Signals and systems

Lab 03

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Problem 1

Solutions:

(a) By using MATLAB, we can find the poles and zeros of the three systems respectively as:

(i)

```
ps1 =

  -1.0000 + 1.4142i
  -1.0000 - 1.4142i

zs1 =

    -5
```

(ii)

```
ps2 =

  -1.0000 + 3.0000i
  -1.0000 - 3.0000i

zs2 =

  -1.2500 + 2.1065i
  -1.2500 - 2.1065i
```

(iii)

```
ps3 =

  -1.0000 + 3.0000i
  -1.0000 - 3.0000i
  -2.0000 + 0.0000i

zs3 =

  -1.2500 + 2.1065i
  -1.2500 - 2.1065i
```

And their pole-zero diagrams are:

(i)

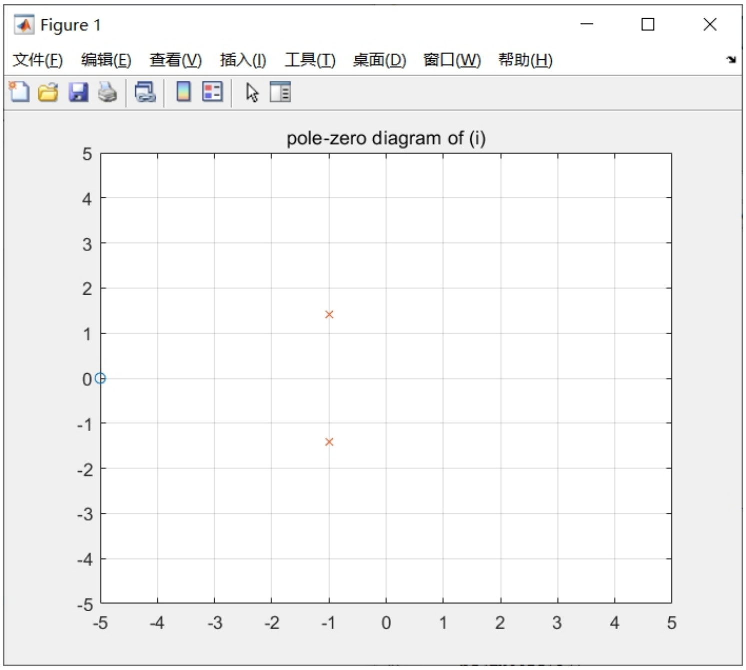

Figure 1.1.1 pole-zero diagram of (i)

(ii)

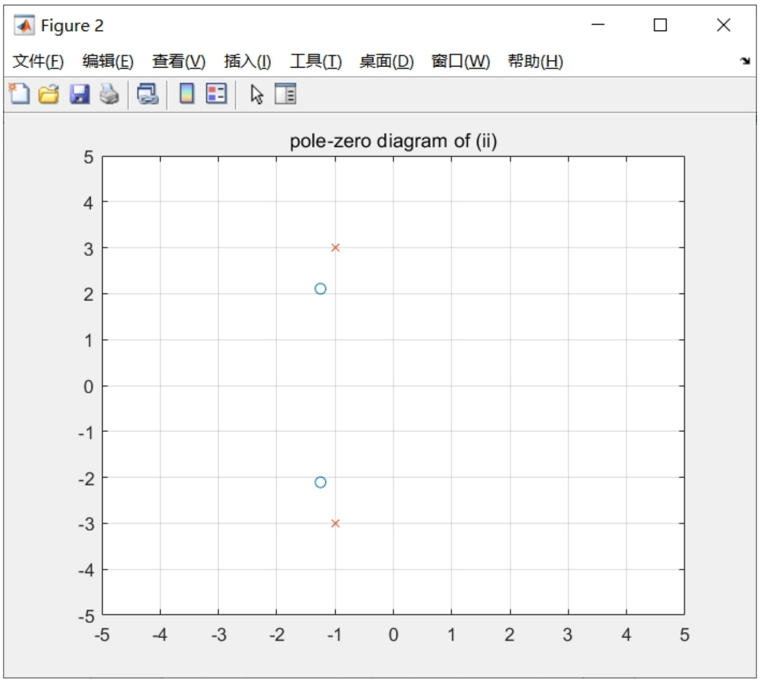

Figure 1.1.2 pole-zero diagram of (ii)

(iii)

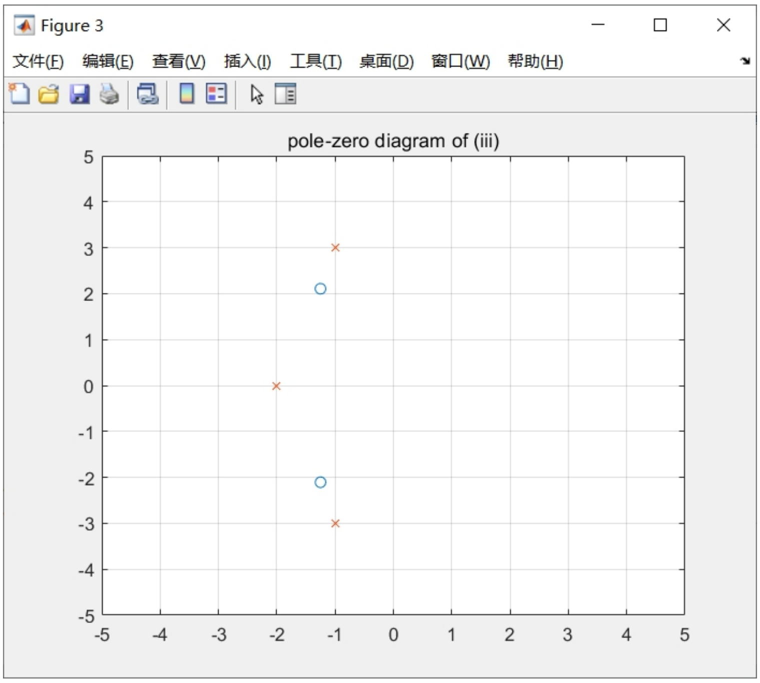

Figure 1.1.3 pole-zero diagram of (iii)

MATLAB code:

```
% prob1a.m

clear;
clc;

% zeros and poles of (i)
b1=[1 5];
a1=[1 2 3];
zs1=roots(b1)
ps1=roots(a1)
% plot the pole-zero diagram
figure;
plot(real(zs1),imag(zs1),'o');
hold on;
plot(real(ps1),imag(ps1),'x');
axis([-5 5 -5 5]);
grid on;
title('pole-zero diagram of (i)');

% zeros and poles of (ii)
b2=[2 5 12];
a2=[1 2 10];
zs2=roots(b2)
```

```
ps2=roots(a2)
% plot the pole-zero diagram
figure;
plot(real(zs2),imag(zs2),'o');
hold on;
plot(real(ps2),imag(ps2),'x');
axis([-5 5 -5 5]);
grid on;
title('pole-zero diagram of (ii)');

% zeros and poles of (iii)
b3=[2 5 12];
a3=conv(a2,[1 2]);
zs3=roots(b3)
ps3=roots(a3)
% plot the pole-zero diagram
figure;
plot(real(zs3),imag(zs3),'o');
hold on;
plot(real(ps3),imag(ps3),'x');
axis([-5 5 -5 5]);
grid on;
title('pole-zero diagram of (iii)');
```

(b) As is known to us, the ROC of a system lies on one side of the pole, and since the systems are stable, their ROC should include the jω axis. From the pole-zero diagrams we obtained in part (a), we can exactly determine the ROC corresponding to the stable system as:

(i) $\mathrm{Re}\{s\} > -1$

(ii) $\mathrm{Re}\{s\} > -1$

(iii) $\mathrm{Re}\{s\} > -1$

(c) By using MATLAB, we can find the poles and zeros of the system as:

```
ps =

     3

zs =

  -1.0000 + 2.0000i
  -1.0000 - 2.0000i
```

And the pole-zero diagram is:

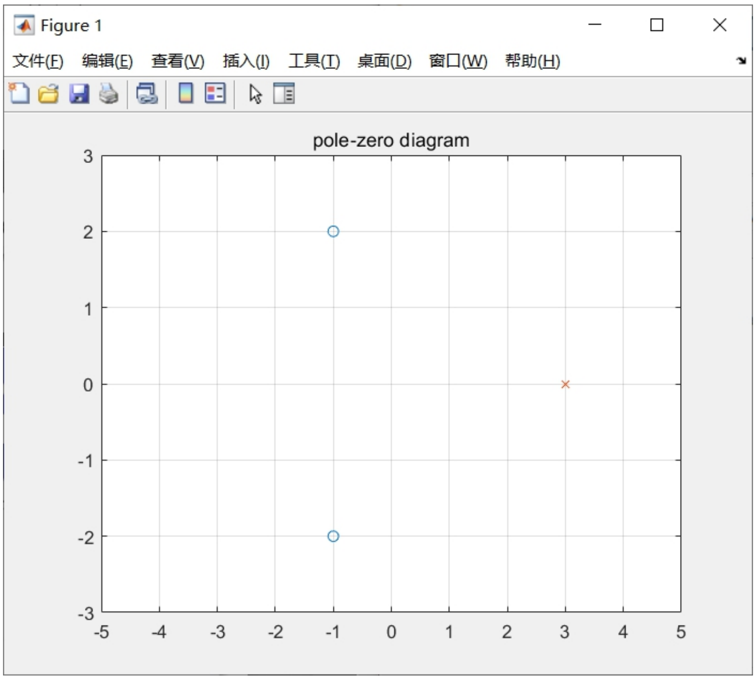

Figure 1.3 pole-zero diagram of the LTI system

MATLAB code:

```
% prob1c.m

clear;
clc;

% the coefficients of the equation
b=[1 2 5];
a=[1 -3];
% the zeros and poles
zs=roots(b)
ps=roots(a)
figure;
plot(real(zs),imag(zs),'o');
```

```
hold on;
plot(real(ps),imag(ps),'x');
axis([-5 5 -3 3]);
grid on;
title('pole-zero diagram');
```

(d) From part (c), we obtained that the pole of the system is 3. Now we use pzplot to get its pole-zero diagram.

According to the title, when we set the third parameter as 1, the function labeled the left side as ROC as follows:

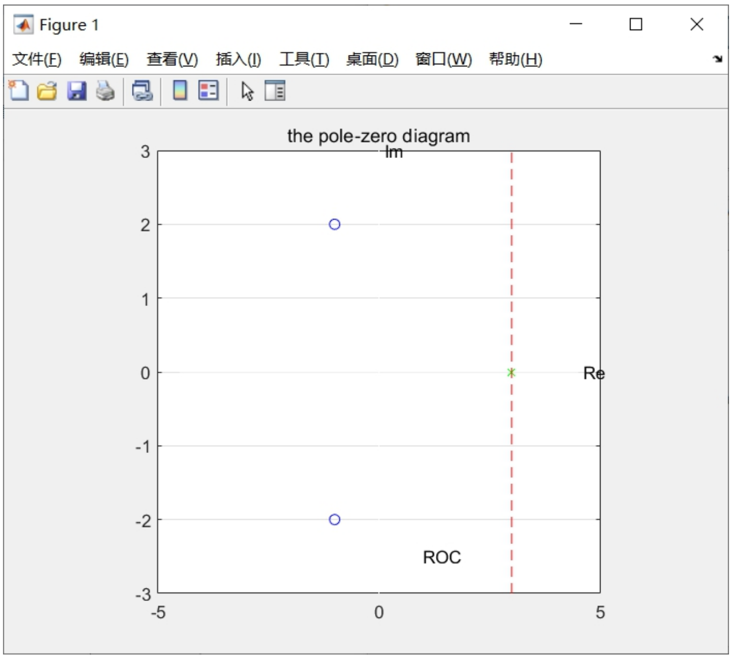

Figure 1.4.1 the left-sided ROC

and when we set the parameter as 5, the ROC is on the right side of the pole as:

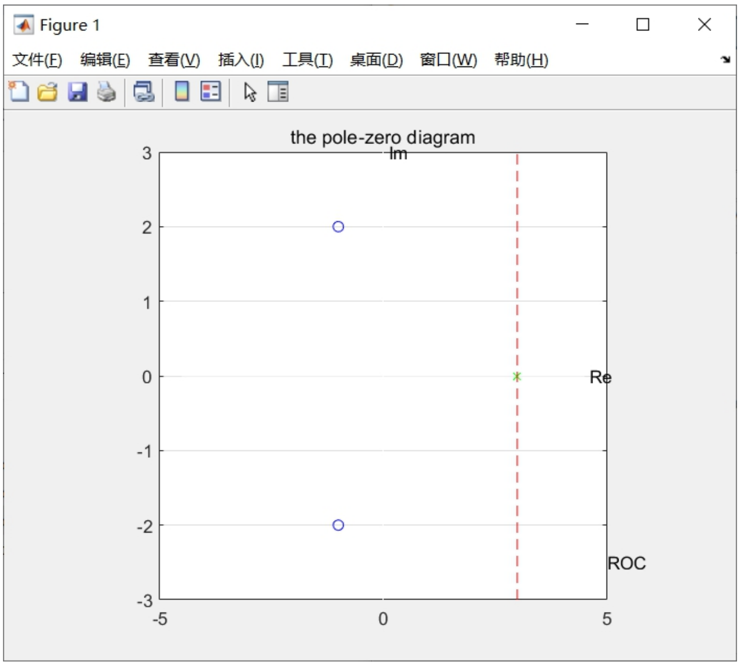

Figure 1.4.2 the right-sided ROC

So it is obvious that the function pzplot determines the ROC of a system by making sure the single point we supplied as the third parameter is in the ROC. Furthermore, if the point is on the boundary of the left and right side, the function will not show us the region of convergence of the system. The following figure is an example with the parameter equals to 3.

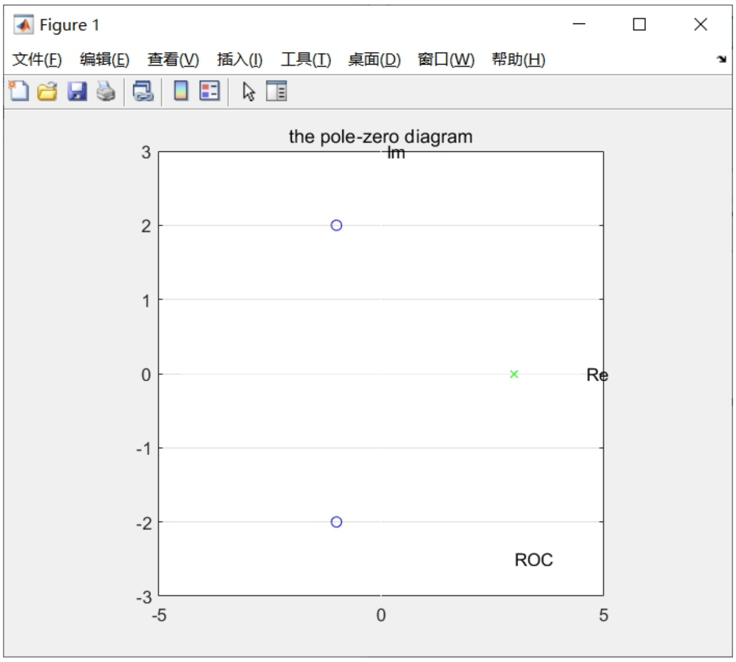

Figure 1.4.3 plot without showing ROC

MATLAB code:

```
% prob1d.m

clear;
clc;

% the coefficients of the equation
b=[1 2 5];
a=[1 -3];

% use pzplot to determine the poles and zeros while
% making the plot
% [ps,zs]=pzplot(b,a,1);
[ps,zs]=pzplot(b,a,5);
% [ps,zs]=pzplot(b,a,3);
title('the pole-zero diagram');
```

Problem 2

Solutions:

(a) By using the MATLAB, we can plot the poles and zeros for $H(z)$ as:

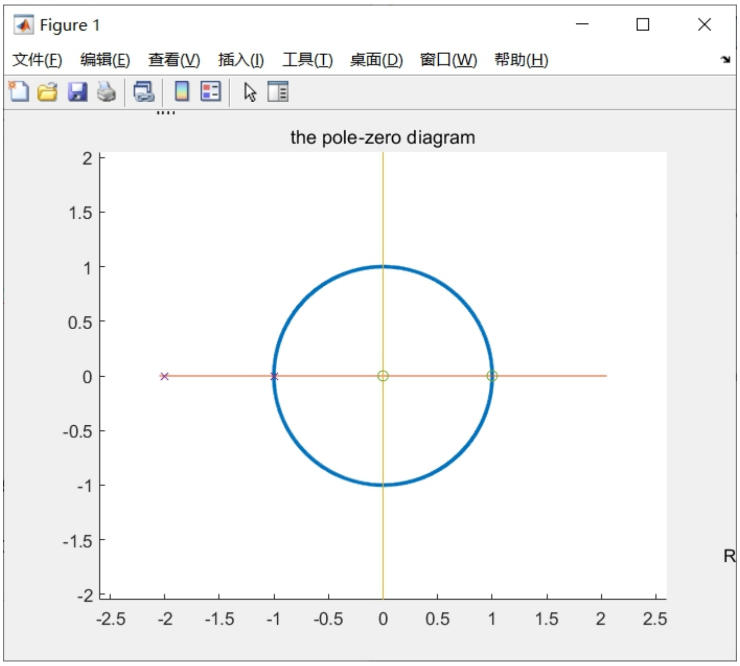

Figure 2.1 pole-zero diagram for $H(z)$

MATLAB code:

```
% prob2a.m
clear;
clc;

% the coefficients of the equation
b=[1 -1];
a=[1 3 2];

% plot the diagram
dpzplot(b,a);
title('the pole-zero diagram');
```

(b) By using the MATLAB, we can plot the poles and zeros for the filter as:

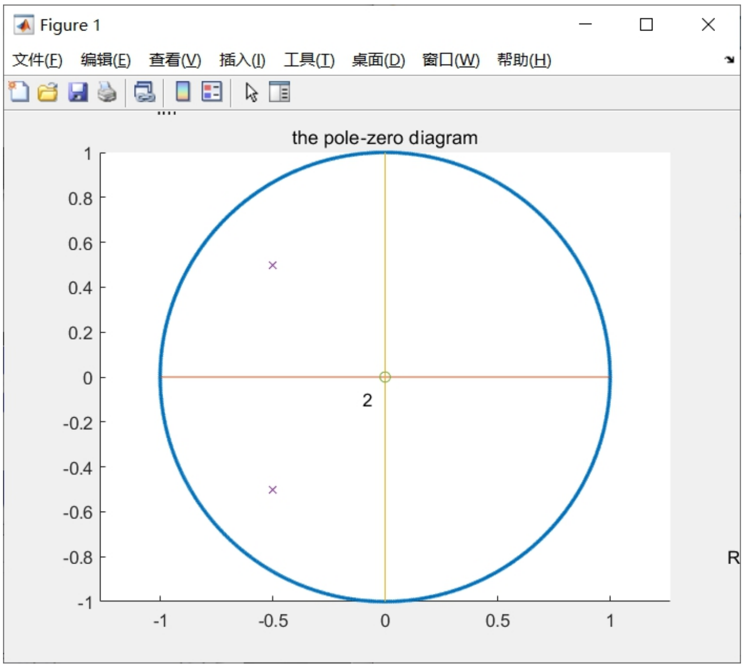

Figure 2.2 pole-zero diagram for the filter

MATLAB code:

```
% prob2b.m
clear;
clc;

% the coefficient vectors
a=[1 1 0.5];
b=[1 0 0];

% draw the diagram
dpzplot(b,a);
title('the pole-zero diagram');
```

(c) By using the MATLAB, we can plot the poles and zeros for the filter as:

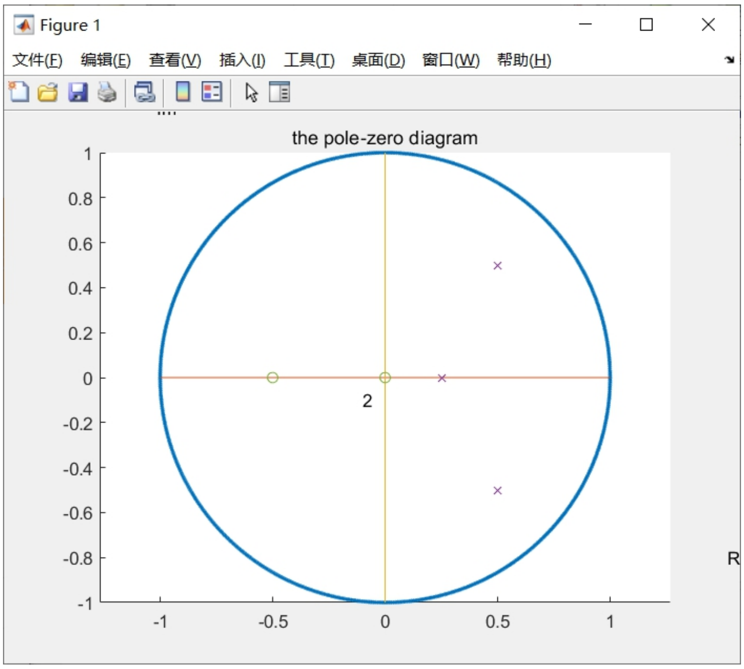

Figure 2.3 pole-zero diagram for the filter

MATLAB code:

```
% prob2c.m
clear;
clc;

% the coefficient vectors
a=[1 -1.25 0.75 -0.125];
b=[1 0.5 0 0];

% draw the diagram
dpzplot(b,a);
title('the pole-zero diagram');
```

Problem 3

Solutions:

(a) According to the title, we have,

$$h_c(t) = h_{ac}(-t)$$

From the scaling property of the Laplace transform, that is,

if $$h(t) <-> H(s)$$

then, $h(at) <-> \frac{1}{|a|}H(\frac{s}{a})$

Since

$$h_c(t) <-> H_c(s)$$

$$h_{ac}(t) <-> H_{ac}(s)$$

we have,

$$h_{ac}(-t) <-> H_{ac}(-s)$$

namely, $h_c(t) <-> H_{ac}(-s)$

therefore, the relationship between the two system functions is:

$$H_c(s) = H_{ac}(-s)$$

and the poles of them are symmetric about the origin in the complex plane.

(b) Take the Laplace transform of both sides of the equation, we have,

$$sY(s) + 2Y(s) = X(s)$$

so,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2}$$

It is obvious that there are two possible ROC of $H(s)$. By take the inverse Laplace transform of $H(s)$, we can obtain the impulse response of the two corresponding systems as:

$$h_c(t) = e^{-2t}u(t), \quad Re\{s\} > -2$$

$$h_{ac}(t) = -e^{-2t}u(-t), \quad Re\{s\} < -2$$

(c) From part (b), it is apparent that $h_c(t)$ is none-zero only for $t \geq 0$ and $h_{ac}(t)$ is none-zero only for $t \leq 0$, so the auxiliary condition for the equation is:

$Re\{s\} >-2$: initial rest conditions

$Re\{s\} <-2$: final rest conditions

(d) By using MATLAB, we can plot the impulse response simulated and the analytic expression of the causal system in a figure as follows:

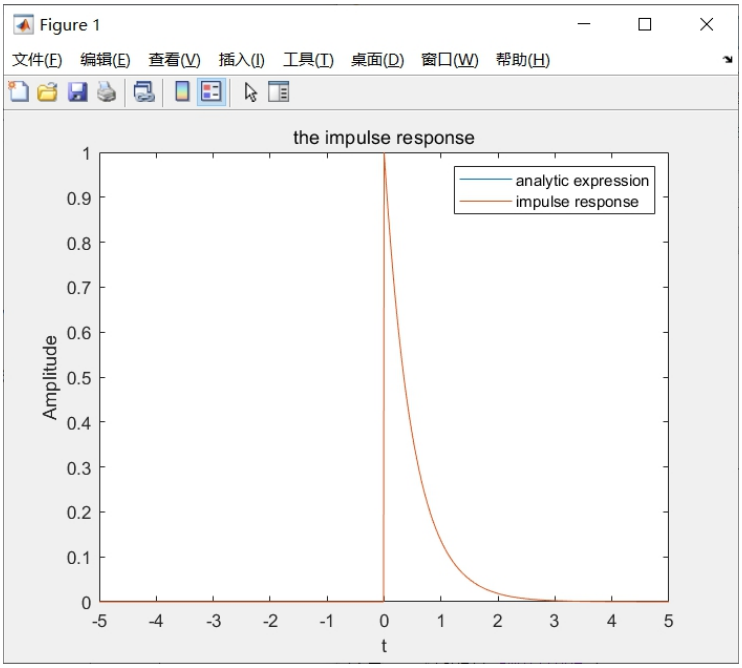

Figure 3.4 figure of the impulse response

From the figure we can verify that the image of analytic expression is the same as the image of impulse response, so it is the right one.

MATLAB code:

```
% prob3d.m
```

```
clear;
clc;

t=-5:0.01:5;
a=[1 2];
b=1;

% the analytic expression
ha=exp(-2.*t).*(t>=0);
% the impulse response
h1=impulse(b,a,t);

% append the result and plot
h=[zeros(length(t)-length(h1),1);h1];
figure;
plot(t,ha);
hold on;
plot(t,h);
title('the impulse response');
xlabel('t');
ylabel('Amplitude');
legend('analytic expression','impulse response');
```

(e) By using MATLAB, we can plot the impulse response simulated and the analytic expression of the anticausal system in a figure as follows:

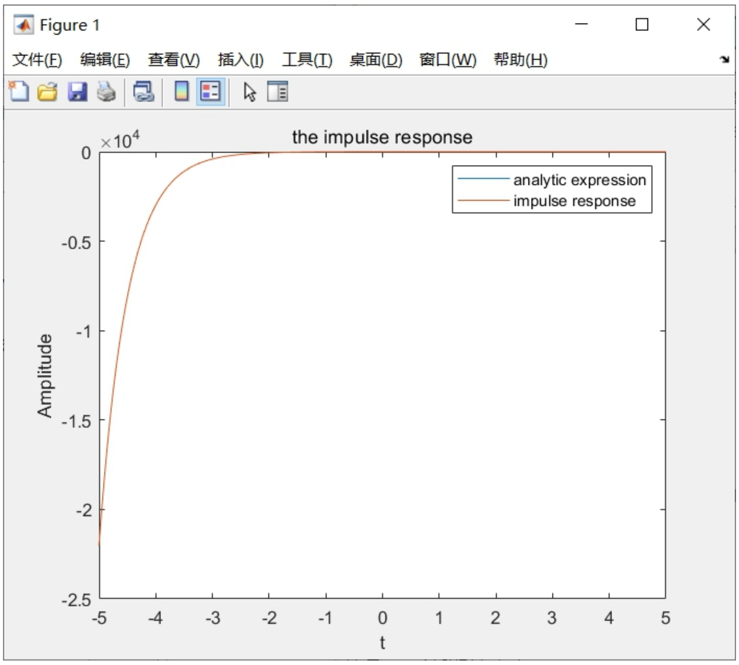

Figure 3.5 figure of the impulse response

It is obvious that the two images are the same, so the expression we computed is right for the anticausal system.

MATLAB code:

```
% prob3e.m
clear;
clc;

t=-5:0.01:5;
ha=-exp(-2*t).*(t<=0);

% the new coefficients
a_new=[-1 2];
b_new=1;
h_c=impulse(b_new,a_new,t);

% append the result
h_c1=[zeros(length(t)-length(h_c),1);h_c];

% flip the impulse response and plot
h_ac=flipud(h_c1);
plot(t,ha);
hold on;
plot(t,h_ac);
title('the impulse response');
xlabel('t');
ylabel('Amplitude');
legend('analytic expression','impulse response');
```

(f) Since the impulse response of the anticausal system is

$$h_{ac}(t) =- e^{-2t}u(- t)$$

we can obtain the corresponding output to $x(t)$ by convolution, that is,

$$y(t) = h_{ac}(t) * x(t) = \int_{-\infty}^{0} -e^{-2\tau} e^{\frac{5}{2}(t-\tau)} u(-t+\tau)\ d\tau$$

if $t > 0,\ y(t) = 0;$

if $t < 0,$

$$y(t) = \int_{t}^{0} -e^{-2\tau} e^{\frac{5}{2}(t-\tau)}\ d\tau = -e^{\frac{5}{2}t} \int_{t}^{0} e^{-\frac{9}{2}\tau}\ d\tau$$

$$= \frac{2}{9} e^{\frac{5}{2}t} e^{-\frac{9}{2}\tau} |_{t}^{0} = \frac{2}{9} e^{\frac{5}{2}t} (1 - e^{-\frac{9}{2}t})$$

$$= \frac{2}{9} e^{\frac{5}{2}t} - \frac{2}{9} e^{-2t}$$

So the output of the system is:

$$y(t) = (\frac{2}{9} e^{\frac{5}{2}t} - \frac{2}{9} e^{-2t}) u(-t)$$

(g) By using MATLAB, we can plot the output signal of the anticausal system simulated as well as the image of the analytic expression in a figure as follows:

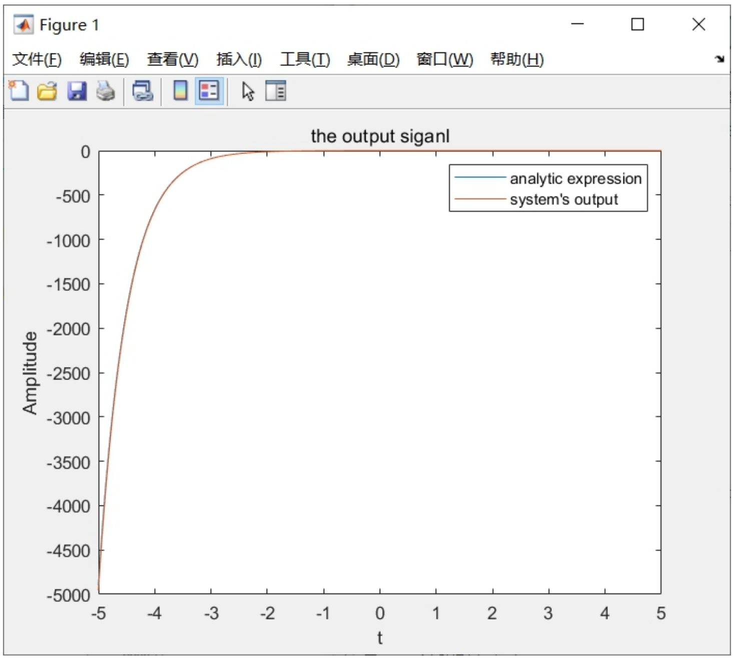

Figure 3.7 image of the output signal

Since the two images are the same, so the result we derived in (f) is exactly the right expression for the output of the anticausal system.

MATLAB code:

```
% prob3g.m

clear;
clc;

% the new coefficient vectors
a_new=[-1 2];
b_new=1;

t=-5:0.01:5;

% the analytic expression
y=2/9*exp(5*t/2).*(1-exp(-9/2*t)).*(t<=0);

x=exp(5*t/2).*(t<=0);

% the causal response
x_reverse=exp(-5*t/2).*(t>=0);
```

```
w1=lsim(b_new,a_new,x_reverse,t);

% time-reverse the simulated response and plot
w=flipud(w1);
plot(t,y);
hold on;
plot(t,w);
title('the output siganl');
xlabel('t');
ylabel('Amplitude');
legend('analytic expression',"system's output")
```

(h) Similarly, taking the Laplace transform of the both sides we then have,

$s^3Y(s) + s^2Y(s) + 24sY(s) - 26Y(s) = s^2X(s) + 7sX(s) + 21X(s)$

Therefore ,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s^2 + 7s + 21}{s^3 + s^2 + 24s - 26}$$

By using the MATLAB, we can obtain the pole-zero diagram of the function as follows:

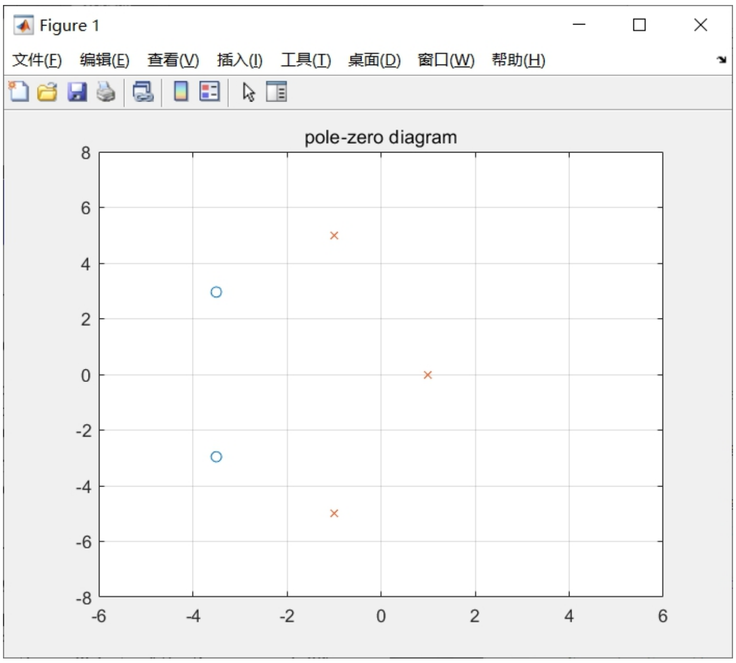

Figure 3.8 the pole-zero diagram

Furthermore, we know the poles of the system are:

```
ps =

  -1.0000 + 5.0000i
  -1.0000 - 5.0000i
   1.0000 + 0.0000i
```

so there are three possible regions of convergence:

① $\mathrm{Re}\{s\} > 1$

② $\mathrm{Re}\{s\} < -1$

③ $-1 < \mathrm{Re}\{s\} < 1$

and since only ③ includes the jω axis, the system is stable when ROC is $-1 < \mathrm{Re}\{s\} < 1$.

MATLAB code:

```
% prob3h.m

clear;
clc;

a=[1 1 24 -26];
b=[1 7 21];

zs=roots(b)
ps=roots(a)

% plot the pole-zero diagram
figure;
plot(real(zs),imag(zs),'o');
hold on;
plot(real(ps),imag(ps),'x');
axis([-6 6 -8 8]);
grid on;
title('pole-zero diagram');
```

```
% figure;
% pzplot(b,a,1);
```

(i) Using function residue, we obtained the following results:

```
r =

   0.0000 - 0.5000i
   0.0000 + 0.5000i
   1.0000 + 0.0000i
p =

  -1.0000 + 5.0000i
  -1.0000 - 5.0000i
   1.0000 + 0.0000i

k =

     []
```

So the partial fraction expansion of $H(s)$ is

$$H(s)=\frac{-0.5i}{s-(-1+5i)}+\frac{0.5i}{s-(-1-5i)}+\frac{1}{s-1}$$

then we have,

Poles	function	left-sided;ROC	right-sided;ROC
$-1+5i$	$e^{(-1+5i)t}$	$e^{(-1+5i)t}u(t);\mathrm{Re}\{s\}>-1$	$-e^{(-1+5i)t}u(-t);\mathrm{Re}\{s\}<-1$
$-1-5i$	$e^{(-1-5i)t}$	$e^{(-1-5i)t}u(t);\mathrm{Re}\{s\}>-1$	$-e^{(-1-5i)t}u(-t);\mathrm{Re}\{s\}<-1$
1	e^{t}	$e^{t}u(t);\mathrm{Re}\{s\}>1$	$-e^{t}u(-t);\mathrm{Re}\{s\}<1$

When ROC is $\mathrm{Re}\{s\}>1$, the impulse response of the system is:

$$h(t)=-0.5ie^{(-1+5i)t}u(t)+0.5ie^{(-1-5i)t}u(t)+e^{t}u(t)$$

$$=[e^{-t}sin(5t)+e^{t}]u(t)$$

when ROC is $\mathrm{Re}\{s\}<-1$, the impulse response is:

$$h(t)=0.5ie^{(-1+5i)t}u(-t)-0.5ie^{(-1-5i)t}u(-t)-e^{t}u(-t)$$

$$=-[e^{-t}sin(5t)+e^{t}]u(-t)$$

when ROC is $-1<\mathrm{Re}\{\mathrm{s}\}<1$, the impulse response is:

$$h(t)=-0.5ie^{(-1+5i)t}u(t)+0.5ie^{(-1-5i)t}u(t)-e^{t}u(-t)$$

$$=e^{-t}sin(5t)u(t)-e^{t}u(-t)$$

MATLAB code:

```
% prob3i.m

clear;
clc;

% the coefficient vectors
b=[1 7 21];
a=[1 1 24 -26];

% use residue to determine the partial fraction
% expansion
[r,p,k]=residue(b,a)
```

(j) Since the system is causal, its analytic expression is

$$h_c(t)=[e^{-t}sin(5t)+e^{t}]u(t)$$

Using the MATLAB, we obtained the figure of the impulse response as follows:

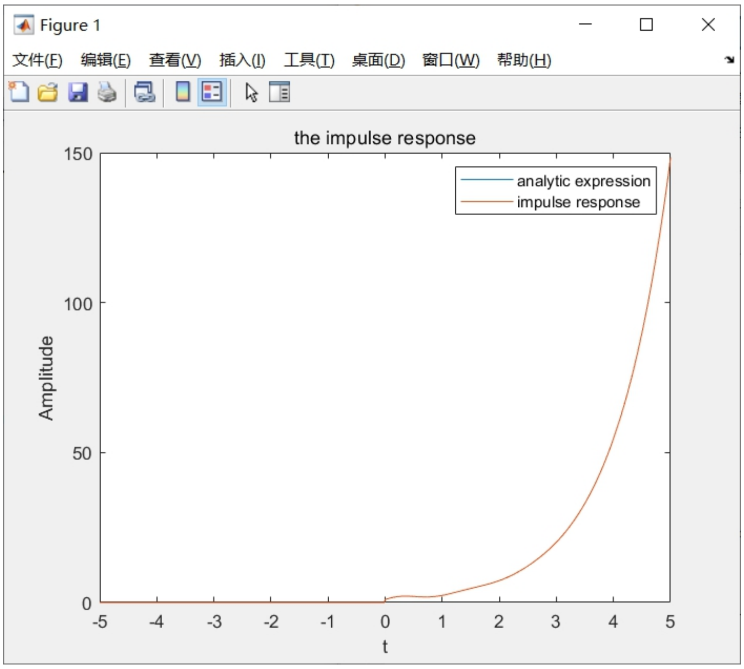

Figure 3.10 the causal impulse response

Since the images of the analytic expression and the simulated impulse response are the same, the corresponding impulse response we computed in (i) is right.

Furthermore, since $h_c(t)$ is none-zero only when $t \geq 0$ for the causal system, the auxiliary conditions on $y(t)$ are initial rest conditions.

MATLAB code:

```
% prob3j.m

clear;
clc;

t=-5:0.01:5;

% the coefficient vectors
a=[1 1 24 -26];
b=[1 7 21];
```

```
ha=(exp(-t).*sin(5*t)+exp(t)).*(t>=0);
h1=impulse(b,a,t);
% append the result and plot
h=[zeros(length(t)-length(h1),1);h1];
plot(t,ha);
hold on;
plot(t,h);
title('the impulse response');
xlabel('t');
ylabel('Amplitude');
legend('analytic expression','impulse response');
```

(k) Similarly, when the system is anticausal, its analytic expression is

$$h_{ac}(t) = -[e^{-t}sin(5t) + e^{t}]u(-t)$$

By using MATLAB, we can plot the impulse response simulated and the analytic expression of the anticausal system in a figure as follows:

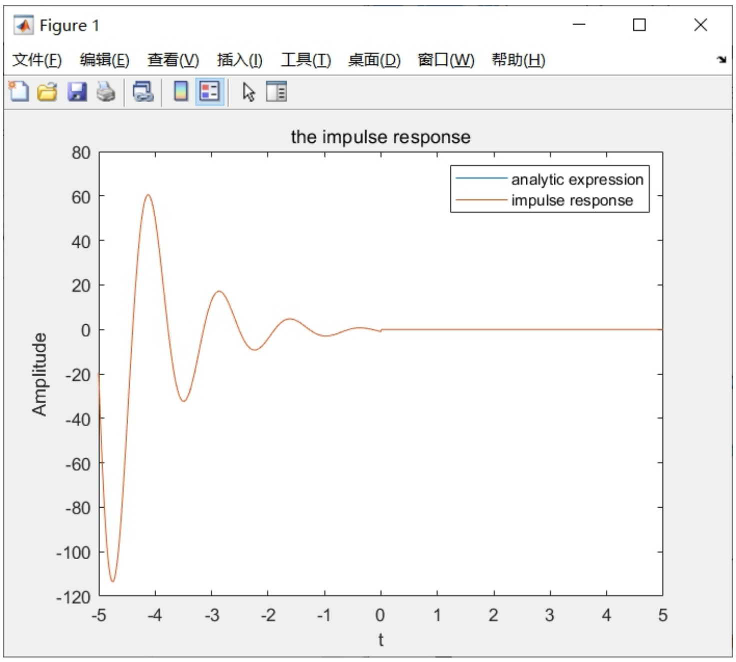

Figure 3.11 the anticausal impulse response

So the two images are identical, which means the expression we computed analytically for the anticausal system in (i) is the same as the simulated result.

In addition, since $h_{ac}(t)$ is none-zero only when $t \leq 0$, the auxiliary conditions on $y(t)$ are final rest conditions.

MATLAB code:

```
% prob3k.m

clear;
clc;

t=-5:0.01:5;

% the new coefficient vectors
a_new=[-1 1 -24 -26];
b_new=[1 -7 21];

% the analytic expression
ha=-(exp(-t).*sin(5*t)+exp(t)).*(t<=0);

% the causal response
h_c1=impulse(b_new,a_new,t);
% append the result
h_c=[zeros(length(t)-length(h_c1),1);h_c1];

% time-reverse the simulated response and plot
h_ac=flipud(h_c);
plot(t,ha);
hold on;
plot(t,h_ac);
title('the impulse response');
xlabel('t');
ylabel('Amplitude');
legend('analytic expression','impulse response');
```

(l) From (i), we have,

$$H(s) = \frac{-0.5i}{s-(-1+5i)} + \frac{0.5i}{s-(-1-5i)} + \frac{1}{s-1}$$

and when the first two fractions have a ROC of $\mathrm{Re}\{s\} > -1$, and the last one's ROC is $\mathrm{Re}\{s\} < 1$, the system will be noncausal, whose corresponding impulse response is:

$$h_s(t) = e^{-t}sin(5t)u(t) - e^{t}u(-t)$$

Above all, we can determine that

$$H_1(s) = \frac{-0.5i}{s-(-1+5i)} + \frac{0.5i}{s-(-1-5i)}, \quad \mathrm{Re}\{s\} > -1$$

$$H_2(s) = \frac{1}{s-1}, \quad \mathrm{Re}\{s\} < 1$$

and in this case, we have $h_1(t) = e^{-t}sin(5t)u(t)$ is causal and $h_2(t) = -e^{t}u(-t)$ is anticausal, which meets the requirement of the title.

(m) As has shown in (i) and (l),

$$h_s(t) = e^{-t}sin(5t)u(t) - e^{t}u(-t), \quad -1 < \mathrm{Re}\{s\} < 1$$

and

$$h_1(t) = e^{-t}sin(5t)u(t), \quad \mathrm{Re}\{s\} > -1$$

$$h_2(t) = -e^{t}u(-t), \quad \mathrm{Re}\{s\} < 1$$

(n) According to the title, $y_1(t)$ is the output of the causal system.

Since

$$H_1(s) = \frac{Y_1(s)}{X_1(s)} = \frac{-0.5i}{s-(-1+5i)} + \frac{0.5i}{s-(-1-5i)}$$

$$= \frac{5}{(s+1)^2+25}, \quad \text{Re}\{s\} > -1$$

we have,

$$[(s+1)^2+25]Y_1(s) = 5X_1(s)$$

Taking the inverse Laplace transform of the both sides, we can find that

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 26y(t) = 5x(t)$$

which is the differential equation corresponding to $H_1(s)$.

Furthermore, because $h_1(t)$ is non-zero only for $t \geq 0$, we can then conclude that the auxiliary conditions on $y_1(t)$ are initial rest conditions.

(o) Similarly, since

$$H_2(s) = \frac{Y_2(s)}{X_2(s)} = \frac{1}{s-1}, \quad \text{Re}\{s\} < 1$$

we have,

$$(s-1)Y_2(s) = X_2(s)$$

Taking the inverse Laplace transform of the both sides, we have,

$$\frac{dy(t)}{dt} - y(t) = x(t)$$

which is the differential equation corresponding to $H_2(s)$.

Besides, since $h_2(t)$ is non-zero only for $t \leq 0$, the auxiliary conditions on $y_2(t)$ are final rest conditions.

(p) Using the MATLAB, we obtained the image of $y_1(t)$, $y_2(t)$ and $y(t)$ as follows:

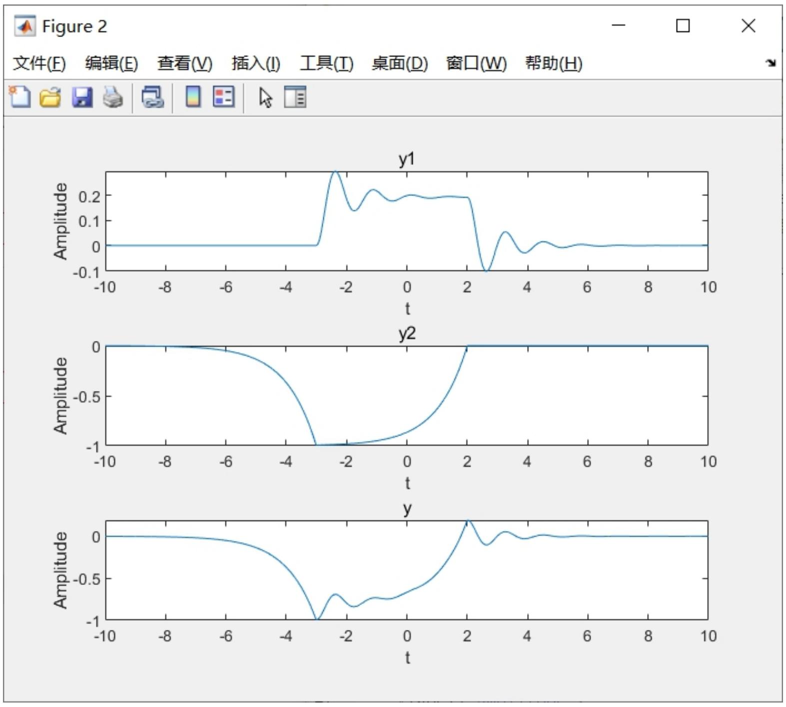

Figure 3.16 image of the outputs

MATLAB code:

```
% prob3p.m

clear;
clc;

t=[-10:0.01:10];

% coefficients of the causal system
a1=[1 2 26];
b1=5;

% coefficients of the anticausal system
a2_new=[-1 -1];
b2_new=1;
```

```
% the input signals
x=(t>=-3).*(t<=2);
% time-inverse
x_reverse=fliplr(x);

% simulate y1
y1_old=lsim(b1,a1,x,t);
y1=[zeros(length(t)-length(y1_old),1);y1_old];

% simulate y2
y2_c=lsim(b2_new,a2_new,x_reverse,t);
y2=[zeros(length(t)-length(y2_c),1);y2_c];
% time-inverse
y2_ac=flipud(y2);

% plot the figure
figure;
subplot(3,1,1);
plot(t,y1);
title('y1');
xlabel('t');
ylabel('Amplitude');
subplot(3,1,2);
plot(t,y2_ac);
title('y2');
xlabel('t');
ylabel('Amplitude');
subplot(3,1,3);
plot(t,y1+y2_ac);
title('y');
xlabel('t');
ylabel('Amplitude');
```