Artificial Intelligence—Spring 2022

Homework 3

Issued: March 21^{th}, 2022 Due: Apr. 2^{nd}, 2022

Problem 1

Solutions:

According to the title, there are 9 digits unknown in this problem and each digits represents a different number except the carry digits. The normal digits are in the range of 0 to 9 and the carry digits have value of 0 or 1. Moreover, the leading digit should not be 0. When using the MRV and least-constraining-value heuristics, we have following solving process:

1. Choose C_3 first, which can be either 0 or 1. Becasue F cannot be 0, we can determine C_3 as 1;
2. Choose F, which has only one option that F = 1;
3. According to MRV, we consider C_2 and C_1. Assume C_2 equals to 0;
4. Then it is C_1's turn to choose a value. Assume C_1 = 0;
5. Now consider the constrains. Because C_3 = 0, C_2 = C_1 = 0, we can know that $O+O=R$, $W+W=U$ and $T+T=O+10$. Furthermore, R、U、O are no bigger than 9, so O is less than 5 and it is even, which makes O the most constrained digit;
6. Assume O = 2(it cannot be 0 because R will be 0 if we use the forward checking), so R = 4 and T = 6;
7. Consider U, which has two options as 0 or 8 while W has tow options as 0 or 3. If U = 8(U should not be 0 for differnt digits represent different numbers), then W = 4, which disobeys the constrains. So trace back to step 6 and assume O = 4;
8. R and T both have one option, so R = 8 and T = 7;
9. Assume U = 6(it cannot be 2 or W will be 1), so W has only one option as 3, which can be a reasonable solution.

All in all, one solution for this question is T=7, W=3, O=4, F=1, U=6, R=8.

Problem 2

Solutions:

When using the AC-3 algorithm with the partial assignment {*WA* = *green*, *V* = *red*}, we can note the arcs *WA—NT*、*WA—SA*、*NT—SA*、*NT—Q*、*SA—Q*、*SA—NSW*、*SA—V*、*Q—NSW*、*NSW—V* as numbers from 1 to 9, and the process will be as follows:

1. pop arc 1, delete *green* in *NT*, enqueue arc 3 and 4;
2. pop arc 2, delete *green* in *SA*, enqueue arc 3、5、6、7;
3. pop arc 3, and no revision needed;
4. pop arc 4, and no revision needed;
5. pop arc 5, and no revision needed;
6. pop arc 6, and no revision needed;
7. pop arc 7, delete *red* in *SA*, enqueue arc 2、3、5、6;
8. pop arc 8, and no revision needed;
9. pop arc 9, delete *red* in *NSW*, enqueue arc 6 and 8;
10. pop arc 3, delete *blue* in *NT*, enqueue arc 4;
11. pop arc 4, delete *red* in *Q*, enqueue arc 5、6、8;
12. pop arc 3, and no revision needed;
13. pop arc 5, delete *blue* in *Q*, enqueue arc 4、6、8;
14. pop arc 6, delete *blue* in *NSW*, enqueue arc 8、9;
15. pop arc 7, and no revision needed;
16. pop arc 2, and no revision needed;
17. pop arc 3, and no revision needed;
18. pop arc 5, and no revision needed;
19. pop arc 6, and no revision needed;
20. pop arc 6, and no revision needed;
21. pop arc 8, delete *green* in *Q*, and no domain remained for *Q*. Inconsistency found!

Problem 3

Solutions:

a. This statement is correct. Because *False* has no model, which means it entails every sentence while *True* is true in all models, which shows it is entailed by every sentence.

b. This statement is uncorrect. Because in every model where *True* is true, *False* is always false.

c. This statement is correct. This is because $A \wedge B$ is true iff A and B are both true. In this case, $A \Leftrightarrow B$ is true as well.

d. This statement is uncorrect. For example, if A and B are both false, we can conclude that $A \Leftrightarrow B$ is true but $A \vee B$ is false.

e. This statement is correct. Because $A \Leftrightarrow B$ is true iff $A \Rightarrow B$ and $B \Rightarrow A$ are both true and $\neg A \vee B$ means $A \Rightarrow B$, every model makes the left-hand side true will make the right-hand side true as well.

f. This statement is correct. This is because $(A \wedge B) \Rightarrow C$ is equivalent to $\neg A \vee \neg B \vee C$, which is what $(A \Rightarrow C) \vee (B \Rightarrow C)$ means too. Hence, every model that makes the left-hand side true will ensure the correctness of right-hand side.

g. This statement is correct. Use the distributivity, we can know that left-hand side is equivalent to $(\neg A \vee C) \wedge (\neg B \vee C)$, which is definitely what right-hand side means.

Problem 4

Solutions:

a. As the table shown below, the sentence is true for all the models, so it is valid.

Food	*Drinks*	*Party*	*Food∧Drinks*	*Food=>Party*	*Drinks =>Party*	*(Food=>Party) ∨(Drinks =>Party)*	*(Food∧Drinks) =>Party*	*[(Food=>Party) ∨(Drinks =>Party)] =>[(Food∧Drinks) =>Party]*
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	1	1
0	1	0	0	1	0	1	1	1
0	1	1	0	1	1	1	1	1
1	0	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

b. The left-hand side:

$(Food => Party) \vee (Drinks => Party)$

$(\neg Food \vee Party) \vee (\neg Drinks \vee Party)$

$\neg Food \vee Party \vee \neg Drinks \vee Party$

$\neg Food \vee Party \vee \neg Drinks$

The right-hand side:

$(Food \wedge Drinks) => Party$

$\neg(Food \wedge Drinks) \vee Party$

$\neg Food \vee \neg Drinks \vee Party$

It is obvious that the CNF of the both sides are the same, so the original sentence is equivalent to the form $\alpha => \alpha$, which is valid for any α

c. Negate the sentence, we have:

$\neg\{[(Food => Party) \vee (Drinks => Party)] => [(Food \wedge Drinks) => Party]\}$

$\neg\{\neg[(Food => Party) \vee (Drinks => Party)] \vee [(Food \wedge Drinks) => Party]\}$

$[(Food => Party) \vee (Drinks => Party)] \wedge \neg[(Food \wedge Drinks) => Party]$

$[(\neg Food \vee Party) \vee (\neg Drinks \vee Party)] \wedge [(Food \wedge Drinks) \wedge \neg Party]$

$(\neg Food \vee \neg Drinks \vee Party) \wedge Food \wedge Drinks \wedge \neg Party$

It is obvious that there will be a empty clause after resolution, so the sentence is true.