6.632 Solution to Problem Set 6

Solution P5.1

(a) Using the notation for TE wave, p $_{12} = 2$.

$$T_{12} = \frac{1}{1+p_{12}} = \frac{2}{3}$$

$$R_{12} = \frac{1-p_{12}}{1+p_{12}} = -\frac{1}{3}$$

Since the wave is incident normally, we could also use TM notation. Then, $p_{01} = 1/2$, $R_{12} = 1/3$, and $T_{12} = 4/3$.
(b)

$$< \overline{S}_i > + < \overline{S}_r >= \hat{z}\frac{1}{2}\frac{k_1}{\omega\mu_1}(1-|R|^2)$$

$$< \vec{S}_t >= \hat{z}\frac{1}{2}\frac{k_2}{\omega\mu_2}|T|^2$$

Power is conserved since we know from application of the boundary conditions $1-|R|^2 = p_{12}|T|^2$.
(c)

$$< \overline{G}_i > + < \overline{G}_r >= \hat{z}\frac{\epsilon_1}{2}\frac{k_1}{\omega}(1-|R|^2)$$

$$< \vec{G}_t >= \hat{z}\frac{\epsilon_2}{2}\frac{k_2}{\omega}|T|^2$$

Momentum is not conserved for the given parameters since $1-|R|^2 \neq 8|T|^2$.
(d) The radiation pressure of an electromagnetic wave is $p = \frac{1}{2}\epsilon|E|^2$. Therefore the pressure on the half space is

$$\bar{F} = \overline{p}_i - \overline{p}_r - \overline{p}_t = \hat{z}\frac{1}{2}\left[\epsilon_1(1+|R|^2) - \epsilon_2|T|^2\right] = -\hat{z}\frac{\epsilon_0}{3}$$

(e) The half space moves toward the incident wave.

Solution P5.2

(a) $\nabla\times\bar{H} = -i\omega\bar{D} = -i\omega(\epsilon_0\bar{E}+\bar{P})$ Therefore, we may isolate the contribution from the material as $\nabla\times\bar{H} + i\omega\epsilon_0\bar{E}$ = -i $\omega\bar{P} \equiv \overline{J}_e$
(b) $\overline{J}_b = -i\omega(\epsilon_R-\epsilon_0)\overline{E}$
(c)

$$\overline{H} = \frac{1}{\omega\mu}\overline{k}\times\overline{E} = \hat{y}\frac{k_R+ik_I}{\omega\mu_0}E_0e^{k_Iz}e^{ik_Rz}$$

(d) $< \overline{f}_b >= -\hat{z}\frac{1}{2}k_I(\epsilon_R-\epsilon_0)|E|^2$, where $|E|^2 = |E_0|^2e^{2k_iz}$.
(e) $< \overline{f}_c >= \hat{z}\frac{1}{2}k_R\epsilon_I|E|^2 = -\hat{z}\frac{1}{2}\frac{n}{c}\mathrm{Re}\{\nabla\cdot\overline{S}\}$ since Poynting's theorem tells us that $\omega\epsilon_I|E|^2 = -\mathrm{Re}\{\nabla\cdot\overline{S}\}$.

Solution P5.3

(a) $\theta_c = 60^\circ$.

(b) The Brewster angle is $\theta_b = 40.9°$.

(c) It is impossible, $\sin\theta < \tan\theta$ for any θ between $0°$ and $90°$.

Solution P5.4

(a) For E layer, $f_p = 2.84\,\mathrm{MHz}$.
For F layer, $f_p = 6.95\,\mathrm{MHz}$.

(b) In E layer, $\theta_t = \sin^{-1}[1.04\sin\theta]$.
In F layer, $\theta_t = \sin^{-1}[1.39\sin\theta]$.

(c) For E layer total reflection happens when $f < \dfrac{2}{\sqrt{3}} f_p = 3.27\,\mathrm{MHz}$.

For F layer total reflection happens when $f < \dfrac{2}{\sqrt{3}} f_p = 8.03\,\mathrm{MHz}$.