Lecture 4.
Analog Communications

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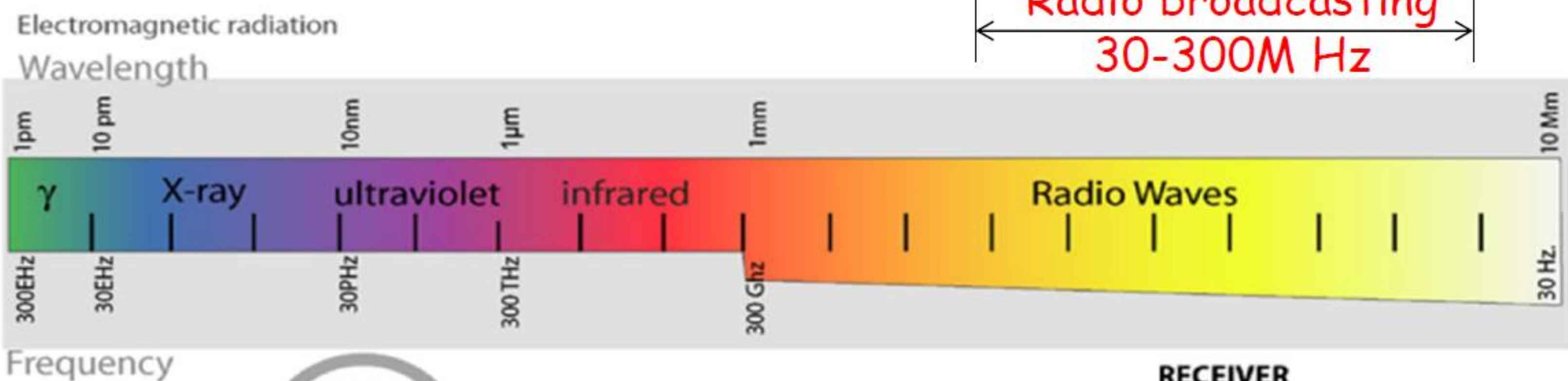
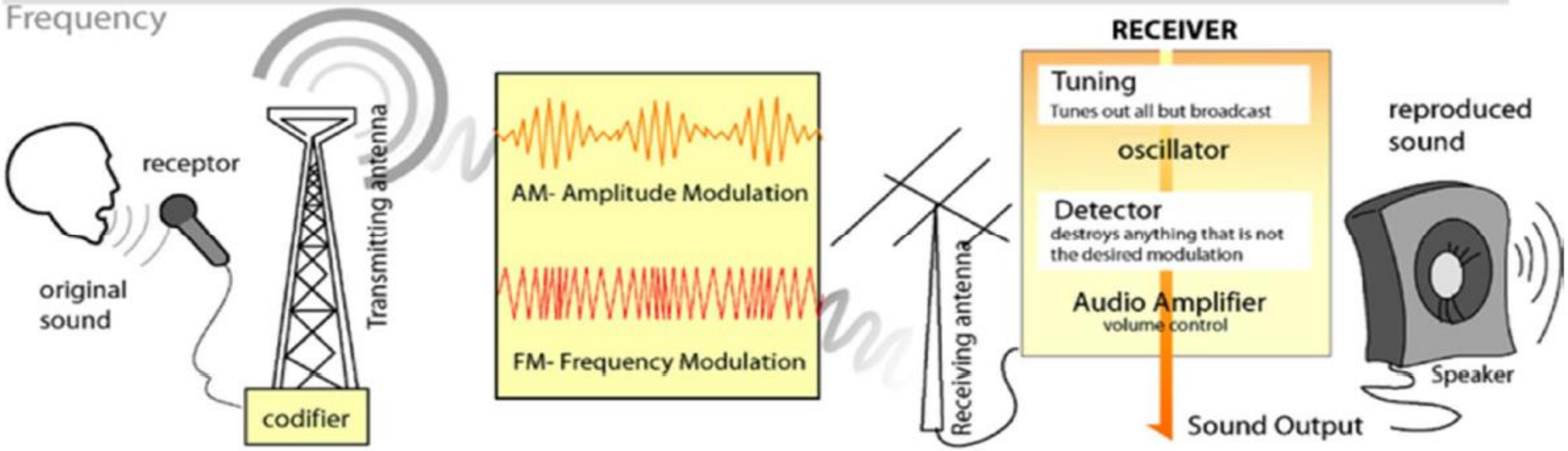
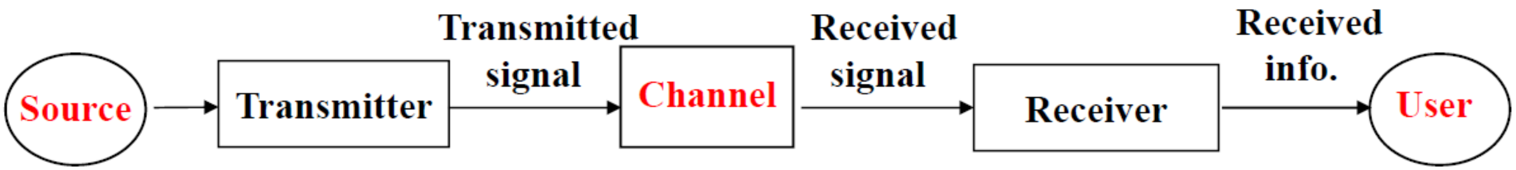
Radio broadcasting
30-300M Hz



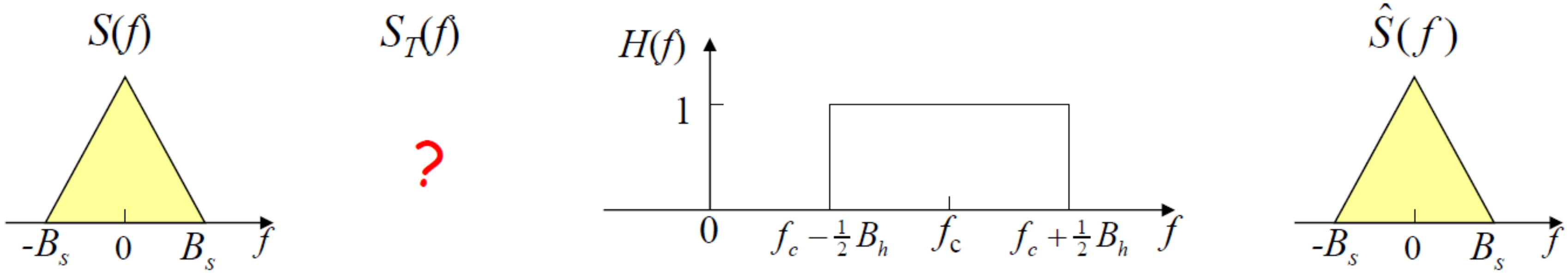


- Analog baseband signal
- Bandpass channel

Block Diagram of Analog Communications



- Analog baseband signal
- Bandpass channel
- Ignore noise



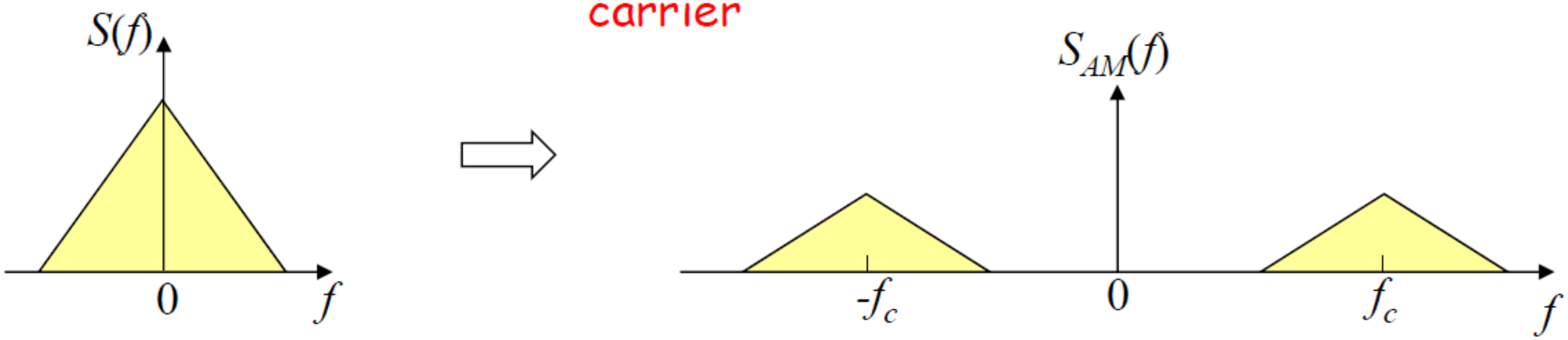
- For reliable communications, i.e., $\hat{S}(f) = S(f)$, all frequency components of the transmitted signal should pass through the channel, which requires:
  - Frequency components of transmitted signal should be centered at f_c;
  - The channel bandwidth B_h should be no smaller than the bandwidth of transmitted signal B_m.

Analog Modulation

Modulation property of Fourier Transform:

$$s(t)\cos(2\pi f_c t) \Leftrightarrow \tfrac{1}{2}[S(f-f_c)+S(f+f_c)]$$

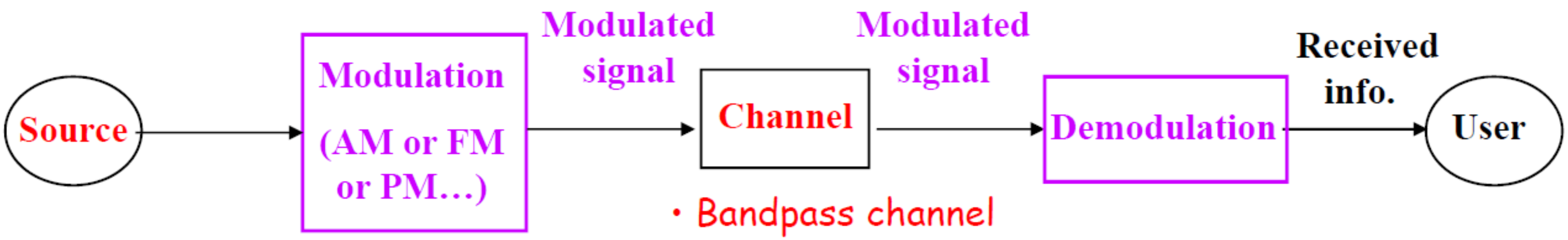
carrier



- Amplitude Modulation (AM) $s_{AM}(t) = A s(t)\cos(2\pi f_c t)$

- Phase Modulation (PM) $s_{PM}(t) = A\cos(2\pi f_c t + \alpha s(t))$
- Frequency Modulation (FM) $s_{FM}(t) = A\cos(2\pi(f_c t + k\int_{-\infty}^{t} s(\tau)d\tau))$

Analog Modulation



- Analog baseband signal
- Bandpass channel
- Ignore noise

- Bandwidth efficiency is an important performance metric, which is defined as:

$$\gamma \triangleq \frac{\text{Information Signal Bandwidth } B_s}{\text{Required Channel Bandwidth } B_h} = \frac{\text{Information Signal Bandwidth } B_s}{\text{Modulated Signal Bandwidth } B_m}$$

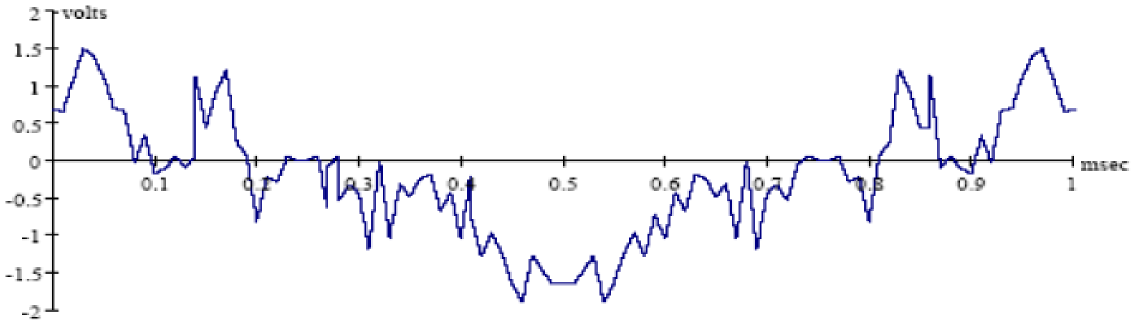
- Required channel bandwidth B_h = Modulated signal bandwidth B_m
- A higher γ indicates a better spectral utilization.

Analog Modulation –
Part I. Amplitude Modulation (AM)

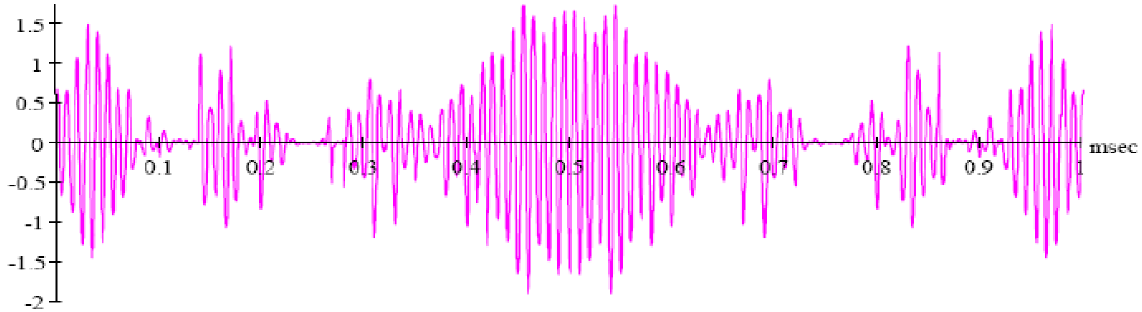
AM-DSB-SC -- Modulation

- Time Domain: $s_{AM-DSB-SC}(t) = As(t)\cos(2\pi f_c t)$

$s(t)$

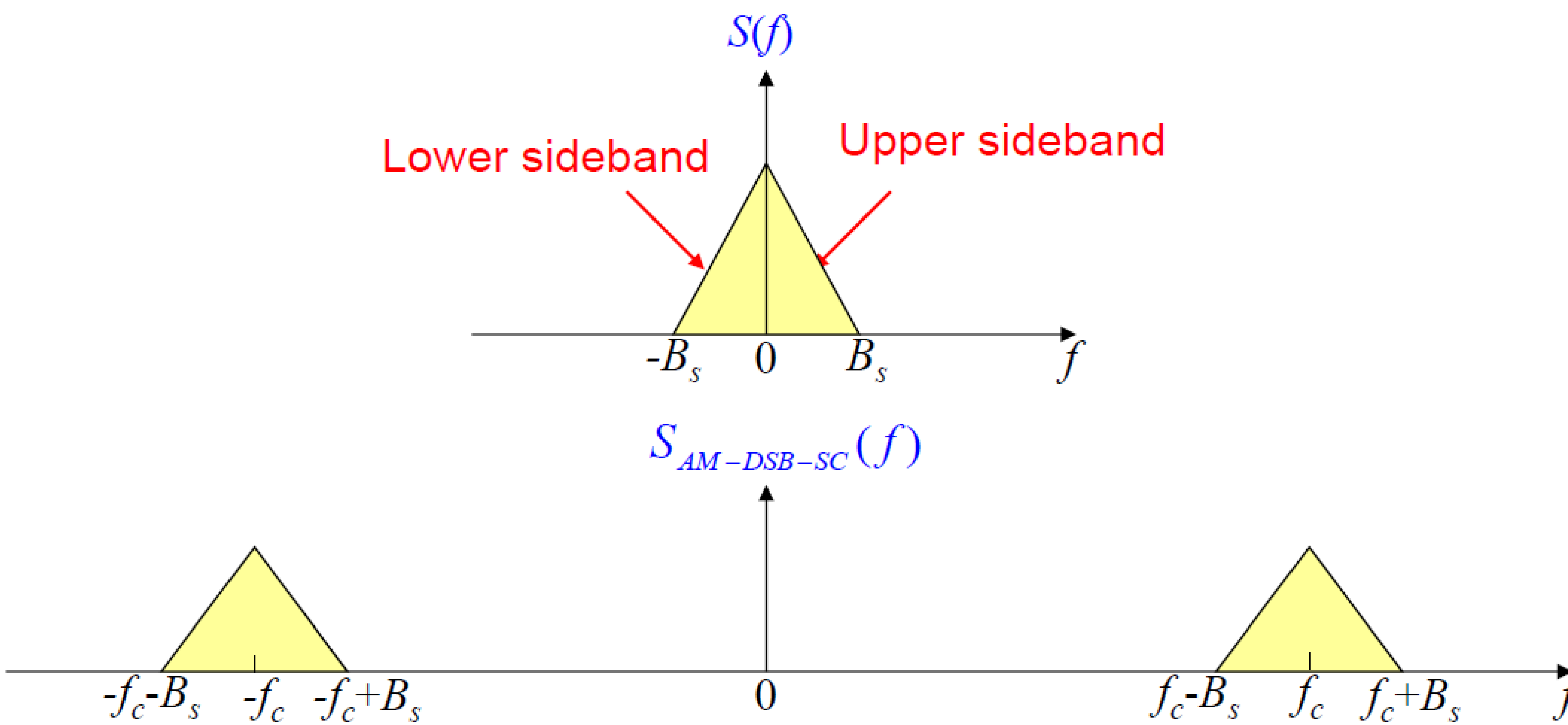


$s_{AM-DSB-SC}(t)$



AM-DSB-SC -- Modulation

- Frequency Domain: $S_{AM-DSB-SC}(f) = \frac{A}{2}[S(f-f_c)+S(f+f_c)]$



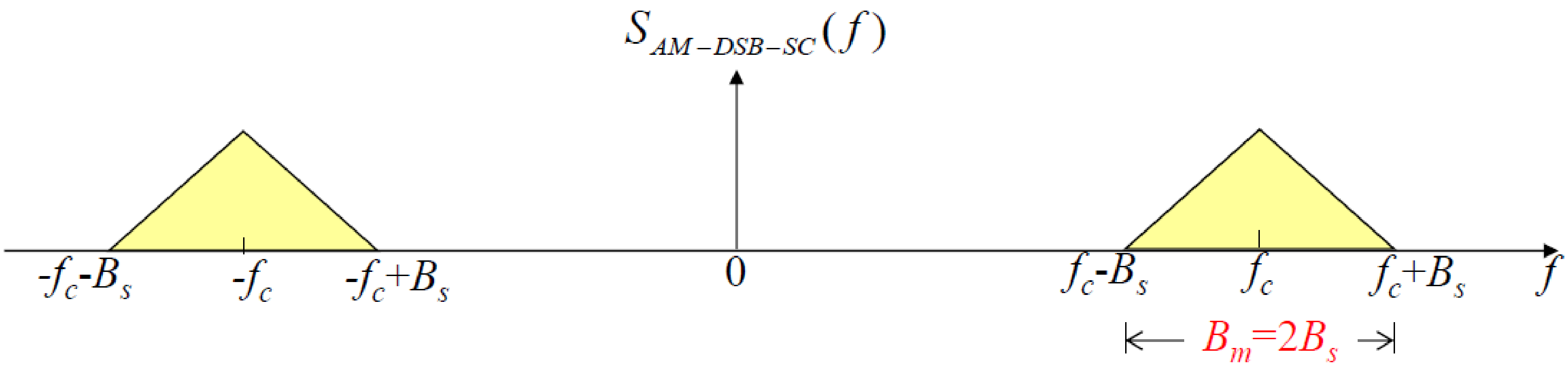
AM-DSB-SC: Amplitude Modulation-Double SideBand-Suppressed Carrier

Bandwidth Efficiency of AM-DSB-SC

- Bandwidth Efficiency :

$$\gamma = \frac{\text{Information Signal Bandwidth } B_s}{\text{Required Channel Bandwidth } B_h} = \frac{\text{Information Signal Bandwidth } B_s}{\text{Modulated Signal Bandwidth } B_m}$$

With AM-DSB-SC :



$$\boxed{\gamma_{AM-DSB-SC} = \frac{B_s}{2B_s} = 50\%}$$

AM-DSB-SC -- Demodulation

- Time Domain: $s_{AM-DSB-SC}(t) \overset{?}{\Rightarrow} s(t)$

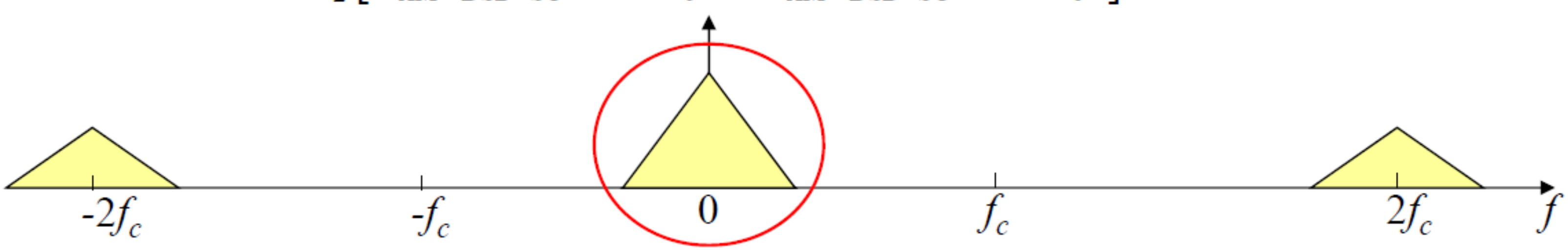
$$s_{AM-DSB-SC}(t)\cos(2\pi f_c t) = As(t)\cos(2\pi f_c t)\cos(2\pi f_c t) = 0.5As(t) + 0.5As(t)\cos(2\pi 2 f_c t)$$

- Frequency Domain: $S_{AM-DSB-SC}(f) \overset{?}{\Rightarrow} S(f)$

$$\tfrac{1}{2}\left[S_{AM-DSB-SC}(f-f_c)+S_{AM-DSB-SC}(f+f_c)\right] = \tfrac{A}{4}[S(f-2f_c)+S(f)] + \tfrac{A}{4}[S(f)+S(f+2f_c)]$$

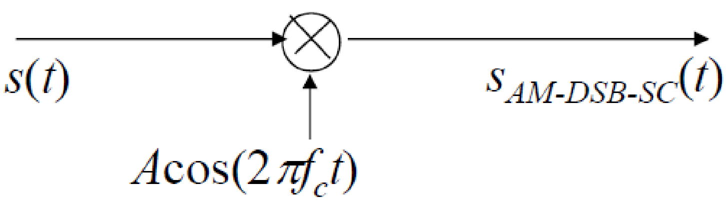
$$= \tfrac{A}{4}[S(f-2f_c)+S(f+2f_c)] + \tfrac{A}{2}S(f)$$

$$\tfrac{1}{2}\left[S_{AM-DSB-SC}(f-f_c)+S_{AM-DSB-SC}(f+f_c)\right]$$



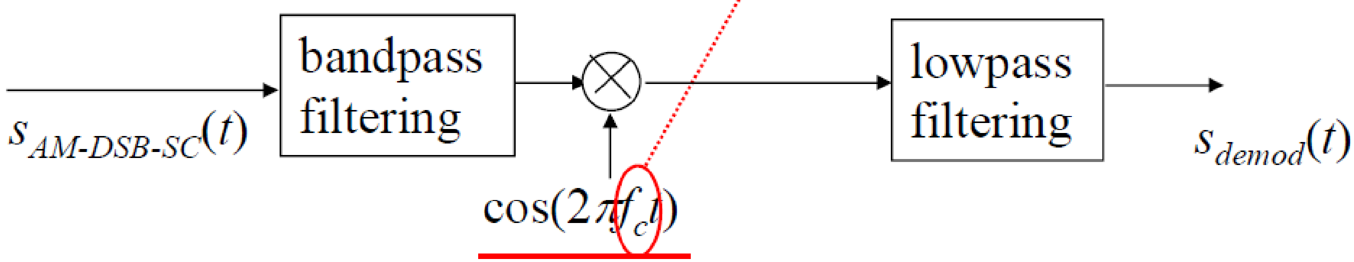
Modulator and Demodulator of AM-DSB-SC

Modulator



What if there is a frequency error?

Demodulator



Coherent Demodulation: the demodulator requires a reference signal which has exactly the same frequency and phase as the carrier signal.

Frequency Error of Coherent Demodulator

Consider that the reference signal has a small frequency error, Δf.

$$w(t) = As(t)\cos(2\pi f_c t)\cos(2\pi(f_c + \Delta f)t)$$
$$= 0.5As(t)\left(\underline{\cos(2\pi\Delta f t)} + \cos(2\pi(2f_c + \Delta f)t)\right)$$

After lowpass filtering, we have

$$0.5As(t)\cos(2\pi\Delta f t)$$

$$\cos(2\pi\Delta f t) = 1 \quad \text{when} \quad \Delta f = 0$$

$$\cos(2\pi\Delta f t) \text{ changes with } t \text{ when } \Delta f \neq 0$$

The performance of AM-DSB-SC is sensitive to the frequency error of the reference signal.

Pros and Cons of AM-DSB-SC

- Straightforward

- Sensitive to frequency and phase error of the reference signal (coherent demodulation)
- Bandwidth inefficient ($\gamma_{AM\text{-}DSB\text{-}SC}$=50%)

AM-DSB-C

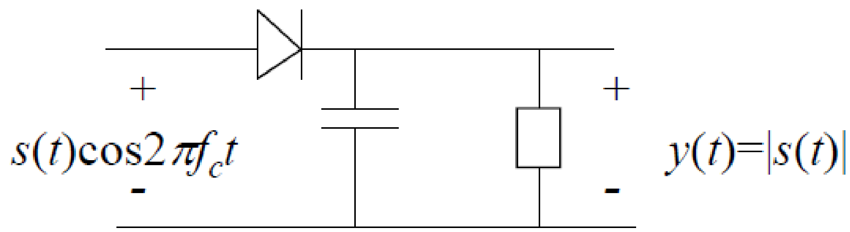
Envelope and Envelope Detector

- Envelope

  Consider a signal $s(t)\cos 2\pi f_c t$. If $s(t)$ varies slowly in comparison with the carrier $\cos 2\pi f_c t$, the envelope of $s(t)\cos 2\pi f_c t$ is $|s(t)|$.

  - The envelope $|s(t)| = s(t)$ if $s(t) \geq 0$.

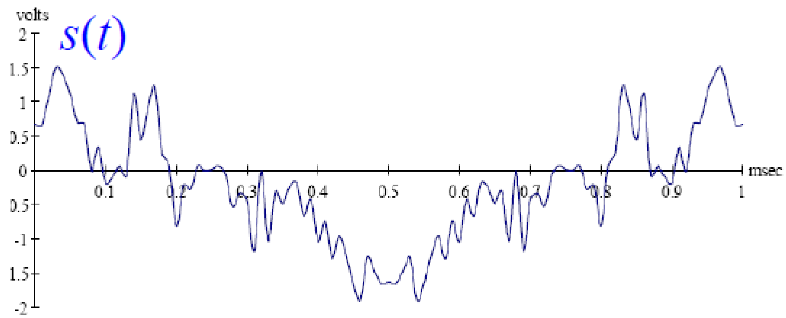
  - Envelope Detector:

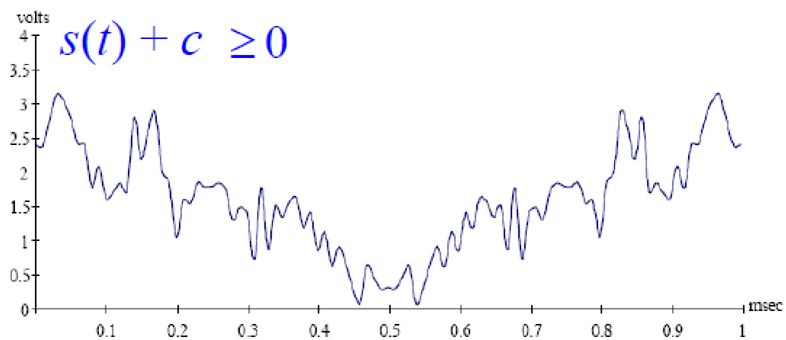


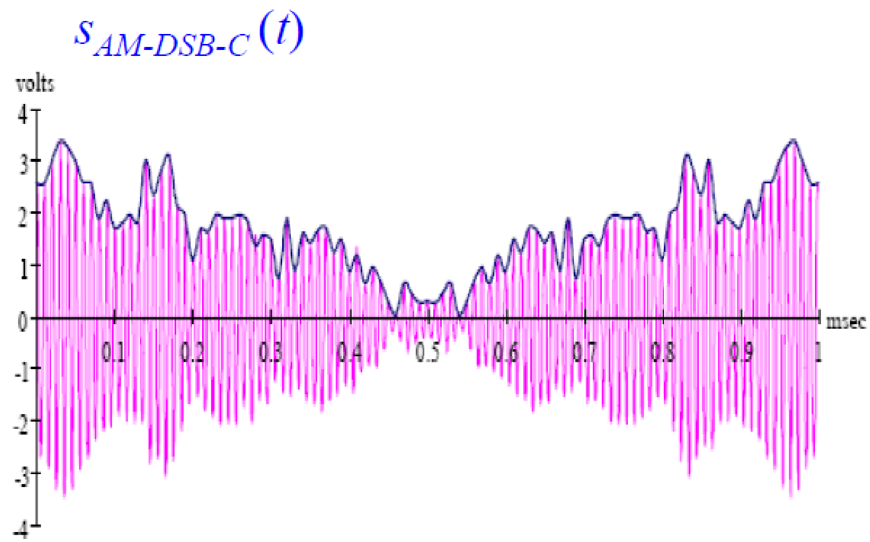
How to apply the Envelope Detector to AM systems?

AM-DSB-C -- Modulation

- Time Domain: $s_{AM-DSB-C}(t) = A\left(s(t)+c\right)\cos(2\pi f_c t)$







c is a dc offset to ensure $s(t)+c \geq 0$ for any time t.

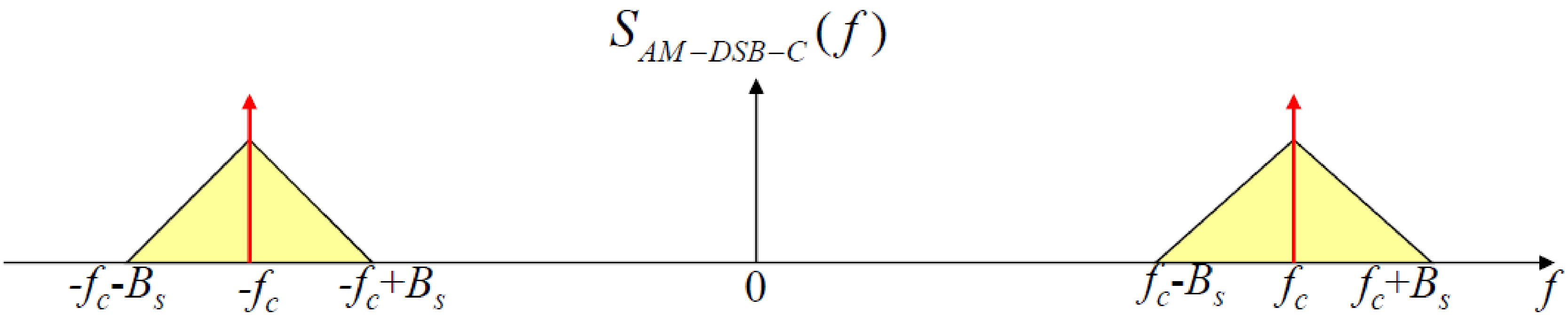
AM-DSB-C -- Modulation

- Frequency Domain:

$$s_{AM-DSB-C}(t) = A\big(s(t)+c\big)\cos(2\pi f_c t) = As(t)\cos(2\pi f_c t) + Ac\cos(2\pi f_c t)$$

$$\Leftrightarrow$$

$$S_{AM-DSB-C}(f) = \tfrac{A}{2}[S(f-f_c)+S(f+f_c)] + \tfrac{Ac}{2}[\delta(f-f_c)+\delta(f+f_c)]$$



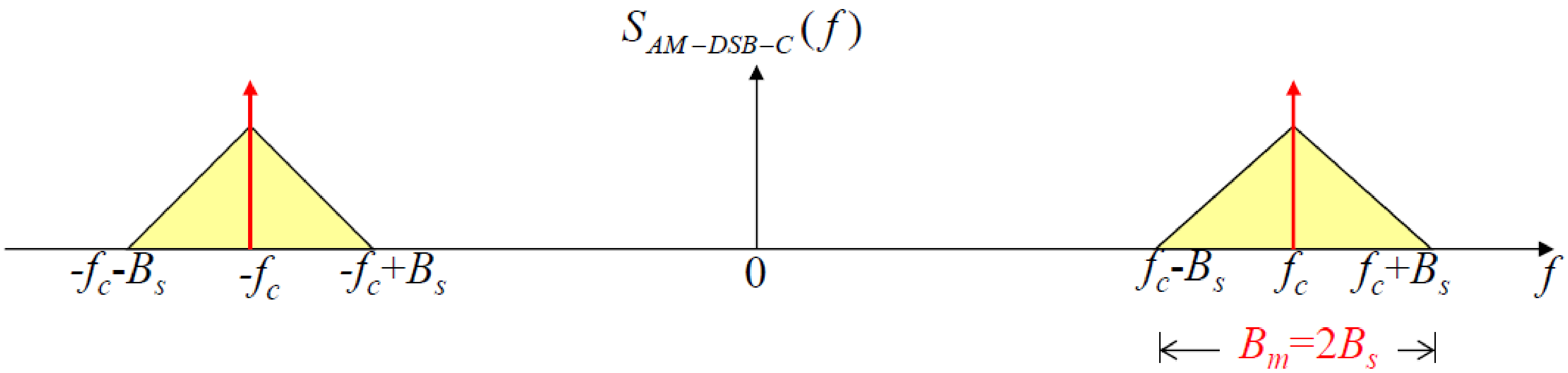
AM-DSB-C: Amplitude Modulation-Double SideBand-Carrier

Bandwidth Efficiency of AM-DSB-C

- Bandwidth Efficiency :

$$\gamma = \frac{\text{Information Signal Bandwidth } B_s}{\text{Required Channel Bandwidth } B_h} = \frac{\text{Information Signal Bandwidth } B_s}{\text{Modulated Signal Bandwidth } B_m}$$

With AM-DSB-C :



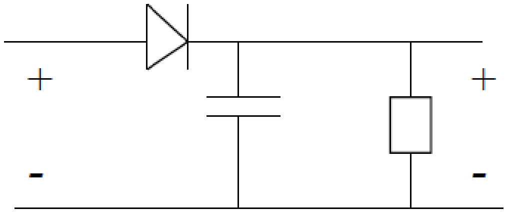
$$\boxed{\gamma_{AM-DSB-C} = \frac{B_s}{2B_s} = 50\% = \gamma_{AM-DSB-SC}}$$

AM-DSB-C -- Demodulation

- Time Domain: $s_{AM-DSB-C}(t) \overset{?}{\Rightarrow} s(t)$

Envelope Detector:

$$s_{AM-DSB-C}(t) = A\left(s(t)+c\right)\cos(2\pi f_c t)$$



$$y(t) = |A\left(s(t)+c\right)| = A(s(t)+c)$$

$$(s(t)+c \geq 0)$$

Non-coherent demodulation (no need to generate a reference signal)

- Apply $s_{AM-DSB-C}(t)$ to an envelope detector.
- Remove the dc offset c.

- Simple
- Robust
- Any price to pay?

More about AM-DSB-C

- Define the power efficiency of an AM-DSB-C system as:

$$\eta = \frac{\text{power of information signal } s(t)}{\text{power of modulating signal } s(t)+c} = \frac{P_s}{c^2 + P_s} \le 50\%$$

  - η increases as the dc offset c decreases.
  - to ensure $s(t)+c \ge 0,\quad c^2 \ge P_s$

- Define the modulation index of an AM-DSB-C system as:

$$m = \frac{\max[s(t)+c]-\min[s(t)+c]}{\max[s(t)+c]+\min[s(t)+c]} = \frac{\max s(t) - \min s(t)}{\max s(t) + \min s(t) + 2c}$$

  - m increases as the dc offset c decreases.

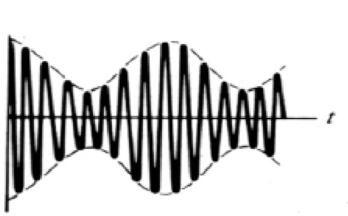
Can m be arbitrarily large?

Modulation Index m of AM-DSB-C

$$m = \frac{\max[s(t)+c]-\min[s(t)+c]}{\max[s(t)+c]+\min[s(t)+c]} = \frac{\max s(t)-\min s(t)}{\max s(t)+\min s(t)+2c}$$

$m < 1$

when $\min(s(t)+c) > 0$



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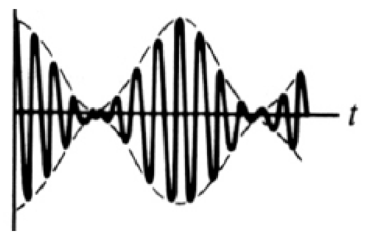
$m = 1$

when $\min(s(t)+c) = 0$



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$m > 1$

when $\min(s(t)+c) < 0$

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m should not exceed 1 to avoid over-modulation.

Pros and Cons of AM-DSB-C

- Simple and robust receiver design (non-coherent demodulation)

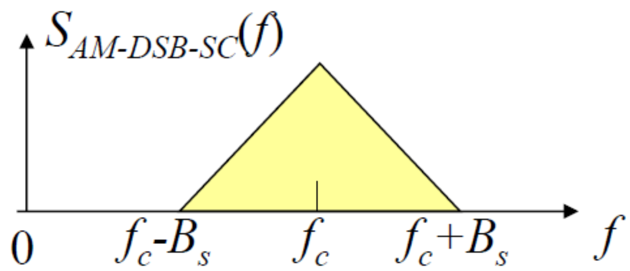
Commercial radio broadcasting

- Cost of power efficiency ($\eta<50\%$)
- Bandwidth inefficient ($\gamma_{AM\text{-}DSB\text{-}C}=\gamma_{AM\text{-}DSB\text{-}SC}=50\%$)

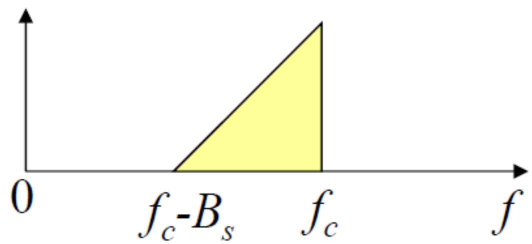
AM-SSB and AM-VSB

How to Improve Bandwidth Efficiency?

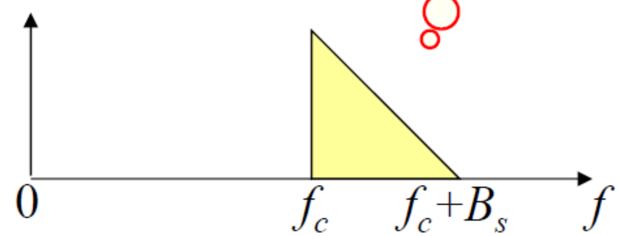
Double sideband (DSB)



Upper sideband carries the same information as the lower sideband!



Lower sideband



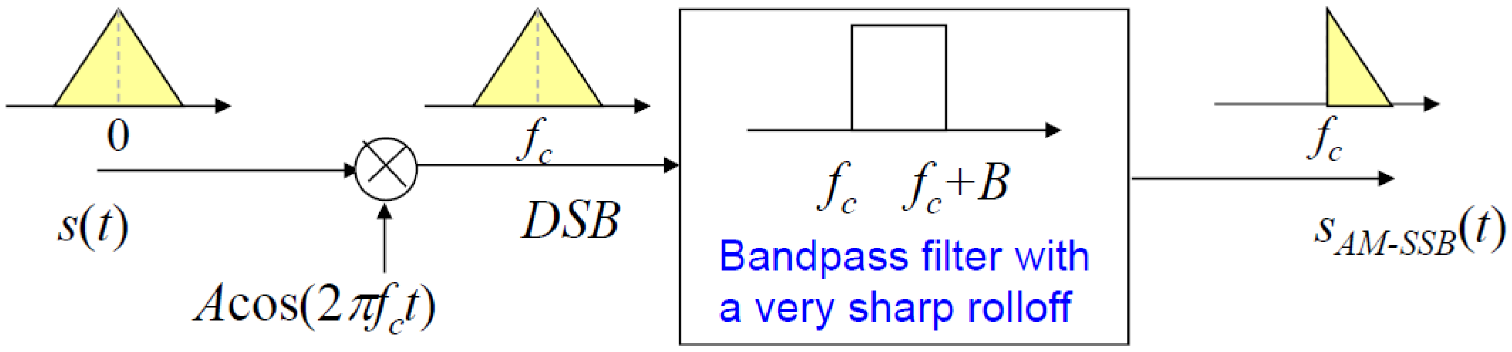
Upper sideband

Use a bandpass filter to select the desired sideband, and only transmit the desired sideband.

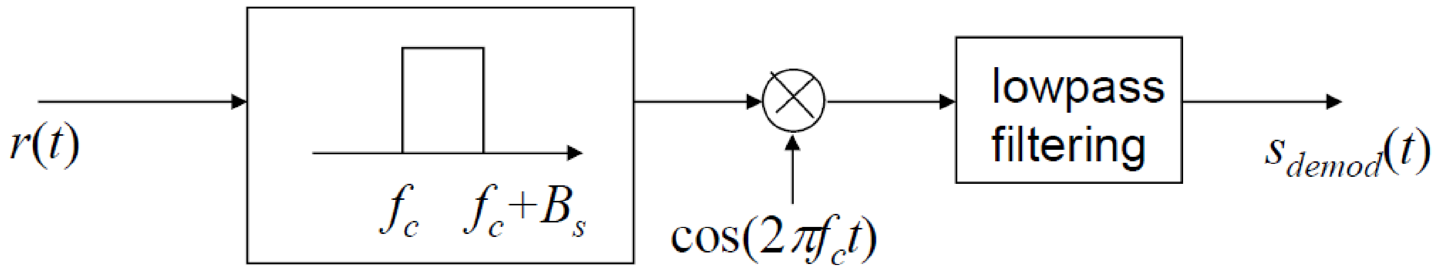
AM-SSB

Amplitude Modulation-Single SideBand (AM-SSB)

Modulation (Frequency Discrimination Method)



Demodulation (coherent)



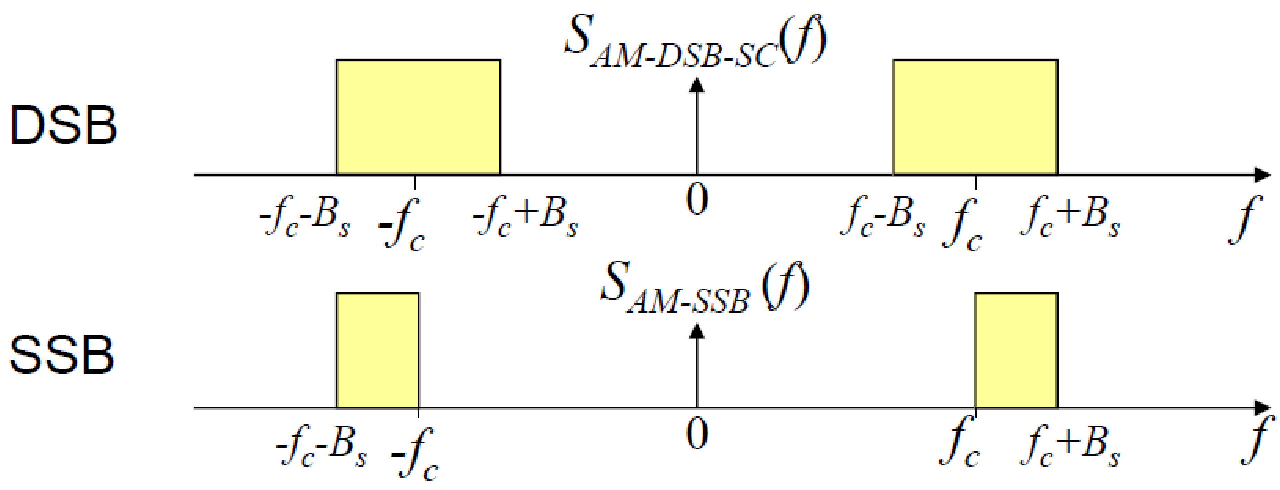
Pros and Cons of AM-SSB

- Bandwidth efficient ($\gamma_{AM\text{-}SSB}$=100%)

Mobile communications, military communications, …

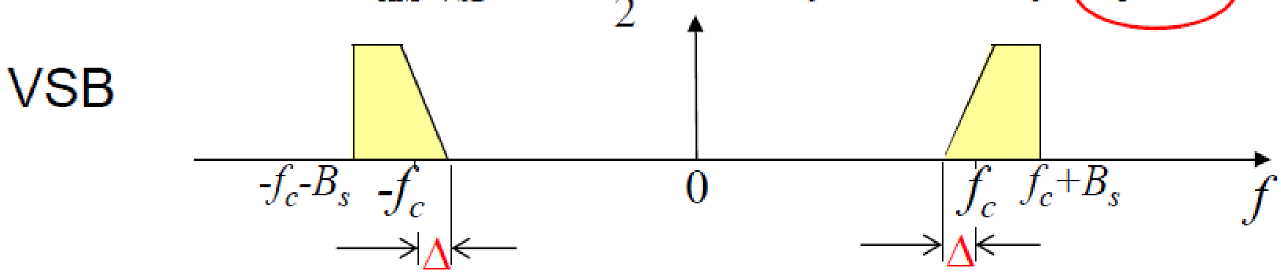
- High requirement on filtering (sharp rolloff)
- Sensitive to frequency and phase error of the reference signal (coherent demodulation)

AM-VSB



Amplitude Modulation-Vestigial SideBand (AM-VSB)

$$S_{AM-VSB}(f) = \frac{1}{2}[S(f-f_c)+S(f+f_c)]H_s(f)$$



Allow a small portion (or *vestige*) of the lower sideband, Δ, along with the upper sideband.

Spectrum of AM-VSB Signal

modulation:

$$S_{AM-VSB}(f)=\frac{1}{2}[S(f-f_c)+S(f+f_c)]H_s(f)$$
$$=\frac{1}{2}S(f-f_c)H_s(f)+\frac{1}{2}S(f+f_c)H_s(f)$$

Demodulation:

$$S_{\text{demod}}(f)=\frac{1}{2}[S_{AM-VSB}(f-f_c)+S_{AM-VSB}(f+f_c)]$$
$$=\frac{1}{4}[S(f-2f_c)+S(f)]H_s(f-f_c)+\frac{1}{4}[S(f)+S(f+2f_c)]H_s(f+f_c)$$
$$=\frac{1}{4}S(f)[H_s(f-f_c)+H_s(f+f_c)]+\frac{1}{4}S(f-2f_c)H_s(f-f_c)+\frac{1}{4}S(f+2f_c)H_s(f+f_c)$$

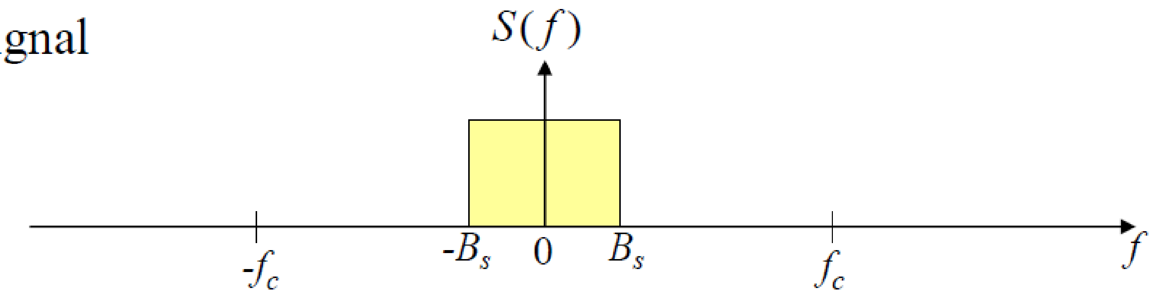
After lowpass filtering:

$$S_{\text{demod}}(f)=S(f)[\underline{H_s(f-f_c)+H_s(f+f_c)}]$$

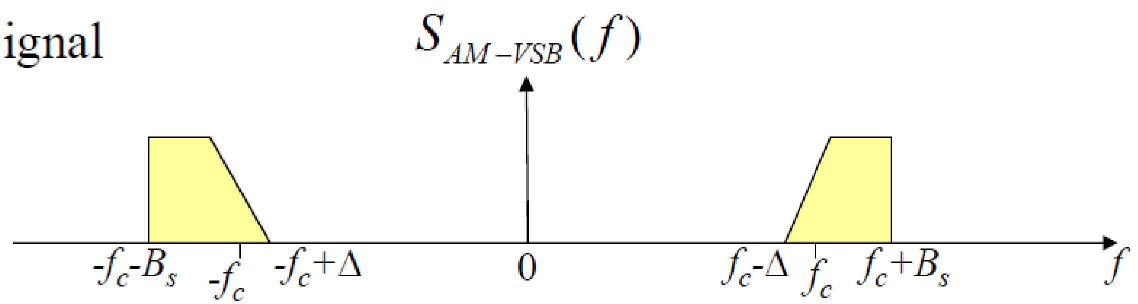
$$H_s(f-f_c)+H_s(f+f_c)=k \qquad 0\le|f|\le B$$

Spectrum of AM-VSB Signal

Baseband signal

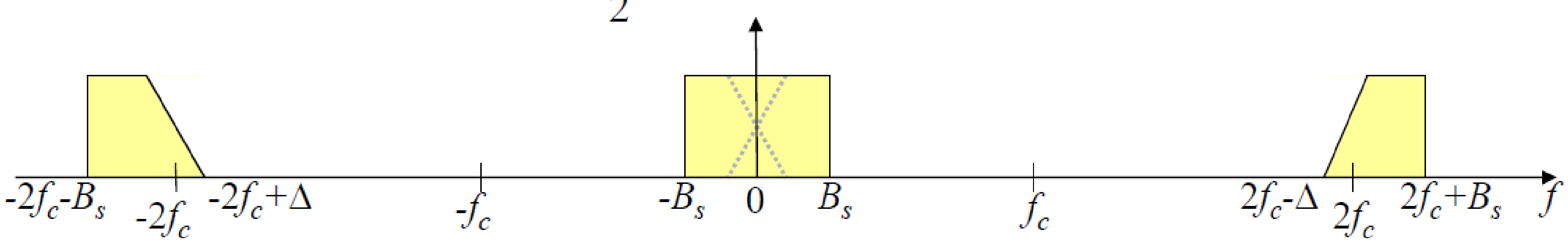


Modulated Signal

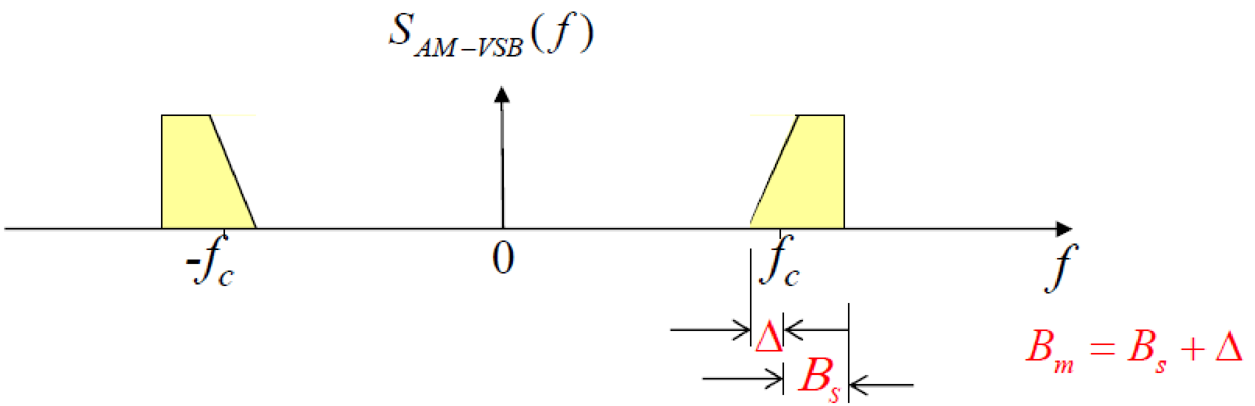


De-modulation

$$S_{\text{demod}}(f) = \frac{1}{2}[S_{AM-VSB}(f - f_c) + S_{AM-VSB}(f + f_c)]$$



Tradeoff between Complexity and Bandwidth Efficiency



$$\boxed{50\% < \gamma_{AM-VSB} = \frac{B_s}{B_s + \Delta} < 100\%}$$

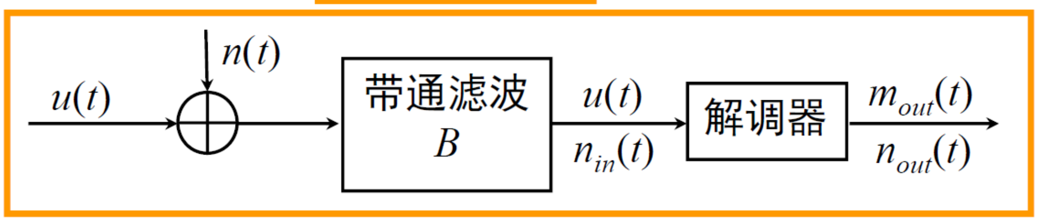
$0 < \Delta < B_s$

- The bandwidth efficiency γ decreases as the vestige Δ increases.
- The larger vestige Δ, the lower receiver complexity.

Signal-to-Noise-Ratio (SNR)

- The performance of analog communications is determined by the output SNR
- The SNR gain G reflects the ability of the system to overcome the noise effect

$$G = \frac{SNR_{out}}{SNR_{in}}$$



$$(SNR)_{in} = \frac{E\left[u^2(t)\right]}{E\left[n_{in}^2(t)\right]} \qquad (SNR)_{out} = \frac{E\left[m_{out}^2(t)\right]}{E\left[n_{out}^2(t)\right]} \qquad G = \frac{SNR_{out}}{SNR_{in}}$$

Summary of AM

AM-DSB-SC	Power efficient	Coherent demodulation	Bandwidth inefficient γ=50%
AM-DSB-C Commercial radio broadcasting	Power inefficient	Non-coherent demodulation (simple and robust)	Bandwidth inefficient γ=50%
AM-SSB Mobile and military communications	Power efficient	Coherent demodulation	Bandwidth efficient γ=100%
AM-VSB Public television systems	Power efficient	Coherent demodulation	Tradeoff between bandwidth and complexity $50\% < \gamma = \frac{B_s}{B_s + \Delta} < 100\%$