Inequality and Ex-Post Fairness in Rationing Mechanisms

By Eric Tang*

In market design settings where agents have differing marginal utilities for money, the socially optimal mechanism may involve rationing agents out of the market. However, less work has studied the optimality of rationing in settings with risk-aversion or wealth effects, such as labor markets. In this work, we analyze the optimal single-price mechanism on the seller side of a market for an indivisible good. We generalize the welfare maximization problem in this setting when agents have general preferences for money, and analyze optimal prices for several distributions using numerical simulation. In our simulation results, we find that rationing can still be optimal when agents have slightly concave preferences, but concave preferences do bring the welfare-maximizing price closer to the competitive price.

I. Main Question

Recent years have seen a proliferation of work on fair allocation in market design, by modeling agents with varying utility functions, and designing allocation and pricing mechanisms to address inequality in these markets. This work spans the fields of computer science, operations research, and economics. Among others, this work includes that of Babaioff, Nisan and Talgam-Cohen (2019), Akbarpour, Dworczak and Kominers (2020) and Budish (2011). These are also natural extensions of the broader work on matching markets in the computer science literature, such as that of Alaei, Jalaly and Tardos (2017).

One work we focus on is Dworczak, Kominers and Akbarpour (2020), who models a two-sided market with buyers and sellers who have differing marginal utilities for a good and for money. This can be viewed as a way to more highly value monetary transfers to agents with high marginal utility for money, who often have lower incomes. They then derive optimal mechanisms to maximize total welfare under these two-sided markets. In cases of significant same-side inequality, the optimal mechanism they derive is a randomized (rationing) mechanism, in which some agents can only trade with a given probability. As the authors point out, this randomization can in practice be harmful to poorer individuals, who may have less tolerance for uncertainty. This raises several possible approaches for addressing these harms in practice.

Our first approach is as follows: we will derive optimal mechanisms in the altered model where agents have risk-averse preferences. This extends the linear

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utility functions of agents which are assumed in the current model. To extend this to risk-averse settings, we will parametrize agents' utility functions for money as one of a family of concave functions, where this parameter differs across agents. We can then use similar techniques to the original work of Dworczak, Kominers and Akbarpour (2020) to derive the optimal mechanism, by determining optimal mechanisms for the seller side, for the buyer side, and finally for the market as a whole. We hypothesize that maximizing social welfare for agents with risk-averse (concave) utility functions will result in optimal mechanisms favoring wedge mechanisms over rationing mechanisms.

We will begin our exploration of this first approach by simulating our existing optimal mechanisms in a setting with agents who have risk-averse utility functions. This will suggest in what cases the optimal mechanisms for risk-neutral agents differ from the optimal mechanisms for risk-averse agents. We will continue our exploration by deriving utility-maximizing mechanisms in the risk-neutral case. As milestones along the way, we will derive mechanisms in the simpler settings which the paper initially lays out: with one price on each side of the market, then up to two prices on each side of the market, and finally, with any possible allocation and pricing mechanism.

Another approach is to change our model of the good. Can we improve on ex-post fairness when the good is divisible? In such a market, the designer would not be constrained solely to either giving each buyer a good or not giving them a good, but could distribute the good more evenly among these buyers. In such a market, instead of randomly choosing which of the rationed buyers are granted access to the market, the market designer might allocate smaller quantities of the good among those buyers, (hopefully) improving ex-post fairness. One might also hope to optimize such market designs based on measures of ex-post fairness, such as the minimal ex-post utility of all agents in the resulting market outcome. Examples of real-world market designs for divisible goods which are motivated by equity could include the provisioning of medical care, or rationing for limited supplies of food.

Section II reviews recent literature in market design for redistribution which is relevant to the proposed work. Section III lays out other open questions related to the two proposed question above, which could be used as extensions to this work.

II. Background: Redistribution through Markets

Price regulations, such as rent control and minimum wage laws, typically face tradeoffs between reducing inequality and maintaining efficient outcomes. In an age of dramatic inequality, it is critical to design markets with equity in mind.

Early work exploring this balance includes Weitzman (1977), who asks whether competitive prices or rationing in a one-commodity market is more effective at ensuring agents with the highest need receive the good. Weitzman focuses primarily on goods which are deemed a societal need, potentially including housing, food, or medical care. As in actual marketplaces, agents have both differing incomes and differing needs for the good. Intuitively, if income distribution is relatively egalitarian and agents' needs vary widely, then competitive prices better identifies those agents with greater need. On the other hand, if income inequality is high compared to the dispersion in need for the good, rationing is more effective.

Formally, Weitzman (1977) reach these results by parametrizing agents' demand functions according to λ and ϵ, where λ represents the agent's marginal value for money (falls as income rises) and ϵ represents the agent's need for the good. Critically, these types are unobservable by the market designer, a problem which will arise again in later work as well.

Weitzman now considers two possible rules for allocating a fixed quantity of the good to each agent. Under competitive prices, the market designer sets a price to ensure market clearing. Under pure rationing, the market designer equally divides the good between all agents. Note that Dworczak, Kominers and Akbarpour (2020) build on this model by, among other improvements, considering more nuanced rationing schemes that allocate to a subset of agents or impose multiple price levels. Weitzman then may compute a loss for each method in meeting actual need, and compare the two schemes' outcomes based on the prior distributions of λ and ϵ in the populace. This formalizes the intuition of when rationing or competitive prices better satisfy needs in the market.

Dworczak, Kominers and Akbarpour (2020) characterize the Pareto frontier of this tradeoff by framing price regulation as a market design problem, with buyers and sellers seeking to buy an indivisible good. They frame the Pareto frontier as determining the welfare-maximizing solution when agents have different valuations for money and for the good. (Roughly speaking, the valuation for money falls with an agent's wealth). They find that when there is significant inequality between buyers and sellers, optimal price controls take the form of a "tax" which charges buyers a lower price than that of sellers, then transfers the surplus to the poorer side of the market. When there is significant inequality between agents on one side of the market, price controls take a more complex form but may still be optimal.

The key obstacle which Dworczak, Kominers and Akbarpour (2020) overcomes, as was faced by earlier models such as Weitzman (1977), is that the market designer does not perfectly identify agents' needs, and hence setting a price at which the commodity is rationed helps to identify these needs.

One interesting novelty of the Dworczak, Kominers and Akbarpour (2020) model is their use of agents' differing marginal values for money, instead of the more standard approach of giving agents different Pareto weights to construct the Pareto frontier. The authors show that these two formulations are actually equivalent. Intuitively, an agent having a higher marginal utility for money is akin to a market designer placing greater weight on the money they are given in the final allocation. They show that agents' actions are fully characterized by their rate of substitution between the good and money.

The bulk of the paper describes the optimal mechanisms to address cross-side inequality (between sellers and buyers) and same-side inequality (between agents on the same side of the market). The paper discusses applications in markets for kidney exchange (rather, the one market which runs in Iran), which is generally an example of same-side inequality among sellers. It also discusses applications in the housing rental market, which is generally an example of cross-side inequality.

The paper raises several interesting extensions. Most immediately, the model only studies an indivisible good without wealth effects. Both of these modeling assumptions could be extended. The model also neglects possible aftermarkets of trade between agents, which could be explicitly modeled. Finally, the rationing mechanism used to address same-side inequality is randomized; work on designing non-random mechanisms could be fruitful. It would also be interesting to find real-world examples where this randomization impacts participant behavior in measurable ways.

Akbarpour, Dworczak and Kominers (2020) approach the problem slightly differently, as one in which a market designer is alllocating a public good of heterogeneous quality to buyers, and additionally deciding on the price. Also notable about the Akbarpour, Dworczak and Kominers (2020) model is that it weights the valuation of generating revenue with its own Pareto weight, reflecting the potential uses of that generated revenue. Finally, the work also introduces observable types which the market designer can condition on: one can think of these as observable demographics which are partially informative about a buyer's need or their marginal utility for money. Akbarpour, Dworczak and Kominers (2020) is meant to be more "applied" than its predecessor, giving policymakers more concrete recommendations, and Piotr Dworczak has mentioned in a seminar that the authors are applying this paper's results to a model for vaccine distribution.

III. Potential Research Ideas

1) Recent years have seen a proliferation of work on fair allocation in market design to address inequality, spanning the theoretical computer science and economics literature. One example is Akbarpour, Dworczak and Kominers (2020), whose proposed mechanism maximizes expected net welfare given buyers and sellers with differing marginal utilities for a good and for money. This can be viewed as a way to more highly weight the utilities of those agents with low marginal utility for money, who often have lower incomes. In cases of significant same-side inequality, the optimal mechanism they derive is a randomized (rationing) mechanism, in which some agents can only trade with a given probability.

   As the authors point out, this randomization can in practice be harmful to poorer individuals, who may have less tolerance for uncertainty. This raises several possible approaches addressing these harms in practice.

   One approach is as follows: can we explicitly model agents as having risk-averse preferences, and continue to derive the optimal mechanism under these circumstances? This should entail extending the linear agent preferences which are assumed in the current model. For example, one could imagine analyzing this mechanism for agents whose marginal utilities for money fall into a family of decreasing functions, parametrized by a parameter which differs across agents.

A second approach could be to consider divisible goods. For example, instead of randomly choosing which of the rationed buyers are granted access to the market, the market designer might allocate smaller quantities of the good among those buyers. One might also hope to optimize such market designs based on measures of ex-post fairness, such as the minimal ex-post utility of all agents in the resulting market outcome. Examples of real-world market designs for divisible goods which are motivated by equity could include the provisioning of medical care, or rationing for limited supplies of food.

Other questions on the theme of ex-post fairness include the following (these are less concrete). Can we model the harm to poorer agents in this model that results from randomization? Can we develop a mechanism that "trades off" efficiency for less randomization? Could these answers change if the good is divisible? If this game is repeated, can we explicitly choose which agents are rationed to reduce ex-post unfairness? Are there established metrics of ex-post fairness which we can use to evaluate this metric?

2) For a fairly technical extension, in the accompanying seminar talk for Akbarpour, Dworczak and Kominers (2020), Piotr Dworczak notes that their work only considers the case when there are a finite number of different label groups i (the observable characteristics), but their work could likely be extended to the continuous case (when Pareto weights are continuous). This could be an interesting technical extension, but I'm not sure what the real-world analogue of interest would be here.

3) Very interesting: taking Akbarpour, Dworczak and Kominers (2020) as a departure point, what if one incorporates externalities of the good's distribution into our optimization? It seems that many public provisioning systems are also partially motivated by positive externalities. One motivating question here is the distribution of vaccines, but one could imagine similar positive externalities in public provision of housing and education.

4) Akbarpour, Dworczak and Kominers (2020) value revenue generation by giving revenue its own Pareto weight, representing the value of revenue to the market designer. This is quite important; in many congestion pricing models, for example, how the revenue is spent almost completely determines whether a congestion toll is progressive or regressive. Can we model revenue redistribution in this model more explicitly?

5) In a larger vein, it remains to model the markets of Dworczak, Kominers and Akbarpour (2020) in conjunction with much of the literature on taxation and macroeconomic redistribution. What might happen if we have a government tax collector who can directly observe incomes, and another market designer (the subject of Akbarpour, Dworczak and Kominers (2020)) who cannot? How do their optimal mechanisms interact?

6) Pathak et al. (2020) develop results in the theory of reserves, when there are various categories of a good in reserve, and agents are eligible for each reserve based on what eligibility categories they fall into. What if goods in the reserves are heterogeneous in quality, and agents have preferences over the quality of goods? This extension would be analogous to the way that Akbarpour, Dworczak and Kominers (2020) extended Condorelli (2013), expanding the work to include items of heterogeneous quality. That said, I'm not quite sure what the real-world analogy would be here. The original model of Pathak et al. (2020) focused on medical supplies such as vaccines and ventilators; I'm not sure how salient preferences over quality are in this domain.

IV. Risk-Averse Agents

V. Economic Intuition

As a motivating example to consider the limitations of Dworczak, Kominers and Akbarpour (2020), consider an example in labor markets, where agents may be quite risk-averse about the income they receive. In our case, consider a group of taxi drivers (sellers) queued up and hoping to sell rides to a set of passengers (buyers). Sellers in this case may be risk-averse, as they may prefer a lower guaranteed income (a transfer) to an uncertain probability of a higher reward (being rationed into the market and making a sale). Thus with risk-averse preferences for money, we might expect that the optimal mechanism is more likely to utilize a competitive price accompanied by lump-sum transfers, instead of rationing.

As rationing is generally used more often in the seller side, we may naturally be more concerned with risk-averse preferences for sellers. As in the case of sellers entering a market to earn their income, this suggests that we should first concern ourselves with agents' risk-averse preferences for money. For example, a taxi driver may be less risk-averse with respect to the time it takes to accept a ride, but may be fairly risk-averse with regards to the income they accumulate in a given day.

VI. Framework

In our modeling framework, we preserve much of the notation from Dworczak, Kominers and Akbarpour (2020). We continue to consider an indivisible good K, a divisible good M representing money, a unit mass of prospective sellers, and a mass μ of prospective buyers. Our main departure will be to model risk-aversion of agents over money.

A. Preferences

Our first departure is our modeling of the preference of agents. Here we wish to capture a world in which buyers and sellers are risk-averse over money, though not necessarily over the good K. However, we would also like to maintain the notion that agents have varying marginal utilities for both money and the good K. Again, let v^K denote an agent's marginal utility for good K and let v^M denote an agent's marginal utility for M.

We then can characterize possible preferences for agents by utility functions of the form

$$u(x^K, x^M; v^K, v^M) = v^K x^K + w(x^M; v^M)$$

where w denotes agents' utility for money, parametrized by their marginal utility for money v^M. To model our risk-aversion scenario, this utility function w should satisfy three criteria. First, it should be concave (representing risk aversion). Second, the agent's marginal utility for money when they have no money should be v^M. Third, agents' utilities for a given amount of money should be increasing in their marginal valuation v^M. We formalize these conditions as

(i) $w'' > 0$ (concavity)

(ii) $w'(0; v^M) = v^M$ (marginal utility)

(iii) For all x^M and all $v^{M'} > v^M$, $w(x^M; v^{M'}) > w(x^M; v^M)$ (increasing in v^M)

In this work, we will specify the utility function for money as an exponential function, which allows us to interpolate between highly concave and mildly concave functions. For a risk-averse agent receiving quantities x^V and x^M of the good and of money, respectively, we may define their utility as

$$u(x^K, x^M; v^K, v^M) = v^K x^K + (v^M x^M + 1)^k$$

for some $k \in (0, 1)$. Note here that our chosen utility for money, $w(x^M; v^M) = (v^M x^M + 1)^k$ satisfies our three desired criteria. Furthemore, k lets us select more concave functions at lower values of k, and nearly linear functions for higher values of k. [1]

VII. Optimality on the Seller Side

We now move to analyzing the optimal mechanism on the seller side, for a fixed quantity Q of the good that the designer would like to purchase, and an existing revenue R that they have.

[1]Other specifications for w are possible and satisfy our criteria, such as $w(x^M; v^M) = \log(v^M x^M + 1)$, but here we focus on our exponential specification.

A. Economic Intuition

Turning again to the three factors that influence the market designer in choosing a price p_s, we may consider again the effects of raising the price p_s. To review, the three effects of raising the price are

- A decrease in allocative efficiency, since we now may be allocating to agents with a lower marginal rate of substitution.
- The market designer must spend more to acquire Q items, reducing the amount of the lump-sum transfer $R - p_S Q$.

\+ Sellers who are rationed into the market receive a higher price for their sale.

In comparison to the case of linear utilities, in this risk-averse scenario, we may find that agents prefer a higher lump-sum transfer to a higher price when they sell their good. This is because the lump-sum transfer is guaranteed to all agents, while agents may be risk-averse about their chance of being rationed into the market in (c).

B. Maximization Problem

The next complication is that since utility in money is non-linear, we can no longer solely characterize agents' utilities using their marginal rate of substitution between the good and money.

Second, the original work of Dworczak, Kominers and Akbarpour (2020) separates the objective maximization into two components: the expected change in utility from transactions for sellers who are able to sell their good, and the expected additional utility from any lump sum to the sellers. However, since our preferences for money are now non-linear, we must now separate our objective maximization into components based on which agents are receiving each transfer. Concretely, we have three categories of sellers: those who are willing to sell and do sell, those who are willing to sell but cannot sell, and those who do not wish to sell.

First, to determine the relative proportion of each group of sellers, we must determine the quantity of sellers who are willing to sell at a price p_S. As utilities are non-linear, sellers' decision to trade also depends on the amount of the lump-sum payment $R - p_S Q$. We denote this quantity of sellers by τ. This quantity is then

$$\tau = \int_{\underline{v}^K, \underline{v}^M}^{\overline{v}^K, \overline{v}^M} \mathbb{1}[w(p_s + R - p_S Q; v^M) \geq w(R - p_S Q; v^M) + v^K] dF(v^K, v^M)$$

For sellers who are able to sell their good, these agents have none of the good, receive the price p_s, and further receive a lump-sum transfer of $R - p_s Q$. Thus the contribution of these sellers is

$$\frac{Q}{\tau}\int_{\underline{v}^K,\underline{v}^M}^{\overline{v}^K,\overline{v}^M} \mathbb{1}[w(p_s+R-p_SQ;v^M) \geq w(R-p_SQ;v^M)+v^K]w(p_s+R-p_SQ;v^M)dF(v^K,v^M) \tag{1}$$

On the other hand, sellers who wish to sell their good but are rationed out of the market receive only the good and the lump sum transfer. Thus the contribution of these sellers is

$$\frac{\tau-Q}{\tau}\int_{\underline{v}^K,\underline{v}^M}^{\overline{v}^K,\overline{v}^M} \mathbb{1}[w(R-p_SQ;v^M) \geq w(R-p_SQ;v^M)+v^K](v^K+w(R-p_SQ;v^M))dF(v^K,v^M) \tag{2}$$

Finally, sellers who do not wish to sell receive only the good and the lump sum transfer as well, making the contribution of a unit of these sellers

$$\int_{\underline{v}^K,\underline{v}^M}^{\overline{v}^K,\overline{v}^M} \mathbb{1}[w(R-p_SQ;v^M) \leq w(R-p_SQ;v^M)+v^K](v^K+w(R-p_SQ;v^M))dF(v^K,v^M) \tag{3}$$

Thus our complete maximization objective is

$$\begin{aligned} &\frac{Q}{\tau}\int_{\underline{v}^K,\underline{v}^M}^{\overline{v}^K,\overline{v}^M} \mathbb{1}[w(p_s+R-p_SQ;v^M) \geq w(R-p_SQ;v^M)+v^K]w(p_s+R-p_SQ;v^M)dF(v^K,v^M) \\ &+\frac{\tau-Q}{\tau}\int_{\underline{v}^K,\underline{v}^M}^{\overline{v}^K,\overline{v}^M} \mathbb{1}[w(R-p_SQ;v^M) \geq w(R-p_SQ;v^M)+v^K](v^K+w(R-p_SQ;v^M))dF(v^K,v^M) \\ &+\int_{\underline{v}^K,\underline{v}^M}^{\overline{v}^K,\overline{v}^M} \mathbb{1}[w(R-p_SQ;v^M) \leq w(R-p_SQ;v^M)+v^K](v^K+w(R-p_SQ;v^M))dF(v^K,v^M) \end{aligned} \tag{4}$$

For a fixed revenue R, quantity Q, and valuation distribution F, we aim to select the p_S which maximizes this welfare objective.

VIII. Numerical Analysis

We are particularly interested in how the gap between the competitive price and welfare-maximizing price changes as agents' preferences become more risk-averse. In our setting, this means analyzing how these two prices change as k moves from 1 (perfectly linear, non-risk-averse preferences) towards 0 (more concave preferences).

Based on our intuition described above, we expect that in settings where the competitive price is optimal with linear utilities for money, the competitive price should still be optimal when agents have risk-averse preferences. Second, we expect that in settings where rationing is optimal with linear utilities for money, the optimal price should shift closer to the competitive price as agents' preferences become more concave. In these settings, the question most of interest to us is whether our results are robust to the introduction of slight concavity. That is, for values of k near 1, it may be that the competitive price is always optimal. Such a result would substantially weaken the theoretical case for the use of price controls and rationing on the seller side, even in cases of high inequality.

Thankfully for the earlier literature, our numerical simulation results suggest that rationing can still be optimal even in settings with concave preferences.[2]

Figure VIII displays a setting in which inequality is low, in the sense of Dworczak, Kominers and Akbarpour (2020). In this setting, when agents' utilities for wealth are linear, the welfare-maximizing price is the competitive price. Figure VIII shows that the welfare-maximizing and competitive prices still coincide when agents have risk-averse preferences.

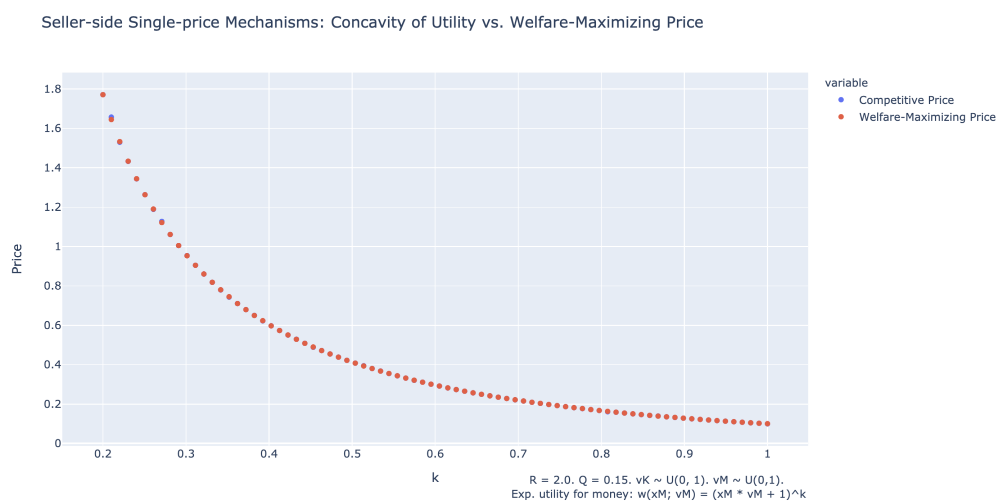

Figure 1. Concavity of Utility (k) vs. Competitive Price and Welfare-Maximizing Price. Low-inequality setting.

Figure VIII displays a setting in which inequality is high. In particular, we consider a setting where agents marginal utilities for the good are distributed as $v^K \sim U(0,1)$ and their marginal utilities for money are distributed as $v^M \sim$ Pareto(1/3). In this setting, when agents have linear utilities for money, the optimal mechanism is a single price above the competitive price, which results in

[2]Code for our numerical simulations can be found at https://github.com/Etang21/redistribution-report-284.

rationing. In our numerical simulation, we find that when preferences are slightly concave, the optimal mechanism still utilizes rationing. When preferences are very concave, however, the optimal price coincides with the competitive price.

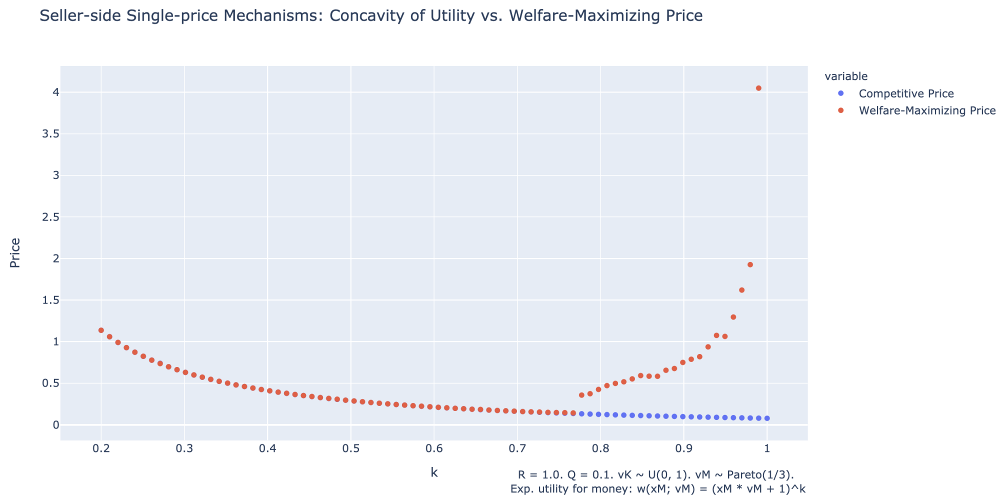

FIGURE 2. CONCAVITY OF UTILITY (K) VS. COMPETITIVE PRICE AND WELFARE-MAXIMIZING PRICE. HIGH-INEQUALITY SETTING.

In both cases, we have only investigated two possible settings for the optimization problem, and we emphasize that it is an open question as to how much these results generalize to other settings.

A. Conjectures

Our work in the above sections suggest two corresponding analytic conjectures. First, our economic intuition in VII.A and our numerical simulations as in VIII suggest that when the competitive price is optimal for the linear utility setting, it is also optimal in the concave-preference settings. Formally,

Conjecture VIII.1. *Suppose that when agent preferences for money are given by $w(x^M; v^M) = x^M v^M$, the optimal price is equal to the competitive price, so $p_S = p_S^C$. Holding all other parameters fixed, if we set agent preferences for money to any w with w concave, increasing in v^M and satisfying $v'(0) = v^M$, then $p_S = p_S^C$ in the new setting.*

We suspect that Conjecture VIII.1 can be solved by comparing the increased utility from higher lump-sum transfers to the decreased utility for higher sale prices of agents rationed in.

Our second conjecture corresponds to the numerical simulation result in VIII.

Conjecture VIII.2. *Consider the family of utility functions parametrized by* k*, with* $w(x^M; v^M, k) = \frac{1}{k}(x^M v^M + 1)^k$*. Suppose that when agent preferences for money are given by* $w(x^M; v^M) = x^M s v^M + 1$*, the optimal mechanism involves rationing, so* $p_S > p_S^C$*. Then there exists some* $\epsilon > 0$ *such that for all* $k \in (1-\epsilon, 1]$*, rationing is optimal in the risk-averse setting with agent preferences for money given by* $w(x^M; v^M, k)$*.*

If true, this conjecture would suggest that the use of rationing as found in Dworczak, Kominers and Akbarpour (2020) is reasonably robust to the presence of risk-aversion among agents. When rationing is optimal for linear preferences, it is also optimal for slightly concave preferences.

IX. Future Work

More work is needed in three key directions: exactly characterizing the optimal mechanism for general concave preferences, characterizing optimality among all mechanisms, and extending this work to both sides of the market.

In the setting displayed in Figure VIII, the welfare-maximizing price approaches the competitive price as preferences become more concave. However, more work is needed to verify exactly what the welfare-maximizing price is for various concave preferences. Alternatively, we could hope to find a threshold k' for a given setting such that when preferences are sufficiently concave ($k < k'$), the competitive price must be optimal.

The second major area of extension is to generalize beyond single-price mechanisms to all possible mechanisms. We have covered only the first analysis of Dworczak, Kominers and Akbarpour (2020), which considers single-price mechanisms as a preliminary case to general mechanisms. It is possible to imagine that concave preferences lead to more complex optimal mechanisms, even though Dworczak, Kominers and Akbarpour (2020) find that optimal mechanism generally take on a simple form. However, with general concave preferences, analytical approaches become particularly difficult.

Finally, it remains to extend this work to a two-sided market, and solve joint optimality in the market. We have analyzed only the seller-side of the market, in which poorer sellers are those more likely to want to sell. In the buyer side, willingness to buy identifies the opposite, identifying buyers with a higher value for money. This could lead to qualitatively different behavior.

X. Conclusion

In this work, we analyzed the optimal single-price mechanism for maximizing total welfare in a setting where agents may have different marginal utilities for a good, and differing concave preferences for money. This setting is particularly relevant when designing price controls for labor markets. For example, when determining an appropriate minimum wage in a labor market, we should be concerned about wealth effects and agents' possible risk-aversion.

Our numerical simulation results validate this caustion, while also showing that the use of rationing can be robust to slight concavity in agents' preferences. We generalize the method of Dworczak, Kominers and Akbarpour (2020) to formulate a welfare-maximization problem for general agent preferences for money. We then numerically simulate how the competitive price and welfare-maximizing price change as preferences become more concave.

Further work is needed to analytically solve for the optimal price under general preferences, to generalize to all mechanisms (not solely single-price mechanisms) and to extend our findings to a two-sided market. More broadly, the literature on redistribution through market design can benefit from a better understanding of how wealth effects shape optimal redistributive mechanisms. More precisely characterizing such mechanisms should yield implementable designs that are more robust to preferences in the wild.

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