The Nine Chapters Analyzer

J.Y. Peng [1,*], Y.F. Shen [1], J.B. Qian [1], Z.A. Feng [1], X.Y. Zhang [1]
Supervisor: Dr. Z.Y. Li

[1] *Computer Science and Technology Program, Division of Science and Technology, BNU-HKBU United International College*
** Corresponding Student Author Tel: +86-18207572289, E-mail: r130031205@mail.uic.edu.cn*

Abstract

This project is intended to provide an interactive learning tool to help users better understand the syntax and operation rules of the nine chapters of arithmetic. As one of the ancient Chinese arithmetic, its unique calculation method and way of thinking play an important role in cultivating logical thinking and mathematical ability. By combining the compiler with interactive learning, we hope to stimulate users' interest in learning in a fun way and improve their understanding and application of the nine chapters of arithmetic. The project has compiler function, which can compile and calculate the results of nine chapters of arithmetic expressions entered by users to achieve automatic calculation.

Introduction

"The Nine Chapters on the Mathematical Art" is an ancient Chinese text that covers practical mathematical topics like ratios, geometry, and equations. It served as a key guide for solving real-world problems in ancient China. A compiler translates high-level code into machine code through stages like lexical analysis, syntax analysis, and code generation. These processes transform human-written code into a form that computers can execute. Both "The Nine Chapters" and compilers bridge the gap between theory and application—whether in ancient calculations or modern computing.

Design Approach

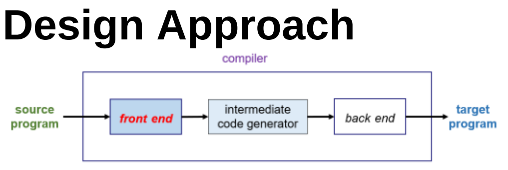

Figure 1: Compiler's sequence of phases

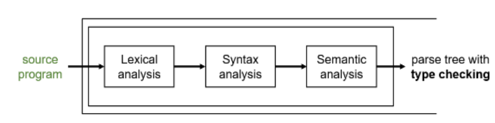

Figure 2: Compiler's analysis

Lexical Analysis

今|又|有
田|圭田|邪田|箕田|圆田|宛田|弧田|环田|广|从|周|径|弦|矢|头广|舌广|踵广|上广|下广|正从|高|袤|深|方|上方|
下方|上周|下周|上袤|下袤|中周|外周|上中周|下中周|立圆径|立方|圆周|积|方|粟|粝米|粺米|糳米|御米|小䵂|
大䵂|粝饭|粺饭|糳饭|御饭|菽|荅|麻|麦|稻|豉|飧|熟菽|糵|米|穿地|坚|壤|城|垣|堤|沟|壍|穿渠|渠|堑|方堡壔|圆
堡壔|方亭|圆亭|圆锥|方锥|堑堵|阳马|刍童|曲池|盘池|冥谷|鳖臑|羡除|刍甍|仓|圆囷|委粟平地|委菽依垣|委米
依垣内角|出|买瓴甓|买竹|买漆|买缣|买羽|买矢簳|勾|股
步|里|尺|寸|匹|丈|亩|顷|斗|升|斛|钱|枚|个|石|钧|斤|两|铢|瑕
问|为|馀|得|各|约之|合之|几何|减多益少|孰多|而|平
分|之|减|其|太|半|少|太半|少半
一|二|三|四|五|六|七|八|九|亿|万|千|百|十
，|。|?|、

Figure 3: Lexemes

First, it is necessary to design a lexical analyzer (Lexer) to convert the user's input in Classical Chinese into tokens. The lexical analyzer can use regular expressions or finite state machines to identify and extract different tokens, such as operators, numbers, and parentheses. Based on the lexical rules of *The Nine Chapters on the Mathematical Art* in Classical Chinese, design and implement a Lexer. Regular expressions or other methods can be employed to carry out the lexical analysis.

Syntax analysis

Rule	Production	Rule	Production	Rule	Production
1	S' → S	17	V → num unit	33	Q → Y， 问 Z
2	S → 今 有 E。 Q ?	18	V → num 分 unit 之 num	34	Y → 欲 为 type
3	S → 又 有 E。 Q ?	19	V → num unit K	35	Y → 欲 unit 率之
4	E → E， D	20	V → K unit	36	Y → 求 type num unit
5	E → E， F	21	V → L unit	37	Z → 为 type 几何
6	E → D	22	U → num	38	Z → type 几何
7	D → T I	23	U → num 分 之 num	39	Z → unit 几何
8	D → I	24	U → num K	40	Z → 孰多， 多 几何
9	D → U	25	U → num L	41	Z → 减多益少， 各 几何而平
10	D → G T I	26	K → 太	42	Z → W 得 几何
11	G → T	27	K → 半	43	W → 各
12	G → T，	28	K → 少	44	W → 馀
13	T → type	29	L → 太半	45	W → 约之
14	T → — type	30	L → 少半	46	W → 合之
15	I → V	31	F → fun 其 U	47	W → type
16	I → V 、 I	32	Q → 问 Z		

Figure 4: Production Rules

Next, a syntax analyzer (Parser) must be designed to perform syntactic analysis of the tokens based on the grammatical rules of The Nine Chapters on the Mathematical Art in Classical Chinese. This will generate the corresponding parsing table. The primary challenges in implementing the parser lie in achieving efficient state transitions during parsing based on the parsing table, updating the stack for state management, and displaying each step of the state transition. Additionally, it is crucial to generate a parse tree at every step of the parsing process to facilitate type checking and evaluation later on.

Semantic analysis

In the semantic analysis phase, the abstract syntax tree is subjected to semantic checks and application of semantic rules. This includes type checking, variable declaration and initialization, and ensuring the correct usage of operators. In the semantic analysis phase, certain tasks such as unit conversions and fraction handling have already been addressed in the lexical analysis, allowing us to simplify this step. By leveraging inverse functions and set theory to optimize the semantic analysis process, we can avoid redundant checks and streamline the analysis of expressions. Specifically, the use of inverse functions helps to reverse operations and ensure consistency in expressions, while set theory facilitates grouping and validating related elements efficiently, particularly for units and types.

Results

Figure 5: Input UI

Figure 6: Output UI

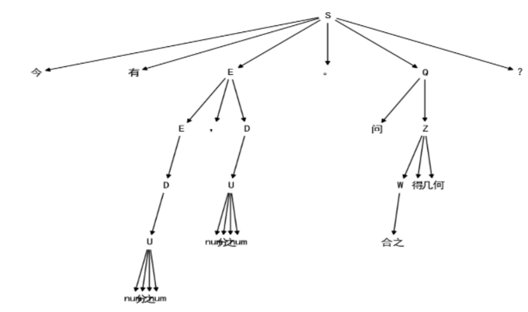

Figure 7: Parse tree

Figure 8: Instruction UI

We demonstrated a complete dynamic process, generating a parse tree and finally returning the calculation result. For users who may not be familiar with how to use the system, we have provided a detailed user manual with more than 20,000 words to guide them through understanding and operating the system effectively.

Discussion

In The Nine Chapters on the Mathematical Art, objects like fields, grain, soil, buildings etc., are used as subjects for mathematical problems involving area, volume calculations, density conversions, and more. In the following content of this document, these objects are collectively referred to as "type nouns." The reason they are termed "type" is that these objects mostly belong to the following categories: construct types, basic types, and other types. Construct types are formed from two or more basic types. For example, in the case of a triangular field 【圭田】, it is constructed from the base 【广】 and height 【正从】. Its area attribute is 【田】. Here, 【广】 and 【正从】 are basic types, while 【圭田】 is a construct type.

These type nouns also correspond to units. If categorized by units, they can be divided into length type nouns, area type nouns, volume type nouns, capacity type nouns, and so on. The corresponding type nouns align with units, such as 【田】, which, as an area type noun, should use units like 【顷】 or 【亩】.

Since they are identical to length units, when defining an area unit, it is necessary to specify at least one of 【亩】 or 【顷】 to indicate that the value is an area unit. For example: 【田一亩、四里】 (one【亩】of field, four 【里】) rather than 【田二百四十步、四里】 (240 【步】of field, four 【里】). An extended notation might be 【田一亩、三里、四步】 (one 【亩】 of field, three 【里】, four【步】). However, 【田三里、四步、一亩】 (three【里】, four【步】, one 【亩】of field) is not acceptable, because 【三里、四步】 would create ambiguity regarding area and length. For volume units, only the same length units should be used to express the volume. For instance, 【积三尺】 means a volume of three cubic 【尺】. It is also acceptable to say 【积二尺、一尺】 to express the same meaning as 【积三尺】. However, using a notation like 【积五尺、四寸】 to describe volume is not permissible, as it would create ambiguity.

Figure 9: Unit conversion and inverse function solving

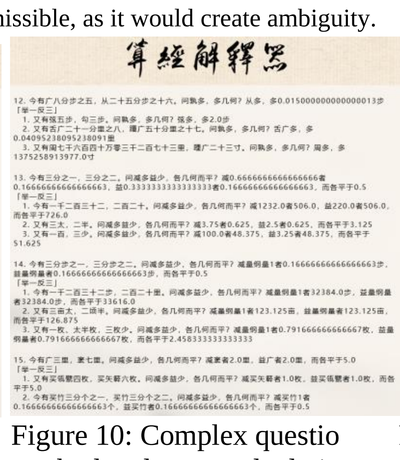

Figure 10: Complex questio method and score calculation

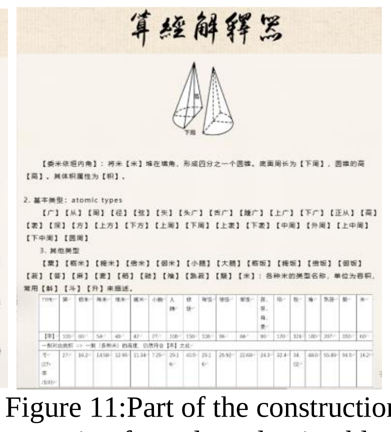

Figure 11:Part of the construction equation formula and unit table

Conclusion

In conclusion, we have successfully built a system that handles unit conversions, Chinese numeral calculations (up to one billion), and supports 11 types of units across 97 different measurement units. Special cases for fraction calculations have been implemented, and both dimensionless and dimensional quantities are handled using a tiered calculation approach. Additionally, the system now includes a method for custom equation solving, allowing users to define equations and solve for specific target values. However, due to the complexity of Classical Chinese, we have only completed implementations for five chapters of *The Nine Chapters on the Mathematical Art*. It must be acknowledged that the intricate and sometimes ambiguous grammatical structures of ancient Chinese texts present significant challenges, adding complexity to the mathematical analysis itself. In the future, there is potential to expand the system to cover additional chapters and further optimize the front-end user interface for a more seamless user experience. The framework developed so far provides a solid foundation for these future enhancements.

References

1. Guo, S. (2021). The Nine Chapters on the Mathematical Art and Liu Hui. In X. Jiang (Ed.), *The origins of sciences in China* (pp. 234-258). Springer. https://doi.org/10.1007/978-981-15-7853-3_16
2. Shen, K., Crossley, J. N., & Lun, A. W.-C. (1999). *The nine chapters on the mathematical art: Companion and commentary*. Oxford University Press. https://academic.oup.com/book/52966