# Artificial Intelligence—Spring 2022

## Homework 5

Issued: Apr. 18th, 2022 Due: Apr. 25th, 2022

## Problem 1

Solutions:

$\because P(y|x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-(\theta_1 x+\theta_2))^2}{2\sigma^2}}$, and the data points are $(x_j, y_j)$

$\therefore L = \Sigma_{j=1}^{N} \log \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y_j-(\theta_1 x_j+\theta_2))^2}{2\sigma^2}} = -\Sigma_{j=1}^{N} \frac{(y_j-(\theta_1 x_j+\theta_2))^2}{2\sigma^2} - N(\log\sqrt{2\pi} + \log\sigma)$

Let the derivatives equal to 0, we have:

$$\frac{\partial L}{\partial \theta_1} = -\Sigma_{j=1}^{N} \frac{x_j(y_j - (\theta_1 x_j + \theta_2))}{\sigma^2} = 0 \tag{1}$$

$$\frac{\partial L}{\partial \theta_2} = -\Sigma_{j=1}^{N} \frac{y_j - (\theta_1 x_j + \theta_2)}{\sigma^2} = 0 \tag{2}$$

$$\frac{\partial L}{\partial \sigma} = \Sigma_{j=1}^{N} \frac{(y_j - (\theta_1 x_j + \theta_2))^2}{\sigma^3} - \frac{N}{\sigma} = 0 \tag{3}$$

Solve Equations (1)-(3), we can obtain that

$$\theta_1 = \frac{N\Sigma_{j=1}^{N} x_j y_j - (\Sigma_{j=1}^{N} y_j)(\Sigma_{j=1}^{N} x_j)}{N\Sigma_{j=1}^{N} x_j^2 - (\Sigma_{j=1}^{N} x_j)^2}$$

$$\theta_2 = \frac{1}{N}\Sigma_{j=1}^{N}(y_j - \theta_1 x_j)$$

$$\sigma = \sqrt{\frac{\Sigma_{j=1}^{N}(y_j - (\theta_1 x_j + \theta_2))^2}{N}}$$

## Problem 2

Solutions:

**a.** $\because P(Y = true) = \pi$

$\therefore P(Y = false) = (1 - \pi)$

$\because$ $p$ of the $N$ samples are positive and $n$ of the $N$ are negative

$\therefore$ the probability of seeing this particular sequence of examples is $l = \pi^p(1-\pi)^n$

and the log likelihood is $L = p\log\pi + n\log(1-\pi)$

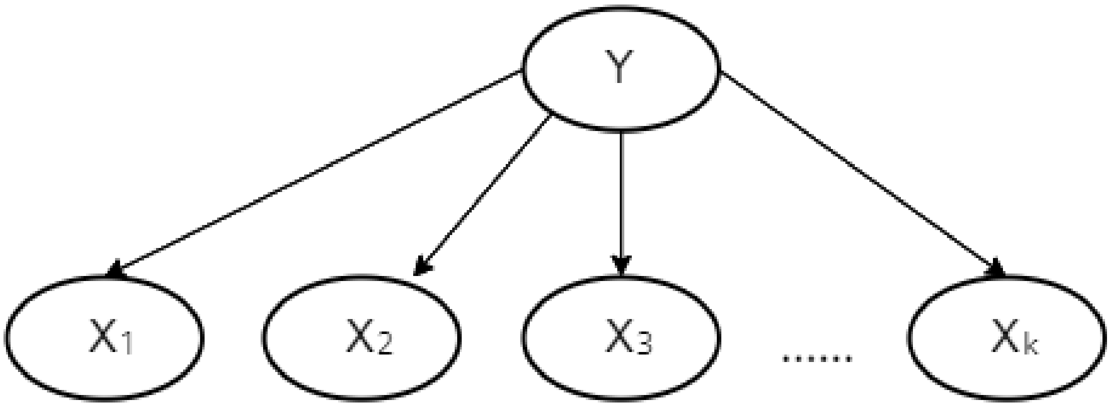
**b.** Let the deriative equal to 0, that is

$$\frac{\partial L}{\partial \pi} = \frac{p}{\pi} + \frac{n}{\pi - 1} = 0$$

and we obtain

$$\pi = \frac{p}{n+p} = \frac{p}{N}$$

**c.** Acoording to the sumption, we have the Bayes network as follows:




**d.** Acoording to the additional notation,

$$P(X_i = true|Y = true) = \alpha_i$$
$$P(X_i = true|Y = false) = \beta_i$$

Therefore, the likelihood for the data including the attributes is:

$$l = \pi^p[\Pi_{i=1}^k \alpha_i^{p_i^+}(1-\alpha)^{n_i^+}] \times (1-\pi)^n[\Pi_{i=1}^k \beta_i^{p_i^-}(1-\beta_i)^{n_i^-}]$$

that is,

$$l = \pi^p(1-\pi)^n \Pi_{i=1}^k \alpha_i^{p_i^+} \beta_i^{p_i^-}(1-\alpha_i)^{n_i^+}(1-\beta_i)^{n_i^-}$$

and the log likelihood is

$$L = p\log\pi + n\log(1-\pi) + \Sigma_{i=1}^k[p_i^+\log\alpha_i + p_i^-\log\beta_i + n_i^+\log(1-\alpha_i) + n_i^-\log(1-\beta_i)]$$

**e.** Let the derivative equal to 0, we have:

$$\frac{\partial L}{\partial \alpha_i} = \frac{p_i^+}{\alpha_i} + \frac{n_i^+}{\alpha_i - 1} = 0$$
$$\frac{\partial L}{\partial \beta_i} = \frac{p_i^-}{\beta_i} + \frac{n_i^-}{\beta_i - 1} = 0$$

and the solution for these two equaltions are:

$$\alpha_i = \frac{p_i^+}{n_i^+ + p_i^+}$$
$$\beta_i = \frac{p_i^-}{n_i^- + p_i^-}$$

In words, the value of $\alpha_i$ reprsents the proportion that $X_i = true$ given $Y = true$ and the value of $\beta_i$ reprsents the proportion that $X_i = true$ given $Y = false$.