6.632 Solution to Problem Set 2

Solution P2.1

For ordinary waves, $<\overline{S}>$ is in the same direction with $\overline{k}$.

For extraordinary waves, the angle θ_2 between $\overline{S}$ and the optic axis is larger than θ_1 except when $\theta_1 = 0$ or $\pi/2$.

Solution P2.2

(a) We can absorb a polarization by assigning z axis to either u or v axis, since the conductivity in z is nonzero. After a sufficiently long distance, this polarization in z is completly absorbed while the other polarization still survives.

(b) This question is equivalent to calculating the penetration depth in this case. So we start with the imaginary part of k. $k_L \approx \frac{1}{2}\omega\sqrt{\mu_o\epsilon_o}\left(\frac{\sigma_z}{\omega\epsilon_z}\right) = \frac{0.1\pi}{\lambda}$. $d = 1/k_I = 3.18\lambda$.

(c) In order to avoid power absorption, the z axis must be assigned to w, and x and y can be assigned to either u and v or v and u. We can choose an incident wave as $\overline{E}_{inc} = \frac{E_o}{\sqrt{2}}(\hat{x}-\hat{y})\cos(k_o z - \omega t)$, where $x, y = u, v$. The x- and y-components of the electric field at $z = 0$ are

$$E_x = \frac{E_o}{\sqrt{2}}\cos(\omega t)\,,\ E_y = -\frac{E_o}{\sqrt{2}}\cos(\omega t)\,.$$

Inside the slab, the x- and y-components of E field propagates with different k vectors, one being k_o while the other being $2\sqrt{3}k_o$. To get a circularly polarized wave at $z = d$, we need $(2\sqrt{3}k_o - k_o)d = \pi/2$. So $d = \frac{\lambda}{4(2\sqrt{3}-1)}$. In this case, the E field at $z = d$ is right-handed circularly polarized.

Solution P2.3

(a) From the expression of $\hat{z}\times\overline{H}_1$, we can obtain that $\overline{\overline{z}} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(b) $\frac{d\overline{M}}{dt} = \frac{d\overline{M}_1}{dt} = -i\omega\overline{M}_1 \approx g\mu_o M_o\hat{z}\times\overline{H}_1 - g\mu_o H_o\hat{z}\times\overline{M}_1 = g\mu_o(M_o\overline{\overline{z}}\cdot\overline{H}_1 - H_o\overline{\overline{z}}\cdot\overline{M}_1)$.

From the above equation we can get the relation between $\overline{H}_1$ and $\overline{M}_1$. From the definition $\overline{B}_1 = \mu_o(\overline{H}_1 + \overline{M}_1) = \overline{\overline{\mu}}\cdot\overline{H}_1$, it can be obtained that

$$\overline{\overline{\mu}} = \begin{bmatrix} \mu & i\mu_g & 0 \\ -i\mu_g & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix}$$

where $\mu = \frac{\omega^2 - g^2\mu_o^2H_o^2 - g^2\mu_o^2H_oM_o}{\omega^2 - g^2\mu_o^2H_o^2}\mu_o$, $\mu_g = \frac{-\omega g\mu_o M_o}{\omega^2 - g^2\mu_o^2H_o^2}\mu_o$, $\mu_z = \mu_o$.

(c) We first get

$$\overline{\overline{\nu}} = \begin{bmatrix} \nu & i\nu_g & 0 \\ -i\nu_g & \nu & 0 \\ 0 & 0 & \nu_z \end{bmatrix}$$

where $\nu = \frac{\mu}{\mu^2 - \mu_g^2}$, $\nu_g = \frac{-\mu_g}{\mu^2 - \mu_g^2}$, $\nu_z = \frac{1}{\mu_o}$. Similar to the derivations in the textbook, we eventually get the wave equation in kDB system as follows

$$\begin{bmatrix} u^2 - \kappa\nu & -i\kappa\nu_g\cos\theta \\ i\kappa\nu_g\cos\theta & u^2 - \kappa(\nu\cos^2\theta + \nu_z\sin^2\theta) \end{bmatrix}\cdot\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = 0$$

.

So

$$\frac{D_2}{D_1} = -\frac{B_1}{B_2} = \frac{(\nu - nu_o)\sin^2\theta \pm \sqrt{(\nu-\nu_o)^2\sin^4\theta + 4\nu_g^2\cos^2\theta^2}}{2i\nu_g\cos\theta}$$

.

Define $\tan 2\psi = \frac{2\nu_g\cos\theta}{(\nu-\nu_z)\sin^2\theta}$, then $\frac{D_2}{D_1} = -i\cot\psi$ or $\frac{D_2}{D_1} = i\tan\psi$. Both are elliptically polarized.

So Faraday rotation exists.