数值分析方法

作业2

Problem 1

解: 由题意: $f(x) = -x^3 - \cos x, f'(x) = -3x^2 + \sin x$

代码如下:

```c
#include<stdio.h>
#include<math.h>

int main()
{
        int i=0;
        double p,p0,delta=1;
        scanf("%lf",&p0);
        while(i<=1){
                p=p0-(-pow(p0,3)-cos(p0))/\
                (-3*pow(p0,2)+sin(p0));
                delta=p-p0;
                if (delta<0){
                        delta=-delta;
                }
                printf("n = %2d, p = %12.8lf, \
                |p-p0| = %.8lf\n",++i,p,delta);
                p0=p;
        }
 }
```

可得:

n	p_n	$\lvert p_n - p_{n-1}\rvert$
1	-0.88033290	0.11966710
2	-0.86568416	0.01464874

$\therefore p_2 = -0.86568416$

$\because f'(0) = 0$

$\therefore$ 不能使用$p_0 = 0$

Problem 2

解: (i) 由题意:

$$f(x) = b - \frac{1}{x}$$

则:

$$f'(x) = \frac{1}{x^2}$$

由牛顿迭代法：

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{b - \frac{1}{x_k}}{\frac{1}{x_k^2}} = 2x_k - bx_k^2$$

所以

$$|\epsilon_{k+1}| = \frac{|x - x_{k+1}|}{|x|} = \frac{\left|\frac{1}{b} - x_{k+1}\right|}{\frac{1}{b}} = |1 - bx_{k+1}| = |b^2x_k^2 - 2bx_k + 1| = (bx_k - 1)^2$$

又因为

$$\epsilon_k = \frac{\frac{1}{b} - x_k}{\frac{1}{b}} = 1 - bx_k$$

所以

$$|\epsilon_{k+1}| = \epsilon_k^2$$

证毕.

(ii) 由(i)得:

$$\left|x_{k+1} - \frac{1}{b}\right| = \left|2x_k - bx_k^2 - \frac{1}{b}\right| = \frac{|2bx_k - b^2x_k^2 - 1|}{b} = \frac{|bx_k - 1|^2}{b}$$

$$\left|x_k - \frac{1}{b}\right| = \frac{|bx_k - 1|}{b}, \quad k = 0, 1, 2 \cdots$$

所以

$$\left|x_{k+1} - \frac{1}{b}\right| = \frac{|bx_k - 1|^2}{b}$$

同理可得

$$\left|x_k - \frac{1}{b}\right| = \frac{|bx_k - 1|}{b} = \frac{|bx_{k-1} - 1|^2}{b}$$

所以

$$|bx_k - 1| = |bx_{k-1} - 1|^2$$

$$\therefore \left|x_{k+1} - \frac{1}{b}\right| = \frac{|bx_k - 1|^2}{b} = \frac{|bx_{k-1} - 1|^4}{b} = \cdots = \frac{|bx_0 - 1|^{2^{k+1}}}{b}$$

由题意:

$$0 < x_0 < \frac{2}{b}$$

则: $|bx_0 - 1| \in [0, 1)$

$$\therefore \lim_{k\to\infty}\left|x_{k+1} - \frac{1}{b}\right| = \lim_{k\to\infty}\frac{|bx_0 - 1|^{2^{k+1}}}{b} = 0$$

即数列$\{x_k\}_{k=0}^{\infty}$收敛到$\frac{1}{b}$, 证毕.

Problem 3

解: a. 由题意, 令

$$f_1 = 3x_1 - \cos(x_2x_3) - \frac{1}{2}$$
$$f_2 = 4x_1^2 - 625x_2^2 + 2x_2 - 1$$
$$f_3 = e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3}$$

则雅可比矩阵

$$\mathbf{J} = \begin{pmatrix} 3 & x_3\sin(x_2x_3) & x_2\sin(x_2x_3) \\ 8x_1 & -1250x_2 + 2 & 0 \\ -x_2e^{-x_1x_2} & -x_1e^{-x_1x_2} & 20 \end{pmatrix}$$

代码如下:

```
#include<stdio.h>
#include<math.h>

#define e 2.718281828459
#define pi 3.1415926535898

int main()
{
        int i=0;
        double f1,f2,f3;
        double x01,x02,x03;
        double x1,x2,x3;
        double y1,y2,y3;
        double j1[3],j2[3],j3[3];
        double jn1[3],jn2[3],jn3[3];
        double det;
        scanf("%lf,%lf,%lf",&x01,&x02,&x03);
        while (i<=1){
                f1=3*x01-cos(x02*x03)-0.5;
                f2=4*pow(x01,2)-625*pow(x02,2)+2*x02-1;
                f3=pow(e,-x01*x02)+20*x03+(10*pi-3)/3;
                j1[0]=3;   //Jacobi Matrix
                j1[1]=x03*sin(x02*x03);
                j1[2]=x02*sin(x02*x03);
                j2[0]=8*x01;
                j2[1]=-1250*x02+2;
                j2[2]=0;
                j3[0]=-x02*pow(e,-x01*x02);
                j3[1]=-x01*pow(e,-x01*x02);
```

```
                    j3[2]=20;
                    det=j1[0]*(j2[1]*j3[2]-j2[2]*j3[1])\
                    -j2[0]*(j1[1]*j3[2]-j1[2]*j3[1])+\
                    j3[0]*(j1[1]*j2[2]-j1[2]*j2[1]);
                    //the Determinant

                    jn1[0]=(j2[1]*j3[2]-j2[2]*j3[1])/det;
                    jn1[1]=(j1[2]*j3[1]-j1[1]*j3[2])/det;
                    jn1[2]=(j1[1]*j2[2]-j1[2]*j2[1])/det;
                    jn2[0]=(j2[2]*j3[0]-j2[0]*j3[2])/det;
                    jn2[1]=(j1[0]*j3[2]-j1[2]*j3[0])/det;
                    jn2[2]=(j2[0]*j1[2]-j1[0]*j2[2])/det;
                    jn3[0]=(j2[0]*j3[1]-j2[1]*j3[0])/det;
                    jn3[1]=(j1[1]*j3[0]-j1[0]*j3[1])/det;
                    jn3[2]=(j1[0]*j2[1]-j2[0]*j1[1])/det;
                    //inverse matrix of the Jacobi Matrix

                    y1=jn1[0]*f1+jn1[1]*f2+jn1[2]*f3;
                    y2=jn2[0]*f1+jn2[1]*f2+jn2[2]*f3;
                    y3=jn3[0]*f1+jn3[1]*f2+jn3[2]*f3;
                    x1=x01-y1;
                    x2=x02-y2;
                    x3=x03-y3;
                    printf("n = %d, x1 = %.8f, x2 = %.8f\
, x3 = %.8f\n",++i,x1,x2,x3);
                    x01=x1;
                    x02=x2;
                    x03=x3;
          }
 }
```

可得:

n	$x_1^{(n)}$	$x_2^{(n)}$	$x_3^{(n)}$
1	0.50000000	0.50000000	-0.52359878
2	0.50016669	0.25080364	-0.51738743

$\therefore \mathbf{x}^{(2)} = (0.50016669,\ 0.25080364,\ -0.51738743)^{\mathbf{T}}$.

b. 由题意, 令

$$f_1 = x_1^2 + x_2 - 37$$

$$f_2 = x_1 - x_2^2 - 5$$

$$f_3 = x_1 + x_2 + x_3 - 3$$

则雅可比矩阵

$$\mathbf{J} = \begin{pmatrix} 2x_1 & 1 & 0 \\ 1 & -2x_2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

代码如下:

```c
#include<stdio.h>
#include<math.h>

int main()
{
        int i=0;
        double f1,f2,f3;
        double x01,x02,x03;
        double x1,x2,x3;
        double y1,y2,y3;
        double j1[3],j2[3],j3[3];
        double jn1[3],jn2[3],jn3[3];
        double det;
        scanf("%lf,%lf,%lf",&x01,&x02,&x03);
        while (i<=1){
                f1=pow(x01,2)+x02-37;
                f2=x01-pow(x02,2)-5;
                f3=x01+x02+x03-3;
                j1[0]=2*x01;
                j1[1]=1;
                j1[2]=0;
                j2[0]=1;
                j2[1]=-2*x02;
                j2[2]=0;
                j3[0]=1;
                j3[1]=1;
                j3[2]=1;
                det=j1[0]*(j2[1]*j3[2]-j2[2]*j3[1])\
                -j2[0]*(j1[1]*j3[2]-j1[2]*j3[1])+\
                j3[0]*(j1[1]*j2[2]-j1[2]*j2[1]);
                //the Determinant

                jn1[0]=(j2[1]*j3[2]-j2[2]*j3[1])/det;
```

```
                        jn1[1]=(j1[2]*j3[1]-j1[1]*j3[2])/det;
                        jn1[2]=(j1[1]*j2[2]-j1[2]*j2[1])/det;
                        jn2[0]=(j2[2]*j3[0]-j2[0]*j3[2])/det;
                        jn2[1]=(j1[0]*j3[2]-j1[2]*j3[0])/det;
                        jn2[2]=(j2[0]*j1[2]-j1[0]*j2[2])/det;
                        jn3[0]=(j2[0]*j3[1]-j2[1]*j3[0])/det;
                        jn3[1]=(j1[1]*j3[0]-j1[0]*j3[1])/det;
                        jn3[2]=(j1[0]*j2[1]-j2[0]*j1[1])/det;
                        //inverse matrix of the Jacobi Matrix

                        y1=jn1[0]*f1+jn1[1]*f2+jn1[2]*f3;
                        y2=jn2[0]*f1+jn2[1]*f2+jn2[2]*f3;
                        y3=jn3[0]*f1+jn3[1]*f2+jn3[2]*f3;
                        x1=x01-y1;
                        x2=x02-y2;
                        x3=x03-y3;
                        printf("n = %d, x1 = %.8f, x2 = %.8f\
, x3 = %.8f\n",++i,x1,x2,x3);
                        x01=x1;
                        x02=x2;
                        x03=x3;
            }
  }
```

可得:

n	$x_1^{(n)}$	$x_2^{(n)}$	$x_3^{(n)}$
1	0.50000000	37.00000000	-39.00000000
2	4.35087719	18.49122807	-19.84210526

$\therefore \mathbf{x}^{(2)} = (4.35087719,\ 18.49122807,\ -19.84210526)^{\mathbf{T}}$.

Problem 4

解: a. 由题意:

$$f_1 = 15x_1 + x_2^2 - 4x_3 - 13$$

$$f_2 = x_1^2 + 10x_2 - x_3 - 11$$

$$f_3 = x_2^3 - 25x_3 + 22$$

令

$$g(\mathbf{x}) = f_1^2 + f_2^2 + f_3^2$$

$$\therefore \nabla g(\mathbf{x}) = \left(\frac{\partial g}{\partial x_1}, \frac{\partial g}{\partial x_2}, \frac{\partial g}{\partial x_3}\right) = 2\begin{pmatrix} 2x_1^3+15x_2^2-2x_1x_3+20x_1x_2+203x_1-60x_3-195 \\ 10x_1^2+2x_2^3-3x_2^5-66x_2^2+74x_2+30x_1x_2-10x_3-110+75x_2^2x_3 \\ -x_1^2-4x_2^2-25x_2^3-60x_1-10x_2+643x_3-487 \end{pmatrix}$$

代码如下:

```c
#include<stdio.h>
#include<math.h>
#include<stdlib.h>

#define e 2.718281828459
#define pi 3.1415926535898

int main()
{
        int i=0;
        double f1,f2,f3;
        double a,a0,a1,a2,a3;
        double h1,h2,h3;
        double g,g0,g1=0,g2,g3;
        double x01,x02,x03;
        double z1,z2,z3,z0;
        double tol,min=10;
        scanf("%lf,%lf,%lf,%lf",&x01,&x02,&x03,&tol);
        while (fabs(min-g1)>=tol){
                f1=15*x01+pow(x02,2)-4*x03-13;
                f2=pow(x01,2)+10*x02-x03-11;
                f3=pow(x02,3)-25*x03+22;
                g1=f1*f1+f2*f2+f3*f3;
                z1=2*(2*x01*x01*x01+15*x02*x02-2*x01*\
                   x02+203*x01-60*x03-195);
                z2=2*(10*x01*x01+2*x02*x02*x02-3*\
                   pow(x02,5)-66*pow(x02,2)+74*x02\
                   +30*x01*x02-10*x03-110+\
                   75*pow(x02,2)*x03);
                z3=2*(-x01*x01-4*x02*x02-25*pow(x02,3)\
                   -60*x01-10*x02+643*x03-487);
                z0=sqrt(pow(z1,2)+pow(z2,2)+pow(z3,2));
                //z0 is the L2-norm of the vector z
                if (z0==0){
                        printf("failed, z0 = 0, n = \
```

```
%d, x1 = %.8f, x2 = %.8f, x3 = %.8f\n",i,x01,x02,x03);
                                exit(0);
                        }
                        z1/=z0;
                        z2/=z0;
                        z3/=z0;
                        a1=0;
                        a3=1;
                        g3=pow((15*(x01-a3*z1)+pow(x02-a3*z2,\
                            2)-4*(x03-a3*z3)-13),2)+pow((pow\
                            (x01-a3*z1,2)+10*(x02-a3*z2)-\
                            (x03-a3*z3)-11),2)+pow((pow\
                            (x02-a3*z2,3)-25*(x03-a3*z3)+22),2);
                        while (g3>=g1){
                                a3/=2;
                                g3=pow((15*(x01-a3*z1)+\
                                    pow(x02-a3*z2,2)-\
                                    4*(x03-a3*z3)-13),2)+\
                                    pow((pow(x01-a3*z1,2)+\
                                    10*(x02-a3*z2)-(x03-a3*z3)\
                                    -11),2)+pow((pow(x02-a3*z2\
                                    ,3)-25*(x03-a3*z3)+22),2);
                                if (a3<0.01*tol){
                                        printf("failed, a3 = \
%.8lf, n = %d, x1 = %.8lf, x2 = %.8lf, x3 = %.8lf\n",\
a3,i,x01,x02,x03);
                                        exit(0);
                                }
                        }
                        a2=a3/2;
                        g2=pow((15*(x01-a2*z1)+pow(x02-a2*z2\
                            ,2)-4*(x03-a2*z3)-13),2)+pow((pow\
                            (x01-a2*z1,2)+10*(x02-a2*z2)-\
                            (x03-a2*z3)-11),2)+pow((pow\
                            (x02-a2*z2,3)-25*(x03-a2*z3)+22),2);
                        h1=(g2-g1)/a2;
                        h2=(g3-g2)/(a3-a2);
                        h3=(h2-h1)/a3;
                        a0=(a2-h1/h3)/2;
                        g0=pow((15*(x01-a0*z1)+pow(x02-a0*z2\
                            ,2)-4*(x03-a0*z3)-13),2)+pow((pow\
```

```
                        (x01-a0*z1,2)+10*(x02-a0*z2)-\
                        (x03-a0*z3)-11),2)+pow((pow\
                        (x02-a0*z2,3)-25*(x03-a0*z3)+22),2);
                    min=g0;
                    if(g3<min){
                            min=g3;
                    }
                    if(min==g0){
                            a=a0;
                    }
                    else {
                            a=a3;
                    }
                    x01-=a*z1;
                    x02-=a*z2;
                    x03-=a*z3;
                    printf("n = %d, x1 = %.8lf, x2 = \
%.8lf, x3 = %.8lf, g = %.8lf\n",++i,x01,x02,x03,min);
        }
}
```

令$\mathbf{x} = (1, 1, 1)^{\mathbf{T}}$,可得:

n	$x_1^{(n)}$	$x_2^{(n)}$	$x_3^{(n)}$	$g^{(n)}$
1	1.05321268	0.99712364	0.91946190	0.71702448
2	1.05481094	0.99718700	0.91937309	0.71640813

$\therefore$ 一个近似解为 $\mathbf{x} = (1.05481094,\ 0.99718700,\ 0.91937309)^{\mathbf{T}}$.

b. 由题意:

$$f_1 = 10x_1 - 2x_2^2 + x_2 - 2x_3 - 5$$

$$f_2 = 8x_2^2 + 4x_3^2 - 9$$

$$f_3 = 8x_2x_3 + 4$$

令

$$g(\mathbf{x}) = f_1^2 + f_2^2 + f_3^2$$

$$\therefore \nabla g(\mathbf{x}) = \left(\frac{\partial g}{\partial x_1}, \frac{\partial g}{\partial x_2}, \frac{\partial g}{\partial x_3}\right) = 2\begin{pmatrix} 100x_1 - 20x_2^2 + 10x_2 - 20x_3 - 50 \\ 136x_2^3 - 6x_2^2 + 128x_2x_3^2 - 40x_1x_2 + 8x_2x_3 + 10x_1 - 123x_2 + 30x_3 - 5 \\ 32x_3^3 + 128x_2^2x_3 + 4x_2^2 - 20x_1 + 30x_2 - 68x_3 + 10 \end{pmatrix}$$

代码如下:

```c
#include<stdio.h>
#include<math.h>
#include<stdlib.h>

#define e 2.718281828459
#define pi 3.1415926535898

int main()
{
        int i=0;
        double f1,f2,f3;
        double a,a0,a1,a2,a3;
        double h1,h2,h3;
        double g,g0,g1=0,g2,g3;
        double x01,x02,x03;
        double z1,z2,z3,z0;
        double tol,min=10;
        scanf("%lf,%lf,%lf,%lf",&x01,&x02,&x03,&tol);
        while (fabs(min-g1)>=tol){
                f1=10*x01-2*pow(x02,2)+x02-2*x03-5;
                f2=8*pow(x02,2)+4*pow(x03,2)-9;
                f3=8*x02*x03+4;
                g1=f1*f1+f2*f2+f3*f3;
                z1=2*(100*x01-20*x02*x02+10*x02\
                   -20*x03-50);
                z2=2*(136*x02*x02*x02-6*x02*x02+128*\
                   x02*x03*x03-40*x01*x02+8*x02*x03\
                   +10*x01-123*x02+30*x03-5);
                z3=2*(32*x03*x03*x03+128*x02*x02*x03+4\
                  *pow(x02,2)-20*x01+30*x02-68*x03+10);
                z0=sqrt(pow(z1,2)+pow(z2,2)+pow(z3,2));
                //z0 is the L2-norm of the vector z
                if (z0==0){
                        printf("failed, z0 = 0, n = \
%d, x1 = %.8f, x2 = %.8f, x3 = %.8f\n",i,x01,x02,x03);
                        exit(0);
                }
                z1/=z0;
                z2/=z0;
                z3/=z0;
                a1=0;
```

```
            a3=1;
            g3=pow((10*(x01-a3*z1)-2*pow((x02-\
               a3*z2),2)+(x02-a3*z2)-2*(x03-a3*z3)\
               -5),2)+pow((8*pow((x02-a3*z2),2)+\
               4*pow((x03-a3*z3),2)-9),2)+pow\
               ((8*(x02-a3*z2)*(x03-a3*z3)+4),2);
            while (g3>=g1){
                    a3/=2;
                    g3=pow((10*(x01-a3*z1)-2*pow(\
                       (x02-a3*z2),2)+(x02-a3*z2)\
                       -2*(x03-a3*z3)-5),2)+pow(\
                       (8*pow((x02-a3*z2),2)+4*\
                       pow((x03-a3*z3),2)-9),2)+\
                       pow((8*(x02-a3*z2)*(x03-\
                       a3*z3)+4),2);
                    if (a3<0.01*tol){
                            printf("failed , a3 = \
%.8lf , n = %d, x1 = %.8lf , x2 = %.8lf , x3 = %.8lf\n",\
a3,i,x01,x02,x03);
                            exit(0);
                    }
            }
            a2=a3/2;
            g2=pow((10*(x01-a2*z1)-2*pow((x02-\
               a2*z2),2)+(x02-a2*z2)-2*(x03-a2*z3)\
               -5),2)+pow((8*pow((x02-a2*z2),2)+\
               4*pow((x03-a2*z3),2)-9),2)+pow(\
               (8*(x02-a2*z2)*(x03-a2*z3)+4),2);
            h1=(g2-g1)/a2;
            h2=(g3-g2)/(a3-a2);
            h3=(h2-h1)/a3;
            a0=(a2-h1/h3)/2;
            g0=pow((10*(x01-a0*z1)-2*pow((x02\
               -a0*z2),2)+(x02-a0*z2)-2*(x03-a0*z3)\
               -5),2)+pow((8*pow((x02-a0*z2),2)+\
               4*pow((x03-a0*z3),2)-9),2)+pow(\
               (8*(x02-a0*z2)*(x03-a0*z3)+4),2);
            min=g0;
            if(g3<min){
                    min=g3;
            }
```

```
                        if(min==g0){
                                a=a0;
                        }
                        else {
                                a=a3;
                        }
                        x01-=a*z1;
                        x02-=a*z2;
                        x03-=a*z3;
                        printf("n = %d, x1 = %.8lf, x2 = \
%.8lf, x3 = %.8lf, g = %.8lf\n",++i,x01,x02,x03,min);
                }
        }
```

令$\mathbf{x}=(1,1,1)^{\mathbf{T}}$,可得:

n	$x_1^{(n)}$	$x_2^{(n)}$	$x_3^{(n)}$	$g^{(n)}$
1	0.92753552	0.49999507	0.57970600	81.62449064
2	0.67589563	0.65390087	0.63774376	69.52080786
3	0.52253096	0.89189684	-0.32133767	7.88536591
4	0.51592829	1.03474947	-0.38996809	0.65404290
5	0.53651718	1.02248306	-0.46597604	0.14174062
6	0.52198314	1.00728128	-0.46735906	0.07238155
7	0.51584441	1.01388912	-0.47501960	0.04200303

$\therefore$ 一个近似解为 $\mathbf{x}=(0.51584441,\ 1.01388912,\ -0.47501960)^{\mathbf{T}}$.