Rendita vitalizia immediata, perpetua, anticipata, in Progressione Aritmetica

Definizione: Pagamento delle rate, la prima $R=1$ e le successive variabili in progressione aritmetica di ragione $\Delta = 1$ all'inizio di ogni anno a partire dall'epoca di stipula, fin quando la testa assicurata è in vita.

$$\tilde{Y} = \begin{cases} v^0, & v^0+2v, & v^0+2v+3v^2, & \dots, & v^0+2v+3v^2+\dots+(\omega-x)v^{\omega-x-1} \\ {}_{0/1}q_x, & {}_{1/1}q_x, & {}_{2/1}q_x, & \dots, & {}_{\omega-x-1/1}q_x \end{cases}$$

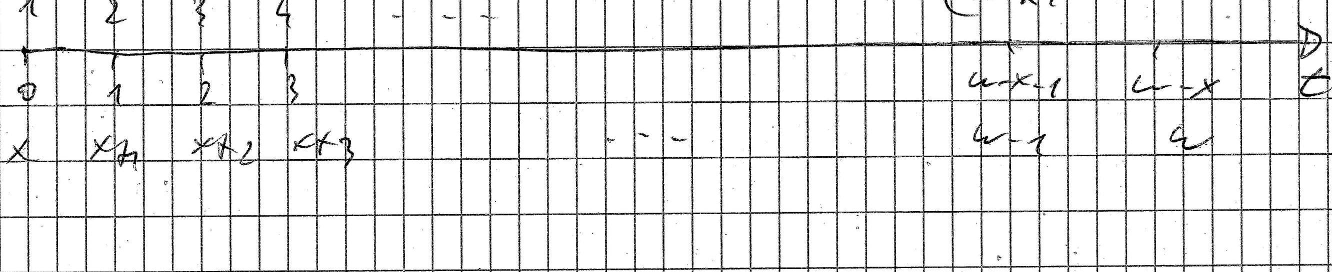
$$V = E(\tilde{Y}) = v^0 \, {}_{0/1}q_x + (v^0+2v)\, {}_{1/1}q_x + \dots + \left[v^0+2v+\dots+(\omega-x)v^{\omega-x-1}\right] {}_{\omega-x-1/1}q_x$$

$$= v^0\left({}_{0/1}q_x + \dots + {}_{\omega-x-1/1}q_x\right) + 2v\left({}_{1/1}q_x + \dots + {}_{\omega-x-1/1}q_x\right) + \dots + (\omega-x)v^{\omega-x-1} \cdot {}_{\omega-x-1/1}q_x$$

$$= v^0 \cdot {}_0p_x + 2v \, {}_1p_x + \dots + (\omega-x)v^{\omega-x-1} \, {}_{\omega-x-1}p_x$$

$$= \sum_{h=0}^{+\infty} (1+h) \cdot {}_hE_x = (I\ddot{a})_x$$

$\rightarrow$ se $R > 1 \rightarrow V = R \cdot (I\ddot{a})_x$



Rendita vitalizia, differita, anticipata, perpetua, in pop. anticipata

$$\begin{array}{cccccccc} & & 1 & 2 & 3 & & & n-x \\ 0 & \dots & m & m+1 & m+2 & \dots & n-x-1 & n-x \\ x & & x+m & & & & & \end{array}$$

$$(I\ddot{a})_{x+m}$$

$${}_mE_x \cdot (I\ddot{a})_{x+m}$$

$$\to U = {}_{m/}(I\ddot{a})_x = {}_mE_x \cdot (I\ddot{a})_x$$

Rendita vitalizia immediata, anticipata, temporanea, in pop. ant.

$$\begin{array}{lcccccccccc} \text{tempi:} & 0 & 1 & 2 & \dots & m-1 & m & m+1 & \dots & n-x-1 & n-x \\ \text{Importi:} & 1 & 2 & 3 & \dots & m & 0 & 0 & \dots & 0 & 0 \end{array}$$

$$\begin{array}{lccccccccc} & 0 & 1 & 2 & \dots & m-1 & m & m+1 & \dots & n-x-1 \\ (I\ddot{a})_x & 1 & 2 & 3 & & m & m+1 & m+2 & & (n-x) \end{array}$$

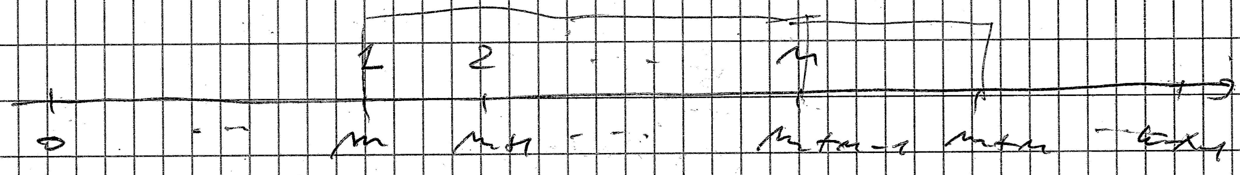
$$\begin{array}{lccccccccc} & 0 & 1 & 2 & \dots & m-1 & m & m+1 & \dots & n-x-1 \\ -{}_{m/}(I\ddot{a})_x & & & & & & -1 & -2 & \dots & -[(n-x)-m] \end{array}$$

$$\begin{array}{lccccccccc} & 0 & 1 & 2 & \dots & m-1 & m & m+1 & \dots & n-x-1 \\ -m \cdot {}_{m/}\ddot{a}_x & & & & & & -m & -m & \dots & -m \end{array}$$

$$\Rightarrow \boxed{{}_{m}(I\ddot{a})_x = (I\ddot{a})_x - {}_{m/}(I\ddot{a})_x - m \cdot {}_{m/}\ddot{a}_x}$$

Rendite vitalizie differite, anticipate, temporanee, in prog. aritm.

$${}_{m/n}(I\ddot{a})_x = {}_mE_x \cdot {}_{/n}(I\ddot{a})_{x+m}$$



/—/

* Rendite in progressione aritmetica POSTICIPATE

- Immediate, perpetue: $(Ia)_x = \sum_{h=1}^{+\infty} h \cdot {}_hE_x$

- Differite, perpetue: ${}_{m/}(Ia)_x = {}_mE_x \cdot (Ia)_{x+m}$

- Immediate, temporanee: ${}_{/n}(Ia)_x = (Ia)_x - {}_{n/}(Ia)_x - n \cdot {}_{n/}a_x$

- Differite, temporanee: ${}_{m/n}(Ia)_x = {}_mE_x \cdot {}_{/n}(Ia)_{x+m}$

Rendite vitalizie in Progressione Aritmetica: Simboli di Commutazione

(1)

$$(I\ddot{a})_x = {}_0E_x + 2\,{}_1E_x + 3\,{}_2E_x + \dots$$

$\boxed{\text{Anticipate}}$

$$= \frac{D_x}{D_x} + 2\frac{D_{x+1}}{D_x} + 3\frac{D_{x+2}}{D_x} + \dots$$

$$= \frac{(D_x + D_{x+1} + D_{x+2} + \dots)}{D_x} + \frac{(D_{x+1} + D_{x+2} + \dots)}{D_x} + \frac{(D_{x+2} + D_{x+3} + \dots)}{D_x} + \dots$$

$$= \frac{N_x + N_{x+1} + N_{x+2} + \dots + N_\omega}{D_x}$$

Si pone $\boxed{S_x = \sum_{h=0}^{+\infty} N_{x+h}}$

Allora $(I\ddot{a})_x = \boxed{\frac{S_x}{D_x}}$

(2) $${}_{m|}(I\ddot{a})_x = {}_mE_x \cdot (I\ddot{a})_{x+m} = \frac{D_{x+m}}{D_x} \cdot \frac{S_{x+m}}{D_{x+m}} = \boxed{\frac{S_{x+m}}{D_x}}$$

(3) $${}_{/n}(I\ddot{a})_x = (I\ddot{a})_x - {}_{n|}(I\ddot{a})_x - n \cdot {}_{n|}\ddot{a}_x = \boxed{\frac{S_x - S_{x+n} - n \cdot N_{x+n}}{D_x}}$$

(4) $${}_{m|n}(I\ddot{a})_x = {}_mE_x \cdot {}_{/n}(I\ddot{a})_{x+m} = \boxed{\frac{S_{x+m} - S_{x+m+n} - n \cdot N_{x+m+n}}{D_x}}$$

$\boxed{\text{Posticipata}}$

(5) $$(Ia)_x = \frac{S_{x+1}}{D_x}$$

(6) $$_{m/}(Ia)_x = \frac{S_{x+m+1}}{D_x}$$

(7) $$_{/n}(Ia)_x = \frac{S_{x+1} - S_{x+n+1} - n \cdot N_{x+n+1}}{D_x}$$

(8) $$_{m/n}(Ia)_x = \frac{S_{x+m+1} - S_{x+m+n+1} - n \cdot N_{x+m+n+1}}{D_x}$$

Rendita vitalizia, immediata, perpetua, anticipata, in progressione aritmetica.

Sia $\boxed{R_1 = R > 0 \,, \quad \Delta \gtrless 1}$

$$\tilde{y} = \begin{cases} R\cdot v^0 \,, & R v^0 + (R+\Delta) v \,, & R v^0 + (R+\Delta) v + (R+2\Delta) v^2 \,, & \dots \\ {}_{0/1}p_x \,, & {}_{1/1}p_x \,, & {}_{2/1}p_x \,, & \dots \end{cases}$$

$\tilde{y} = \tilde{y}_1 + \tilde{y}_2$, dove :

$$\tilde{y}_1 = \begin{cases} R v^0 \,, & R(v^0 + v) \,, & R(v^0 + v + v^2) \,, & \dots \\ {}_{0/1}p_x \,, & {}_{1/1}p_x \,, & {}_{2/1}p_x \,, & \dots \end{cases}$$

$$\tilde{y}_2 = \begin{cases} 0 \,, & \Delta v \,, & \Delta(v + 2v^2) \,, & \Delta(v + 2v^2 + 3v^3) \,, & \dots \\ {}_{0/1}p_x \,, & {}_{1/1}p_x \,, & {}_{2/1}p_x \,, & {}_{3/1}p_x \,, & \dots \end{cases}$$

$$E(\tilde{y}) = E(\tilde{y}_1) + E(\tilde{y}_2) = R\cdot \ddot{a}_x + \Delta \cdot (Ia)_x$$

$$\begin{array}{cccccc} R & R+\Delta & R+2\Delta & R+3\Delta & R+4\Delta & \dots \\ \hline 0 & 1 & 2 & 3 & 4 & \dots \end{array}$$

Rendite vitalizie immediate posticipate, anticipate, in Progressione Geometrica.

Definizione: Pagamento delle rate, inizialmente unitarie, variabili in progressione geometrica di ragione $q = 1+\alpha$, all'inizio di ogni anno, finché la testa assicurata è in vita.

1	$(1+\alpha)$	$(1+\alpha)^2$	$(1+\alpha)^3$	$\dots$	$(1+\alpha)^{\omega-x-1}$
0	1	2	3	$\dots$	$\omega-x-1$

$$\tilde{Y} = \begin{cases} 1\cdot v^0, & [1+(1+\alpha)v], \quad [1+(1+\alpha)v+(1+\alpha)^2v^2], \quad \dots, \quad [1+\dots+(1+\alpha)^{\omega-x-1}v^{\omega-x-1}] \\ {}_{0/1}q_x, & {}_{1/1}q_x, \quad {}_{2/1}q_x, \quad \dots, \quad {}_{\omega-x-1/1}q_x \end{cases}$$

$$U = E(\tilde{Y}) = v^0\,{}_{0/1}q_x + [1+(1+\alpha)v]\,{}_{1/1}q_x + [1+(1+\alpha)v+(1+\alpha)^2v^2]\,{}_{2/1}q_x + \dots$$

$$= v^0({}_{0/1}q_x + {}_{1/1}q_x + \dots) + (1+\alpha)v({}_{1/1}q_x + {}_{2/1}q_x + \dots) + (1+\alpha)^2v^2({}_{2/1}q_x + {}_{3/1}q_x + \dots) + \dots$$

Sia $\alpha \leq i \Rightarrow (1+\alpha)\cdot v = \frac{1+\alpha}{1+i} \leq 1$

Posto $(1+\alpha)v = \frac{1}{1+\eta} = v^{[\eta]}$ allora:

$$U = v^{0[\eta]}\,{}_0p_x + v^{[\eta]}\,{}_1p_x + v^{2[\eta]}\,{}_2p_x + \dots$$

$$= {}_0E_x^{[\eta]} + {}_1E_x^{[\eta]} + {}_2E_x^{[\eta]} + \dots = \sum_{h=0}^{+\infty} {}_hE_x^{[\eta]} = \ddot{a}_x^{[\eta]}$$

Rendita vitalizia con prefissato numero di rate iniziali certe

Definizione. Pagamento di una rendita certa, unitaria, posticipata, temporanea, e successivamente, in caso di sopravvivenza alla fine dell'anno n, pagamento di una rendita unitaria, posticipata finché la testa assicurata è in vita.

$$\begin{array}{ccccccccccc} & 1 & 1 & 1 & 1 & & 1 & 1 & 1 & \cdots & 1 & \\ \hline 0 & 1 & 2 & 3 & 4 & \cdots & n-1 & n & n+1 & \cdots & \omega-x-1 & \omega-x \end{array}$$

$$\tilde{Y} = \begin{cases} a_{\overline{n}|}, & a_{\overline{n}|}, & \cdots, & a_{\overline{n}|}, & a_{\overline{n}|}, & a_{\overline{n}|}+v^{n+1}, & a_{\overline{n}|}+v^{n+1}+v^{n+2}, & \cdots \\ {}_{0/1}q_x, & {}_{1/1}q_x, & \cdots, & {}_{n-1/1}q_x, & {}_{n/1}q_x, & {}_{n+1/1}q_x, & {}_{n+2/1}q_x, & \cdots \end{cases}$$

$$V = E(\tilde{Y}) = \boxed{a_{\overline{n}|} + {}_{n/}a_x}$$