MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 5	6.632 Electromagnetic Wave Theory	Issued: 03/08/07 R
Spring Term 2007		Due: 03/15/07 R

4.1 1.3

Reading assignment: Section ~~3.3, 3.4~~ J. A. Kong, "*Electromagnetic Wave Theory*"
Quiz 1: March 22, 3:00 - 5:00 P.M., Rm. 26-314

Problem P5.1

Consider an electromagnetic wave impinging normally upon a dielectric half space (Region 2) with permittivity ϵ_2 from a medium (Region 1) with permittivity ϵ_1.

(a) Let $\epsilon_1 = \epsilon_o$ and $\epsilon_2 = 4\epsilon_o$. What are the reflection coefficient R_{12} and the transmission coefficient T_{12}?
(b) What is the sum of Poynting power of the wave on either side of the interface? Do they conserve?
(c) What is the sum of momentum density of the wave on either side of the interface? Do they conserve?
(d) Find the radiation pressure exerted on both sides of the boundary. Do they match?
(e) Will the half space move towards the incident wave or away from it?

Problem P5.2

The constitutive relation for a lossy dielectric with $\epsilon = \epsilon_R + i\epsilon_I$ and $\mu = \mu_0$ can be written as a material contribution in excess of the "free space" part $\epsilon_0 \bar{E}$ by defining the polarization $\bar{P} = \bar{D} - \epsilon_0 \bar{E}$.
(a) Show that the material contribution in maxwell equations can be contained within an effective electric current density $\bar{J}_e = -i\omega \bar{P}$.
(b) In a conductor, a conduction current $\bar{J}_c = \sigma \bar{E}$ is responsible for the loss of electromagnetic energy. More generally, we can write $\bar{J}_c = \omega \epsilon_I \bar{E}$. If we define the free current density by $\bar{J}_c$ and a bound current density $\bar{J}_b$ such that $\bar{J}_e \equiv \bar{J}_c + \bar{J}_b$, what is $\bar{J}_b$?

Now consider an electromagnetic wave $\bar{E} = \hat{x} E_0 e^{-k_I z} e^{ik_R z}$ propagating in a lossy dielectric with $k = k_R + ik_I = \omega\sqrt{\mu_0 \epsilon}$.
(c) What is the magnetic field of the electromagnetic wave?
(d) Show that the time-average Lorentz force density $< \bar{f}_b >= \frac{1}{2}\mathrm{Re}\{\bar{J}_b \times \bar{B}^*\}$ on bound currents reduces to

$$< \bar{f}_b >= -\hat{z}\frac{1}{2}k_{zI}(\epsilon_R - \epsilon_0)|E|^2,$$

where the negative sign indicates that the force on bound currents is opposite to the incident wave propagation direction when $\epsilon_R > \epsilon_0$.

This radiation attraction was observed by Poynting and Barlow in 1910 when an absorbing disk was initially attracted toward an incident light due to the heating and expulsion of occluded gas from the unilluminated side of the disk. To understand why the gas gained a momentum larger than the incident light, it is necessary to evaluate $< \bar{f}_c >$.

(e) Show that the time-average Lorentz force density on free currents is

$$< \bar{f}_c >= \hat{z}\frac{1}{2}k_R\epsilon_I|E|^2 = -\frac{1}{2}\mathrm{Re}\left\{\frac{n}{c}\nabla\cdot\bar{S}\right\},$$

where $n = ck_R/\omega$ is the index of refraction and $\bar{S} = \bar{E}\times\bar{H}^*$ is the complex Poynting vector resulting from the application of Poynting's theorem to the second equality.

Thus, the momentum transfer to free currents due to absorption of electromagnetic energy is proportional to the index of refraction. This dependence upon n has been observed in the Photon drag measurements [Gibson et. al., 1980] and the absorption of photons in a dilute gas of atoms [Campbell et. al., 2005].

Problem P5.3

A plane wave of angular frequency ω is incident on a plasma medium with permeability μ_o and permittivity $\epsilon = \epsilon_o\left(1 - \omega_p^2/\omega^2\right)$, where ω_p is the plasma frequency and $\omega = 2\omega_p$.

(a) Calculate the critical angle θ_C such that the incident wave is totally reflected.

(b) Calculate the Brewster angle θ_B such that TM waves are totally transmitted.

(c) In general for any two isotropic media, can you find an incident angle θ such that $\theta = \theta_B > \theta_C$? If you can, give an example. If you cannot, explain why not.

Problem P5.4

The ionosphere extends from approximately 50 km above the earth to several earth radii (mean earth radius is about 6371 km) with the maximum in ionization density at about 300 km. For simplicity, assume that the ionosphere consists of a 40 km thick E layer with electron density $N = 10^{11}\ \mathrm{m}^{-3}$ below a 200 km thick F layer with $N = 6\times10^{11}\ \mathrm{m}^{-3}$.

(a) What are the plasma frequencies of the E and F layers ?

(b) Consider a plane wave of 10 MHz incident at an angle θ upon the ionosphere from below the E layer, what is the angle θ_t of the wave in the ionospheric E layer ?

(c) Let $\theta = 30°$, below what frequency will the wave be totally reflected by the E layer and below what frequency will it be totally reflected by the F layer ?