6.630 Solution to Problem Set 4

Solution P4.1

Combine $\overline{\overline{\epsilon}}$ and $\overline{\overline{\sigma}}$ to form a new permittivity tensor

$$\overline{\overline{\epsilon_c}} = \begin{pmatrix} \epsilon + i\frac{\sigma}{\omega} & 0 & 0 \\ 0 & \epsilon + i\frac{\sigma}{\omega} & 0 \\ 0 & 0 & \epsilon_z + i\frac{\sigma_z}{\omega} \end{pmatrix}$$

This is a uniaxial medium. For a wave propagating in $\hat{x}$ direction, we need to consider two cases: (1) Oridanary wave, $k^{(o)} = \omega\sqrt{\mu_o(\epsilon + i\frac{\sigma}{\omega})}$; (2) Extraordinary wave, $k^{(e)} = \omega\sqrt{\mu_o(\epsilon_z + i\frac{\sigma_z}{\omega})}$.

The extraordinary wave is polarized in $\hat{z}$ direction while the ordinary wave is polarized in $\hat{y}$ direction. Since σ_z is much larger than σ, extraordinary wave will decay much faster. So the wave coming out of a piece of polaroid will be only $\hat{y}$ polarized.

Solution P4.2

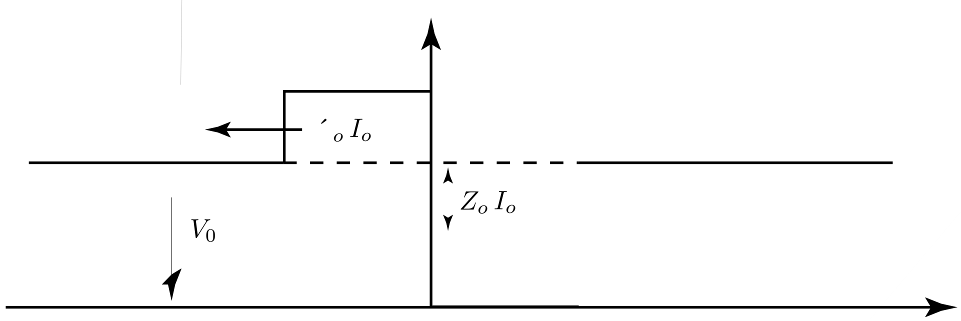
(a) $\alpha = 1$, $\beta = \pi/2$
(b) The $\hat{x}$ polarized wave is an ordinary wave. $k_y = \omega\sqrt{\mu_o 4\epsilon_o} = 2k_o$
(c) Let $(k^{(o)} - k^{(e)})d = \pi + 2n\pi$. The minimum d to satisfy this condition is $\lambda_o/2$

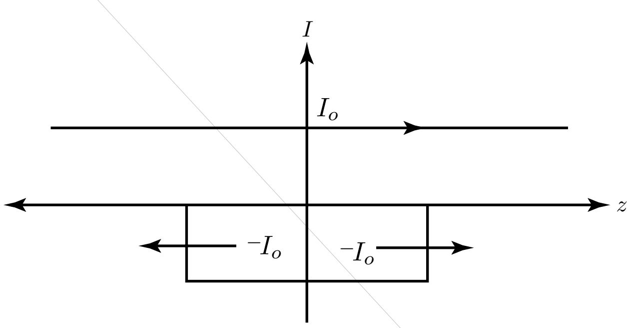
Solution P4.3

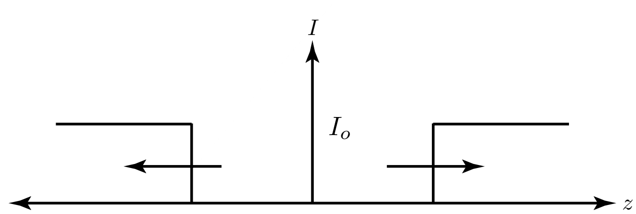
(a) **Method 1**

Fisrt, consider the transmission line right of the break point. When the break occurs, $I_r = I_0 + I_+ = 0$, where I_+ is the generated current. So, $I_+ = -I_0$, So, $V_+ = I_+ Z_o = -I_o Z_o$

Then, consider the transmission line left of the break point. When the break occurs, $I_l = I_0 + I_- = 0$, where I_- is the generated current. So, $I_- = -I_0$, So, $V_- = -I_- Z_o = I_o Z_o$







(a) **Method 2**

Consider the situation before the break occurs. In general there are two waves on the line, one propagating in the positive z-direction and one propagating in the negative z-direction.

$$V(z) = V_o = V_+ + V_-$$
$$I(z) = I_o = \frac{1}{Z_o}(V_+ - V_-)$$

Solving for V_+ and V_- we get,

$$V_+ = \frac{1}{2}(V_o + Z_o I_o)$$
$$V_- = \frac{1}{2}(V_o - Z_o I_o)$$

At the point of the break there is an open circuit at which the V_+ and V_- waves will be reflected generating $V_-' = \Gamma V_+$ and $V_+' = \Gamma V_-$ respectively, where $\Gamma = 1$.

To determine the current, we can use the above analysis to relate, V_+' to I_+' and V_-' to I_-'.

(b) The current, I_o can be found from

$$P_{dc} = I_o V_o = 1 \times 10^9 W$$

$$\Rightarrow I_o = \frac{1 \times 10^9}{6 \times 10^5} = \frac{1}{6} \times 10^4 A$$

The peak voltage is then given by,

$$V_{peak} = V_o + Z_o I_o = 1433 \text{ kV}$$

Solution P4.4

(a) We can find the length of the transmission by,

$$\ell = \frac{(3 \times 10^8 \text{ m/s})(10^{-8} \text{ s})}{2} = 1.5\text{m}$$

(b) Since the transmission line extends to infinity, we can terminate the line with an impedance of Z_0 without introducing reflection. The terminal load, instead of R_f, becomes $Z_L = R_f || Z_0$

$$\text{Since} \quad V = V_+(1 + \Gamma_L)$$

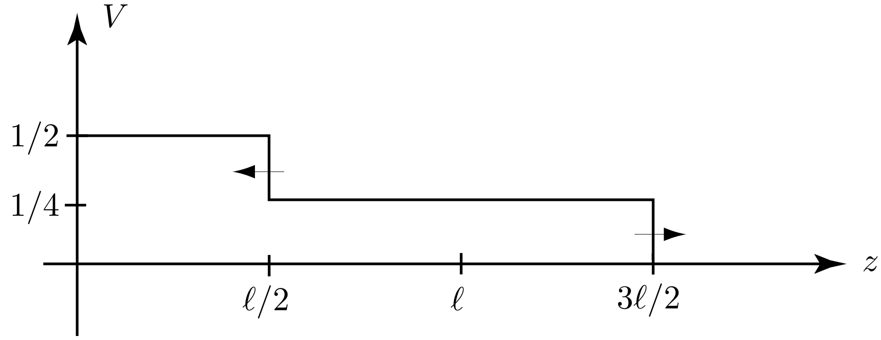
$$V = 0.25\text{V} \quad V_+ = 0.5\text{V}$$

$$\text{Hence} \quad \Gamma_L = -\frac{1}{2} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

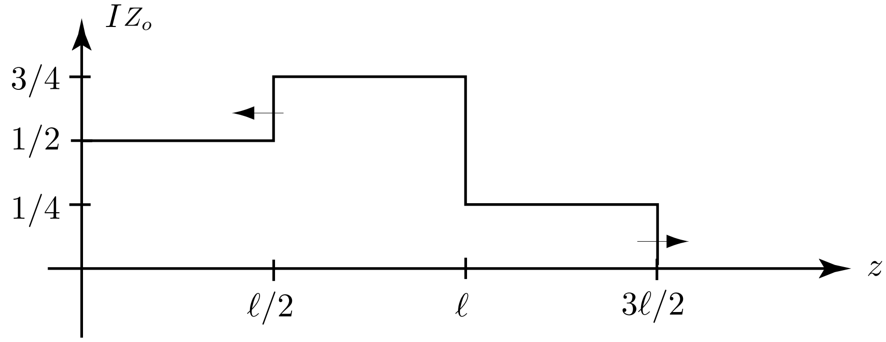
$$Z_L = \frac{1}{3} Z_0, \quad \frac{R_L Z_0}{R_L + Z_0} = \frac{Z_0}{3}$$

$$R_L = \frac{Z_0}{2}$$

(c)



V
1/2
1/4
z
$\ell/2$
ℓ
$3\ell/2$



IZ_o
3/4
1/2
1/4
z
$\ell/2$
ℓ
$3\ell/2$