WEAK FIELD OF A MOVING MASS

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Consider a spherically symmetric mass distribution moving through Minkowski space. We wish to find the linear perturbation $h_{\mu\nu}$ to the metric induced by this mass. First off we have the weak field Einstein equation which reads:

$$\partial_\alpha \partial^\alpha \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu} \tag{1}$$

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\bar{h}\eta_{\mu\nu} \tag{2}$$

We can invert equation (1) via the use of Fourier transforms. The resulting equation reads:

$$\bar{h}_{\mu\nu} = 4G \int d^3\vec{x}' \frac{T_{\mu\nu}(\vec{x}', t_r)}{|\vec{x} - \vec{x}'|} \tag{3}$$

$$t_r = t - |\vec{x} - \vec{x}'|$$

Furthermore, the stress tensor for the mass moving along the x-direction reads

$$\mathbf{T} = \rho(r) \begin{bmatrix} 1 & v & 0 & 0 \\ v & v^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

Plugging (4) into (3) we have:

$$\bar{\mathbf{h}} = I(\vec{x} - \vec{v}t) \begin{bmatrix} 1 & v & 0 & 0 \\ v & v^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

$$I(\vec{x} - \vec{v}t) = 4G \int d^3\vec{x}' \frac{\rho(\vec{x}', t_r)}{|\vec{x} - \vec{x}'|} \tag{6}$$

For a classical source only the $\bar{h}_{00}$ and $\bar{h}_{0i}$ components contribute to the perturbed metric.