Artificial Intelligence—Spring 2022

Homework 4

Issued: Apr. 2nd, 2022 Due: Apr. 18th, 2022

Problem 1

Solutions:

a. Yes, *Burglary* and *Earthquake* are independent if no evidence is observed.

From numerical semantics: According to Figure 14.2 and the equation that $P(B,E)=\Sigma_i P(B)P(E)P(A_i|B,E)$, we can conclude the following probabilities:

B	E	$P(B)$	$P(E)$	$P(a\|B,E)$	$P(B,E)$
t	t	0.001	0.002	0.95	0.000002
t	f	0.001	0.998	0.94	0.000998
f	t	0.999	0.002	0.29	0.001998
f	f	0.999	0.998	0.001	0.997002

which shows that the joint distribution of *B* and *E*, *P(B,E)*, is the multiplication result of *B* and *E*'s marginal distributions, that is, *P(B,E)=P(B)P(E)*, so *B* and *E* are independent.

From topological semantics: Because *B* and *E* are d-separated by *A*, they are independent.

b. No, *Burglary* and *Earthquake* are NOT independent if we observe *Alarm=true*. From the probability shown in the figure and the Bayesian theorem, we can compute the following probabilities:

B	E	$P(B\|a)$	$P(E\|a)$	$P(B,E\|a)$
t	t	0.3736	0.2311	0.0008
t	f	0.3736	0.7690	0.3728
f	t	0.6265	0.2311	0.2303
f	f	0.6265	0.7690	0.3962

So it is obvious that $P(B|a)P(E|a) \neq P(B,E|a)$, which means *B* and *E* are not conditionally independent given *A*.

Problem 2

Solutions:

a. (c) is the figure that claims the equation. The equation needs G_{father},G_{mother} and G_{child} to be independent from each other, which means no links should exist between the three genes.

b. (a) and (b) are consistent with the hypothesis, while (c) presents a contradictory independence of genes.

c. (a) is the best description of the hypothesis. Because the links between handedness are not mentioned in the theory.

d. According to the title, G_{child} node's CPT is as follows:

G_{father}	G_{mother}	$P(G_{child}=l)$	$P(G_{child}=r)$
l	l	$1-m$	m
l	r	0.5	0.5
r	l	0.5	0.5
r	r	m	$1-m$

e. $\because P(G_{father}=l)=P(G_{mother}=l)=q$

$$
\begin{aligned}
\therefore P(G_{child}=l) &= \Sigma_{g_f,g_m} P(G_{child}=l|g_f,g_m)P(g_f,g_m) \\
&= \Sigma_{g_f,g_m} P(G_{child}=l|g_f,g_m)P(g_f)P(g_m) \\
&= q^2(1-m)+0.5\times 2\times q\times(1-q)+(1-q)^2 m \\
&= q+m-2qm
\end{aligned}
$$

f. According to genetic equilibrium, we have $P(G_{child}=l)=q+m-2qm=q$, so $q=0.5$. However, as what we have known about human handedness, the right-handedness is relevantly dominant, so the hypothesis described at the beginning must be wrong.

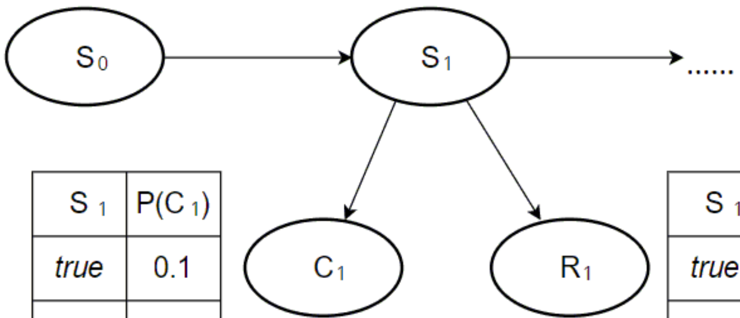
Problem 3

Solutions:

According to the professor's theory, we note random variables S_t、C_t、R_t to present whether a student get enough sleep on night t, whether a student sleep in class on the t th day, and whether a student have red eyes on the t th day. So the dynamic Bayesian network can be formulated as follows:

S_0	$P(S_1)$
true	0.8
false	0.3

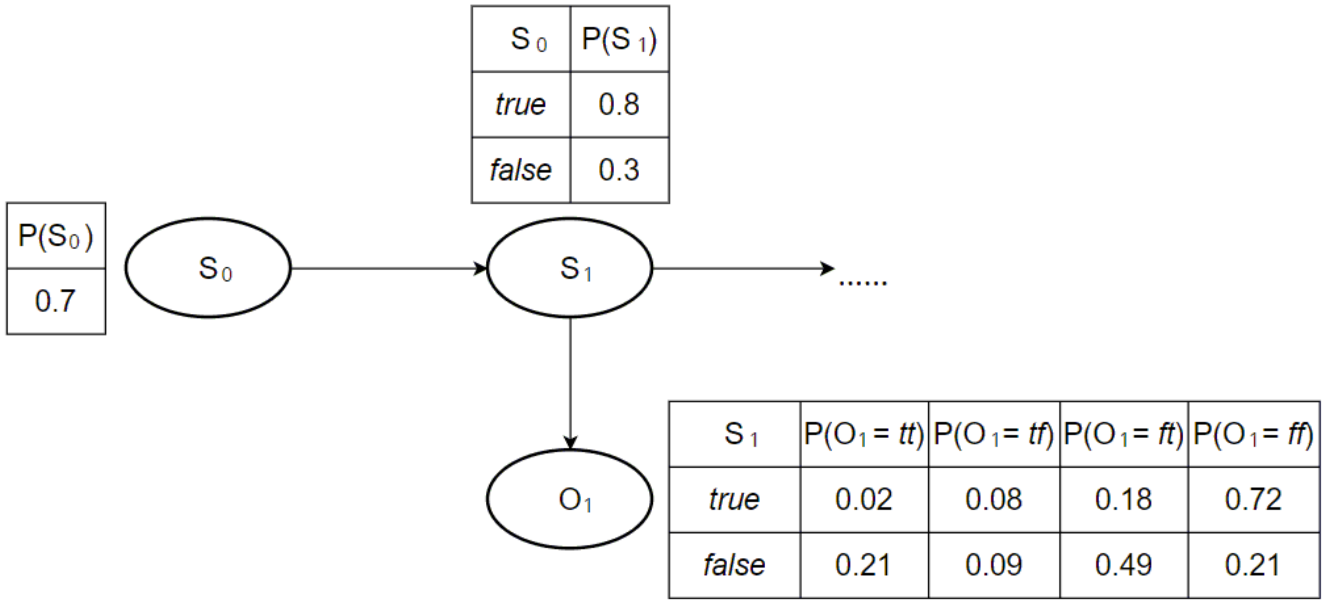
$P(S_0)$
0.7



S_1	$P(C_1)$
true	0.1
false	0.3

S_1	$P(R_1)$
true	0.2
false	0.7

Combining the random variables C_t、R_t into one single random variable, O_t={C_t,R_t}, we can get the hidden Markov model with complete probability tabels as follows:



S_0	$P(S_1)$
true	0.8
false	0.3

$P(S_0)$
0.7

S_1	$P(O_1 = tt)$	$P(O_1 = tf)$	$P(O_1 = ft)$	$P(O_1 = ff)$
true	0.02	0.08	0.18	0.72
false	0.21	0.09	0.49	0.21

Problem 4

Solutions:

a. According to the DBN and HMM we have otained in Problem 3, we can compute the probabilities as follows:

$P(S_0) = 0.7$

$P(S_1) = \Sigma_{s0} P(S_1|s_0)P(s_0) = 0.7 < 0.8, 0.2 > +0.3 < 0.3, 0.7 >=< 0.65, 0.35 >$

$P(S_1|e_1) = \alpha P(e_1|S_1)P(S_1) = \alpha < 0.72, 0.21 >< 0.65, 0.35 >=< 0.8643, 0.1357 >$

$P(S_2|e_1) = \Sigma_{s1} P(S_2|s_1)P(s_1|e_1) = 0.8643 < 0.8, 0.2 > +0.1357 < 0.3, 0.7 >= < 0.7322, 0.2679 >$

$P(S_2|e_{1:2}) = \alpha P(e_2|S_2)P(S_2|e_1) = \alpha < 0.18, 0.49 >< 0.7322, 0.2679 >= < 0.5010, 0.4990 >$

$P(S_3|e_{1:2}) = \Sigma_{s2} P(S_3|s_2)P(s_2|e_{1:2}) = 0.5010 < 0.8, 0.2 > +0.4990 < 0.3, 0.7 >= < 0.5505, 0.4495 >$

$P(S_3|e_{1:3}) = \alpha P(e_3|S_3)P(S_3|e_{1:2}) = \alpha < 0.02, 0.21 >< 0.5505, 0.4495 >= < 0.1045, 0.8955 >$

b. Similarly, we have:

$P(e_3|S_3) =< 0.02, 0.21 >$

$P(e_3|S_2) = \Sigma_{s3} P(e_3|s_3)P(s_3|S_2) =< 0.8 \times 0.02 + 0.2 \times 0.21, 0.3 \times 0.02 + 0.7 \times 0.21 >=< 0.058, 0.153 >$

$P(e_{2:3}|S_1) = \Sigma_{s2} P(e_2|s_2)P(e_3|s_2)P(s_2|S_1) =< 0.8 \times 0.058 \times 0.18 + 0.2 \times 0.153 \times 0.49, 0.3 \times 0.058 \times 0.18 + 0.7 \times 0.153 \times 0.49 >=< 0.0233, 0.0556 >$

$P(S_1|e_{1:3}) = \alpha P(e_{2:3}|S_1)P(S_1|e_1) = \alpha < 0.0233, 0.0556 >< 0.8643, 0.1357 >= < 0.7275, 0.2725 >$

$P(S_2|e_{1:3}) = \alpha P(e_3|S_2)P(S_2|e_{1:2}) = \alpha < 0.058, 0.153 >< 0.5010, 0.4990 >= < 0.2757, 0.7243 >$

$P(S_3|e_{1:3}) =< 0.1045, 0.8955 >$

C. Obviously, the smoothed probabilities integrated future observation values to refine the results, so it provides more accurate estimations than filtered analysis.