6.630 Solution to Problem Set 10

Solution P10.1

(a) $f_{c10} = \frac{\omega_{c10}}{2\pi} = \frac{c}{2\pi}\left(\frac{\pi}{a}\right) = \frac{3\times10^8}{2\pi} \times \frac{\pi}{6.55} = 22.9\,(MHz) < f < f_{c01} = \frac{c}{2\pi}\left(\frac{\pi}{b}\right) = \frac{3\times10^8}{2\pi} \times \frac{\pi}{4.19} = 35.8\,(MHz)$

(b) An AM radio operates in the range of 500 to 1600 (KHz) is below the cutoff frequency of the fundamental mode TE_{10}. Therefore AM signals can not be received in the tunnel.

(c) FM singals operate in the range of 88 to 108 (MHz) can be received in the tunnel.

Solution P10.2

In the region of $z > 0$,

$$H_y = \Sigma_{m=1}^{\infty} H_m^{(1)} \cos(m\pi x/d) e^{ik_z z},$$

$$E_x = \Sigma_{m=1}^{\infty} \frac{k_z H_m^{(1)}}{\omega\epsilon} \cos(m\pi x/d) e^{ik_z z}.$$

In the region of $z < 0$,

$$H_y = \Sigma_{m=1}^{\infty} H_m^{(2)} \cos(m\pi x/d) e^{-ik_z z},$$

$$E_x = \Sigma_{m=1}^{\infty} \frac{-k_z H_m^{(1)}}{\omega\epsilon} \cos(m\pi x/d) e^{-ik_z z}.$$

At the boundary $z = 0$,

$$\Sigma_{m=1}^{\infty}(H_m^{(2)} - H_m^{(1)}) \cos\frac{m\pi x}{d} = J_s \cos\frac{3\pi x}{d},$$

$$\Sigma_{m=1}^{\infty} \frac{k_z H_m^{(1)}}{\omega\epsilon} \cos\frac{m\pi x}{d} = \Sigma_{m=1}^{\infty} \frac{-k_z H_m^{(2)}}{\omega\epsilon} \cos\frac{m\pi x}{d}.$$

So $m = 3$ and $H_m^{(2)} = -H_m^{(1)} = J_s/2$.
In the regin of $z > 0$, $H_y = -\frac{J_s}{2}\cos\frac{3\pi x}{d} e^{ik_z z}$, $E_x = -\frac{k_z J_s}{2\omega\epsilon}\cos\frac{3\pi x}{d} e^{ik_z z}$ and $E_z = \frac{iJ_s}{2\omega\epsilon}(\frac{3\pi}{d})\sin\frac{3\pi x}{d} e^{ik_z z}$.
In the regin of $z < 0$, $H_y = \frac{J_s}{2}\cos\frac{3\pi x}{d} e^{-ik_z z}$, $E_x = -\frac{k_z J_s}{2\omega\epsilon}\cos\frac{3\pi x}{d} e^{-ik_z z}$ and $E_z = \frac{-iJ_s}{2\omega\epsilon}(\frac{3\pi}{d})\sin\frac{3\pi x}{d} e^{-ik_z z}$.

Solution P10.3

(a) Only TE_{10} mode is propagating. $t = \ell/v_g = 1.8\ \mu\mathrm{s}$.

(b) Three modes, TE_{10}, TE_{01} and TE_{20}, whose cutoff frequencies are 5 GHz, 10GHz, and 10GHz, respectively.
For TE_{10}, $t = 1.14\ \mu\mathrm{s}$. For TE_{01} and TE_{20}, $t = 3.28\ \mu\mathrm{s}$.