6.630 Solution to Problem Set 7

Solution P7.1

$k_x = k_o \sin\theta\,,\ k_z = \sqrt{\omega^2\mu_o(\epsilon_o + i\sigma/\omega) - k_x^2} \approx \sqrt{\omega\mu_o\sigma}(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}})$

$\theta_t = \tan^{-1}(k_x/k_{zR}) = tan^{-1}(\frac{\sqrt{2}k_x}{\sqrt{\omega\mu_o\sigma}}) \approx 0$

Solution P7.2

For a TEM wave, we can treat it as either a TE or a TM wave. Though the reflection coefficients R^{TE} and R^{TM} have different forms, they represent the same field and can be converted to each other. Therefore for normal incidence, no mather using R^{TE} or R^{TM}, we should get the same field and the same reflectivity and transmissivity.

Solution P7.3

$\overline{E}_r = (-\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{z} - i\hat{y})e^{i\frac{1}{\sqrt{2}}k_o x - i\frac{1}{\sqrt{2}}k_o z}$

The incident wave is L. H. C. P. The reflected wave is R. H. C. P.

Solution P7.4

(a) $\phi = 2\theta_2 - 2(\theta_1 - \theta_2) = 2(2\theta_2 - \theta_1)$

(b) $\frac{d\phi}{d\theta_1} = \frac{d}{d\theta_1}\left[2\sin^{-1}(\frac{\sin\theta_1}{n}) - \theta_1\right] = 0 \Rightarrow \sin\theta_1 = \sqrt{(4-n^2)/3}$

For $n = 4/3 = 1.33$, $\sin\theta = 0.86 \Rightarrow \phi = 2(2\theta_2 - \theta_1) \simeq 42^\circ$.

(c) For red light, $\theta_1 = 59.58^\circ$ $\theta_2 = 40.42^\circ$, $\phi_{\max} = 42.52^\circ$, and $\theta_s = 137.5^\circ$; for violet light $\theta_1 = 58.89^\circ$, $\theta_2 = 39.64^\circ$, $\phi_{\max} = 40.78^\circ$, and $\theta_s = 139^\circ$. The outer arc of the rainbow will be red, and violet will be on the inner arc of the rainbow.