized measure, we define the *coherence* of C as:

$$\mu_{\mathrm{coherence}}(C) = \frac{TC(C)}{\sum_{x \in \mathrm{Props}(C)} H(x)},$$

which takes values in $[0, 1]$. A value near 1 indicates that the properties of C are highly interdependent (i.e., coherent), while a value near 0 indicates that the properties are nearly independent.

7.2 Multiobjective Blending

Suppose we feel that a desirable blend should not only exhibit emergent properties – that is, have $\mu_{\text{hq-blend}}(C; A, B)$ as high as possible – but also be internally coherent, as measured by $\mu_{\text{coherence}}(C)$. We might then propose a multiobjective quality function for a blend:

$$\text{Quality}(C) = \Big(\mu_{\text{hq-blend}}(C; A, B),\ \mu_{\text{coherence}}(C)\Big).$$

A desirable blend is one that simultaneously maximizes both objectives.

7.3 Heuristic Search with Multiobjective GA or EDA

To find candidate blends that optimize both emergent quality and internal coherence, one may apply a multiobjective optimization algorithm such as a multiobjective Genetic Algorithm (GA) or an Estimation of Distribution Algorithm (EDA). In these approaches:

- **Representation:** Each candidate blend C is represented as a vector of property degrees (e.g., $d(x, C)$ for each $x \in \text{Props}(C)$).
- **Evaluation:** For each candidate, compute both $\mu_{\text{hq-blend}}(C; A, B)$ and $\mu_{\text{coherence}}(C)$.
- **Selection:** The multiobjective algorithm uses methods such as Pareto dominance or weighted aggregation to select candidates that offer a favorable trade-off between emergence and coherence.
- **Crossover and Mutation (or Sampling):** Standard GA operators (or probabilistic sampling in EDAs) generate new candidate blends, potentially recombining properties from high-quality candidates.
- **Iteration:** The process iterates until convergence or a satisfactory blend is found.

This multiobjective approach allows us to systematically search for blends that not only incorporate additional, emergent properties beyond the sources but also exhibit strong internal interdependencies among their features – both of which are key hallmarks of a concept's quality and coherence.

In summary, by introducing $\mu_{\text{coherence}}(C)$ as an additional objective function, we enrich our framework for assessing blend quality. Using multiobjective GA or EDA techniques, we can optimize both the emergent information (relative to the sources) and the internal coherence (interaction among properties) of the blend, leading to concepts that are not only novel but also internally well-integrated.

7.4 Multiobjective Music Synthesis

In the music synthesis context briefly touched above, the use of multiobjective optimization would potentially allow us to combine emergence-based quality measure and a coherence measure to product musical passages that is are only novel (in the sense of providing additional information beyond the source solos) but also internally well-integrated. Multiobjective GA or EDA methods could effectively explore the complex higher-dimensional property space, yielding candidate blends that serve as precise targets for music synthesis potentially offering robust pathway for generating innovative and stylistically coherent AI compositions.

8 Conclusion

We have developed a formal framework for assessing blends using fuzzy-set membership functions in higher-order predicate logic. The blend membership function $\mu_{\text{blend}}(C; A, B)$ measures the extent to which the properties of an entity C are derived from source entities A and B. The high-quality blend membership $\mu_{\text{hq-blend}}(C; A, B)$ captures the degree of emergent properties—those features that are enhanced in the blend relative to the sources. The degree of coherence, measured using interaction information among properties, provides an additional desirable quality for blends to possess.

By grounding these ideas in cognitive science and AI perspectives on concept blending, our approach provides a quantitative tool for evaluating creative and innovative outcomes. We have given detailed examples from coffee, music, material design and character design explicitly demonstrating how our formal definitions can distinguish between lower- and higher-quality blends, and assess concept coherence. We have argued that heuristic algorithms, such GAs and EDAs, may offer practical methods for exploring the vast space of potential blends, mirroring creative processes observed in human cognition. These considerations offer multiple clear directions for ongoing research exploration.

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