数值分析方法

作业4

Problem 1

解: a. $\because x_0 = 0,\ x_1 = 0.3,\ x_2 = 0.6$

$\therefore$ 可得系数多项式

$$L_0(x) = \frac{(x-0.3)(x-0.6)}{(0-0.3)(0-0.6)} = \frac{50(x-0.3)(x-0.6)}{9}$$

$$L_1(x) = \frac{(x-0)(x-0.6)}{(0.3-0)(0.3-0.6)} = -\frac{100x(x-0.6)}{9}$$

$$L_2(x) = \frac{(x-0)(x-0.3)}{(0.6-0)(0.6-0.3)} = \frac{50x(x-0.3)}{9}$$

$\because f(x) = e^{2x}\cos 3x$

$\therefore f(x_0) = 1,\ f(x_1) = e^{0.6}\cos 0.9,\ f(x_2) = e^{1.2}\cos 1.8$

且 $f'(x) = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$

$f''(x) = -5e^{2x}\cos 3x - 12e^{2x}\sin 3x$

$f'''(x) = -46e^{2x}\cos 3x - 9e^{2x}\sin 3x$

$\therefore P_2(x) = \prod_{i=0}^{2} f(x_i)L_i(x)$

$= \frac{50(x-0.3)(x-0.6)}{9} - e^{0.6}\cos 0.9\frac{100x(x-0.6)}{9} + e^{1.2}\cos 1.8\frac{50x(x-0.3)}{9}$

$= -11.22x^2 + 3.81x + 1.00$

且余项为:

$$R = \frac{f'''(\xi)}{3!}(x-x_0)(x-x_1)(x-x_2)$$
$$= -\left(\tfrac{23e^{2\xi}\cos 3\xi}{3} + \tfrac{3e^{2\xi}\sin 3\xi}{2}\right)x(x-0.3)(x-0.6)$$

其中, $\xi \in [0, 0.6]$

所以绝对误差 $|R| = \left|\left(\tfrac{23e^{2\xi}\cos 3\xi}{3} + \tfrac{3e^{2\xi}\sin 3\xi}{2}\right)x(x-0.3)(x-0.6)\right| \le 0.113712$

即绝对误差限为 0.113712.

b. $\because x_0 = 2.0,\ x_1 = 2.4,\ x_2 = 2.6$

$\therefore$ 可得系数多项式

$$L_0(x) = \frac{(x-2.4)(x-2.6)}{(2.0-2.4)(2.0-2.6)} = \frac{25(x-2.4)(x-2.6)}{6}$$

$$L_1(x) = \frac{(x-2.0)(x-2.6)}{(2.4-2.0)(2.4-2.6)} = -\frac{25(x-2.0)(x-2.6)}{2}$$

$$L_2(x) = \frac{(x-2.0)(x-2.4)}{(2.6-2.0)(2.6-2.4)} = \frac{25(x-2.0)(x-2.4)}{3}$$

$\because f(x) = \sin(\ln x)$

$\therefore f(x_0) = \sin(\ln 2.0),\ f(x_1) = \sin(\ln 2.4),\ f(x_2) = \sin(\ln 2.6)$

且 $f'(x)=\frac{\cos(\ln x)}{x}$

$f''(x)=-\frac{\sin(\ln x)+\cos(\ln x)}{x^2}$

$f'''(x)=\frac{\cos(\ln x)+3\sin(\ln x)}{x^3}$

$\therefore P_2(x)=\prod_{i=0}^{2}f(x_i)L_i(x)$

$=\sin(\ln 2)\frac{25(x-2.4)(x-2.6)}{6}-\sin(\ln 2.4)\frac{25(x-2)(x-2.6)}{2}+\sin(\ln 2.6)\frac{25(x-2)(x-2.4)}{3}$

$=-0.131x^2+0.897x-0.632$

且余项为:

$$R=-\Big[\frac{\cos(\ln\xi)+3\sin(\ln\xi)}{6\xi^3}\Big](x-2)(x-2.4)(x-2.6)$$

其中, $\xi\in[2.0,2.6]$

所以绝对误差 $|R|=\Big|\frac{\cos(\ln\xi)+3\sin(\ln\xi)}{6\xi^3}(x-2)(x-2.4)(x-2.6)\Big|\leq 9.458\times10^{-4}$

即绝对误差限为 9.458×10^{-4}.

Problem 2

解: 由题意, 系数多项式为:

$$L_0(x)=\frac{(x-0.5)(x-1)(x-2)}{(0-0.5)(0-1)(0-2)}=-(x-0.5)(x-1)(x-2)$$

$$L_1(x)=\frac{x(x-1)(x-2)}{0.5(0.5-1)(0.5-2)}=\frac{8x(x-1)(x-2)}{3}$$

$$L_2(x)=\frac{x(x-0.5)(x-2)}{1(1-0.5)(1-2)}=-2x(x-0.5)(x-2)$$

$$L_3(x)=\frac{x(x-0.5)(x-1)}{2(2-0.5)(2-1)}=\frac{x(x-0.5)(x-1)}{3}$$

$\therefore$ 拉格朗日内插多项式为:

$$P_3(x)=\prod_{i=0}^{3}f(x_i)L_i(x)=\frac{8yx(x-1)(x-2)}{3}-6x(x-0.5)(x-2)+\frac{2x(x-0.5)(x-1)}{3}$$

则立方项系数 $\frac{8y}{3}-6+\frac{2}{3}=6$

解得: $y=4.25$

Problem 3

解: a. 由表可得: $P_{0,1,2,3}=3.016$

又

$$P_{0,1,2,3}=\frac{(x-x_3)P_{0,1,2}-(x-x_0)P_{1,2,3}}{x_0-x_3}$$

$$\therefore\frac{(0.4-0.75)P_{0,1,2}-2.96\times0.4}{-0.75}=3.016$$

解得: $P_{0,1,2}=3.08$

同理:

$$P_{0,1,2} = \frac{(x-x_0)P_{1,2} - (x-x_2)P_{0,1}}{x_2 - x_0} = \frac{0.4P_{1,2} + 0.1 \times 2.6}{0.5}$$

解得: $P_{0,1,2} = 3.2$

又

$$P_{1,2} = \frac{(x-x_2)P_1 - (x-x_1)P_2}{x_1 - x_2} = \frac{-0.1 \times 2 - (0.4 - 0.25)P_2}{0.25 - 0.5}$$

可得: $P_2 = f(0.5) = 4$

b. 由题意:

$$P_{0,1}(2.5) = 6,\ P_{0,2}(2.5) = 3.5,\ P_{1,2,3}(2.5) = 3$$

$$\therefore P_{0,1,2}(2.5) = \frac{(x-x_2)P_{0,1}(2.5) - (x-x_1)P_{0,2}(2.5)}{x_1 - x_2}$$

$$= \frac{0.5 \times 6 - 1.5 \times 3.5}{-1} = 2.25$$

又$P_{1,2,3}(2.5) = 3$

$$\therefore P_{0,1,2,3}(2.5) = \frac{(x-x_3)P_{0,1,2}(2.5) - (x-x_0)P_{1,2,3}(2.5)}{x_0 - x_3}$$

$$= \frac{-0.5 \times 2.25 - 2.5 \times 3}{-3} = 2.875$$

Problem 4

解: 由表可得:

$$f[x_0, x_1, x_2] = \frac{50}{7},\ f[x_1, x_2] = 10,\ f[x_2] = 6$$

$$\therefore f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{10 - f[x_0, x_1]}{0.7}$$

解得:

$$f[x_0, x_1] = 5$$

又

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{6 - f[x_1]}{0.3}$$

解得:

$$f[x_1] = 3$$

同理:

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{3 - f[x_0]}{0.4} = 5$$

解得:

$$f[x_0] = 1$$

综上: $f[x_0] = 1,\ f[x_1] = 3,\ f[x_0, x_1] = 5.$

Problem 5

解: a. 在区间$[0,1]$上, 设$S_0(x) = a_0 + b_0 x + c_0 x^2 + d_0 x^3$

在区间$[1,2]$上, 设$S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3$
则:

$$S_0(0) = a_0 = 0,\ S_0(1) = a_0 + b_0 + c_0 + d_0 = 1$$

$$S_1(1) = a_1 = 1,\ S_1(2) = a_1 + b_1 + c_1 + d_1 = 2$$

且有:

$$S_0'(1) = b_0 + 2c_0 + 3d_0 = S_1'(1) = b_1$$

$$S_0''(1) = 2c_0 + 6d_0 = S_1''(1) = 2c_1$$

由自然边界条件:

$$S_0''(0) = 2c_0 = 0,\ S_1''(2) = 2c_1 + 6d_1 = 0$$

解得: $a_0 = 0,\ b_0 = 1,\ c_0 = 0,\ d_0 = 0;\ a_1 = 1,\ b_1 = 1,\ c_1 = 0,\ d_1 = 0$
$\therefore S_0(x) = x,\ x \in [0,1];\ S_1(x) = x,\ x \in [1,2]$
故: $S(x) = x,\ x \in [0,2]$

b. 同理, 设$S_0(x) = a_0 + b_0 x + c_0 x^2 + d_0 x^3$, $x \in [0,1]$
$S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3$, $x \in [1,2]$
有:

$$S_0(0) = a_0 = 0,\ S_0(1) = a_0 + b_0 + c_0 + d_0 = 1$$

$$S_1(1) = a_1 = 1,\ S_1(2) = a_1 + b_1 + c_1 + d_1 = 2$$

且有:

$$S_0'(1) = b_0 + 2c_0 + 3d_0 = S_1'(1) = b_1$$

$$S_0''(1) = 2c_0 + 6d_0 = S_1''(1) = 2c_1$$

由边界条件:

$$S_0'(0) = b_0 = 1,\ S_1'(2) = b_1 + 2c_1 + 3d_1 = 1$$

解得: $a_0 = 0,\ b_0 = 1,\ c_0 = 0,\ d_0 = 0;\ a_1 = 1,\ b_1 = 1,\ c_1 = 0,\ d_1 = 0$
$\therefore S_0(x) = x,\ x \in [0,1];\ S_1(x) = x,\ x \in [1,2]$
故: $S(x) = x,\ x \in [0,2]$