You have **2 hours** to complete this exam.

1. **[38]** The following dataset reports lifetimes of lightbulbs since their first use (measured in days).

3	33	48
12	34	54
17	35	55
28	39	57
29	48	115

   (a) **[8]** Calculate an estimate of the survival function.
   (b) **[8]** Assuming that the data are exponentially distributed, give an estimate of the scale parameter based on the method of moments. Can you justify this assumption?
   (c) **[6]** Estimate the variance of this estimator and give a 95% confidence interval.
   (d) **[6]** Test the hypothesis H_0: *the expected lifetime is 6 weeks.*
   (e) **[4]** Give the definition of right-censored data.
   (f) **[6]** How would you estimate the scale parameter if there were right-censored entries? Justify.

2. **[40]** The survival times of 227 lung cancer patients have been recorded, along with their age (centered on 62), their gender and their ECOG score (it measures the level of disability, a score of 0 corresponds to a fully able person). A Cox proportional hazards model has been fitted to these data.

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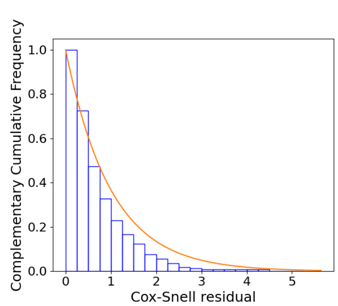
Coefficients                Covariance Matrix

covariate  value                          age       female         ecog

age         0.01            age      0.000086     0.000004    -0.000182
female     -0.55            female   0.000004     0.028136    -0.000930
ecog        0.46            ecog    -0.000182    -0.000930     0.012900
```

   (a) **[4]** Give the definition of the Cox proportional hazards model.
   (b) **[8]** Derive a numerical method to estimate the parameters of the model.
   (c) **[4]** What are the characteristics of the baseline subject?
   (d) **[8]** Analyse the effect of the covariates and their significance. Give a 95% confidence interval for the coefficients.
   (e) **[8]** Calculate the hazard ratio between a 50 years old female and a 50 years old male with an identical ECOG score. Give a 95% confidence interval.
   (f) **[8]** The distribution of the Cox-Snell residuals is given in the figure below. How should you interpret this plot? Demonstrate your statement.



3. **[22]** 200 steel specimens have been subjected to cycles of a given stress amplitude. For each specimen the stress amplitude and the number of cycles until failure have been recorded in kilopound per square inch (kpsi) and in 1000 cycles respectively. A Weibull accelerated failure time model has been fitted.

	coef	se	p-value
Stress Amplitude	0.416196	0.02	<0.005
Intercept	-20.546640	0.85	<0.005
Gamma	0.758578	0.05	<0.005

(a) **[4]** Give the definition of the general accelerated failure time model.

(b) **[4]** In general, how can we choose between the Weibull and the log-logistic baseline?

(c) **[6]** Calculate the acceleration factor and give a 95% confidence interval.

(d) **[8]** Calculate the median failure time for a specimen subjected to a stress amplitude of 35 kpsi.

Table 1: Standard normal distribution table

	0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Table 2: Chi-Square distribution table with 1 degree of freedom

0.95	0.9	0.5	0.2	0.1	0.05	0.025	0.02	0.01	0.005	0.002	0.001
0.00393	0.0158	0.455	1.642	2.706	3.841	5.024	5.412	6.635	7.879	9.550	10.828

The End.