STRESS ENERGY OF ISOTROPIC AND HOMOGENOUS PHOTON GAS

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The Electromagnetic stress energy tensor reads

$$T_{\mu\nu} = F_{\mu\alpha}F^{\alpha}_{\ \nu} - \frac{1}{4}\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \tag{1}$$

In component form this breaks up into the electromagnetic energy density (2), the Poynting flux (3) and the Maxwell stress tensor (4):

$$T_{00} = \frac{1}{2}(|\vec{E}|^2 + |\vec{B}|^2) \tag{2}$$

$$T_{0i} = \epsilon_{ijk}E_jB_k \tag{3}$$

$$T_{ij} = \frac{1}{2}(|\vec{E}|^2 + |\vec{B}|^2)\delta_{ij} - E_iE_j - B_iB_j \tag{4}$$

We can break down the electric(magnetic) field into components:

$$E_x(B_x) = E(B)\sin\theta\cos\phi$$
$$E_y(B_y) = E(B)\sin\theta\sin\phi$$
$$E_z(B_z) = E(B)\cos\theta$$

Since we're dealing with an isotropic photon gas we should take the average of the stress tensor over all directions. This is accomplished by integrating over all solid angles:

$$\langle T_{\mu\nu}\rangle = \frac{1}{4\pi}\int d\Omega\ T_{\mu\nu} \tag{5}$$

We find then the components of the stress-energy tensor:

$$\langle T_{00}\rangle = \frac{1}{8\pi}(E^2 + B^2) \tag{6}$$

$$\langle T_{0i}\rangle = 0 \tag{7}$$

$$\langle T_{ij}\rangle = -\frac{1}{24\pi}(E^2 + B^2)\delta_{ij} \tag{8}$$

This tensor is diagonal, so we can equate it now to the stress energy of a perfect fluid and find the energy density and pressure of an isotropic and homogenous photon gas:

$$\rho_\gamma = \frac{1}{8\pi}(E^2 + B^2) \tag{9}$$

$$P_\gamma = \frac{1}{24\pi}(E^2 + B^2) \tag{10}$$

From these expressions we also get the equation of state for the photon gas:

$$\frac{P}{\rho} = \frac{1}{3} \tag{11}$$