Homework 2

Chapter 3.

3.1 $\because x_c(t) = A_c m(t)\cos(2\pi f_c t + \phi_0)$

and the demodulation carrier is $2\cos[2\pi f_c t + \theta(t)]$.

$\therefore d(t) = 2A_c m(t)\cos(2\pi f_c t + \phi_0)\cos[2\pi f_c t + \theta(t)]$

$= A_c m(t)[\cos(4\pi f_c t + \phi_0 + \theta(t)) + \cos(\phi_0 - \theta(t))]$

$\because y_D(t)$ is the output by filtering $d(t)$ with a Lowpass filter, that is.

$y_D(t) = Lp[d(t)] = A_c m(t)\cos[\phi_0 - \theta(t)]$.

(a) if $A_c = 1$ and $\theta(t) = \theta_0$.

$\therefore y_D(t) = m(t)\cos(\phi_0 - \theta_0)$

$\therefore$ the mean-square error between $m(t)$ and $y_D(t)$ is:

$\varepsilon^2(t) = \langle [m(t) - y_D(t)]^2 \rangle = \langle m^2(t)[1-\cos(\phi_0-\theta_0)]^2 \rangle$

$\because \theta_0$ is a constant.

$\therefore \varepsilon^2(t) = \langle m^2(t) \rangle [1-\cos(\phi_0 - \theta_0)]^2$

where $\langle \cdot \rangle$ denotes the time average value.

(b) if $A_c = 1$ and $\theta_0 = 2\pi f_0 t$. and $m(t)$ is slowly varying relavantly,

$\therefore \varepsilon^2(t) = \langle m^2(t)[1-\cos(\phi_0 - 2\pi f_0 t)]^2 \rangle = \langle m^2(t)[1+\cos^2(\phi_0 - 2\pi f_0 t) - 2\cos(\phi_0 - 2\pi f_0 t)] \rangle$

$= \langle m^2(t) \rangle \langle 1 + \cos^2(\phi_0 - 2\pi f_0 t) - 2\cos(\phi_0 - 2\pi f_0 t) \rangle$

$= \langle m^2(t) \rangle (1 + \frac{1}{2} - 0) = \frac{3}{2}\langle m^2(t) \rangle$.

where $\langle \cdot \rangle$ denotes the time average value.

3.5 $\because x_c(t) = A_c[1 + a m_n(t)]\cos(2\pi f_c t)$.

$\therefore$ the envelope of $x_c(t)$ is $A_c[1 + a m_n(t)]$.

$\because$ The message signal is a waveform having ~~no~~ zero DC value.

and $40 - 25 = 25 - 10 = 15$. as well as $m_n(t) = \frac{m(t)}{|\min[m(t)]|}$

$\therefore$ we can infer that $m_n(t)$ has a minimum value of -1 and maximum value of 1

According to Figure 3.33. we have: $\begin{cases} A_c(1+a) = 40. \\ A_c(1-a) = 10. \end{cases}$

$\therefore A_c = 25$. $a = 0.6$

$\therefore x_c(t) = 25[1 + 0.6\, m_n(t)]\cos(2\pi f_c t)$

and the carrier power is $\frac{1}{2}A_c^2 = \frac{1}{2}\times 625 = 312.5$ W.

$\because \langle m_n^2(t)\rangle = \frac{4}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{t^2}{T^2}\,dt = \frac{1}{3}$ W.

$\therefore E_{ff} = \frac{a^2\langle m_n^2(t)\rangle}{1 + a^2\langle m_n^2(t)\rangle} = \frac{0.36\times\frac{1}{3}}{1 + 0.36\times\frac{1}{3}} = 10.71\%$

the sideband power is $\frac{1}{2}A_c^2 a^2 \langle m_n^2(t)\rangle = \frac{1}{2}\times 625 \times 0.36 \times \frac{1}{3} = 37.5$ W.

Above all, $a = 0.6$, carrier power is 312.5 W. the efficiency is 10.71% and the power in the sidebands is 37.5 W.

3.8 (a) $\because m(t) = 9\cos(20\pi t) - 8\cos(60\pi t) = 9\cos(20\pi t) + 24\cos(20\pi t) - 32\cos^3(20\pi t)$.

$= 33\cos(20\pi t) - 32\cos^3(20\pi t)$

Let $x(t) = \cos(20\pi t) \in [-1, 1]$.

$\therefore m(t) = 33x - 32x^3$.

$\therefore \frac{dm}{dx} = 33 - 96x^2$.

when $\frac{dm}{dx} = 0$, we have $x = \pm \frac{\sqrt{11}}{4\sqrt{2}}$

$\therefore m(t)$ will ~~increase~~ decrease in the interval of $[-1, -\frac{\sqrt{11}}{4\sqrt{2}}]$ and $[\frac{\sqrt{11}}{4\sqrt{2}}, 1]$, while

indecrease in the interval of $[-\frac{\sqrt{11}}{4\sqrt{2}}, \frac{\sqrt{11}}{4\sqrt{2}}]$, for x.

$\because 33\times(\pm1) - 32\times(\pm1)^3 = \pm33 \mp 32 = \pm1$.

$-33\times\frac{\sqrt{11}}{4\sqrt{2}} + 32\times(\frac{\sqrt{11}}{4\sqrt{2}})^3 = -\frac{11\sqrt{11}}{2\sqrt{2}} < 1$.

$\therefore \min[m(t)] = -\frac{11\sqrt{11}}{2\sqrt{2}}$.

$\therefore m_n(t) = \frac{m(t)}{|\min[m(t)]|} = \frac{2\sqrt{2}}{11\sqrt{11}}[33\cos(20\pi t) - 32\cos^3(20\pi t)]$.

(b) $\langle m_n^2(t)\rangle = \frac{8}{1331}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} 81\cos^2(20\pi t) + 64\cos^2(60\pi t) - 144\cos(20\pi t)\cos(60\pi t)\,dt$

$= \frac{8}{1331}\times(\frac{81}{2} + \frac{64}{2} + 0) = 0.4358$ W.

(c) $\because a = 0.8$.

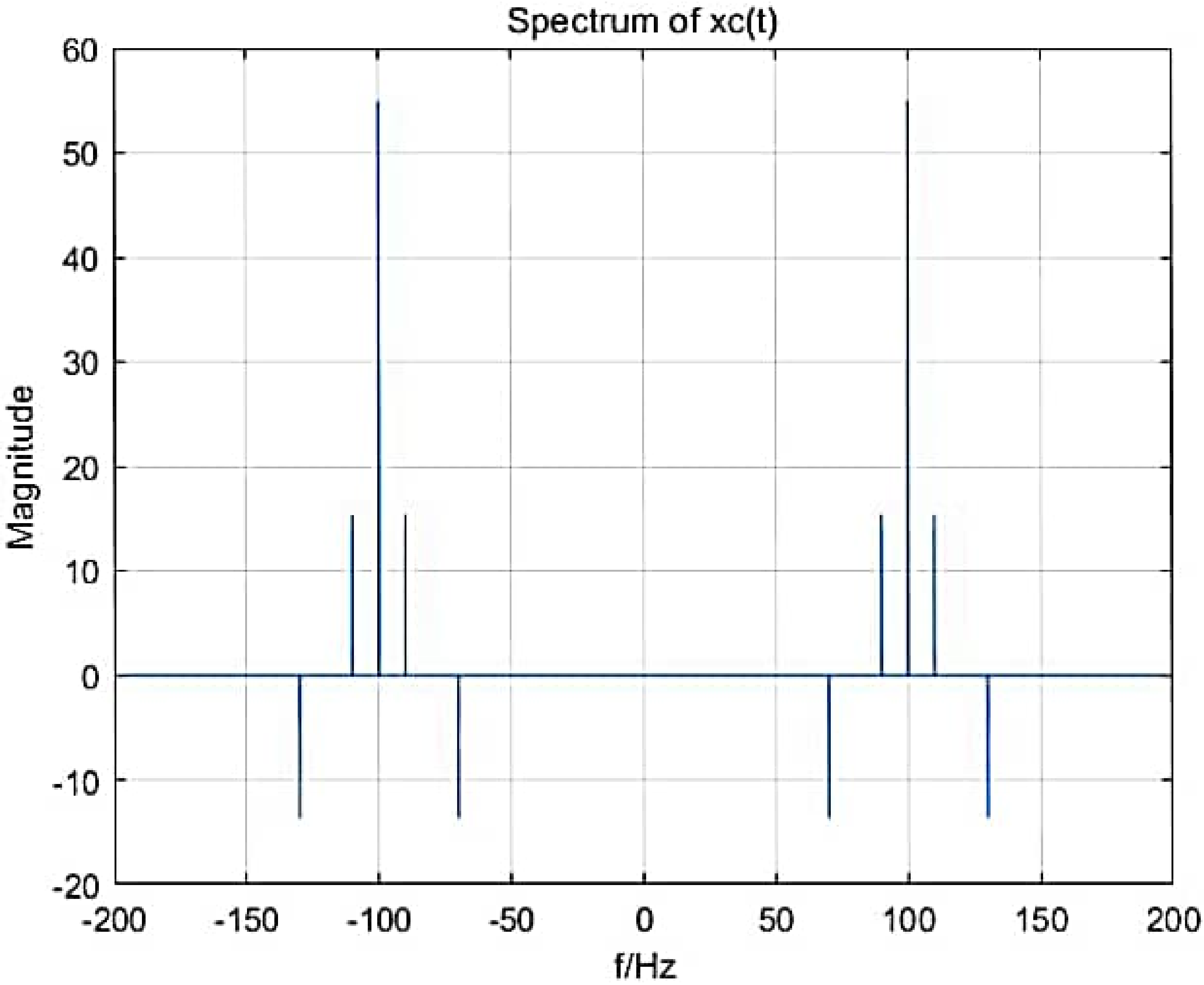
$\therefore E_{ff} = \frac{a^2\langle m_n^2(t)\rangle}{1+a^2\langle m_n^2(t)\rangle} = \frac{0.64\times0.4358}{1+0.64\times0.4358} = 21.81\%$

(d) $\because x_c(t) = 110[1 + 0.8\, m_n(t)]\cos(2\times 100\pi t)$

$\therefore X_c(f) = F[x_c(t)] = 55[\delta(f-100) + \delta(f+100)] + \overset{198}{\cancel{396}} \times (\frac{2}{11})^{\frac{3}{2}}[\delta(f-110) + \delta(f+110) + \delta(f-90)$

$+ \delta(f+90)] - 176 \times (\frac{2}{11})^{\frac{3}{2}}[\delta(f-130) + \delta(f+130) + \delta(f-70) + \delta(f+70)]$

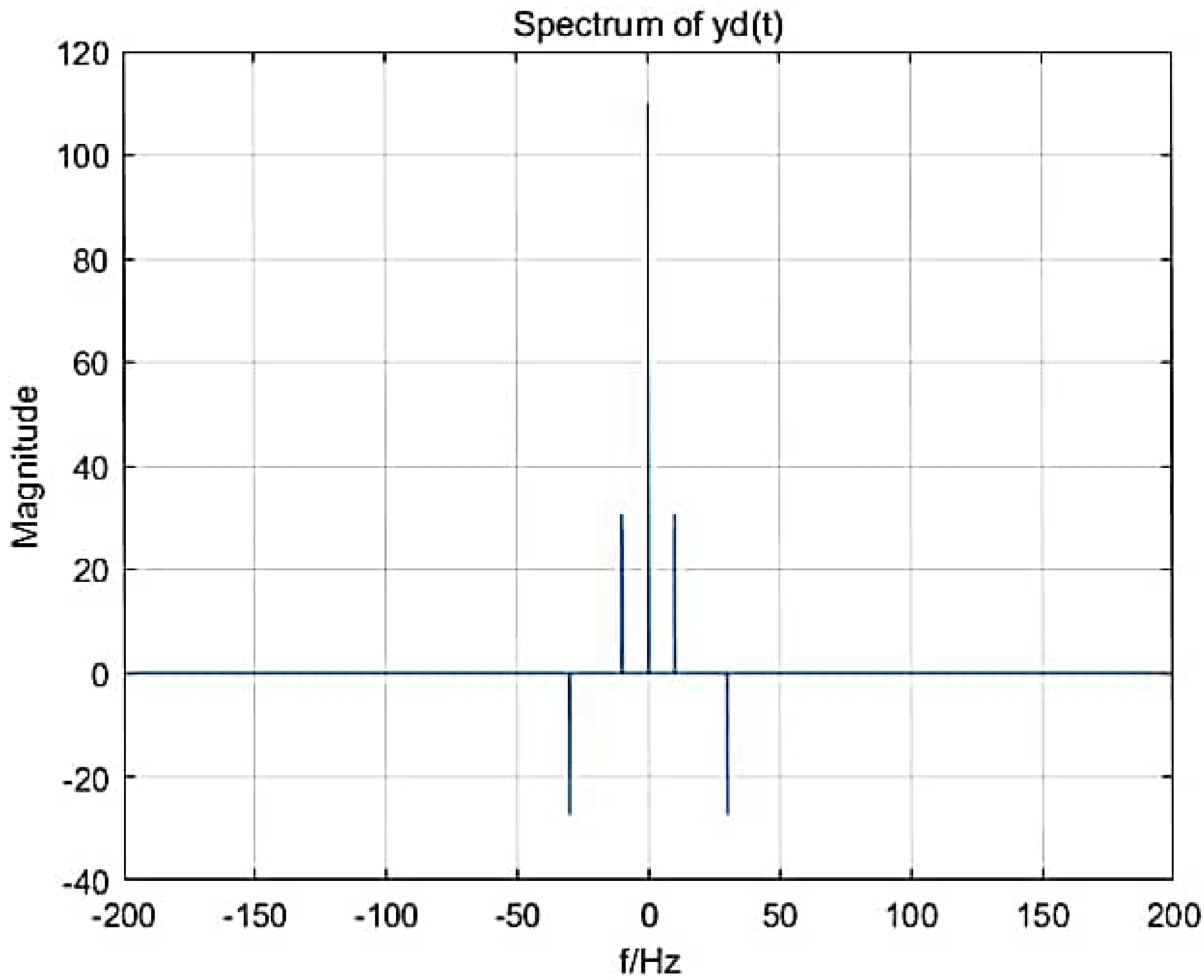
$\therefore$ the double-sided spectrum of $x_c(t)$ is as follows:


Spectrum of xc(t)
Magnitude
f/Hz
60
50
40
30
20
10
0
-10
-20
-200
-150
-100
-50
0
50
100
150
200

$\because y_D(t) = 110[1 + 0.8\, m_n(t)]$

$\therefore Y_D(f) = F[y_D(t)] = 110\delta(t) + 396 \times (\frac{2}{11})^{\frac{3}{2}}[\delta(f-10) + \delta(f+10)] - 352 \times (\frac{2}{11})^{\frac{3}{2}}[\delta(f-30) + \delta(f+30$

$\therefore$ its spectrum is as follows:


Spectrum of yd(t)
Magnitude
f/Hz
120
100
80
60
40
20
0
-20
-40
-200
-150
-100
-50
0
50
100
150
200

3.15 $\because m(t) = 4\cos(2\pi f_m t) + \cos(4\pi f_m t)$

$\therefore$ The Hilbert transform of $m(t)$ is:

$$\hat{m}(t) = 4\sin(2\pi f_m t) + \sin(4\pi f_m t)$$

$$\therefore x_c(t) = \frac{1}{2}A_c[4\cos(2\pi f_m t) + \cos(4\pi f_m t)]\cos(2\pi f_c t) \pm \frac{1}{2}A_c[4\sin(2\pi f_m t) + \sin(4\pi f_m t)]\sin(2\pi f_c t)$$

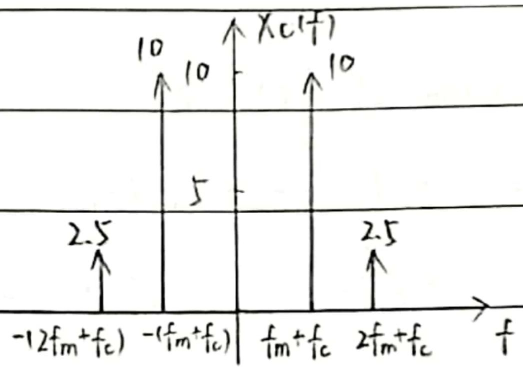
$$= \frac{1}{2}A_c[4\cos(2\pi f_m t)\cos(2\pi f_c t) \pm 4\sin(2\pi f_m t)\sin(2\pi f_c t) + \cos(4\pi f_m t)\cos(2\pi f_c t) \pm \sin(4\pi f_m t)\sin(2\pi f_c t)]$$

$$= \frac{1}{2}A_c\{4\cos[2\pi(f_m \pm f_c)t] + \cos[2\pi(2f_m \pm f_c)t]\} \quad (A_c = 10)$$

when we take the algebraic sign as plus, we have:

$$X_c(f) = \overset{10}{\cancel{A_c}}[\delta(f+f_m+f_c) + \delta(f-f_m-f_c)] + \underset{2.5}{A_c}[\delta(f+2f_m+f_c) + \delta(f-2f_m-f_c)].$$

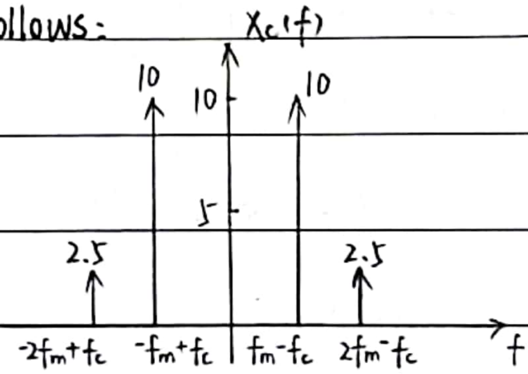
its sketch is as follows:



when we take the algebraic sign as minus. we have.

$X_c(f) = 10[\delta(f+f_m-f_c)+\delta(f-f_m+f_c)] + 2.5[\delta(f+2f_m-f_c)+\delta(f-2f_m+f_c)]$

its sketch is as follows:



As we can see. the first result is upper-sideband while the second result is lower-sideband. SSB.

3.19. According to Example 3.3. the message signal is: $m(t)=A\cos(2\pi f_1 t)+B\cos(2\pi f_2 t)$ and the filter's amplitude and phase responses are:

$H(f_c-f_2)=0$. $H(f_c-f_1)=\epsilon e^{-j\theta_a}$, $H(f_c+f_1)=(1-\epsilon)e^{-j\theta_b}$, $H(f_c+f_2)=1e^{-j\theta_c}$

$\because x_{DSB}(t)=Re[(\frac{A}{2}(e^{-j2\pi f_1 t}+e^{j2\pi f_1 t})+\frac{B}{2}(e^{-j2\pi f_2 t}+e^{j2\pi f_2 t}))e^{j2\pi f_c t}]$

$\therefore x_c(t)=Re[(\frac{A}{2}\epsilon e^{-j(2\pi f_1 t+\theta_a)}+\frac{A}{2}(1-\epsilon)e^{j(2\pi f_1 t-\theta_b)}+\frac{B}{2}e^{j(2\pi f_2 t-\theta_c)})e^{j2\pi f_c t}]$

By multiplying $x_c(t)$ by $2e^{-j2\pi f_c t}$ and taking the real part, we have:

$e(t) = A\epsilon\cancel{B}\cos(2\pi f_1 t + \theta_a) + A(1-\epsilon)\cos(2\pi f_1 t - \theta_b) + B\cos(2\pi f_2 t - \theta_c)$

$\therefore$ the output of the demodulator is definitely real.

Chapter 4

4.4 $\because x_{c1}(t) = A_c\cos[2\pi f_c t + \beta\sin(2\pi f_m t)] = \mathrm{Re}[A_c e^{j\beta\sin(2\pi f_m t)} e^{j2\pi f_c t}]$

and $e^{j\beta\sin(2\pi f_m t)} = \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{j2\pi n f_m t}$

$\therefore x_{c1}(t) = \mathrm{Re}[A_c \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t}] = A_c \sum_{n=-\infty}^{+\infty} J_n(\beta)\cos(2\pi n f_m t + 2\pi f_c t)$

$= \frac{A_c}{2}\sum_{n=-\infty}^{+\infty} J_n(\beta)[e^{j2\pi(nf_m+f_c)t} + e^{-j2\pi(nf_m+f_c)t}]$.

Similarly, $\because x_{c2}(t) = A_c\cos[2\pi f_c t + \beta\cos(2\pi f_m t)] = \mathrm{Re}[A_c e^{j\beta\cos(2\pi f_m t)} e^{j2\pi f_c t}]$

and $e^{j\beta\cos(2\pi f_m t)} = e^{j\beta\sin(2\pi f_m t + \frac{\pi}{2})} = \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{j(2\pi f_m t + \frac{\pi}{2})n}$

$\therefore x_{c2}(t) = \mathrm{Re}[A_c \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{j(2\pi f_m t + \frac{\pi}{2})n} e^{j2\pi f_c t}] = A_c \sum_{n=-\infty}^{+\infty} J_n(\beta)\cos(2\pi n f_m t + 2\pi f_c t + \frac{n}{2}\pi)$

$= \frac{A_c}{2}\sum_{n=-\infty}^{+\infty} J_n(\beta)[e^{j[2\pi(nf_m+f_c)t + \frac{n}{2}\pi]} + e^{-j[2\pi(nf_m+f_c)t + \frac{n}{2}\pi]}]$

$\therefore$ ~~th~~ the amplitude spectrum of $x_1(t)$ and $x_2(t)$ are identical

and their phase spectrama are different as n varys.

With respect to the periodic of exponent function, whose period is 2π and $\frac{n}{2}\pi = 2\pi\cdot\frac{n}{4}$, the ranges of $x_{c2}(t)$'s phase spectrum's value are $[-1, 1]$.

4.9 (a) $\because f_c = 1000$, and the instantaneous phase $\theta_i = 2\pi f_c t + 40\sin(5t^2)$.

$\therefore$ the phase deviation $\phi(t) = |40\sin(5t^2)| = 40|\sin(5t^2)|$

$\because \cancel{\frac{d\theta_i}{dt}}$ $f_i(t) = \frac{1}{2\pi}\frac{d\theta_i}{dt} = f_c + \frac{1}{2\pi} 10t\cos(5t^2) \times 40 = f_c + \frac{200}{\pi} t\cos(5t^2)$

$\therefore$ the frequency deviation is $\frac{200}{\pi}|t\cos(5t^2)|$. Hz.

(b) $\because x_c(t) = \cos[2\pi(600)t] = \cos[2\pi f_c t - 2\pi(400)t]$

$\therefore$ the instantaneous phase $\theta_i = 2\pi f_c t - 2\pi(400)t$

$\therefore$ the phase deviation $\phi(t) = |-2\pi(400)t| = 800\pi|t|$

$\because f_i(t) = \frac{1}{2\pi}\frac{d\theta_i}{dt} = f_c - 400.$

$\therefore$ the frequency deviation is $|-400| = 400$ Hz.

4.11 (a) $\because x_c(t) = 100\cos[2\pi f_c t + 2\pi f_d \int^t m(\alpha)d\alpha]$

$\therefore$ the phase deviation is $\phi(t) = 2\pi f_d \int^t m(\alpha)d\alpha$.

$\because m(t) = 4\Pi[\frac{1}{8}(t-4)]$.

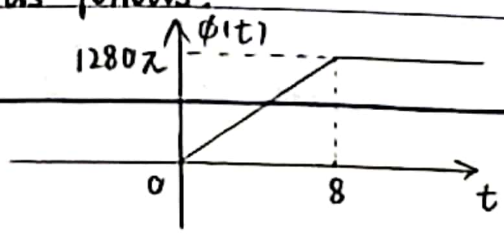
① if $t \le 0$, $\phi(t) = 0$.

② if $0 < t \le 8$, $\phi(t) = 2\pi f_d \times 4 \times t = 8\pi \times 20t = 160\pi t$

③ if $t > 8$, $\phi(t) = 2\pi f_d \times 4 \times 8 = 1280\pi$.

$$\therefore \phi(t) = \begin{cases} 0, & t \le 0 \\ 160\pi t, & 0 < t \le 8 \\ 1280\pi, & t > 8. \end{cases}$$

its sketch is as follows:

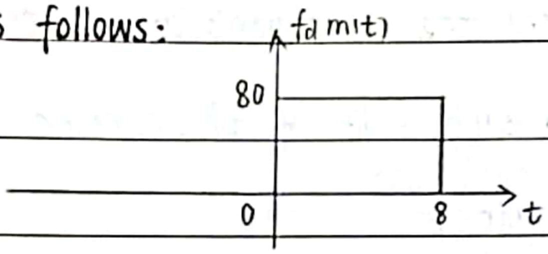


(b) $\because$ the instantaneous phase $\theta_i = 2\pi f_c t + 2\pi f_d \int^t m(\alpha)\,d\alpha$

$\therefore$ the instantaneous frequency $f_i(t) = \frac{1}{2\pi}\frac{d\theta_i}{dt} = f_c + f_d m(t)$.

$\therefore$ the frequency deviation is $f_d m(t) = 80\,\Pi\left[\frac{1}{8}(t-4)\right] = \begin{cases} 80, & 0 \le t \le 8 \\ 0, & \text{otherwise.} \end{cases}$

its sketch is as follows:



(c) From (b), we can know the peak frequency deviation is 80 Hz.

(d) From (a), we can know the peak phase deviation is 1280π rad.

(e) $\because A_c = 100$.

$\therefore$ the power at the modulator ouptput is:

$\langle x_c^2(t) \rangle = \frac{1}{2}A_c^2 = 5000$ W.

4.18 (a) According to the title, $f_d = 8$ Hz/V

$\therefore$ the peak frequency deviation is $\max[f_d m(t)] = 8 \times 10 = 80$ Hz.

(b) Similarly, the peak phase deviation is:

$2\pi f_d \int^t m(\alpha)\,d\alpha = 16\pi \int^t 10\cos(20\pi\alpha)\,d\alpha = 8\sin(20\pi t)$.

$\therefore$ its peak value is $\max[8\sin(20\pi t)] = 8$ rad.

(c) the modulation index $\beta = \frac{f_d A}{f_m} = \frac{8 \times 10}{10} = 8$.

(d) At the filter input, ~~这~~ because $A_c = 10$.

$\therefore$ the power is $\frac{1}{2}A_c^2 = 50$ W.

$\because f_m = 10$ Hz, $f_c = 500$ Hz and the filter has a band-width of 70 Hz.

$\therefore k = \lfloor \frac{70}{20} \rfloor = 3$.

$\therefore$ the filter passes the component at the carrier frequency and three components on each side of the carrier.

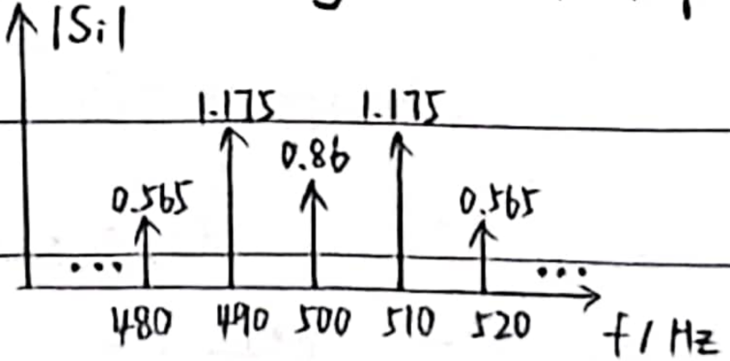
so the power ratio is:

$$P_r = J_0^2(8) + 2[J_1^2(8) + J_2^2(8) + J_3^2(8)] = 0.172^2 + 2[0.235^2 + (-0.113)^2 + (-0.291)^2]$$

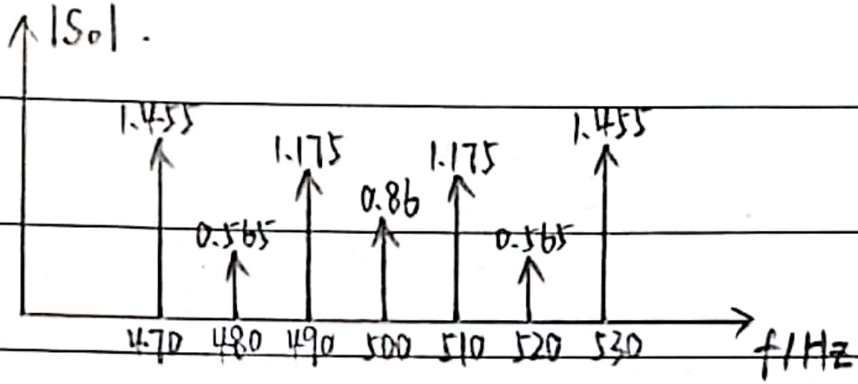
$$= \cancel{0.34332}\ 0.3349.$$

$\therefore$ the ouptput ~~a~~ power is $50 \times P_r = 16.75$ W.

(e) At the filter input, the single-sided amplitude spectrum is as follows:



At the filter output, the sigle-sided ampliatude spectrum is as follows:



4.20 $\because D_1 = 0.05,\ D_2 = 20.$

$\therefore n = \frac{D_2}{D_1} = 400$

$\because f_0 = 110$ kHz, $f_c = 100$ MHz.

while $f_c = |nf_0 + f_{LO}|$ or $f_c = |nf_0 - f_{LO}|$

$\therefore f_{LO} = 56$ MHz or $f_{LO} = 144$ MHz.

Obviously, the center frequency of the bandpass filter is 100 MHz.

using Carson's rule, we have

$B = 2(D_2+1)W = 2 \times (20+1) \times 10k = 420$ kHz.