Inequality and Ex-Post Fairness in Rationing Mechanisms

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1. Main Question

Recent years have seen a proliferation of work on fair allocation in market design, by modeling agents with varying utility functions, and designing allocation and pricing mechanisms to address inequality in these markets. This work spans the fields of computer science, operations research, and economics. Among others, this work includes that of Babaioff, Nisan and Talgam-Cohen (2019), Akbarpour, Dworczak and Kominers (2020) and Budish (2011). These are also natural extensions of the broader work on matching markets in the computer science literature, such as that of Alaei, Jalaly and Tardos (2017).

One work we focus on is Dworczak, Kominers and Akbarpour (2020), who models a two-sided market with buyers and sellers who have differing marginal utilities for a good and for money. This can be viewed as a way to more highly value monetary transfers to agents with high marginal utility for money, who often have lower incomes. They then derive optimal mechanisms to maximize total welfare under these two-sided markets. In cases of significant same-side inequality, the optimal mechanism they derive is a randomized (rationing) mechanism, in which some agents can only trade with a given probability. As the

authors point out, this randomization can in practice be harmful to poorer individuals, who may have less tolerance for uncertainty. This raises several possible approaches for addressing these harms in practice.

Our first approach is as follows: we will derive optimal mechanisms in the altered model where agents have risk-averse preferences. This extends the linear utility functions of agents which are assumed in the current model. To extend this to risk-averse settings, we will parametrize agents' utility functions for money as one of a family of concave functions, where this parameter differs across agents. We can then use similar techniques to the original work of Dworczak, Kominers and Akbarpour (2020) to derive the optimal mechanism, by determining optimal mechanisms for the seller side, for the buyer side, and finally for the market as a whole. We hypothesize that maximizing social welfare for agents with risk-averse (concave) utility functions will result in optimal mechanisms favoring wedge mechanisms over rationing mechanisms.

We will begin our exploration of this first approach by simulating our existing optimal mechanisms in a setting with agents who have risk-averse utility functions. This will suggest in what cases the optimal mechanisms for risk-neutral agents differ from the optimal mechanisms for risk-averse agents. We will continue our exploration by deriving

utility-maximizing mechanisms in the risk-neutral case. As milestones along the way, we will derive mechanisms in the simpler settings which the paper initially lays out: with one price on each side of the market, then up to two prices on each side of the market, and finally, with any possible allocation and pricing mechanism.

Another approach is to change our model of the good. Can we improve on ex-post fairness when the good is divisible? In such a market, the designer would not be constrained solely to either giving each buyer a good or not giving them a good, but could distribute the good more evenly among these buyers. In such a market, instead of randomly choosing which of the rationed buyers are granted access to the market, the market designer might allocate smaller quantities of the good among those buyers, (hopefully) improving ex-post fairness. One might also hope to optimize such market designs based on measures of ex-post fairness, such as the minimal ex-post utility of all agents in the resulting market outcome. Examples of real-world market designs for divisible goods which are motivated by equity could include the provisioning of medical care, or rationing for limited supplies of food.

Section 2 reviews recent literature in market design for redistribution which is relevant to the proposed work. Section 3 lays out other open questions related to the two proposed question above, which could be used as extensions to this work.

2. Background: Redistribution through Markets

Price regulations, such as rent control and minimum wage laws, typically face tradeoffs between reducing inequality and maintaining efficient outcomes. In an age of dramatic inequality, it is critical to design markets with equity in mind.

Early work exploring this balance includes Weitzman (1977), who asks whether competitive prices or rationing in a one-commodity market is more effective at ensuring agents with the highest need receive the good. Weitzman focuses primarily on goods which are deemed a societal need, potentially including housing, food, or medical care. As in actual marketplaces, agents have both differing incomes and differing needs for the good. Intuitively, if income distribution is relatively egalitarian and agents' needs vary widely, then competitive prices better identifies those agents with greater need. On the other hand, if income inequality is high compared to the dispersion in need for the good, rationing is more effective.

Formally, Weitzman (1977) reach these results by parametrizing agents' demand functions according to λ and ϵ, where λ represents the agent's marginal value for money (falls as income rises) and ϵ represents the agent's need for the good. Critically, these types are unobservable by the market designer, a problem which will arise again in later work as well.

Weitzman now considers two possible rules for allocating a fixed quantity of the good to each agent. Under competitive prices, the market designer sets a price to ensure market clearing. Under pure rationing, the market designer equally divides the good between all agents. Note that Dworczak, Kominers and Akbarpour (2020) build on this model by, among other improvements, considering more nuanced rationing schemes that allocate to a subset of agents or impose multiple price levels. Weitzman then may compute a loss for each method in meeting actual need, and compare the two schemes' outcomes based on the prior distributions of λ and ϵ in the populace. This formalizes the intuition of

when rationing or competitive prices better satisfy needs in the market.

Dworczak, Kominers and Akbarpour (2020) characterize the Pareto frontier of this tradeoff by framing price regulation as a market design problem, with buyers and sellers seeking to buy an indivisible good. They frame the Pareto frontier as determining the welfare-maximizing solution when agents have different valuations for money and for the good. (Roughly speaking, the valuation for money falls with an agent's wealth). They find that when there is significant inequality between buyers and sellers, optimal price controls take the form of a "tax" which charges buyers a lower price than that of sellers, then transfers the surplus to the poorer side of the market. When there is significant inequality between agents on one side of the market, price controls take a more complex form but may still be optimal.

The key obstacle which Dworczak, Kominers and Akbarpour (2020) overcomes, as was faced by earlier models such as Weitzman (1977), is that the market designer does not perfectly identify agents' needs, and hence setting a price at which the commodity is rationed helps to identify these needs.

One interesting novelty of the Dworczak, Kominers and Akbarpour (2020) model is their use of agents' differing marginal values for money, instead of the more standard approach of giving agents different Pareto weights to construct the Pareto frontier. The authors show that these two formulations are actually equivalent. Intuitively, an agent having a higher marginal utility for money is akin to a market designer placing greater weight on the money they are given in the final allocation. They show that agents' actions are fully characterized by their rate of substitution between the good and money.

The bulk of the paper describes the optimal mechanisms to address cross-side inequality (between sellers and buyers) and same-side inequality (between agents on the same side of the market). The paper discusses applications in markets for kidney exchange (rather, the one market which runs in Iran), which is generally an example of same-side inequality among sellers. It also discusses applications in the housing rental market, which is generally an example of cross-side inequality.

The paper raises several interesting extensions. Most immediately, the model only studies an indivisible good without wealth effects. Both of these modeling assumptions could be extended. The model also neglects possible aftermarkets of trade between agents, which could be explicitly modeled. Finally, the rationing mechanism used to address same-side inequality is randomized; work on designing non-random mechanisms could be fruitful. It would also be interesting to find real-world examples where this randomization impacts participant behavior in measurable ways.

Akbarpour, Dworczak and Kominers (2020) approach the problem slightly differently, as one in which a market designer is allocating a public good of heterogeneous quality to buyers, and additionally deciding on the price. Also notable about the Akbarpour, Dworczak and Kominers (2020) model is that it weights the valuation of generating revenue with its own Pareto weight, reflecting the potential uses of that generated revenue. Finally, the work also introduces observable types which the market designer can condition on: one can think of these as observable demographics which are partially informative about a buyer's need or their marginal utility for money. Akbarpour, Dworczak and Kominers (2020) is meant to be more

"applied" than its predecessor, giving policymakers more concrete recommendations, and Piotr Dworczak has mentioned in a seminar that the authors are applying this paper's results to a model for vaccine distribution.

3. Potential Research Ideas

1) Recent years have seen a proliferation of work on fair allocation in market design to address inequality, spanning the theoretical computer science and economics literature. One example is Akbarpour, Dworczak and Kominers (2020), whose proposed mechanism maximizes expected net welfare given buyers and sellers with differing marginal utilities for a good and for money. This can be viewed as a way to more highly weight the utilities of those agents with low marginal utility for money, who often have lower incomes. In cases of significant same-side inequality, the optimal mechanism they derive is a randomized (rationing) mechanism, in which some agents can only trade with a given probability.

   As the authors point out, this randomization can in practice be harmful to poorer individuals, who may have less tolerance for uncertainty. This raises several possible approaches addressing these harms in practice.

   One approach is as follows: can we explicitly model agents as having risk-averse preferences, and continue to derive the optimal mechanism under these circumstances? This should entail extending the linear agent preferences which are assumed in the current model. For example, one could imagine analyzing this mechanism for agents whose marginal utilities for money fall into a family of decreasing functions, parametrized by a parameter which differs across agents.

   A second approach could be to consider divisible goods. For example, instead of randomly choosing which of the rationed buyers are granted access to the market, the market designer might allocate smaller quantities of the good among those buyers. One might also hope to optimize such market designs based on measures of ex-post fairness, such as the minimal ex-post utility of all agents in the resulting market outcome. Examples of real-world market designs for divisible goods which are motivated by equity could include the provisioning of medical care, or rationing for limited supplies of food.

   Other questions on the theme of ex-post fairness include the following (these are less concrete). Can we model the harm to poorer agents in this model that results from randomization? Can we develop a mechanism that "trades off" efficiency for less randomization? Could these answers change if the good is divisible? If this game is repeated, can we explicitly choose which agents are rationed to reduce ex-post unfairness? Are there established metrics of ex-post fairness which we can use to evaluate this metric?

2) For a fairly technical extension, in the accompanying seminar talk for Akbarpour, Dworczak and Kominers (2020), Piotr Dworczak notes that their work only considers the case when there are a finite number of different label groups i (the observable characteristics), but their

work could likely be extended to the continuous case (when Pareto weights are continuous). This could be an interesting technical extension, but I'm not sure what the real-world analogue of interest would be here.

3) Very interesting: taking Akbarpour, Dworczak and Kominers (2020) as a departure point, what if one incorporates externalities of the good's distribution into our optimization? It seems that many public provisioning systems are also partially motivated by positive externalities. One motivating question here is the distribution of vaccines, but one could imagine similar positive externalities in public provision of housing and education.

4) Akbarpour, Dworczak and Kominers (2020) value revenue generation by giving revenue its own Pareto weight, representing the value of revenue to the market designer. This is quite important; in many congestion pricing models, for example, how the revenue is spent almost completely determines whether a congestion toll is progressive or regressive. Can we model revenue redistribution in this model more explicitly?

5) In a larger vein, it remains to model the markets of Dworczak, Kominers and Akbarpour (2020) in conjunction with much of the literature on taxation and macroeconomic redistribution. What might happen if we have a government tax collector who can directly observe incomes, and another market designer (the subject of Akbarpour, Dworczak and Kominers (2020)) who cannot? How do their optimal mechanisms inter-act?

6) Pathak et al. (2020) develop results in the theory of reserves, when there are various categories of a good in reserve, and agents are eligible for each reserve based on what eligibility categories they fall into. What if goods in the reserves are heterogeneous in quality, and agents have preferences over the quality of goods? This extension would be analogous to the way that Akbarpour, Dworczak and Kominers (2020) extended Condorelli (2013), expanding the work to include items of heterogeneous quality. That said, I'm not quite sure what the real-world analogy would be here. The original model of Pathak et al. (2020) focused on medical supplies such as vaccines and ventilators; I'm not sure how salient preferences over quality are in this domain.

4. Framework

In our modeling framework, we preserve much of the notation from Dworczak, Kominers and Akbarpour (2020). We continue to consider an indivisible good K, a divisible good M representing money, a unit mass of prospective sellers, and a mass μ of prospective buyers.

Our first departure is our modeling of the preference of agents. Here we wish to capture a world in which buyers and sellers are risk-averse over money, though not necessarily over the good K. However, we would also like to maintain the notion that agents have varying marginal utilities for both money and the good K. Again, let v^K denote an agent's marginal utility for good K and let v^M denote an agent's marginal utility for M. Then one possible model for the utility for a risk-averse agent receiving quantities x^K and x^M of the two goods, respectively, is

$$u(x^K, x^M; v^k, v^M) = v^K x^K + \log(v^M x^M + 1).$$

Note here that the marginal utility for good K is indeed v^K, and when the agent receives no money, the marginal utility for money is v^M.

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