

## 6.632 Solution to Problem Set 5

### Solution P5.1

At the cutoff, the Goos-Hanchen shift at  $x = d$  is zero. Combing guidance condition and dispersion relations, we can get

$$k_{cm} = \frac{m\pi + \tan^{-1} \frac{\epsilon_1 \sqrt{\epsilon_2 - \epsilon_0}}{\epsilon_0 \sqrt{\epsilon_1 - \epsilon_2}}}{d \sqrt{1 - \epsilon_2 / \epsilon_1}}$$

### Solution P5.2

- (a)  $\bar{J}_s = \hat{z} \frac{2k_0}{\omega\mu} e^{i\sqrt{3}k_0 z/2}$   
 (b) The waves radiated by the surface current sheet are

$$\bar{E}_+ = (\hat{x}\sqrt{3}/2 - \hat{z}1/2) e^{ik_0 x/2} e^{ik_0 z\sqrt{3}/2}$$

$$\bar{E}_- = (-\hat{x}\sqrt{3}/2 - \hat{z}1/2) e^{-ik_0 x/2} e^{ik_0 z\sqrt{3}/2}$$

which cancel the incident wave in region  $x > 0$  and form the reflected wave in region  $x < 0$ .

### Solution P5.3

- (a)

$$\frac{d^2 S_\rho}{d\ell d\lambda} = \frac{d^2 S_\rho}{d\ell d\omega} \frac{d\omega}{d\lambda} = \frac{\mu q^2}{4\pi} \omega \left(1 - \frac{1}{n^2 \beta^2}\right) \frac{2\pi c}{\lambda^2}$$

$$\frac{d^2 N}{d\ell d\lambda} = \frac{q^2 c}{2\lambda^2 \hbar} \mu \left(1 - \frac{1}{n^2 \beta^2}\right)$$

Since  $1 - \frac{1}{n^2 \beta^2} = \sin^2 \theta$ ,  $\frac{dN}{d\ell} \propto \frac{d\lambda}{\lambda^2} \sin^2 \theta$ .

- (b)  $\theta = 3.6^\circ$   
 (c)  $\ell = 0.528$  m