MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 7	6.630 Electromagnetics	Issued: 20081023R
Fall Term 2008		Due: 20081030R

Reading assignment: Section 1.7 A-C, 4.1 A-C ; J. A. Kong, "*Electromagnetic Wave Theory*," EMW Publishing, 2008.

Problem P7.1

A plane wave in free space is incident at an angle θ on a conducting half-space. For large $\sigma/\omega\epsilon_o$, show that the transmitted wave is almost perpendicular to the boundary by finding $\theta_t = \tan^{-1}(k_x/k_{zR})$.

Problem P7.2

When the incident k vector is normal to a plane boundary, a TE wave becomes a TEM wave; a TM wave also becomes a TEM wave. Compare the reflection and transmission coefficients for TE and TM waves at normal incidence. Do both TE and TM results reduce to the same number? If not, why? Do the reflectivities and transmissivities for TE and TM waves at normal incidence reduce to the same result?

Problem P7.3

A plane wave is incident from free space ($z > 0$) on a perfect electric conductor ($z < 0$) with the incident electric field

$$\overline{E}_i = (\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{z} + i\hat{y})e^{i\frac{1}{\sqrt{2}}k_o x - i\frac{1}{\sqrt{2}}k_o z}.$$

What is the reflected electric field $\overline{E}_r$? Compare the handedness of polarizations of $\overline{E}_i$ and $\overline{E}_r$.

Problem P7.4

Rainbow arc often appears when sunlight shines on water droplets after a brief shower late in the afternoon. When a sun ray is refracted as it enters the raindrop, total internally reflected from inside the drop, and refracted again as it leaves the drop and passes to the observer.

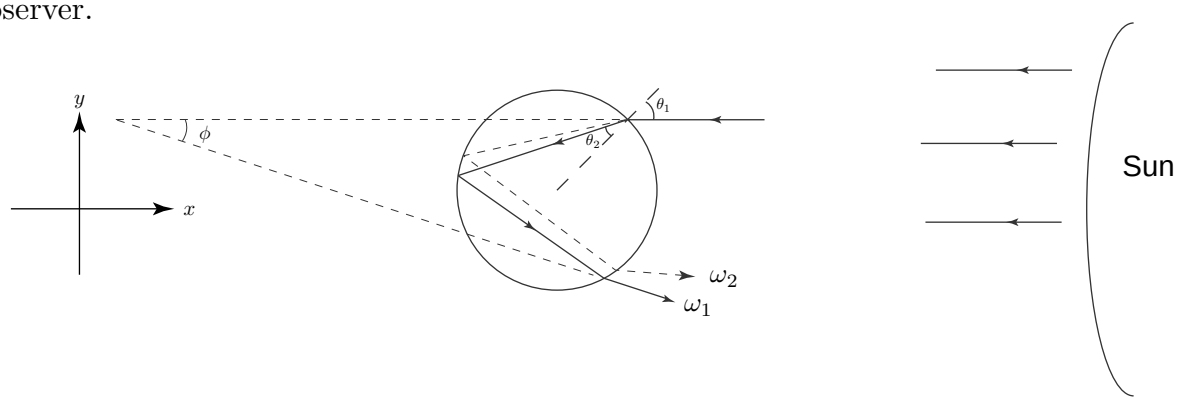

Fig. 1

(a) Consider the ray path with only one internal reflection. Show that the scattering angle ϕ between the incident ray and the exit ray is $2(2\theta_2 - \theta_1)$, where θ_1 is the incident angle and θ_2 is the refracted angle.

(b) For a sphere with a radius a where $ka \gg 1$, the direction where the scattering angle is stationary ($\frac{d\phi}{d\theta_1} = 0$) corresponds to the least cancellation between different rays and hence a large scattering amplitude. Show that the maximum scattering angle ($\phi_{\max}$) occurs at $\theta_1 = \sin^{-1}\sqrt{(4-n^2)/3}$ and $\phi_{max} \approx 42^\circ$ for $n = 4/3$, with the scattering angle between the incident ray and scattered ray $\theta_s = 138^\circ$.

(c) The refractive index for a raindrop is $n = 1.330$ for red light ($\lambda = 0.7\mu\text{m}$), $n = 4/3 = 1.333$ for orange light, and $n = 1.342$ for violet light ($\lambda = 0.4\mu\text{m}$). Determine the scattering angles for the red and violet light rays. What are the relative positions of the different color bands in a rainbow?