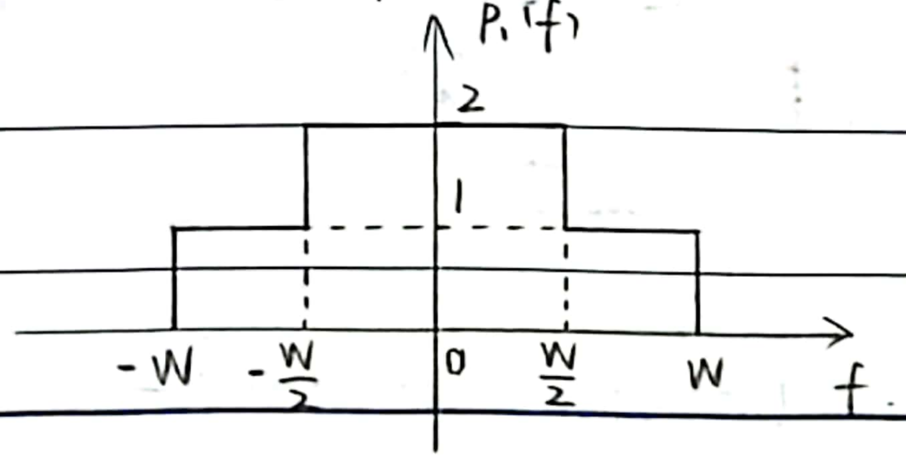
Chapter 5

5.11 (a) $\because P_1(f) = \Pi(\frac{f}{2W}) + \Pi(\frac{f}{W})$.

$\therefore$ the spectrum is as follows:



it is apparent that $P_1(f)$ satisfies Nyquist's pulse-shape criterison.

$\because \sum_{k=-\infty}^{+\infty} P_1(f+\frac{k}{T}) = T, \quad |f| \le \frac{1}{2T}.$

$\therefore$ An appropriate sample interval may satisfy:

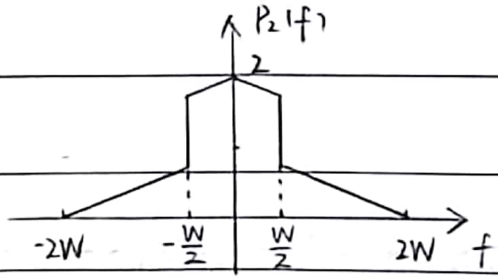
$\frac{1}{T} = \frac{W}{2} + W.$

$\therefore T = \frac{2}{3W}$

and the pulse-shape function: $p_1(t) = 2W\operatorname{sinc}(2Wt) + W\operatorname{sinc}(Wt)$

(b) $\because P_2(f) = \Lambda(\frac{f}{2W}) + \Pi(\frac{f}{W}).$

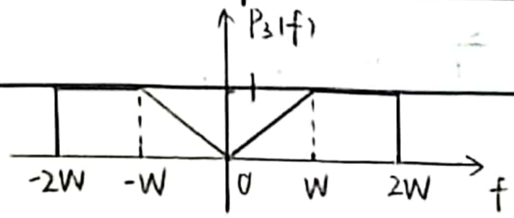
$\therefore$ the spectrum is as follows:



and $P_2(f)$ doesn't satisfy Nyquist's pulse-shape criterion.

(c) $\because P_3(f) = \Pi(\frac{f}{4W}) - \Lambda(\frac{f}{W})$

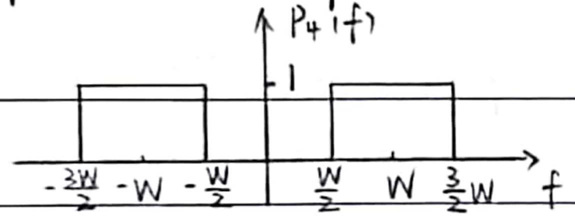
$\therefore$ the spectrum is as follows:



and $P_3(f)$ does not satisfy Nyquist's pulse-shape criterion.

(d) $\because P_4(f) = \Pi\left(\frac{f-W}{W}\right) + \Pi\left(\frac{f+W}{W}\right)$.

$\therefore$ the spectrum is as follows:



It is apparent that $P_4(f)$ satisfies Nyquist's pulse-shape criterion.

$\because$ An appropriate sample interval may satisfy:

$\frac{1}{T} = W$

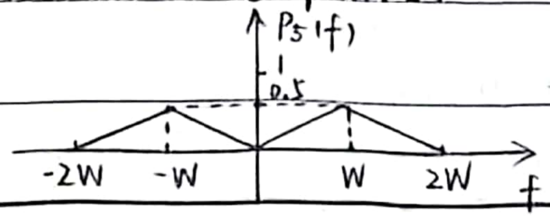
$\therefore T = \frac{1}{W}$.

and the pulse-shape function: ~~$P_4(t) =$~~

$p_4(t) = W\operatorname{sinc}(Wt)e^{j2\pi Wt} + W\operatorname{sinc}(Wt)e^{-j2\pi Wt} = 2W\operatorname{sinc}(Wt)\cos(2\pi Wt)$.

(e) $\because P_5(f) = \Lambda\left(\frac{f}{2W}\right) - \Lambda\left(\frac{f}{W}\right)$.

$\therefore$ the spectrum is as follows:



It is apparent that $P_5(f)$ satisfies Nyquist's pulse-shape criterion.

$\because$ An appropriate sample interval may satisfy:

$\frac{1}{T} = W$

$\therefore T = \frac{1}{W}$

and the pulse-shape function: $p_5(t) = 2W\,\text{sinc}^2(2Wt) - W\,\text{sinc}^2(Wt)$

5.13 $\because$ the bandwidth of the channel is 7 kHz.

while: the bandwidth of $V_{RC}(f)$ is $\frac{1+\beta}{2\tau}$

$\therefore \frac{1+\beta}{2\tau} = 7$ kHz

On the other hand, data are transmitted at 9 kbps.

$\therefore R_b = \frac{1}{\tau} = 9$ kbps.

$\therefore$ ~~β~~ $\beta = \frac{5}{9}$

Chapter 9.

9.6 (a) According to the title, the duration is T and receiver theshold is 0.

$\therefore$ There are two ways in which erros occur. If A_1 is transmitted, an error occurs if $A_1T + N < 0$, that is $N < -A_1T$.

$\because$ the pdf of N is: $f_N(\eta) = \frac{e^{-\frac{\eta^2}{N_0T}}}{\sqrt{\pi N_0 T}}$

$\therefore \Pr(\text{error} \mid A_1 \text{ sent}) = \int_{-\infty}^{-A_1T} \frac{e^{-\frac{\eta^2}{N_0T}}}{\sqrt{\pi N_0T}}\, d\eta \overset{u \triangleq \sqrt{\frac{2}{N_0T}}\eta}{=\!=} \int_{\sqrt{\frac{2A_1^2T}{N_0}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du = Q\left(\sqrt{\frac{2A_1^2T}{N_0}}\right)$

$\because$ The other way in which an error occurs if $-A_2$ is transmitted and $-A_2T + N > 0$. that is, $N > A_2T$.

$$\therefore Pr(\text{error} \mid A_2 \text{ sent}) = \int_{A_2T}^{\infty} \frac{e^{-\frac{\eta^2}{N_0T}}}{\sqrt{\pi N_0 T}} d\eta = Q\left(\sqrt{\frac{2A_2^2T}{N_0}}\right).$$

$$\therefore P_E = P(A_1 \text{ sent}) P(\text{error} \mid A_1 \text{ sent}) + P(A_2 \text{ sent}) P(\text{error} \mid A_2 \text{ sent})$$

$$= \frac{1}{2} Q\left(\sqrt{\frac{2A_1^2T}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{2A_2^2T}{N_0}}\right).$$

$$\because E = \frac{A_1^2 + A_2^2}{2} T = \frac{A_1^2 T}{2}(1+\rho^2). \quad \rho = \frac{A_2}{A_1}$$

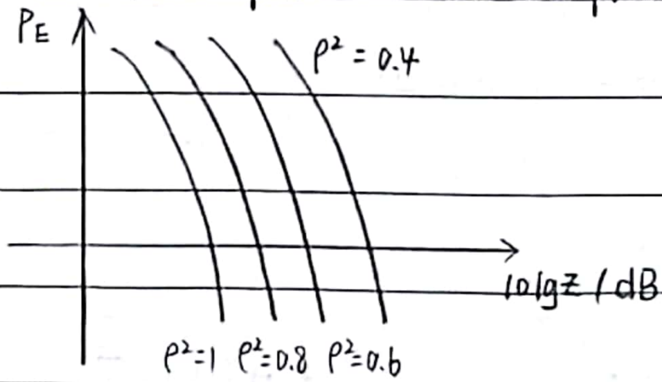
$$\therefore A_1^2 T = \frac{2E}{1+\rho^2}, \quad A_2^2 T = \frac{2E\rho^2}{1+\rho^2}$$

$$\therefore P_E = \frac{1}{2} Q\left(\sqrt{\frac{2}{1+\rho^2}\frac{2E}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{2\rho^2}{1+\rho^2}\frac{2E}{N_0}}\right)$$

$$\because z = \frac{E}{N_0}$$

$$\therefore P_E = \frac{1}{2} Q\left(\sqrt{\frac{4z}{1+\rho^2}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{2\rho^2(2z)}{1+\rho^2}}\right).$$

(b) According to the result in part (a), we can plot as follows:



when $P_E = 10^{-6}$, the degradations are: $10\lg z = \begin{cases} 10.5, & \rho^2 = 1 \\ 10.8, & \rho^2 = 0.8 \\ 11.5, & \rho^2 = 0.6 \\ 13, & \rho^2 = 0.4 \end{cases}$ (dB).

9.11 (a) Note the input as $y(t) = s(t) + n_0(t)$ where $n_0(t)$ is white noise at the instant time of t_0, the SNR:

$$SNR = \frac{s^2(t)}{E\{n_0^2(t)\}}\Big|_{t=t_0} = \frac{\left|\int_{-\infty}^{+\infty} H(f)S(f)e^{j2\pi f t_0}df\right|^2}{\frac{1}{2}N_0\int_{-\infty}^{+\infty}|H(f)|^2 df}$$

using Schwarz's inequality, we have:

$$SNR = \frac{2}{N_0}\frac{\left|\int_{-\infty}^{+\infty}H(f)S(f)e^{j2\pi f t_0}df\right|^2}{\int_{-\infty}^{+\infty}|H(f)|^2 df} \le \frac{2}{N_0}\frac{\int_{-\infty}^{+\infty}|H(f)|^2df\int_{-\infty}^{+\infty}|S(f)|^2df}{\int_{-\infty}^{+\infty}|H(f)|^2df} = \frac{2}{N_0}\int_{-\infty}^{+\infty}|S(f)|^2df$$

The equality holds if and only if: $H(f) = S^*(f)e^{-j2\pi t_0 f}\cdot\alpha$.

Since α just fixes the gain of the filter, we can set it to 1.

$\therefore H_m(f) = S^*(f)e^{-j2\pi f t_0}$

(b) $\because H_m(f) = S^*(f)e^{-j2\pi f t_0}$

$$\therefore h_m(t) = \int_{-\infty}^{+\infty}S^*(f)e^{-j2\pi f t_0}e^{j2\pi f t}df = \int_{-\infty}^{+\infty}S(-f)e^{-j2\pi(t_0-t)f}df$$

$$= s(t_0-t)$$

(c) According to the title, $h_{mr}(t) = \begin{cases} s(t_0-t), & t\geqslant 0 \\ 0, & t<0\end{cases}$

and $s(t) = A\Pi[(t-T/2)/T]$

① if $t_0 = 0$, then: $h_{mr}(t) = \begin{cases} s(-t), & t\geqslant 0\\ 0, & t<0\end{cases} = A\delta(t)$.

② if $t_0 = \frac{T}{2}$, then:

$$h_{mr}(t) = \begin{cases} s(\frac{T}{2}-t), & t\geqslant 0\\ 0, & t<0\end{cases} = A\Pi\left(\frac{t-\frac{T}{4}}{T/2}\right).$$

③ if $t_0 = T$, then:

$$h_{mr}(t) = \begin{cases} s(T-t), & t\geqslant 0\\ 0, & t<0\end{cases} = A\Pi\left(\frac{t-\frac{T}{2}}{T}\right).$$

④ if $t_0 = 2T$, then:

$$h_{mr}(t) = \begin{cases} s(2T-t), & t\geqslant 0\\ 0, & t<0\end{cases} = A\Pi\left(\frac{t-\frac{3}{2}T}{T}\right)$$

(d) Assume: $\because s(t) = \begin{cases} A, & 0 \le t \le T. \\ 0, & \text{otherwise}. \end{cases}$

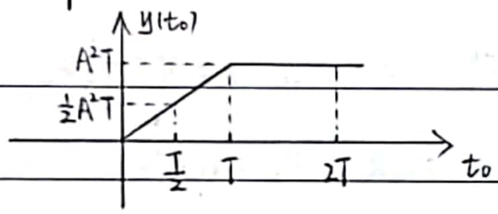
$\therefore$ the response of the filter to $s(t)$ is:

$y(t) = h_{mr}(t) * s(t)$ and the peak output signal occurs at $t = t_0$.

~~$\therefore y(t_0) = h_{mr}(t_0) * s(t_0) =$~~

$$\therefore y(t_0) = h_{mr}(t) * s(t)\big|_{t=t_0} = \begin{cases} 0, & t_0 = 0 \\ \frac{1}{2}A^2T, & t_0 = \frac{T}{2} \\ A^2T, & t_0 = T, 2T. \end{cases}$$

The plot is as follows:



Obviously, when $t_0 \ge T$, we can meet the requirement of causality.

9.19. Firstly, if $Q(x) = 10^{-5}$, we can know that $x = 4.26489$.

(a) For binary ASK, $P_E = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q(\sqrt{z})$.

$\therefore z = 4.26489^2 = 18.18929$.

(b) For BPSK, $P_E = Q\left[\sqrt{2(1-m^2)z}\right]\Big|_{m=0} = Q(\sqrt{2z})$.

$\therefore z = \frac{1}{2} \times 4.26489^2 = 9.09464$.

(c) For binary FSK, $P_E = Q(\sqrt{z})$

$\therefore z = 4.26489^2 = 18.18929$.

(d) For BPSK with a phase error of 5 degrees, $P_E = Q(\sqrt{2z\cos^2\phi})$

$\therefore z = \frac{1}{2\cos^2\phi} \times 4.26489^2 = 9.16426$

(e) For PSK with $m=\sqrt{\frac{1}{2}}$, $P_E = Q(\sqrt{2(1-m^2)z}) = Q(\sqrt{z})$

$\therefore z = 4.26489^2 = 18.18929$.

(f) For PSK with $m=\frac{1}{\sqrt{2}}$ and with a phase error of 5 degrees,

$P_E = Q[\sqrt{2(1-m^2)z\cos^2\phi}] = Q(\sqrt{z\cos^2\phi})$.

$\therefore z = \frac{1}{\cos^2\phi} \times 4.26489^2 = 18.32851$

9.22 (a) Firstly, if $Q(x) = 10^{-6}$, we have $x = 4.75342$

(i) For binary ASK, $B = 2R$, $P_E = Q(\sqrt{z})$.

$\therefore$ the bandwidth $B = 2R =$ ~~100 Hz~~ 100 kHz.

and $\frac{E_b}{N_0} = z = 4.75342^2 = 13.54012$ dB.

(ii) For BPSK, $B = 2R$, $P_E = Q(\sqrt{2z})$

$\therefore B = 2R = 100$ kHz

and $\frac{E_b}{N_0} = z = \frac{1}{2} \times 4.75342^2 = 10.52982$ dB.

(iii) For binary FSK, $B = \frac{1}{T} + \frac{1}{2T} + \frac{1}{T} = 2.5R$, $P_E = Q(\sqrt{z})$.

$\therefore B = 2.5R = 125$ kHz.

$\frac{E_b}{N_0} = z = 4.75342^2 = 13.54012$ dB

(b) Similarly, if $Q(x) = 10^{-5}$, we have $x = 4.26489$

(i) For binary ASK, $B = 2R = 1$ MHz

and $\frac{E_b}{N_0} = z = 4.26489^2 = 12.59816$ dB

(ii) For BPSK, $B = 2R = 1MHz$.

and $\frac{E_b}{N_0} = z = \frac{1}{2} \times 4.26489^2 = 9.58786\ dB$

(iii) For binary FSK, $B = 2.5R = 1.25\ MHz$.

and $\frac{E_b}{N_0} = z = 4.26489^2 = 12.59816\ dB$.

9.32 According to the title, the ~~demi~~ decimal numbers' Gray ~~s~~ code ~~are~~ can be constructed as follows.

Firstly, there are totally 32 numbers so the code should have 5 bits.

$\therefore$ 0's Gray code is 00000.

Further more, let's take 2 as an example.

$\because 2 = (00010)_2$

$\therefore g_1 = 0,\ g_2 = 0,\ g_3 = 0,\ g_4 = 1,\ g_5 = 1$.

$\therefore$ 2's Gray code is 00011.

$\therefore$ The answers are shown in following table:

Decimal numbers	Binary numbers	Gray codes	Decimal numbers	Binary numbers	Gray codes
0	00000	00000	16	10000	11000
1	00001	00001	17	10001	11001
2	00010	00011	18	10010	11011
3	00011	00010	19	10011	11010
4	00100	00110	20	10100	11110
5	00101	00111	21	10101	11111
6	00110	00101	22	10110	11101
7	00111	00100	23	10111	11100
8	01000	01100	24	11000	10100
9	01001	01101	25	11001	10101
10	01010	01111	26	11010	10111
11	01011	01110	27	11011	10110
12	01100	01010	28	11100	10010
13	01101	01011	29	11101	10011
14	01110	01001	30	11110	10001
15	01111	01000	31	11111	10000

9.34 (a) $\because \overset{B}{\cancel{R_b}} = \frac{1}{\tau} = \cancel{20\ kbps}\ 5\ kHz$. $R_b = \frac{\log_2 M}{\tau} = 20\ kbps$

$\therefore \log_2 M = 4$.

$\therefore M = 2^4 = 16$.

(b) $\because P_b = \frac{2(M-1)}{M\log_2 M} Q\left(\sqrt{\frac{6\log_2 M E_b}{(M^2-1)N_0}}\right) = \frac{30}{16\times 4} Q\left(\sqrt{\frac{6\times 4 E_b}{255 N_0}}\right) = \frac{15}{\underset{32}{\cancel{16}}} Q\left(\sqrt{\frac{24 E_b}{255 N_0}}\right)$

if $P_b = 10^{-6}$, then: $\sqrt{\frac{24E_b}{255N_0}} = 4.59795$.

$\therefore \frac{E_b}{N_0} = 224.6246572 = 23.51457\ dB$.

if $P_b = 10^{-5}$, then: $\sqrt{\frac{24E_b}{255N_0}} = 4.09254$.

$\therefore \frac{E_b}{N_0} = 177.9568888 = 22.50315\ dB$.

9.35. (a) $\because B = 100\ KHz$, $R_b = 50\ Kbps$.

$\therefore \gamma = \frac{R_b}{B} = 0.5\ bps/Hz$.

$\therefore$ BPSK can be applied.

Assume the probability of error $P_E = 10^{-6}$.

that is: $Q(\sqrt{2z}) = Q(\sqrt{\frac{E_b}{N_0}\times 2}) = 10^{-6}$

$\therefore \frac{E_b}{N_0} = z = \frac{1}{2}\times 4.75342^2 = 10.52982\ dB$.

(b) $\because B = 100\ KHz$. $R_b = 100\ kbps$.

$\therefore \gamma = 1.\ bps/Hz$.

$\therefore$ QPSK can be applied.

Similarly, if $P_E = Q(\sqrt{\frac{2E_b}{N_0}}) = 10^{-6}$.

$\therefore \frac{E_b}{N_0} = z = \frac{1}{2}\times 4.75342^2 = 10.52982\ dB$.

(c) $\because B = 100\ KHz$. $R_b = 150\ kbps$.

$\therefore \gamma = 1.5\ bps/Hz$.

$\therefore$ 8PSK can be applied.

In this case, if $P_E = \cancel{Q}\ \frac{2}{\log_2 8} Q\left(\sqrt{2z\log_2 8}\,\sin(\frac{\pi}{8})\right) = 10^{-6}$.

$\therefore \frac{E_b}{N_0} = z = \frac{1}{6}\left[\frac{4.67082}{\sin(\pi/8)}\right]^2 = 24.82880 = 13.94956\ dB$.

(d) $\because B = 100\ kHz$. $R_b = 200\ kbps$.

$\therefore \gamma = 2\ bps/Hz$.

$\therefore$ 16 PSK can be applied.

$\therefore P_E = \frac{2}{\log_2 16} Q\left(\sqrt{2z\log_2 16}\,\sin(\frac{\pi}{16})\right) = 10^{-6}$.

$\therefore \frac{E_b}{N_0} = z = \frac{1}{8}\left[\frac{4.611382}{\sin(\pi/16)}\right]^2 = 69.83945 = 18.44101\ dB$

(e) $\because B = 100\ kHz$. $R_b = 250\ kbps$.

$\therefore \gamma = 2.5\ bps/Hz$

$\therefore$ 32 PSK can be applied.

$\because P_E = \frac{2}{\log_2 32} Q\left(\sqrt{2z\log_2 32}\,\sin(\frac{\pi}{32})\right) = 10^{-6}$

$\therefore \frac{E_b}{N_0} = z = \frac{1}{10}\left[\frac{4.56479}{\sin(\pi/32)}\right]^2 = 216.88880 = 23.36237\ dB$.

9.39 (a) According to the title, $G_n(f) = \frac{N_0}{2} = 10^{-11}\ W/Hz$.

and $T = \frac{1}{9600}\ s$, $|H_c(f)| = \frac{1}{1+j\frac{f}{4800}}$

$\therefore P(f) = \frac{T}{2}\left[1+\cos(2\pi T|f|)\right] = T\cos^2(\frac{2\pi T}{2}|f|) = \frac{1}{9600}\cos^2\left(\frac{\pi|f|}{19200}\right)$

$\therefore |H_R(f)|_{opt} = \frac{K^{\frac{1}{2}}P^{\frac{1}{2}}(f)}{10^{-\frac{1}{4}}|H_c(f)|^{\frac{1}{2}}} = 10^{\frac{1}{4}}K^{\frac{1}{2}} \times \sqrt{\frac{1}{9600}}\left|\cos\left(\frac{\pi|f|}{19200}\right)\right|\left[1+\left(\frac{f}{4800}\right)^2\right]^{\frac{1}{4}}$

$= \frac{562.34}{97.980}K^{\frac{1}{2}}\left|\cos\left(\frac{\pi|f|}{19200}\right)\right|\left[1+\left(\frac{f}{4800}\right)^2\right]^{\frac{1}{4}}$. $|f| \in [0, 9600]\ (Hz)$

$|H_T(f)|_{opt} = \frac{AP^{\frac{1}{2}}(f)10^{-\frac{1}{4}}}{K^{\frac{1}{2}}|H_c(f)|^{\frac{1}{2}}} = AK^{-\frac{1}{2}}10^{-\frac{1}{4}} \times \sqrt{\frac{1}{9600}}\left|\cos\left(\frac{\pi|f|}{19200}\right)\right|\left[1+\left(\frac{f}{4800}\right)^2\right]^{\frac{1}{4}}$.

$= \frac{A}{55098.0K^{\frac{1}{2}}}\left|\cos\left(\frac{\pi|f|}{19200}\right)\right|\left[1+\left(\frac{f}{4800}\right)^2\right]^{\frac{1}{4}}$. $|f| \in [0, 9600]\ (Hz)$

$\therefore K^{\cancel{\$}} = 0.17424.$ $A = 35098.0$, and the magnitudes are $\frac{1}{\sqrt{K}} = 0.41742$

(b) $\therefore$ KF $\because P_E = Q(\frac{A}{\sigma}) = 10^{-4}$

$\therefore \frac{A}{\sigma} = 3.71902$

(c) $\because (\frac{A}{\sigma})^2 = \frac{1}{E_b}\left[\int_{-\infty}^{+\infty} \frac{G_n^{\frac{1}{2}}(f)P(f)}{|H_c(f)|} df\right]^2$

and $\int_{-\infty}^{+\infty} \frac{G_n^{\frac{1}{2}}(f)P(f)}{|H_c(f)|} df = \int_{-9600}^{9600} \frac{10^{-\frac{11}{2}} \times \frac{1}{9600}\cos^2(\frac{\pi|f|}{19200})}{[1+(\frac{f}{4800})^2]^{-1/2}} df = 3.29 \times 10^{-10} \times \cancel{14618.53310}$ 11644.84881

$\therefore \cancel{A_b} = \cancel{3.71902^2} = \cancel{4.815 \times 10^{-6}}$ 3.83116×10^{-6}

$\therefore E_b = \frac{(3.83 \times 10^{-6})^2}{3.71902^2} = 1.06 \times 10^{-12}$ J