Redistributive Mechanisms under Risk Aversion

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Overview

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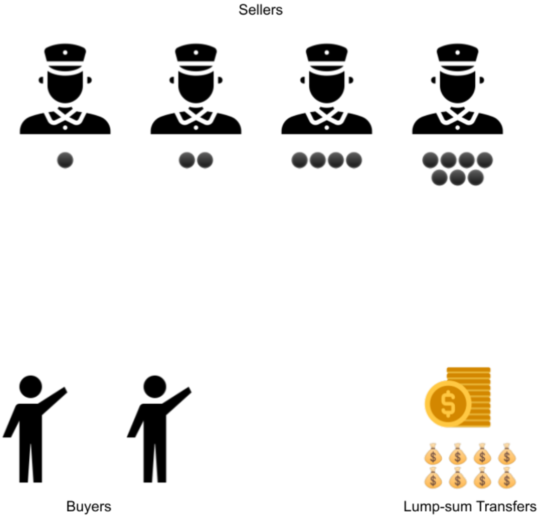
Motivating Question

In seller-side markets with inequality, to what extent does agents' **risk-aversion** change whether price controls and rationing mechanisms are optimal mechanisms?

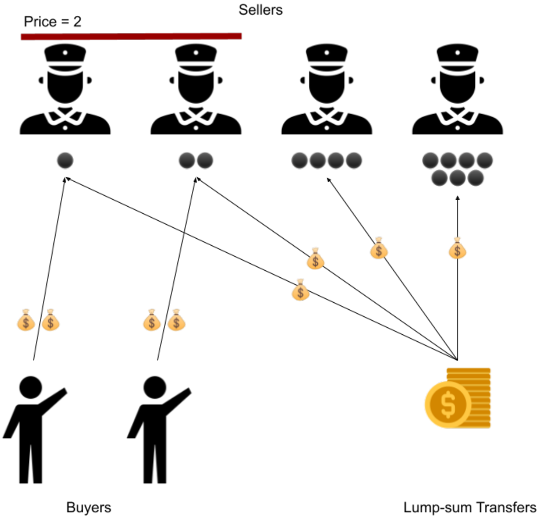
Prior Literature

- Weitzman (1977)
- Condorelli (2013)
- **Dworczak, Kominers and Akbarpour (2020)**
- Akbarpour, Dworczak and Kominers (2020)

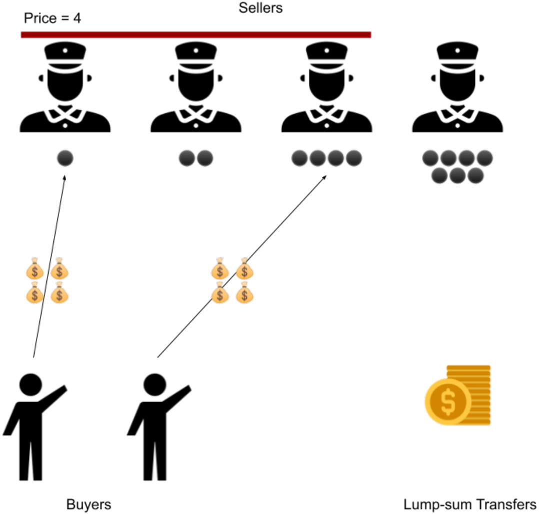
Example



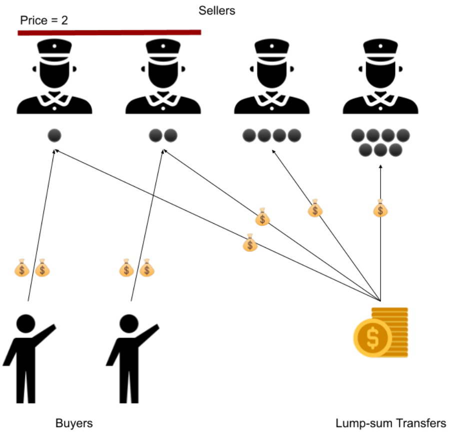
Example



Example



Rationing



Three effects of rationing:

1. (-) Reduced allocative efficiency.
2. (-) Reduced lump-sum transfer for all agents.
3. (+) Higher price received for agents rationed into market.

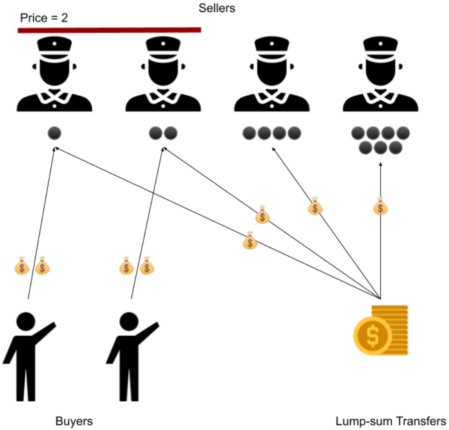
Rationing

Dworczak, Kominers, Akbarpour (2020) determine the optimal single-price mechanism in a setting with linear utility functions.

Proposition 1 (Dworczak, Kominers, Akbarpour (2020))

When seller-side inequality is low, it is optimal to choose $p_s = p_s^C$ (i.e., the competitive mechanism is optimal). When seller-side inequality is high, there exists a non-decreasing function $Q(R) \in [0, 1)$ (strictly positive for high enough R) such that rationing at a price $p_s > p_s^C$ is optimal if and only if $Q \in (0, Q(R))$. Setting $p_S = p_s^C$ (i.e., using the competitive mechanism) is optimal otherwise.

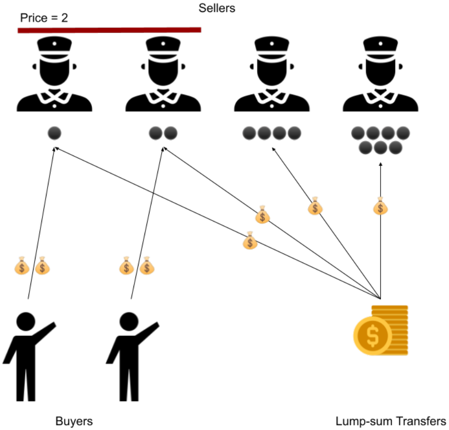
Risk-Aversion



What if agents are risk-averse?

- Motivation: labor markets with risk-aversion over income.
- Motivation: poorer agents may have less tolerance for variable income.

Risk-Aversion



What if agents are risk-averse?
Three effects of rationing:

1. (-) Reduced allocative efficiency.
2. **(-) Reduced lump-sum transfer for all agents.**
3. **(+) Higher price received for agents rationed into market.**

Model I: Design

- Unit mass of sellers.
- R: Revenue raised by market designer.
- Q: Quantity of good to be procured by market designer.
- v^K, v^M: Agent's marginal utilities for good and money, respectively.
- x^K, x^M: Agent's quantity of good and money, respectively.
- $F(v^K, v^M)$: Joint distribution of types v^K, v^M.
- p_s: Single price set by market designer on seller-side of market.

Model II: Preferences

Agents' preferences are given as

$$u(x^K, x^M; v^K, v^M) = v^K x^K + w(x^M; v^M)$$

where w is concave, increasing in v^M, and satisfies $w'(0) = v^M$.
We choose the form

$$w(x^M; v^M) = \frac{1}{k}(v^M x^M + 1)^k$$

where k parametrizes the level of risk-aversion for our agents.

Optimization Problem

We can no longer characterize agents solely by their marginal rates of substitution.

Our approach:

- Determine proportion of agents who want to sell.
- Maximize sum of welfare for three groups of agents:
    - Agents who want to sell and are rationed *in*.
    - Agents who want to sell and are rationed *out*.
    - Agents who do not want to sell.

Optimization Problem: Sales

An agent who sells receives price p_s and lump-sum transfer $R - p_s Q$, for utility:

$$w(p_s + R - p_s Q; v^M)$$

An agent who does not sell receives utility:

$$w(R - p_s Q; v^M) + v^K$$

We then compute the total quantity of agents who wish to sell as

$$\tau := \int_{\underline{v}^K, \underline{v}^M}^{\overline{v}^K, \overline{v}^M} \mathbb{1}[w(p_s + R - p_S Q; v^M) \geq w(R - p_S Q; v^M) + v^K] dF(v^K, v^M)$$

Optimization Problem: Objective

Welfare contribution from agents who wish to sell and do sell:

$$\frac{Q}{\tau}\int_{\underline{v}^K,\underline{v}^M}^{\overline{v}^K,\overline{v}^M} \mathbb{1}[w(p_s + R - p_S Q; v^M) \geq w(R - p_S Q; v^M) + v^K] w(p_s + R - p_S Q; v^M) dF(v^K, v^M)$$

Optimization Problem: Objective

Welfare contribution from agents who wish to sell and do sell:

$$\frac{Q}{\tau}\int_{\underline{v}^K,\underline{v}^M}^{\overline{v}^K,\overline{v}^M}\mathbb{1}[w(p_s+R-p_SQ;v^M)\geq w(R-p_SQ;v^M)+v^K]w(p_s+R-p_SQ;v^M)dF(v^K,v^M)$$

From agents who wish to sell but are rationed out:

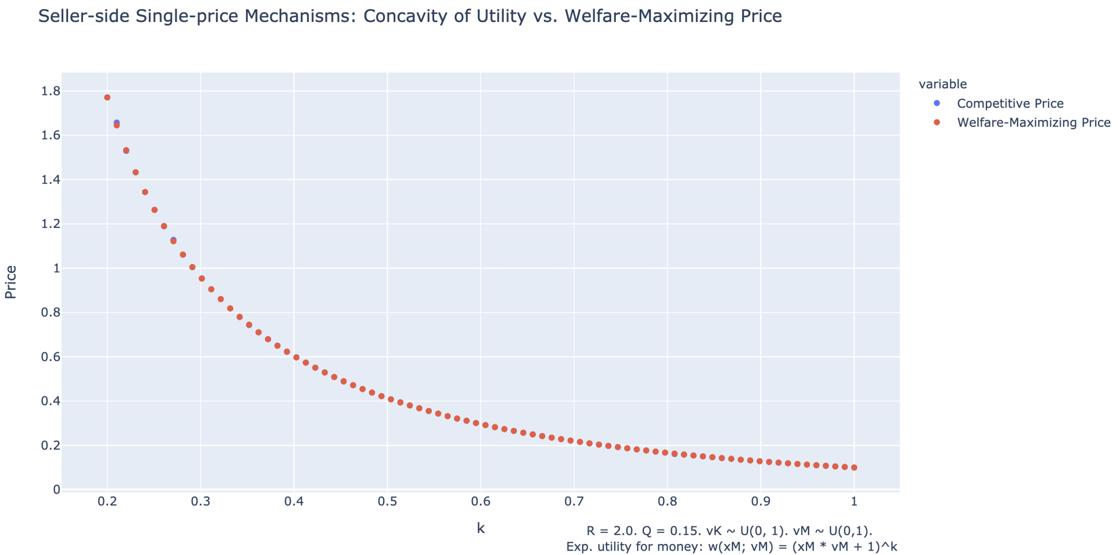
$$\frac{\tau-Q}{\tau}\int_{\underline{v}^K,\underline{v}^M}^{\overline{v}^K,\overline{v}^M}\mathbb{1}[w(R-p_SQ;v^M)\geq w(R-p_SQ;v^M)+v^K](v^K+w(R-p_SQ;v^M))dF(v^K,v^M)$$

From agents who do not wish to sell:

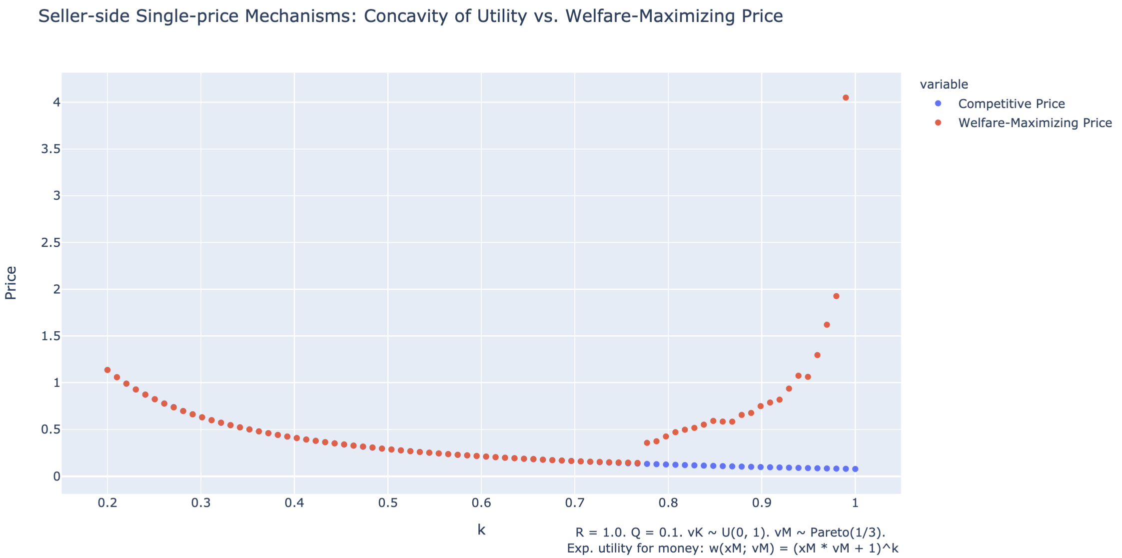
$$\int_{\underline{v}^K,\underline{v}^M}^{\overline{v}^K,\overline{v}^M}\mathbb{1}[w(R-p_SQ;v^M)\leq w(R-p_SQ;v^M)+v^K](v^K+w(R-p_SQ;v^M))dF(v^K,v^M)$$

For a fixed R, Q, F, k, choose the price p_s that maximizes the sum of these welfare terms.

Results: Low Inequality



Results: High Inequality



Future Work

Conjecture 1

When the competitive price is optimal in the linear utility setting ($k = 1$), the competitive price is optimal on the seller-side for any concave preferences satisfying our conditions for utility function $w(x^M; v^M)$.

Conjecture 2

When rationing is optimal in the linear utility setting ($k = 1$), there exists some $\epsilon > 0$ such that for all $k \in (1 - \epsilon, 1]$, rationing is optimal in the risk-averse setting with agent preferences parametrized by k.

- Can we exactly characterize the optimal mechanisms for general concave preferences? Can we extend this to the two-sided market with both buyers and sellers?
- If we have concave preferences on both sides of the market, do we obtain more complex preferences than those in Dworczak, Kominers and Akbarpour (2020)?

Thank You!

Open for questions and comments.