

Matemática Atuarial

- TeleSistech -

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Rendite vitalizie perpetue (anticipate) immobiliari,  $R=1$

Definizione: Pagamento delle rate all'inizio di ogni anno,  
o partire dell'epoca di stipula, fin quando  
la testa umane è in vita.

$$\tilde{Y} = \begin{cases} v^0, v^0 + v, v^0 + v + v^2, \dots, v^0 + v + v^2 + \dots + v^{w-x-1} \\ q_x, q_1 q_x, q_2 q_x, \dots, \frac{v^{w-x-1}}{q_x} \end{cases}$$

$$U = E(\tilde{Y}) = v^0 q_x + (v^0 + v) q_1 p_x + (v^0 + v + v^2) q_2 p_x + \dots + (v^0 + v + \dots + v^{w-x-1}) q_{w-x} p_x$$

$$= v^0 (q_x + q_1 p_x + \dots + q_{w-x} p_x) + v (q_1 p_x + \dots + q_{w-x} p_x) + \dots + v^{w-x-1} q_{w-x} p_x$$

$$= v^0 \cdot 1 + v (1 - q_x) + v^2 (1 - q_1 p_x) + \dots + v^{w-x-1} (1 - q_{w-x} p_x)$$

$$= v^0 p_x + v^1 p_x + v^2 p_x + \dots + v^{w-x-1} p_x$$

$$= {}_0 E_x + {}_1 E_x + {}_2 E_x + \dots + {}_{w-x} E_x$$

$$= \sum_{k=0}^{\infty} {}_k E_x = \ddot{a}_x \quad \text{Se RZ1} \rightarrow U = R \cdot \ddot{a}_x$$

① ② ③ ④ ⑤

0 1 2 3 4 ...

X x+1 x+2 x+3 x+4 ...

①

w-x-1 w-x

w-1 w

stessa  
rate

1.  $E_x$

1.  $E_x$

1.  $E_x$   
anz  
1.  
 $\ddot{a}_x$

## Rendite vitalizie, anticipate, diffuse, $R=1$

Definizione: Pagamenti sulla rete all'inizio si svolgono, se no il giorno  $t$  si soffrono, e finché l'ente esiste è in vita.

$$\tilde{Y} = \begin{cases} 0, & \text{se } V^m, (V^m + V^{m+1}), (V^m + V^{m+1} + V^{m+2}), \dots, (V^m + V^{m+1} + \dots + V^{m+x-1}) \\ q_x, & \text{se } V^m q_x, V^{m+1} q_x, \dots, V^{m+x-1} q_x \end{cases}$$

$$\begin{aligned} U &= E(\tilde{Y}) = V^m \cdot \frac{m}{m} p_x + (V^m + V^{m+1}) \frac{m+1}{m+1} p_x + \dots + (V^m + V^{m+1} + \dots + V^{m+x-1}) \frac{m+x}{m+x} p_x \\ &= V^m (m p_x + m^2 p_x + \dots + m^x p_x) + V^{m+1} (m^2 p_x + \dots + m^x p_x) + \dots + V^{m+x-1} (m^x p_x) \\ &= V^m \cdot (1 - m p_x) + V^{m+1} \cdot (1 - \frac{m}{m+1} p_x) + V^{m+2} \cdot (1 - \frac{m}{m+2} p_x) + \dots + V^{m+x-1} \cdot (1 - \frac{m}{m+x-1} p_x) \\ &= V^m m p_x + V^{m+1} m^2 p_x + V^{m+2} m^3 p_x + \dots + V^{m+x-1} m^x p_x \\ &= \sum_{k=m}^{+\infty} k E_X = m \ddot{e}_X \quad \rightarrow \text{Se } R \geq 1 \rightarrow [U = R \cdot m \ddot{e}_X] \end{aligned}$$

$$= m E_X \cdot \frac{\partial}{\partial x + m}$$

0	0	0	0	1	1	1	1	
0	1	2	3	...	$m$	$m+1$	$m+2$	
$x$	$x+1$	$x+2$	$x+3$	...	$x+m$	$x+m+1$	$x+m+2$	

scheide  
etwa

$$\frac{\partial}{\partial x + m}$$

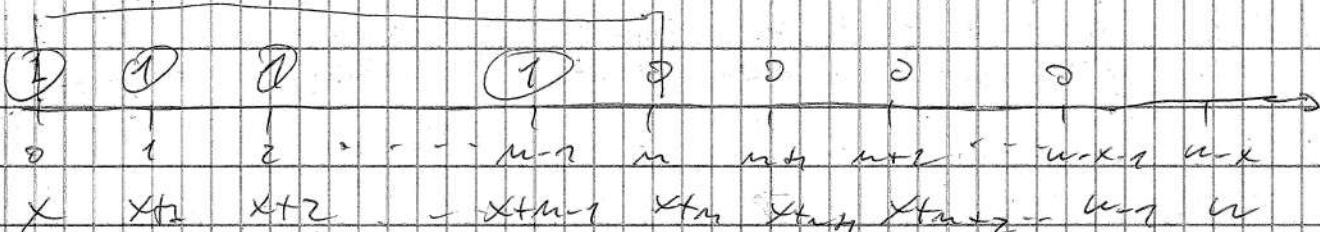
$$\frac{F}{m} x \cdot \frac{\partial}{\partial x + m}$$

Rendite vitellizie anticipate, immediate, temprese,  $R=1$

Definizione: Rendita delle rette all'inizio di ogni anno, a partire dalle stigie delle polte, fin quando le feste esaurite è un  $N^{\text{ta}}$ , e il più fino alle desunte delle polte.

$$\tilde{y} = \begin{cases} v^0, (v^0 + v), (v^0 + v + v^2), \dots, (v^0 + v + \dots + v^{n-1}), \dots, (v^0 + v + \dots + v^{m-1}) \\ q_1 p_x, q_2 p_x, q_3 p_x, \dots, q_{n-1} p_x, \dots, q_m p_x \end{cases}$$

$$\begin{aligned} U &= E(\tilde{y}) = v^0 q_1 p_x + (v^0 + v) q_2 p_x + (v^0 + v + v^2) q_3 p_x + \dots + (v^0 + v + \dots + v^{m-1}) q_m p_x + \dots \\ &\quad + \dots + (v^0 + v + \dots + v^{m-1}) q_{n-1} p_x \\ &= v^0 (q_1 p_x + q_2 p_x + \dots + q_{n-1} p_x) + v (q_1 p_x + \dots + q_{n-1} p_x) + \dots + v^{m-1} (q_1 p_x + \dots + q_{n-1} p_x) \\ &= v^0 p_x + v^{m-1} p_x + \dots + v^{m-1} p_x \\ &= \sum_{k=0}^{m-1} v^k E x = \tilde{m}^2 x \rightarrow \text{Se } R > 1 \rightarrow U = R \cdot \tilde{m}^2 x \end{aligned}$$



Rendite mitelizie, anticette, differente, temporanea,  $R=1$

Definizione. I segmenti delle rette ellittiche di ogni anno, dopo il punto di sfidamento, finché l'incertezza è in vita e di cui per mesi

$$\tilde{Y} = \begin{cases} 0, & V^m, (V^m + V^{m+1}) \\ & \vdots \\ & (V^m p_x, V^{m+1} p_x, \dots, V^{m+n-1} p_x) \end{cases}$$

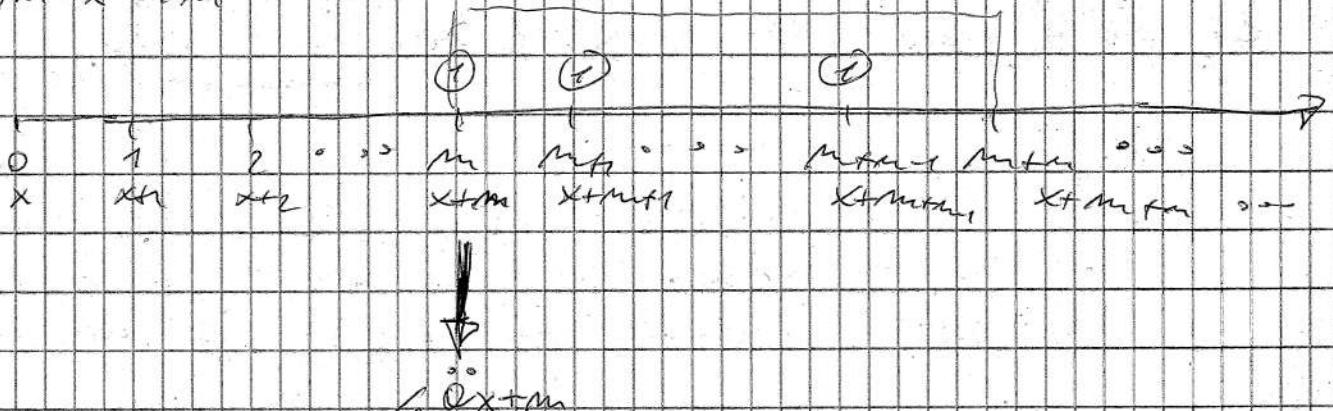
$$U = E(\tilde{Y}) = V^m p_x + (V^m + V^{m+1}) p_x + \dots + (V^m + \dots + V^{m+n-1}) p_x$$

$$= V^m (p_x + p_{x+1} + \dots + p_{x+n-1}) + \dots + V^{m+n-1} (p_x + \dots + p_{x+n-1})$$

$$= V^m p_x + V^{m+1} p_x + \dots + V^{m+n-1} p_x$$

$$= \sum_{h=m}^{m+n-1} h E_x = m \bar{x}_x \rightarrow \text{Se } R \geq 1 \rightarrow U = R \cdot m \bar{x}_x$$

$$= m E_x \cdot \bar{x}_{x+m}$$



$$E_x = \frac{m}{n} \bar{x}_{x+m}$$

Rendite mitiche preferenze, partecipate immediate,  $R=1$

Definizione: Pagamenti sulle rette alle fine di ogni anno e sotto la stessa scistale, per quei 6 le tante onorevoli e in vita.

$$\tilde{Y} = \begin{cases} 0, \sqrt{v}, \sqrt{v+v^2}, \sqrt{v+v^2+v^3}, \dots, \sqrt{v+v^2+v^3+\dots+v^{u-x-1}} \\ (\sqrt{v}x, \sqrt{v}x, \sqrt{v}x, \sqrt{v}x, \dots, \sqrt{v}x) \end{cases}$$

$$\begin{aligned} U &= E(\tilde{Y}) = \sqrt{v}p_x + (\sqrt{v+v^2})p_x + (\sqrt{v+v^2+v^3})p_x + \dots + (\sqrt{v+v^2+\dots+v^{u-x-1}})p_x \\ &= \sqrt{v}(p_x + 2p_x + \dots + \sqrt{v}p_x) + \sqrt{v^2}(p_x + 3p_x + \dots + \sqrt{v}p_x) + \dots + \sqrt{v^{u-x}}p_x \\ &= \sqrt{v}_1 p_x + \sqrt{v^2}_2 p_x + \dots + \sqrt{v^{u-x}}_{u-x} p_x \\ &= \sum_{k=1}^{+\infty} \sqrt{v}^k p_x = \varrho_x \rightarrow \text{Se } R \geq 1 \rightarrow U = R \cdot \varrho_x \end{aligned}$$



Osservazione:  $\tilde{\varrho}_x = \sum_{k=0}^{+\infty} \sqrt{v}^k p_x = 1 + \sum_{k=1}^{+\infty} \sqrt{v}^k p_x = 1 + \varrho_x$

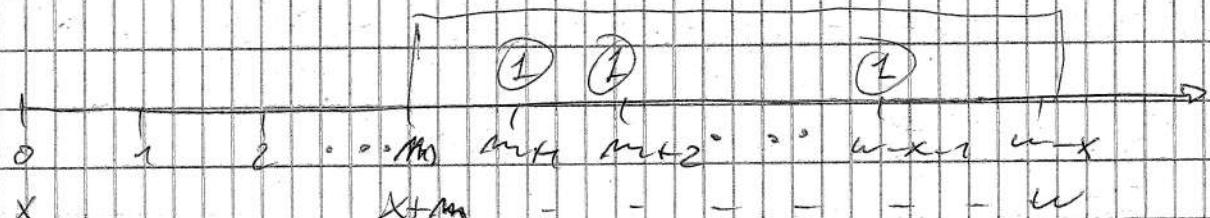
$$\rightarrow \boxed{\tilde{\varrho}_x = \varrho_x + 1}$$

Rendite vittoriose, diverse partecipate, per tutte, R=1

Definizione: Referenza delle rette alle fine di ogni anno  
sopra il possibile raffiguramento, e finché l'andamento  
è in vita

$$\tilde{Y}_x = \begin{cases} 0, & p \\ 1, & \sqrt{m^x} (r^{m+1} + r^{m+2}) + \dots + (\sqrt{m^x} + \dots + \sqrt{m^{x-1}}) \\ \sqrt{m^x} p_x, & \frac{m^x}{1} p_x \\ \dots, & \dots \\ \sqrt{m^x} p_x & \end{cases}$$

$$U = E(\tilde{Y}) = \sum_{t=m+1}^{+\infty} q_t^x r^t \rightarrow U = R \cdot \tilde{Y} Q X$$



$$q_x^x \cdot Q_{x+m}$$



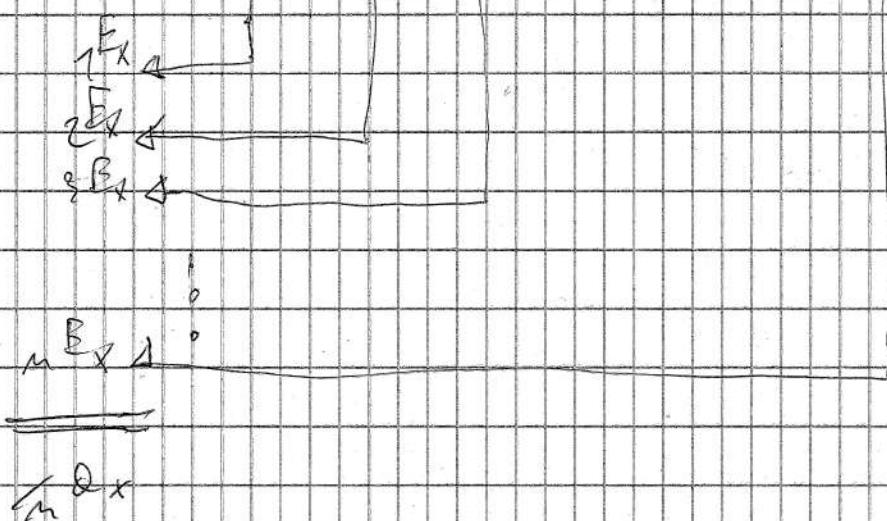
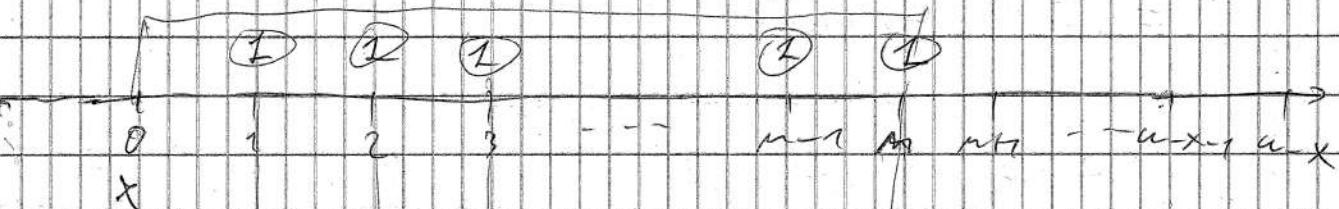
$$U = R \cdot \frac{E_x}{m^x} \cdot Q_{x+m}$$

Rendite vitalizie immediate, partite per temponee,  $R=1$

Definizione. Pagamenti sulla rete alle fine di ogni anno, e partite dell'anno si stende sulla polita, se l'entrate è in vita e al più per  $m$ -anni

$$\tilde{y} = \begin{cases} 0, & \text{se } (V+N^1), \dots, (V+N^2 + \dots + N^m) \\ n \times (1/p_x, 2/p_x, \dots, m/p_x, \dots, (n+1)/p_x) \end{cases}$$

$$U = E(\tilde{y}) = \sum_{x=1}^m q_x E_x = \frac{q}{m} x \rightarrow \text{se } R \geq 1 \rightarrow U = R \cdot \frac{q}{m} x$$



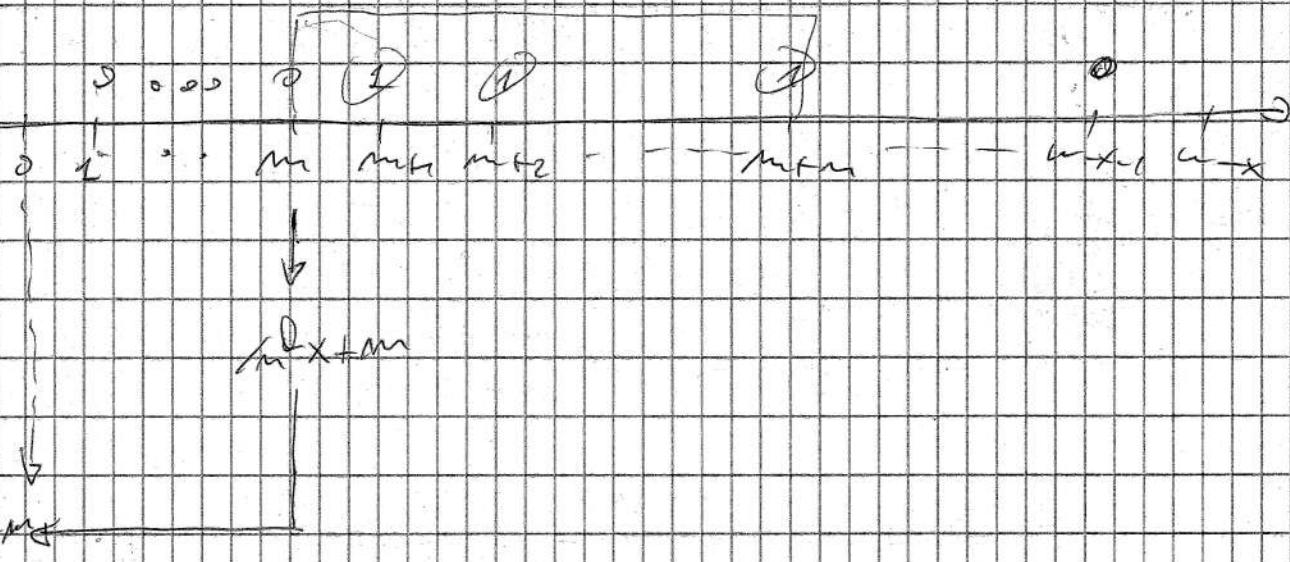
Reuslate with fine, stiffened, partially open, tempered mesh, R=1

Definizione. Pagamento delle rate alle fine di ogni anno  
se il prezzo si riferisce a finche l'acquisto  
è in vita e di più per mani.

$$y = \begin{cases} 0 & 0 \\ 0 & \sqrt{m+1} \left( \sqrt{m+1} + \sqrt{m+2} \right) \\ \vdots & \vdots \\ 0 & (\sqrt{m+1} + \dots + \sqrt{m+n}) \end{cases}$$

$$V = E(\tilde{y}) = \sum_{\ell=1}^{m+m} \ell x = m \ell x \rightarrow V = R \circ m \circ x$$

$$= m \mathbb{E} X \cdot f_m(x+m)$$



Rendite vitalistiche, immediate, perpetue, anticipate, presentate, R=1

Definizione. Per quanto delle reti emesse, cominciate anticipatamente e presentate nell'anno, e per tutte quelle stipulate sulle polizze e finché l'annuncio è in vita.

Sia  $K = n^o$  presenti delle reti  $\rightarrow R = 1 = \frac{1}{K} \times K$

Eseminiamo sullo stesso tempo/importo le rendite:

$$\begin{array}{ccccccc}
 \frac{1}{K} & \frac{1}{K} & \frac{1}{K} & \cdots & \frac{1}{K} & & \\
 \hline
 0 & 1 & 2 & \ddots & K-1 & 1 & \cdots \\
 & \frac{1}{K} & \frac{2}{K} & \ddots & \frac{K-1}{K} & \frac{1}{K} & \\
 \hline
 \frac{1}{K} & \frac{1}{K} & \frac{1}{K} & \cdots & \frac{1}{K} & & \\
 \hline
 1 & 1+\frac{1}{K} & 1+\frac{2}{K} & \cdots & 1+\frac{K-1}{K} & 2 & \cdots \\
 \vdots & \vdots & \vdots & & \vdots & \vdots & \\
 \frac{1}{K} & \frac{1}{K} & \frac{1}{K} & \cdots & \frac{1}{K} & & \\
 \hline
 (w-x_1) & (w-x_1+\frac{1}{K}) & (w-x_1+\frac{2}{K}) & \cdots & (w-x_1)+\frac{K-1}{K} & w-x & 
 \end{array}$$

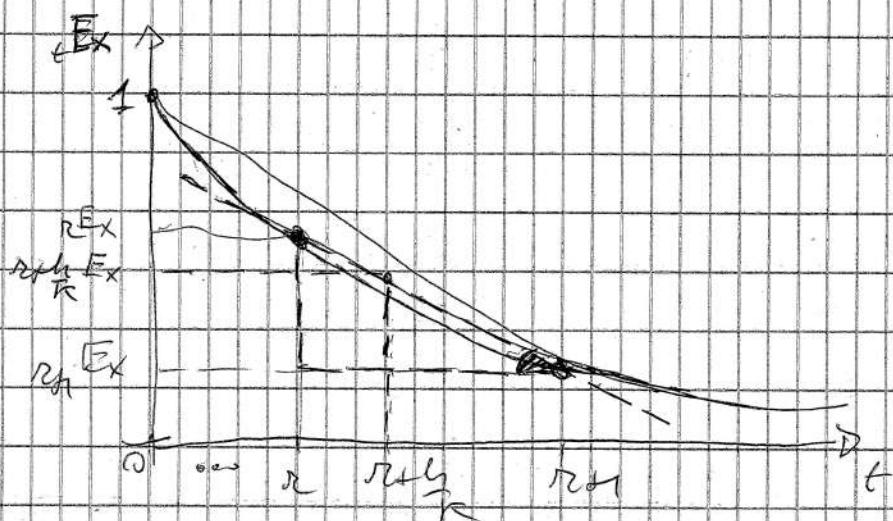
Applichiamo il principio di componibilità dei contratti:

$$\begin{aligned}
 D_x^{(K)} = & \frac{1}{K} \left[ \left( \frac{0}{K} E_x + \frac{1}{K} E_{x+\frac{1}{K}} + \frac{2}{K} E_{x+\frac{2}{K}} + \cdots + \frac{K-1}{K} E_{x+\frac{K-1}{K}} \right) + \right. \\
 & + \left( 1+\frac{1}{K} E_x + 1+\frac{2}{K} E_x + \cdots + 1+\frac{K-1}{K} E_x \right) + \\
 & + \left( 2+\frac{1}{K} E_x + 2+\frac{2}{K} E_x + \cdots + 2+\frac{K-1}{K} E_x \right) + \cdots \\
 & \left. + \left( (w-x_1) E_x + (w-x_1)+\frac{1}{K} E_x + \cdots + (w-x_1)+\frac{K-1}{K} E_x \right) \right]
 \end{aligned}$$

$$\ddot{\varrho}_x^{(K)} = \frac{1}{K} \cdot \sum_{R=0}^{w-x-1} \sum_{k=0}^{K-1} R + \frac{k}{K} E_x$$

Formule approximante (interpolation linéaire) :

$$R + \frac{k}{K} E_x \approx R E_x - \frac{k}{K} (R E_x - \underline{r}_K E_x)$$



$$\ddot{\varrho}_x^{(K)} \approx \frac{1}{K} \sum_{R=0}^{+\infty} \sum_{k=0}^{K-1} \left[ R E_x - \frac{k}{K} (R E_x - \underline{r}_K E_x) \right] = \frac{1}{K} \sum_{R=0}^{+\infty} \left[ \sum_{k=0}^{K-1} R E_x - \frac{1}{K} (E_x - \underline{r}_K E_x) \sum_{k=0}^{K-1} k \right]$$

$$\approx \frac{1}{K} \sum_{R=0}^{+\infty} \left[ K \cdot R E_x - \frac{K-1}{2} (E_x - \underline{r}_K E_x) \right] = \frac{1}{2K} \sum_{R=0}^{+\infty} \left[ (K+1) R E_x + (K-1) \underline{r}_K E_x \right]$$

$$\approx \frac{1}{2K} \left[ (K+1) \sum_{R=0}^{+\infty} R E_x + (K-1) \sum_{R=0}^{+\infty} \underline{r}_K E_x \right]$$

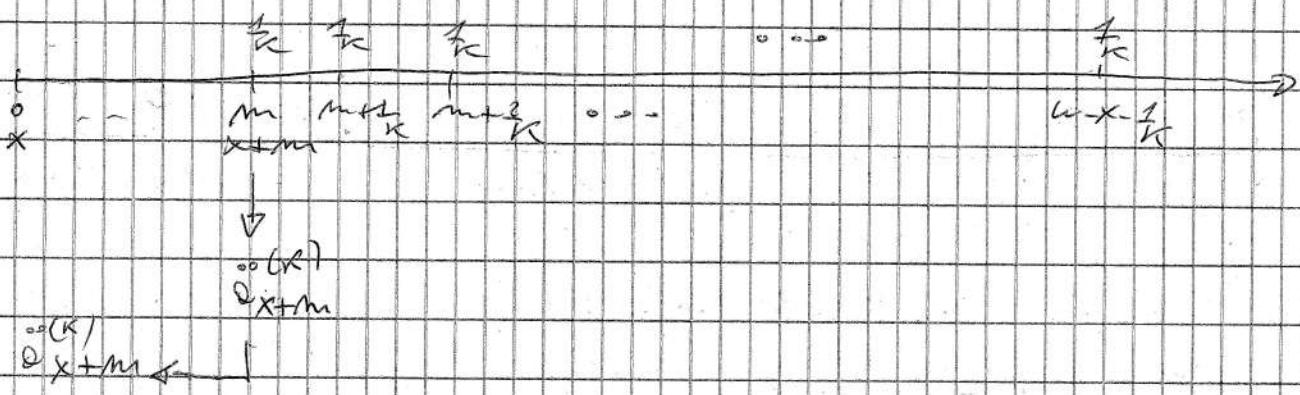
$$\approx \frac{1}{2K} \left[ (K+1) \ddot{\varrho}_x + (K-1) (\ddot{\varrho}_x - 1) \right]$$

$$\approx \ddot{\varrho}_x - \frac{K-1}{2K} \rightarrow \boxed{\ddot{\varrho}_x^{(K)} = \ddot{\varrho}_x - \frac{K-1}{2K}}$$

$$\boxed{U = R \cdot \ddot{\varrho}_x^{(K)}} \rightarrow U = R \cdot \left( \ddot{\varrho}_x - \frac{K-1}{2K} \right).$$

Résultat bimétale, anticyclote, à deux pôles, R=1

Déformations possibles :

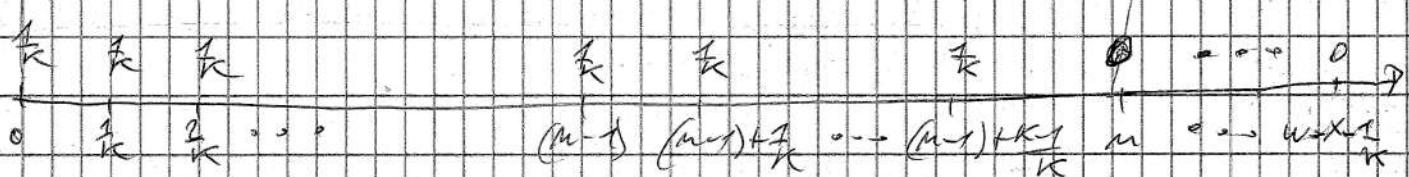


$$m\ddot{x} - \frac{\ddot{x}}{2}x + m\ddot{x}$$

$$\begin{aligned} U &= R \cdot m \ddot{x} = R \cdot m \ddot{E}_X \cdot \frac{\ddot{x}}{2}x + m = R \cdot \frac{m}{2} \ddot{E}_X \left[ \frac{\ddot{x}}{2}x + m - \frac{K-1}{2K} \right] \\ &= R \cdot \left[ m \ddot{E}_X - \frac{K-1}{2K} \frac{m}{m} \ddot{E}_X \right] \end{aligned}$$

Rendite frontière, anticipée, immédiate, temporelle, R=1

Définition personnelle :



Principe de compensation :

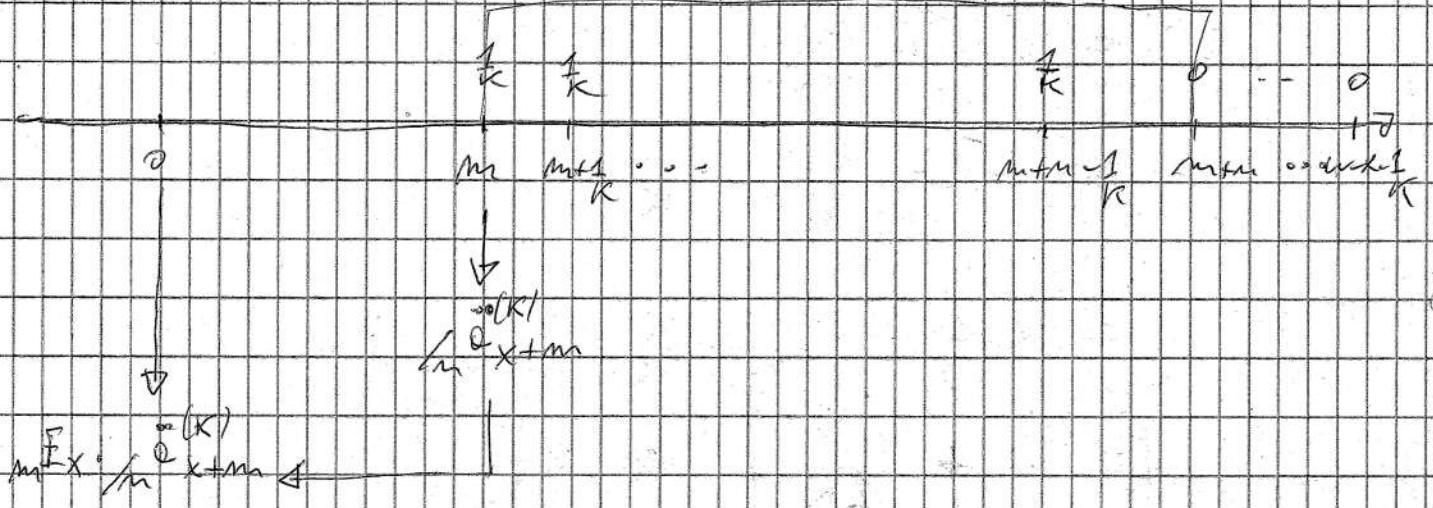
$$\begin{aligned} \frac{\partial^{\circ}(K)}{\partial x} &= \frac{\partial^{\circ}(K)}{\partial x} - \gamma \frac{\partial^{\circ}(K)}{\partial x} \\ &= \left( \frac{\partial^{\circ}}{\partial x} - \frac{K-1}{2K} \right) + \left( \gamma \frac{\partial^{\circ}}{\partial x} - \frac{K-1}{2K} E_x \right) \\ &= \left( \frac{\partial^{\circ}}{\partial x} - \gamma \frac{\partial^{\circ}}{\partial x} \right) - \frac{K-1}{2K} \left( 1 - \frac{E_x}{m} \right) \\ &= \boxed{\frac{\partial^{\circ}}{\partial x} - \frac{K-1}{2K} \left( 1 - \frac{E_x}{m} \right)} \end{aligned}$$

Donc :

$$\begin{aligned} \frac{\partial^{\circ}}{\partial x} &= \frac{\partial^{\circ}(K)}{\partial x} + \gamma \frac{\partial^{\circ}}{\partial x} \\ \frac{\partial^{\circ}(K)}{\partial x} &= \frac{\partial^{\circ}(K)}{\partial x} + \gamma \frac{\partial^{\circ}(K)}{\partial x} \end{aligned}$$

Rendite freiesmete anticipate, differente, Temperaturen,  $R=1$

Differenzne per perioden



$$\text{Allere: } \frac{m \Delta(x)}{m} = \frac{E_x}{m} \cdot \frac{\Delta(x)}{m}$$

$$= m E_x \left[ \frac{\Delta(x)}{m} - \frac{\Delta(x)}{m} \right]$$

$$= m E_x \left[ \left( \frac{\Delta(x+m) - K_1}{2K} \right) - \left( \frac{\Delta(x+m) - K_1}{2K} - \frac{m E_x + m}{2K} \right) \right]$$

$$= m E_x \left[ \frac{\Delta(x+m) - K_1}{2K} \left( 1 - \frac{m E_x + m}{2K} \right) \right]$$

$$= \left[ \frac{m \Delta(x)}{m} - \frac{K_1}{2K} \left( m E_x - \frac{m E_x + m}{2K} \right) \right]$$

$$V = R \cdot \frac{m \Delta(x)}{m}$$

Rendite fraccionaria, partijete, impositiva, pura,  $R=1$

Tasse di premio nullo fu re:

$$\begin{aligned} q_x^{(K)} &= \bar{e}_x - \frac{1}{K} \\ &= \left( \bar{e}_x - \frac{K-1}{2K} \right) - \frac{1}{K} \\ &= \left( \bar{e}_x + \frac{K-1}{2K} \right) - \bar{e}_x \end{aligned}$$

$$V = R \cdot \left( \bar{e}_x + \frac{K-1}{2K} \right)$$

$$\begin{array}{c} \bullet \quad \frac{1}{K} \quad \frac{1}{K} \\ \bullet \quad \frac{1}{K} \quad \frac{2}{K} \quad \cdots \\ \times \end{array} \quad \rightarrow$$

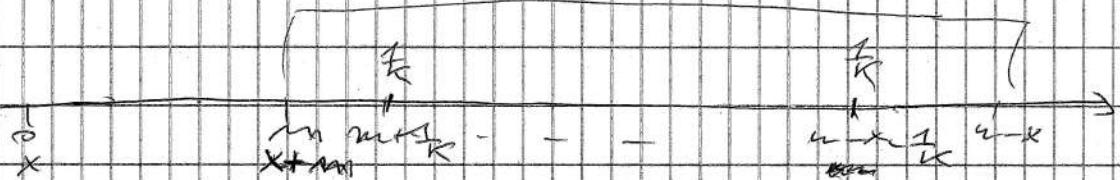
$$\begin{array}{c} \frac{1}{K} \\ \vdots \\ \frac{1}{K} \quad \frac{1}{K} \quad \cdots \\ \text{axiax} \quad \text{ax} \end{array}$$

Osservazione:

$$q_x^{(K)} \geq \bar{e}_x$$

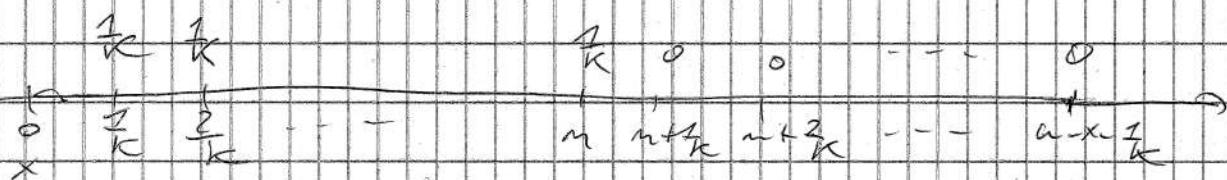
Renzite fractionate, participate, affinite, prefative,  $R=1$

$$m \overset{(K)}{\alpha}_x = \frac{E_x}{m} \cdot \overset{(K)}{\alpha}_{x+m} = \frac{E_x}{m} \left( \overset{(K)}{\alpha}_{x+m} + \frac{R-1}{2K} \right) = m \overset{(K)}{\alpha}_x + \frac{R-1}{2K} \frac{E_x}{m}$$



Renzite fractionate, participate, non-selective, temperature,  $R=1$

$$\begin{aligned} \frac{Q^{(K)}}{m} &= \overset{(K)}{\alpha}_x - \frac{Q^{(K)}}{m} \overset{(K)}{\alpha}_x = \\ &= \frac{Q_x}{m} + \frac{R-1}{2K} \left( 1 - \frac{E_x}{m} \right) \end{aligned}$$



Renzite fractionate, participate, affinite, temperature,  $R=1$

$$m \overset{(K)}{\alpha}_x = \frac{E_x}{m} \cdot \overset{(K)}{\alpha}_{x+m} = \frac{m \overset{(K)}{\alpha}_x}{m} + \frac{R-1}{2K} \left( \frac{E_x}{m} - \frac{E_x}{m+m} \right)$$

