6.630 Solution to Problem Set 9

Solution P9.1

$k_c = \pi/d\,,\ f_c = 1.88\ \text{kHz}.$

Solution P9.2

(a) $m < \frac{d\omega\sqrt{\mu_o\epsilon_o}}{\pi} = \frac{d\omega}{\pi c} = \frac{30\times10^9\times2\sqrt{3}\pi\times10^{-2}}{3\times10^8\pi} = 3.46$.
The possible guided modes are TM_m ($m = 0, 1, 2, 3$) and TE_m ($m = 1, 2, 3$).

(b) For TM_2 mode,

$$\overline{E} = \frac{H_o}{\epsilon_o\omega}\left[\hat{x}k_z\cos\frac{2\pi x}{d}e^{ik_z z} - \hat{z}\frac{2\pi i}{d}\sin\frac{2\pi x}{d}e^{ik_z z}\right]$$

(c) $v_p = \frac{\omega}{k_z} = \frac{30\times10^9}{100\sqrt{2/3}} = 3.67\times10^8 (m/s).$

$v_g = \frac{d\omega}{dk_z} = \frac{k_z}{\omega\mu_o\epsilon_o} = \frac{c^2k_z}{\omega} = \frac{c^2}{v_p} = \frac{(3\times10^8)^2}{3.67\times10^8} = 2.45\times10^8 (m/s).$

(d) Since $\epsilon_1 = 3\epsilon_o > \epsilon_0$, there is no total reflection for any modes

(e) The Brewster angle for the medium is given by $\theta_B = \tan^{-1}\sqrt{3} = 60^\circ$. Now let's see if there is some mode that is incident at 60° . We find that,

$$k_x = \frac{m\pi}{d} = 100K_o\sin 60^\circ \Rightarrow \frac{100m\pi}{\sqrt{3}} = 100(2\pi)\frac{\sqrt{3}}{2} \Rightarrow m = 3$$

So the TM_3 wave can be totally transmitted.

Solution P9.3

Since cutoff occurs when $k_z = k_o$, i.e. Goos-Hanchen shift is zero, the guidance condition now is $k_x d = m\pi = \sqrt{k_{cm}^2 - k_o^2}d$. Noting $k_o^2 = k_{cm}^2\frac{\mu_o\epsilon_o}{\mu\epsilon}$, we can get $k_{cm} = \frac{m\pi}{d\sqrt{1-\frac{\mu_o\epsilon_o}{\mu\epsilon}}}$. Let $k > k_{cm}$, we can get $m = 0, 1, 2$. The propagating modes are TE_0, TM_0 , TE_1, TM_1 , TE_2, TM_2 . Their corresponding cutoff frequencies are 0, 0, 12 GHz, 12 GHz, 24 GHz, 24 GHz, respectively. Note that the cutoff frequencies for TE and TM are degenerate. Within the frequency range 0-12GHz, only TE_0 and TM_0 exist.