数值分析方法

作业5

Problem 1

解： 对端点：

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_0+h) - f(x_0+2h)}{2h}$$

对中点：

$$f'(x_0) = \frac{f(x_0-h) + f(x_0+h)}{2h}$$

a.

$$f'(1.1) = \frac{-3f(1.1) + 4f(1.2) - f(1.3)}{2 \times 0.1} = \frac{-3 \times 9.025013 + 4 \times 11.02318 - 13.46374}{2 \times 0.1} = 17.769705$$

$$f'(1.2) = \frac{f(1.3) - f(1.1)}{2 \times 0.1} = \frac{13.46374 - 9.025013}{2 \times 0.1} = 22.193635$$

$$f'(1.3) = \frac{f(1.4) - f(1.2)}{2 \times 0.1} = \frac{16.44465 - 11.02318}{2 \times 0.1} = 27.10735$$

$$f'(1.4) = \frac{-3f(1.4) + 4f(1.3) - f(1.2)}{2 \times (-0.1)} = \frac{-3 \times 16.44465 + 4 \times 13.46374 - 11.02318}{2 \times (-0.1)} = 32.51085$$

故：$f'(1.1) = 17.769705,\ f'(1.2) = 22.193635,\ f'(1.3) = 27.10735,\ f'(1.4) = 32.51085$

b.

$$f'(8.1) = \frac{-3f(8.1) + 4f(8.3) - f(8.5)}{2 \times 0.2} = \frac{-3 \times 16.94410 + 4 \times 17.56492 - 18.19056}{2 \times 0.2} = 3.09205$$

$$f'(8.3) = \frac{f(8.5) - f(8.1)}{2 \times 0.2} = \frac{18.19056 - 16.94410}{2 \times 0.2} = 3.11615$$

$$f'(8.5) = \frac{f(8.7) - f(8.3)}{2 \times 0.2} = \frac{18.82091 - 17.56492}{2 \times 0.2} = 3.139975$$

$$f'(8.7) = \frac{-3f(8.7) + 4f(8.5) - f(8.3)}{2 \times (-0.2)} = \frac{-3 \times 18.82091 + 4 \times 18.19056 - 17.56492}{2 \times (-0.2)} = 3.169525$$

故：$f'(8.1) = 3.09205,\ f'(8.3) = 3.11615,\ f'(8.5) = 3.139975,\ f'(8.7) = 3.169525$

Problem 2

解： 由题意：

$$M = N(h) + K_1h^2 + K_2h^4 + O(h^6) \tag{1}$$

$$M = N(\frac{h}{3}) + \frac{1}{9}K_1h^2 + \frac{1}{81}K_2h^4 + O(h^6) \tag{2}$$

$$M = N(\frac{h}{9}) + \frac{1}{81}K_1h^2 + \frac{1}{9^4}K_2h^4 + O(h^6) \tag{3}$$

以(2) × 9 − (1), 可得：

$$8M = -N(h) + 9N(\frac{h}{3}) - \frac{8}{9}K_2h^4 + O(h^6) \tag{4}$$

以(3) × 81 − (1), 可得：

$$80M = -N(h) + 81N(\frac{h}{9}) - \frac{80}{81}K_2h^4 + O(h^6) \tag{5}$$

以(5) − (4) × $\frac{10}{9}$, 可得：

$$\frac{640}{9}M = \frac{1}{9}N(h) - 10N(\frac{h}{3}) + 81N(\frac{h}{9}) + O(h^6) \tag{6}$$

即：

$$M = \frac{1}{640}N(h) - \frac{9}{64}N(\frac{h}{3}) + \frac{729}{640}N(\frac{h}{9}) + O(h^6) \tag{7}$$

Problem 3

解： a. (1)梯形：

$$\int_{-0.25}^{0.25}(\cos x)^2dx = \frac{0.5}{2}[(\cos 0.25)^2 + (\cos(-0.25))^2] = 0.469396$$

(2)Simpson:

$$\int_{-0.25}^{0.25}(\cos x)^2dx = \frac{0.25}{3}[2(\cos 0.25)^2 + 4 \times \cos 0] = 0.489799$$

b. (1)梯形：

$$\int_{-0.5}^{0} x\ln(x+1)dx = \frac{0.5}{2}[-0.5\ln 0.5 + 0] = \frac{\ln 2}{8} = 0.086643$$

(2)Simpson:

$$\int_{-0.5}^{0} x\ln(x+1)dx = \frac{0.25}{3}[-0.5\ln 0.5 + 4 \times (-0.25\ln 0.75) + 0] = 0.052855$$

c. (1)梯形：令$f(x) = (\sin x)^2 - 2x\sin x + 1$, 则:

$$\int_{0.75}^{1.3}[(\sin x)^2 - 2x\sin x + 1]dx = \frac{0.55}{2}[f(0.75) + f(1.3)] = -0.03702425$$

(2)Simpson:

$$\int_{0.75}^{1.3}[(\sin x)^2 - 2x\sin x + 1]dx = \frac{0.55}{6}[f(0.75) + 4 \times f(1.025) + f(1.3)] = -0.02027159$$

d. (1)梯形：

$$\int_{e}^{e+1} \frac{1}{x\ln x}dx = \frac{1}{2}[\frac{1}{e} + \frac{1}{(e+1)\ln(e+1)}] = 0.2863342$$

(2)Simpson:

$$\int_{e}^{e+1} \frac{1}{x\ln x}dx = \frac{1}{6}\Big[\frac{1}{e} + \frac{4}{(e+0.5)\ln(e+0.5)} + \frac{1}{(e+1)\ln(e+1)}\Big] = 0.2726704$$

Problem 4

解： a. 代码如下：

```
#include<stdio.h>
#include<math.h>

double f(double x);

int main()
{
        double a=-1,b=1,h=2;
        double r[3][3];
        int i,j;
        r[0][0]=h/2*(f(a)+f(b));
        r[1][0]=h/4*(f(a)+2*f(0)+f(b));
        r[2][0]=h/8*(f(a)+2*(f(-0.5)+f(0)+f(0.5))+\
                f(b));
        for (i=1;i<3;i++){
                for (j=1;j<3;j++){
                        r[j][i]=r[j][i-1]+(r[j]\
                [i-1]-r[j-1][i-1])/(pow(4,i)-1);
                }
        }
        printf("%.8lf",r[2][2]);
 }

double f(double x)
 {
        return pow(cos(x),2);
 }
```

可得: $R_{3,3} = 1.45281435$.

b. 代码如下：

```
#include<stdio.h>
#include<math.h>

double f(double x);

```

```
int main()
{
        double a=-0.75,b=0.75,h=1.5;
        double r[3][3];
        int i,j;
        r[0][0]=h/2*(f(a)+f(b));
        r[1][0]=h/4*(f(a)+2*f(0)+f(b));
        r[2][0]=h/8*(f(a)+2*(f(-0.375)+f(0)+\
                        f(0.375))+f(b));
        for (i=1;i<3;i++){
                for (j=1;j<3;j++){
                        r[j][i]=r[j][i-1]+(r[j]\
                [i-1]-r[j-1][i-1])/(pow(4,i)-1);
                }
        }
        printf("%.8lf",r[2][2]);
 }

double f(double x)
 {
        return x*log(x+1);
 }
```

可得: $R_{3,3} = 0.32795861$.

c. 代码如下:

```
#include<stdio.h>
#include<math.h>

double f(double x);

int main()
{
        double a=1,b=4,h=3;
        double r[3][3];
        int i,j;
        r[0][0]=h/2*(f(a)+f(b));
        r[1][0]=h/4*(f(a)+2*f(2.5)+f(b));
        r[2][0]=h/8*(f(a)+2*(f(1.75)+f(2.5)+f(3.25)\
```

```
                                )+f(b));
        for (i=1;i<3;i++){
                for (j=1;j<3;j++){
                        r[j][i]=r[j][i-1]+(r[j]\
                [i-1]-r[j-1][i-1])/(pow(4,i)-1);
                }
        }
        printf("%.8lf",r[2][2]);
 }

double f(double x)
 {
        return pow(sin(x),2)-2*x*sin(x)+1;
 }
```

可得: $R_{3,3} = 1.38706251$.

d. 代码如下：

```
#include<stdio.h>
#include<math.h>

#define E 2.718281828459

double f(double x);

int main()
{
        double a=E,b=2*E,h=E;
        double r[3][3];
        int i,j;
        r[0][0]=h/2*(f(a)+f(b));
        r[1][0]=h/4*(f(a)+2*f(1.5*E)+f(b));
        r[2][0]=h/8*(f(a)+2*(f(1.25*E)+f(1.5*E)+\
                        f(1.75*E))+f(b));
        for (i=1;i<3;i++){
                for (j=1;j<3;j++){
                        r[j][i]=r[j][i-1]+(r[j]\
                [i-1]-r[j-1][i-1])/(pow(4,i)-1);
                }
```

```
        }
        printf("%.8lf",r[2][2]);
 }

double f(double x)
 {
        return 1/(x*log(x));
 }
```

可得: $R_{3,3} = 0.52681555$.

Problem 5

解： a. 代码如下：

```
#include<stdio.h>
#include<math.h>

double f(double x1,double x2);

int main()
{
        double a,b,a0,t,y,h;
        int i,n;
        scanf("%lf,%lf,%lf,%lf",&a,&b,&h,&a0);
        n=(b-a)/h;
        y=a0;
        t=a;
        printf("(%.8lf, %.8lf)\n",t,y);
        for (i=1;i<=n;i++){
                y+=h*f(t,y);
                t=a+h*i;
                printf("(%.8lf, %.8lf)\n",t,y);
        }
}

double f(double x1,double x2)
{
        return x2/x1-pow((x2/x1),2);
}
```

可得:

n	t_n	y_n
0	1.00000000	1.00000000
1	1.10000000	1.00000000
2	1.20000000	1.00826446
3	1.30000000	1.02168947
4	1.40000000	1.03851473
5	1.50000000	1.05766819
6	1.60000000	1.07846109
7	1.70000000	1.10043216
8	1.80000000	1.12326205
9	1.90000000	1.14672360
10	2.00000000	1.17065157

b. 代码如下：

```c
#include<stdio.h>
#include<math.h>

double f(double x1,double x2);

int main()
{
        double a,b,a0,t,y,h;
        int i,n;
        scanf("%lf,%lf,%lf,%lf",&a,&b,&h,&a0);
        n=(b-a)/h;
        y=a0;
        t=a;
        printf("(%.8lf, %.8lf)\n",t,y);
        for (i=1;i<=n;i++){
                y+=h*f(t,y);
                t=a+h*i;
                printf("(%.8lf, %.8lf)\n",t,y);
        }
}

double f(double x1,double x2)
{
        return 1+x2/x1+pow((x2/x1),2);
}
```

可得：

n	t_n	y_n
0	1.00000000	0.00000000
1	1.20000000	0.20000000
2	1.40000000	0.43888889
3	1.60000000	0.72124276
4	1.80000000	1.05203803
5	2.00000000	1.43725115
6	2.20000000	1.88426081
7	2.40000000	2.40226959
8	2.60000000	3.00283716
9	2.80000000	3.70060070
10	3.00000000	4.51427743

Problem 6

解： 由题意，设$\sin x$的（6,6）级帕德逼近为

$$r(x) = \frac{p_0 + p_1 x + p_2 x^2 + p_3 x^3 + p_4 x^4 + p_5 x^5 + p_6 x^6}{1 + q_1 x + q_2 x^2 + q_3 x^3 + q_4 x^4 + q_5 x^5 + q_6 x^6}$$

而$\sin x$的12阶泰勒级数为：

$$f(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 - \frac{1}{39916800}x^{11}$$

则：

$$a_0 = 0, a_1 = 1, a_2 = 0, a_3 = -\frac{1}{6}, a_4 = 0, a_5 = \frac{1}{120}, a_6 = 0, a_7 = -\frac{1}{5040},$$

$$a_8 = 0, a_9 = \frac{1}{362880}, a_{10} = 0, a_{11} = -\frac{1}{39916800}, a_{12} = 0$$

于是有程序：

```c
#include<stdio.h>
#include<math.h>
#include<stdlib.h>

//compute the factorial
double fact(double x);

double b[13][14];
double diff[18];

int main()
```

```
{
        double q[7]={1},p[7]={0};
        double a[13];
        double nume[13]
        {0,1,0,-1,0,1,0,-1,0,1,0,-1,0};
        double deno[13];
        double x,temp,sum;
        int m=6,n=6,i,j,k,flag=0;
        int N=12;
        for (i=0;i<=N;i++){
                deno[i]=fact(i);
        }
        for (i=0;i<=N;i++){
        //coefficients of the taylor polynomial
                a[i]=nume[i]/deno[i];
        }
        //get the matrix
        for (i=1;i<=N;i++){
                if (i<=n){
                        b[i][i]=1;
                }
                for (j=1;j<=i;j++){
                        if (j<=m){
                                b[i][n+j]=-a[i-j];
                        }
                }
                b[i][N+1]=a[i];
        }
        k=n+1;
        for (i=n+1;i<=N-1;i++){
                for (j=i;j<=N;j++){
                        if (fabs(b[k][i])\
                        <fabs(b[j][i])){
                                k=j;
                        }
                }
                if (b[k][i]==0){
                        printf("Algorithm failed");
                        exit(0);
                }
                if (k!=i){
```

```
                        //exchange the entry
                        for (j=i;j<=N+1;j++){
                                temp=b[i][j];
                                b[i][j]=b[k][j];
                                b[k][j]=temp;
                        }
                }
                for (j=i+1;j<=N;j++){
                        x=b[j][i]/b[i][i];
                        for (k=i+1;k<=N+1;k++){
                                b[j][k]-=x*b[i][k];
                        }
                        b[j][i]=0;
                }
        }
        if (b[N][N]==0){
                printf("Algorithm failed");
                exit(0);
        }
        //start to compute
        q[m]=b[N][N+1]/b[N][N];
        for (i=N-1;i>n;i--){
                sum=0;
                for (j=i+1;j<N+1;j++){
                        sum+=b[i][j]*q[j-n];
                }
                q[i-n]=(b[i][N+1]-sum)/b[i][i];
        }
        for (i=n;i>=1;i--){
                sum=0;
                for (j=n+1;j<=N;j++){
                        sum+=b[i][j]*q[j-n];
                }
                p[i]=b[i][N+1]-sum;
        }
        for (i=0;i<=n;i++){
                printf("p[%d]=%.8lf\n",i,p[i]);
        }
        for (i=0;i<=m;i++){
                printf("q[%d]=%.8lf\n",i,q[i]);
        }
```

```
        for (i=0;i<=17;i++){
                sum=0;
                for (j=0;j<=i&&j<=12;j++){
                        if (i-j<=6&&i-j>=0){
                                sum+=a[j]*q[i-j];
                        }
                }
//compute the coefficients of the difference
                if (i<=6){
                        diff[i]=p[i]-sum;
                }
                else {
                        diff[i]=sum;
                }
        }
        for (i=0;i<=17;i++){
        //considering the error of the conputer
                if (diff[i]>10e-9){
                        flag=1;
                        break;
                }
        }
        if (flag==0){
                printf("The formulas are eaqual.");
        }
}

double fact(double x)
{
        if (x==0||x==1){
                return 1;
        }
        else return x*fact(x-1);
}
```

可得: $p_0 = 0.00000000, p_1 = 1.00000000, p_2 = 0.00000000, p_3 = -0.12995655,$
$p_4 = 0.00000000, p_5 = 0.00290358, p_6 = 0.00000000, q_0 = 1.00000000,$
$q_1 = 0.00000000, q_2 = 0.03671011, q_3 = 0.00000000, q_4 = 0.00068860,$
$q_5 = 0.00000000, q_6 = 0.00000726$
且程序输出有"*The formulas are eaqual.*"

$$\therefore r(x)=\frac{x-0.12995655x^3+0.00290358x^5}{1+0.03671011x^2+0.00068860x^4+0.00000726x^6}$$

与正确答案相同, 且其与$\sin x$的12阶泰勒级数完全相同。

Problem 7

解： a. 由题意：$m=4$, 设多项式为$P(x)=a_0+a_1x$, 则：

$$E=\sum_{i=1}^{4}[y_i-P(x_i)]^2$$

$$=(6-a_0)^2+(8-a_0-2a_1)^2+(14-a_0-4a_1)^2+(20-a_0-5a_1)^2$$

$$\therefore \frac{\partial E}{\partial a_0}=48-4a_0-11a_1=0$$

同理可得：

$$11a_0+45a_1=172$$

可得：
$$\begin{pmatrix}a_0\\a_1\end{pmatrix}=\begin{pmatrix}4.54237288\\2.71186441\end{pmatrix}$$

$\therefore P(x)=4.54237288+2.71186441x$

b. 由a得:

$$P(0)=a_0=4.54237288,\ P(2)=9.96610169,$$

$$P(4)=15.38983051,\ P(5)=18.10169492$$

$$\therefore E=\sum_{i=1}^{4}[y_i-p(x_i)]^2=[6-P(0)]^2+[8-P(2)]^2+[14-P(4)]^2+[20-P(5)]^2$$

$$=11.52542373$$