MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 8	6.630 Electromagnetics	Issued: 20081030R
Fall Term 2008		Due: 20081106R

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Reading assignment: Section 4.1(D-G) ; J. A. Kong, "*Electromagnetic Wave Theory*," EMW Publishing, 2008.

Problem P8.1

That the exhausted travelers in desert often see an unreal "lake" in the distance is called the phenomenon of mirage. The hot sand in desert heats the air close to the ground and thus forms a layer of air with less density as well as less refractive index. The refractive index of air in the visible region is about 1.000271374 at the temperature of $20°C$, while about 1.000257559 at $35°C$. Consider the two air layers with different temperatures as shown in Fig. 1, what is the critical angle θ_c of the wave in this case? Try to explain the mirage using this model.

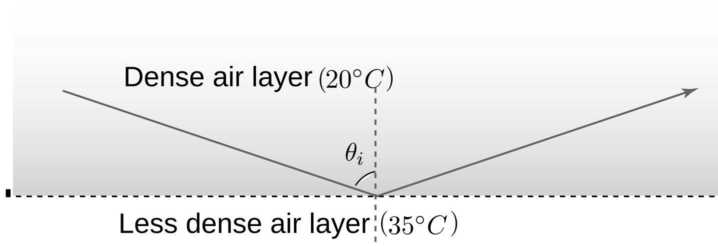

Fig.1

Problem P8.2

Sun light glares caused by reflections from plane surfaces are partially linearly polarized.

(a) Determine the Brewster angle for $\epsilon_t = 9\epsilon_0$ The Brewster angle, θ_B, is also called the polarization angle because at θ_B the reflected wave is entirely perpendicularly polarized (TE polarized).

(b) Your polaroid glasses absorb one linear component of incident light. To minimize sun glare, what component, perpendicularly or parallelly polarized (TM polarized), reaches your eyes after passing through the glasses? Explain why.

Problem P8.3

By comparing the phenomena of total reflection for $\theta > \theta_C$ and total transmission for $\theta = \theta_B$ at an isotropic dielectric interface, we can draw the following conclusions. (1) Total reflection occurs at a range of incident angles larger than the critical angle θ_C; total transmission of TM waves occurs only at the Brewster angle θ_B. (2) Total reflection occurs only when the incident medium is denser than the transmitted medium. The Brewster angle occurs for any two media. (3) When an unpolarized wave is totally reflected, the reflected wave is still unpolarized. When the TM wave components of an unpolarized wave are totally transmitted, the reflected wave contains only TE waves.

Suppose a TM wave is incident at an angle θ such that $\theta = \theta_B > \theta_C$. Then the wave is totally transmitted and at the same time it is totally reflected. Is that possible? Explain.

Problem P8.4

The gas laser depicted in Fig. 2 uses "Brewster angle" quartz windows on the gas discharge tube in order to minimize reflection losses. Determine the angle θ if the index of refraction for quartz at the wavelength of interest is $n = 1.46$. Because of these windows, the laser output is almost completely linearly polarized. What is the direction of polarization, i.e., is $\overline{E}$ parallel or perpendicular to the paper? Why?

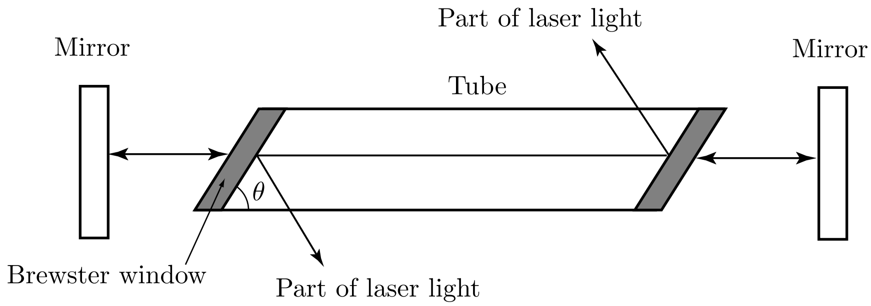

Fig. 2. A gas laser with Brewster windows.

Problem P8.5

Consider a plane wave incident from a dielectric region with permittivity $\epsilon = 3\epsilon_o$ upon a halfspace with $\epsilon = \epsilon_o$ as shown in Fig. 3.

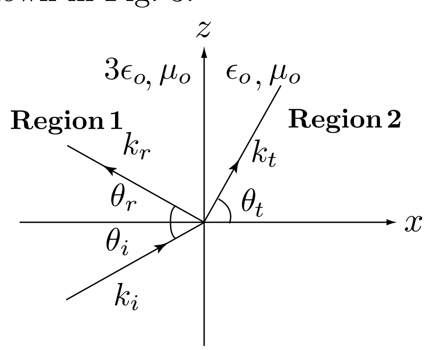

Fig. 3.

(a) Find the Brewster angle for Region 1.

(b) Suppose the transmitted electric field is given by

$$\overline{E}_t = \hat{y}\frac{E_0}{\sqrt{2}}e^{ik_{tx}x+ik_{kz}z} + E_0\frac{\hat{z}-\hat{x}\sqrt{3}}{2\sqrt{2}}e^{ik_{tx}x+ik_{tz}z-i\frac{\pi}{2}}$$

(i) Determine the incident and transmitted angles, θ_i and θ_t.

(ii) What is the polarization of the transmitted field? Be sure to specify the handedness (left or right) if necessary.

(iii) What is the polarization of the reflected wave? Be sure to specify the handedness (left or right) if necessary.

(iv) Give an expression for the incident electric field, $\overline{E}_i$, and the reflected electric field, $\overline{E}_r$.

(c) For this part, assume that the incident wave is a TE wave with magnitude, $|\overline{E}| = E_0$ and $\theta_i = \theta_c$ where θ_c is the critical angle.

(i) Find the Goos-Hänchen phase shift of the reflected wave.

(ii) Make a sketch of the electric field amplitude, E_y, for $x > 0$.