MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 2	6.632 Electromagnetic Wave Theory	Issued: 02/15/07 R
Spring Term 2007		Due: 02/22/07 R

Reading assignment: Section 3.2 J. A. Kong, "*Electromagnetic Wave Theory*"

Problem P2.1

(a) The complex permittivity for bottom round steak is about $\epsilon = 40(1 + i0.3)\epsilon_o$ at the operating frequency (2.5 GHz) of a microwave oven. What is the penetration depth?

(b) Calculate loss tangents and skin depths for sea water at frequencies 60 Hz and 10 MHz. Sea water can be characterized by conductivity $\sigma = 4\,\text{mho/m}$, permittivity $\epsilon = 80\epsilon_0$, and permeability $\mu = \mu_o$ at those frequencies.

(c) A 100-Hz electromagnetic wave is propagating down into the sea water with an electric field intensity E of 1 V/m just beneath the sea surface. What is the intensity of E at a depth of 100 m? What are the time-average Poynting's power densities just beneath the surface and at a depth of 100 m?

Problem P2.2

Superconductivity was first observed by Kamerlingh Onnes in 1911. In 1933 Meissner and Ochsenfeld discovered that superconducting metals cannot be penetrated by magnetic fields. Magnetic fields are expelled from a normal metal when it is cooled to the superconducting state. The macroscopic theory of superconductivity was developed by London and London in 1935 followed by the microscopic theory of Bardeen, Cooper and Schrieffer in 1957.

A simple model of superconductivity calls for an electron plasma with a very high electron density N.

(a) Show that the penetration depth of a plasma with very large N takes the form

$$d_p = \sqrt{\frac{m}{Ne^2\mu_0}}$$

(b) Letting $N = 7 \times 10^{28}\,\text{m}^{-3}$, calculate d_p.

(c) Compare the above result with the skin depth of a good conductor. Explain why a very slowly varying magnetic field can penetrate a good conductor but not a superconductor.

Problem P2.3

An electron plasma becomes anisotropic when an external dc magnetic field $\overline{B}_0$ is applied. Assume that $\overline{B}_0$ is in the direction. The magnetic field enters the constitutive parameters via the cyclotron frequency $\omega_c = qB_0/m$. Show that the permittivity tensor is

$$\overline{\overline{\epsilon}} = \begin{pmatrix} \epsilon & i\epsilon_g & 0 \\ -i\epsilon_g & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix}$$

with the constitutive parameters defined by

$$\epsilon = \epsilon_0 \left[1 - \frac{\omega_p^2}{(\omega^2 - \omega_c^2)}\right]$$

$$\epsilon_g = \epsilon_0 \left[\frac{-\omega_p^2 \omega_c}{\omega(\omega^2 - \omega_c^2)}\right]$$

$$\epsilon_z = \epsilon_0 \left[1 - \frac{\omega_p^2}{\omega^2}\right]$$

Write the constitutive relation as $\overline{E} = \overline{\overline{\kappa}} \cdot \overline{D}$ with

$$\overline{\overline{\kappa}} = \overline{\overline{\epsilon}}^{-1} = \begin{pmatrix} \kappa & i\kappa_g & 0 \\ -i\kappa_g & \kappa & 0 \\ 0 & 0 & \kappa_z \end{pmatrix}$$

Determine the constitutive parameters κ, κ_g, and κ_z.

Problem P2.4

Show that a linearly polarized wave can be decomposed into a right-hand circularly polarized wave and a left-hand circularly polarized wave.