6.630 Solution to Problem Set 3

Solution P3.1

$mv^2/R = Ze^2/4\pi\epsilon R^2$ gives $R = 4\pi\epsilon n^2\hbar^2/Zme^2 \approx 0.52\,n^2 \times 10^{-10}\,\mathrm{m}$ for $Z = 1$.

Solution P3.2

The wave number for the attenuation term is

$$k_I = \omega\sqrt{\mu_o\epsilon}\left[\frac{1}{2}\left(\sqrt{1+\frac{\sigma^2}{\epsilon^2\omega^2}}-1\right)\right]^{1/2}.$$

For a good conductor,

$$\frac{\sigma^2}{\epsilon^2\omega^2} \gg 1,$$

we have

$$k_I \approx \left(\frac{\sigma\mu_o\omega}{2}\right)^{1/2},$$

thus

$$d_p = \frac{1}{k_I} \approx \sqrt{\frac{2}{\omega\mu_o\sigma}}.$$

For the material with low conductivity,

$$\frac{\sigma^2}{\epsilon^2\omega^2} \ll 1,$$

we can approximate

$$\sqrt{1+\frac{\sigma^2}{\epsilon^2\omega^2}} \approx 1+\frac{\sigma^2}{2\epsilon^2\omega^2}.$$

Thus

$$k_I \approx \frac{\sigma\eta}{2},$$

and

$$d_p = \frac{1}{k_I} \approx \frac{2}{\sigma\eta}$$

(a) Since the term,

$$\frac{\sigma}{\epsilon\omega} = \frac{2}{40 \times 8.85 \times 10^{-12} \times 2\pi \times 2.5 \times 10^9} = 0.36,$$

we get

$$k_I = \omega\sqrt{\mu_o\epsilon}\left[\frac{1}{2}\left(\sqrt{1+\frac{\sigma^2}{\epsilon^2\omega^2}}-1\right)\right]^{1/2}$$

$$= \frac{2\pi \times 2.5 \times 10^9}{3 \times 10^8}\sqrt{40}\left[\frac{1}{2}\left(\sqrt{1+0.36^2}-1\right)\right]^{1/2} = 58.7(m^{-1}),$$

thus

$$d_p = \frac{1}{k_I} = \frac{1}{58.7}(m) = 1.7(cm).$$

For polystyrene foam,

$$\frac{\sigma}{\epsilon\omega} = \frac{4 \times 10^{-6}}{1.03 \times 8.85 \times 10^{-12} \times 2\pi \times 2.5 \times 10^{9}} = 2.79 \times 10^{-5} \ll 1,$$

we get

$$d_p \approx \frac{2}{\sigma\eta} = \frac{2\sqrt{1.03}}{4 \times 10^{-6} \times 377} = 1346.0(m)$$

(b) For good conductor,

$$\frac{\epsilon\omega}{\sigma} < 0.1$$

thus

$$\omega < 0.1 \times \frac{\sigma}{\epsilon} = 0.1 \times \frac{5 \times 10^{-3}}{10 \times 8.85 \times 10^{-12}} = 5.65(rad/sec),$$

or

$$f_{\max} = \frac{\omega}{2\pi} = 0.899(MHz)$$

(c) Since $\frac{\sigma}{\epsilon\omega} \gg 1$, thus

$$d_p \approx \sqrt{\frac{2}{\omega\mu_o\sigma}} = \sqrt{\frac{2}{2\pi \times 10^{8} \times 4\pi \times 10^{-7} \times 3.54 \times 10^{7}}} = 8.46 \times 10^{-6}(m).$$

So the thickness of the aluminum layer is $5d_p = 4.24 \times 10^{-5}(m)$. The thickness of ordinary aluminum foil is

$$10^{-3}(inch) = 2.54 \times 10^{-5}(m) < 4.24 \times 10^{-5}(m),$$

which is not thick enough.

(d)

(1) For $f = 100(Hz)$,

$$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi \times 100 \times 80 \times 8.85 \times 10^{-12}} = 8.99 \times 10^{6},$$

and skin depth

$$d_p \approx \sqrt{\frac{2}{\omega\mu_o\sigma}} = \sqrt{\frac{2}{2\pi \times 100 \times 4\pi \times 10^{-7} \times 4}} = 25.2(m).$$

(2) For $f = 5(MHz)$, the loss tangent is

$$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi \times 5 \times 10^{6} \times 80 \times 8.85 \times 10^{-12}} = 180,$$

and the skin depth is

$$d_p \approx \sqrt{\frac{2}{\omega\mu_o\sigma}} = \sqrt{\frac{2}{2\pi \times 5 \times 10^{6} \times 4\pi \times 10^{-7} \times 4}} = 0.11(m).$$

(e) The penetration depth at frequency $f = 1.0(kHz)$ is

$$d_p \approx \sqrt{\frac{2}{\omega\mu_o\sigma}} = \sqrt{\frac{2}{2\pi \times 1000 \times 4\pi \times 10^{-7} \times 4}} = 7.96(m) = \frac{1}{k_I}.$$

The attenuation term for the radiation power is

$$e^{-2k_I z} = e^{-2\times\frac{100}{7.96}} = 1.22\times 10^{-11} = -109.1(dB).$$

Solution P3.3

(a)

$$\overline{T} = \hat{r}\times\overline{F} = \frac{1}{2}l_b\hat{x}\times(\hat{z}\times\hat{x}Il_zB_0) - \frac{1}{2}l_b\hat{x}\times(-\hat{z}\times\hat{x}Il_zB_0) = \hat{z}IAB_0$$

From $\overline{M} = -\hat{y}IA$, and $\overline{B} = \hat{x}B_0$, we find that, $\overline{T} = \overline{M}\times\overline{B} = \hat{z}IAB_0$.

(b) The magnetic moment, $\overline{M}$ of the current loop can be found by applying $\overline{M} = \hat{n}A_0I_l = \frac{1}{\sqrt{2}}(\hat{x}-\hat{y})A_0I_l$.

To find the magnetic field, $\overline{H}$, at the position of the loop due to the straight wire carrying current, I_0, we use the integral form of Ampere's Law, that is,

$$\oint_C \overline{H}\cdot d\overline{l} = \int_s \overline{J}\cdot ds = I_0$$

By symmetry the $\overline{H}$ field will be of uniform amplitude at a fixed radius and in the $\hat{\phi}$ direction yielding,

$$\oint_C \overline{H}\cdot d\overline{l} = \int_0^{2\pi} H_\phi d\ d\phi = 2\pi d H_\phi$$

which implies that at the loop's postion

$$\overline{B} = \hat{\phi}\frac{I_0\mu_0}{2\pi d} = \hat{x}\frac{I_0\mu_0}{2\pi d}$$

where the fact that $\overline{B} = \mu_0\overline{H}$ was used. To calculate the torque, we apply,

$$\overline{T} = \overline{M}\times\overline{B} = \hat{z}\frac{A_0I_lI_0\mu_0}{2\sqrt{2}\pi d}$$

which means that the current loop will move about the z-axis in a counter-clockwise direction.