Exercise #2

$T(n) = 7T\left(\frac{n}{2}\right) + bn^2$

$= 7\cdot\left(7T\left(\frac{n}{2^2}\right) + b\frac{n^2}{2^2}\right) + bn^2 = 7^2 T\frac{n}{2^2} + 7b\frac{n^2}{2^2} + bn^2$

$= 7^2\left[7T\left(\frac{n}{2^3}\right) + b\left(\frac{n^2}{2^4}\right)\right] + 7b\frac{n^2}{2^2} + bn^2$

$= 7^3 T\left(\frac{n}{2^3}\right) + 7^2 b\frac{n^2}{2^4} + 7b\frac{n^2}{2^2} + bn^2$

$= 7^3\left(7T\frac{n}{2^4} + b\frac{n^2}{2^6}\right) + 7^2 b\frac{n^2}{2^4} + 7b\frac{n^2}{2^2} + bn^2$

$= 7^4 T\left(\frac{n}{2^4}\right) + 7^3 b\frac{n^2}{2^6} + 7^2 b\frac{n^2}{2^4} + 7b\frac{n^2}{2^2} + bn^2$

$= 7^i T\left(\frac{n}{2^i}\right) + bn^2 \sum_{j=0}^{i-1} \left(\frac{7}{2^2}\right)^i$

$= 7^i T\left(\frac{n}{2^i}\right) + bn^2\left(\frac{r^i - 1}{r-1}\right)$

$= 7^i T\left(\frac{n}{2^i}\right) + bn^2\left(\frac{\left(\frac{7}{2^2}\right)^i - 1}{\left(\frac{7}{2^2}\right) - 1}\right)$

$= 7^i T\left(\frac{n}{2^i}\right) + \frac{4}{3}bn^2\left[\left(\frac{7}{2^2}\right)^i - 1\right]$

When $\frac{n}{2^i} = 1$, let $i = c$ $\quad \frac{n}{2^c} = 1 \quad n = 2^c \quad c = \log_2 n$

$= 7^{\log_2 n} T(1) + \frac{4}{3}bn^2\left[7^{\log_2 n}\left(2^{-2}\right)^{\log_2 n} - 1\right.$

$= 7^{\log_2 n} T(1) + \frac{4}{3}bn^2\left[7^{\log_2 n} n^{-2} - 1\right]$

$= 7^{\log_2 n} T(1) + \frac{4}{3}b - n^2$

$= n^{\log_2 7} a + \frac{4}{3}b - n^2$ Note!

$= O\left(n^{\log_2 7}\right) \longrightarrow O\left(n^{\log_2 7}\right) < O\left(n^{\overset{3}{\log_2 8}}\right)$