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# Efficient Update-Aware Inference via Uncertainty Prediction-Based Input Filtering

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## Abstract

Under the trend of rapid iteration in edge-side intelligent applications, technologies represented by model pre-labeling have been widely adopted (e.g., smart photo albums). However, constrained by the limited performance of on-device hardware, complete re-inference on all samples is required after each model update, resulting in substantial resource consumption and prolonged update durations. Existing re-inference strategies neglect large historical samples with stable annotations, resulting in significant redundant computation. This highlights the urgent need to implement intelligent input filtering mechanisms to achieve efficient re-inference. We propose a lightweight, easily trainable, and budget-aware filtering router that reuses previous inference results. Extensive experiments demonstrate significant reductions in re-inference overhead while preserving high consistency in the efficiency–consistency frontier over representative baselines across 6 multimodal benchmarks and a real-world smart album system involving diverse architectures from CNNs to LLMs, suggesting a more budget-aware paradigm for sustainable edge-side model evolution.

## 1. Introduction

With the rising demand for on-device intelligence(Bangash, 2023; Jia et al., 2024b) and the rapid growth of edge computing(Barbuto et al., 2023), AI preprocessing has become increasingly vital(Liu & Guan, 2024). Many mobile applications, such as smart photo albums, rely on preprocessing tasks like face clustering, object detection, and scene classification(Abdelhalim & Salem, 2017).

However, when deployed models are updated, existing systems typically do re-inference on all samples (Mishra et al., 2025; Ilic et al., 2024)—a process that is both computationally expensive and time-consuming. This actually proves inadequate to meet users’ needs, creating a major bottleneck in system responsiveness. A representative example is the smart photo albums (Park et al., 2022), where the very

process of a single full re-inference update can take dozens of hours to complete. Worse still, as a low-priority background task, the re-inference is confined to the increasingly narrow window of screen-off charging periods, pushing the total update cycle to an unacceptably long span of weeks. Therefore, there is an urgent need for efficient re-inference strategies on resource-constrained edge devices(Yin et al., 2024; Xu et al., 2023; Jia et al., 2024a; Li et al., 2024).

Prior work in inference acceleration generally follows two directions: (1) reducing model-side redundancy(Pan et al., 2025; Kurtic et al., 2023), and (2) filtering redundant inputs(Yuan et al., 2024). Both directions have proven to be highly effective in practice, and they are orthogonal and complementary. While both are promising, this work will specifically pursue the latter.

Despite the considerable advancement that the latter has seen, most existing methods rely on reinforcement learning(Yuan et al., 2020), which demands long training times and struggles with deployment on edge devices.

This raises a key research question: *How can we design a lightweight, efficient, and deployable filtering router for edge scenarios?*

An existing approach is to train a model to predict whether a sample’s inference result will change after the model update. However, such approaches are limited. They cannot adapt to runtime resource constraints or adjust filtering rates based on time budgets. More importantly, they lack coordination across multiple models—typically deciding only whether to re-infer, without selecting the most appropriate model for execution. To address these challenges, our work makes two key contributions:

- We identify a task that is prevalent in practical applications but has been neglected in prior research: how to leverage historical inference results to achieve more efficient re-inference after a deployed model is updated, rather than recomputing all samples from scratch.
- We introduce a lightweight, easily trainable, and deployable filtering router based on historical inference results, which we name PURE (Predicting Uncertainty Re-inference Efficiency). PURE consists of only a few layers of a sim-

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**Table 1.** Feature comparison of our method and complementary baseline methods for efficient model inference

	Multi-model Scheduling	Budget Aware	Use Past records
Active Model Selection	✓	✗	✗
Mlink	✓	✗	✓
Adaptive Scheduling	✓	✗	✗
Dynamic KD	✗	✓	✗
CEMA	✗	✓	✗
Our Method	✓	✓	✓

ple neural network, enabling efficient decision-making with low training data requirements, minimal memory footprint, and fast inference, making it well-suited for edge deployment. Furthermore, it can dynamically adjust the filtering rate based on current system constraints, thereby enabling

## 2. Related Work

**Efficient Inference and Selection.** Model compression has evolved towards dynamic adaptation (Li, 2025). Techniques like pruning (Han et al., 2016; Liu et al., 2017) and quantization (Jacob et al., 2018; Frantar et al., 2022) reduce redundancy, while knowledge distillation (KD) (Hinton et al., 2015) remains a core methodology. Inspired by active learning (Settles, 2009), dynamic KD (Li et al., 2021) leverages entropy to adjust model interactions. **These methods primarily target static model optimization. Our proposed method can be seamlessly integrated with these compression techniques, extending them to the dynamic maintenance of deployed models.** In parallel, active model selection (Matsuura & Hara, 2023; Sawade et al., 2012) effectively identifies globally optimal models. While powerful, these approaches generally operate under the assumption of uniform data distributions, leaving the potential of sample-specific model assignment in complex edge scenarios less explored.

**Dynamic Model Scheduling.** Our work builds upon scheduling paradigms designed to optimize multi-model efficiency (Jin et al., 2014; Zhu et al., 2020). Seminal works like MLink (Yuan et al., 2022) and CEMA (Chen et al., 2024) have made significant strides in output mapping and handling distribution shifts, while adaptive scheduling (Yuan et al., 2020) has demonstrated the utility of reinforcement learning. Distinct from these approaches, our work specifically addresses the *model update* phase. As summarized in Table 1, we aim to complement existing methods by simultaneously coordinating multiple models and leveraging historical records under budget constraints, offering a lightweight alternative for edge deployment.

## 3. Motivation

Our routing strategy is motivated by two key insights derived from learning dynamics (see Appendix A) and empirical verification.

On the one hand, the feasibility of routing is grounded in model heterogeneity. Rather than attributing misclassification primarily to intrinsic sample ambiguity (Li et al., 2023; Elgaar & Amiri, 2023), we reveal that difficulty is also shaped by training data and is thus model-specific.

On the other hand, analysis based on learning dynamics demonstrates a positive correlation between classifier confidence and prediction accuracy. We empirically verify this correlation across multiple standard architectures and datasets. Furthermore, even if raw model outputs suffer from miscalibration, standard post-hoc techniques such as Temperature Scaling (Guo et al., 2017) can be employed to ensure confidence serves as a reliable proxy for accuracy (see Figure 123; experimental setup and calibration details are provided in Appendix C2).

Notably, unlike accuracy, confidence is an intrinsic property determined solely by the model’s internal state, independent of external ground truth labels. This renders it a more tractable and predictable target, allowing the routing task to be performed by lightweight models, thereby meeting the stringent resource constraints of edge computing scenarios.

## 4. Methodology

### 4.1. Minimizing Objective

Conditional entropy is employed as the measure of model confidence. The following discussion outlines its specific definition and provides a theoretical justification for its validity as a superior alternative to accuracy for the routing criterion.

We begin by formalizing the architecture of a classifier mathematically.

Given a dataset  $D = \{(\mathbf{x}_i, y_i)\}_{i=1}^m$ , where  $\mathbf{x}_i \in \mathbb{R}^l$  and  $y_i \in \{c_1, c_2, \dots, c_k\}$  is the corresponding class label with  $k$  denoting the total number of classes. Let the model parameters be  $\theta \in \mathbb{R}^d$ . For a sample  $\mathbf{x}$ , denote by  $\mathbf{z}$  the model logits and by  $\mathbf{p}$  the output probabilities, of which the  $j$ -th component  $p_j$  represents the predicted probability that the input sample  $\mathbf{x}$  belongs to the  $j - th$  class. The core objective of the classification task is to learn a mapping function  $F : \mathbb{R}^l \rightarrow \mathbb{R}^k$  from the training data. For an unseen sample  $x$ , the model predicts its class  $\hat{y}$  by  $\hat{y} = \arg \max_j f_j(x)$ , that is, selecting the class corresponding to the maximum value in the vector  $\mathbf{f}(x)$ .

Formally, let  $X$  denote the input random variable and  $Y$  the output variable spanning  $k$  possible classes. The conditional

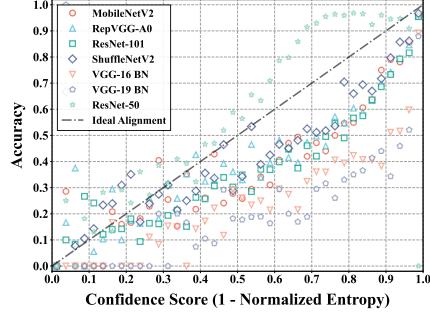


Figure 1. Alignment Between Confidence and Accuracy (Uncalibrated)

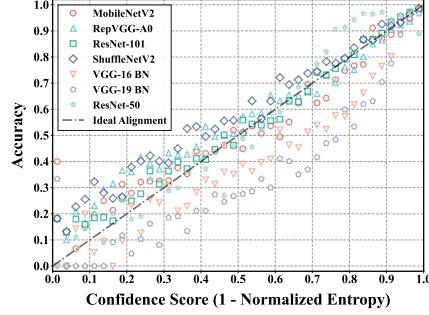


Figure 2. Alignment Between Confidence and Accuracy (Calibrated)

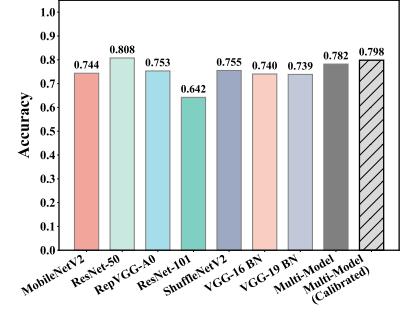


Figure 3. Model Performance Comparison

entropy  $H(Y|X)$ , which measures the average uncertainty of  $Y$  given  $X$ , is defined as:

$$H(Y|X) = - \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \log p(y|x) \quad (1)$$

By **Fano's Inequality** (Gerchinovitz et al., 2020), we derive the following equation (see Appendix F.2 for the detailed derivation) :

$$G(Acc) := Acc - \frac{H(1 - Acc)}{\log(k - 1)} \leq 1 - \frac{H(Y|X)}{\log(k - 1)} \quad (2)$$

where  $Acc$  denotes the probability of correct classification.

Under normal operating conditions where  $Acc > 0.5, k \geq 3$ , it follows that the function  $G(Acc)$  is monotonically increasing.

Therefore, this derived formula unveils a profound insight: the upper limit of  $G(Acc)$  is fundamentally constrained by the conditional accuracy. A higher value of  $H(Y|X)$  implies greater uncertainty in the classification problem.

Clearly, we can indirectly characterize the accuracy of individual samples by minimizing the entropy.

## 4.2. Formalization of Entropy-Decrease-Based Selection

Given a set of samples  $\mathcal{X} = \{x_1, x_2, \dots, x_m\}$ , an initial model  $F_0$ , and  $n$  candidate models  $\{F_1, F_2, \dots, F_n\}$ , we define a selection function  $S : \mathcal{X} \rightarrow \{0, 1, \dots, n\}$  where  $S(x_i)$  indicates which model  $(F_{S(x_i)})$  should be used to make the prediction for  $x_i$ . Our initial optimization goal is to minimize the total conditional entropy over all samples:

$$\min_S \sum_{i=1}^m H(F_{S(x_i)}(x_i)) \quad (3)$$

Let  $\Delta H(F_j(x_i))$  represent the change in conditional entropy when using model  $F_j$  instead of the original model  $F_0$  for sample  $x_i$ :

$$\Delta H(F_j(x_i)) := H(F_0(x_i)) - H(F_j(x_i)) \quad (4)$$

Clearly,  $\sum_{i=1}^m H(F_0(x_i))$  is a constant with respect to the function  $S$ . Thus:

$$\begin{aligned} & \min_S \sum_{i=1}^m H(F_{S(x_i)}(x_i)) \\ &= \sum_{i=1}^m H(F_0(x_i)) - \max_S \sum_{i=1}^m \Delta H(F_{S(x_i)}(x_i)) \end{aligned} \quad (5)$$

Therefore, our task is transformed into maximizing the total decrease in entropy across all samples.

Considering resource constraints and time budget, we also have a constraint on the function  $S$ . Let  $k\%$  be the filtering ratio. Thus a restriction is imposed on the number of samples that select a non-original model:

$$\frac{|\{i : S(x_i) \neq 0\}|}{m} \leq k\% \quad (6)$$

By adjusting the value of  $k$ , our approach can accommodate dynamic resource-constrained scenarios, which significantly enhance its operational robustness.

Overall, in this section, we formalize the optimization problem representing our approach:

$$\begin{aligned} & \max_S \sum_{i=1}^m \Delta H(F_{S(x_i)}(x_i)) \\ & \text{s.t. } \frac{|\{i : S(x_i) \neq 0\}|}{m} \leq k\% \end{aligned} \quad (7)$$

## 4.3. The Uncertainty Prediction-Base Router

Following the formal analysis above, we train a set of predictors, denoted as  $h_0, h_1, \dots, h_n$ , to estimate the entropy decrease  $\Delta \hat{H}(F_j(x_i))$

Notably, our predictors operate on the probability distribution generated by the original model  $F_0$  rather than the raw sample. This distribution acts as a distilled feature set, capturing essential information, particularly the model's decision tendencies. Furthermore, as a by-product of previous inference steps, this distribution is readily available and offers a low-dimensional, lightweight alternative to processing raw sample features.

Based on the predictors, we can estimate the appropriate function  $S_0$  that maximizes the total entropy decrease while

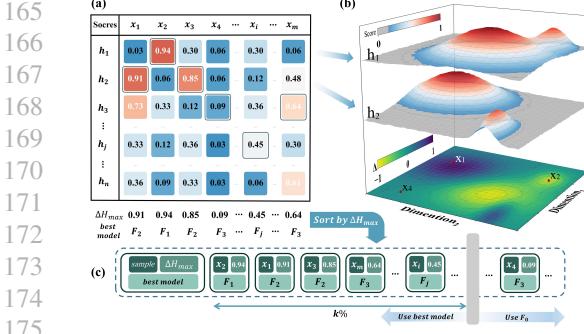


Figure 4. Overview of the Proposed Framework

Figure 5. Predict Entropy Decrease Difference on the t-SNE manifold

Figure 6. Real Entropy Decrease Difference on the t-SNE manifold

**Algorithm 1:** Adaptive Model Selection Strategy

**Require:** Trained predictors  $\{h_i\}$ , dataset  $\mathcal{D}'$ , models  $\{F_0, F_1, \dots, F_n\}$ , previous outputs array  $\mathbf{p}_0$ , threshold ratio  $k\%$

**Ensure:** Updated prediction distributions

```

1: Initialize scores  $\leftarrow []$ , indices  $\leftarrow []$ 
2: for  $m = 0$  to  $|\mathcal{D}'| - 1$  do
3:    $x \leftarrow \mathcal{D}'[m]$ ;  $p_{\text{prev}} \leftarrow \mathbf{p}_0[m]$ ;
4:    $\Delta H_{\max} \leftarrow -\infty$ ,  $i_{\text{best}} \leftarrow 0$ ;
5:   for  $i = 1$  to  $n$  do
6:      $\Delta H_i \leftarrow f_i(p_{\text{prev}})$ ;
7:     if  $\Delta H_i > \Delta H_{\max}$  then
8:        $\Delta H_{\max} \leftarrow \Delta H_i$ ;  $i_{\text{best}} \leftarrow i$ ;
9:     end if
10:   end for
11:   scores  $\leftarrow$  scores  $\cup \{\Delta H_{\max}\}$ ;
12:   indices  $\leftarrow$  indices  $\cup \{i_{\text{best}}\}$ 
13: end for
14:  $\tau \leftarrow \text{TopPercentile}(\text{scores}, k\%)$ ;
15: output_distribution  $\leftarrow \mathbf{p}_0$ ;
16: for  $m = 0$  to  $|\mathcal{D}'| - 1$  do
17:   if scores[m]  $\geq \tau$  then
18:      $i_{\text{sel}} \leftarrow \text{indices}[m]$ ;  $x \leftarrow \mathcal{D}'[m]$ ;
19:      $p_{\text{update}} \leftarrow F_{i_{\text{sel}}}(x)$ ;
20:     output_distribution[m]  $\leftarrow p_{\text{update}}$ 
21:   end if
22: end for

```

satisfying the constraints. The estimation follows the procedure below, just as shown in Figure 4.

- Firstly, for each sample  $x_i$ , we calculate the output results predicted by each predictor  $h_j$  with the result  $h_j(x_i)$  denoted as  $a_{ij}$ . Subsequently, we further obtain the maximum entropy decrease  $b_i$  and the index of the most suitable model predicted for  $x_i$  denoted as  $j^{(i)}$ :  $b_i := \max_j a_{ij}$ ,  $j^{(i)} := \arg \max_j h_j(x_i)$
- Secondly, We sort the set  $\{b_i\}_{i=1}^m$  in descending order and then select the top  $\lfloor k\% \cdot m \rfloor$  elements  $b_{i_1}, b_{i_2}, \dots, b_{i_{\lfloor k\% \cdot m \rfloor}}$

. This yields the index set  $\mathcal{I} := \{i_t\}_{t=1}^{\lfloor k\% \cdot m \rfloor}$ .

- Lastly, we update the selected samples and leave the labels of all others unaltered. For each updated sample  $x_i$ , let  $F_0^{(1)}(x_i)$  denote the updated model. Then:

$$F_0^{(1)}(x_i) = \begin{cases} F_{j^{(i)}} & , i \in \mathcal{I} \\ F_0 & , \text{otherwise} \end{cases} \quad (8)$$

Consequently, we determine the model used for each sample after the update, which will output the updated class label of each sample. That is, we obtain the values of  $S_0(x_i)$ , for  $i = 1, 2, \dots, m$ , estimating the appropriate function  $S_0$ .

#### 4.4. Extension to Multi-Round Updates

In practical applications, model updates are rarely isolated events; rather, they typically occur sequentially. This section extends our framework to the scenario of multi-round updates.

##### 4.4.1. ITERATIVE UPDATE FORMULATION

Each update round strictly adheres to the single-round protocol detailed in the preceding section. Specifically, for the  $(t + 1) - th$  round, the optimization is initialized with the model resulting from the  $t - th$  round, denoted as  $F_0^{(t)}(x_i)$ . Let  $j_{t+1}^{(i)}$  denote the index of the most suitable model predicted for  $x_i$  and let  $\mathcal{I}^{t+1}$  represent the set of indices designated for updating. We thus establish the following recurrence relation:

$$F_0^{(t+1)}(x_i) = \begin{cases} F_{j_{t+1}^{(i)}} & , i \in \mathcal{I}^{t+1} \\ F_0^{(t)}(x_i) & , \text{otherwise} \end{cases} \quad (9)$$

In particular, for the initial update round, this relationship reduces to the recurrence relation established for the single-round scenario, as given in Equation (12).

Notably, for a candidate model  $F_j$  in the  $(t + 1) - th$  round, the associated predictor  $h_j$  takes the probability distribution output by the global initial model  $F_0$  as input, rather than that of the current round's initialization, sample-specific  $F_0^{(t)}$ . Consequently,  $\Delta \hat{H}(F_j(x_i))$ , the entropy reduction for  $x_0$  predicted by  $h_j$ , is intrinsically measured relative to  $F_0$ .

220 4.4.2. MULTI-STAGE SCREENING AND  
 221 RE-CLASSIFICATION FOR OMITTED SAMPLES  
 222  
 223 Ideally, all misclassified samples should be identified and  
 224 updated. In a single-round update, omitting valid candidates  
 225 is an acceptable trade-off due to computational constraints.  
 226 However, if such samples remain uncorrected after multi-  
 227 ple update rounds, this indicates a systemic deficiency in  
 228 our selection strategy. For a given sample  $x_0$ , the me-  
 229 chanisms that lead to this persistent omission are categorized  
 230 into two primary scenarios: Marginal Exclusion caused by  
 231 limited computational resources, and the Overconfidence  
 232 Trap resulting from the model's high confidence in incorrect  
 233 answers.

234 To rectify persistent sample omission, we propose a two-  
 235 stage screening mechanism to identify high-risk samples  
 236 that necessitate rigorous re-classification via multi-model  
 237 voting. Notably, to maintain a manageable computational  
 238 budget, we adopt an adaptive Top- $K$  selection strategy.  
 239

240 **Stage 1: Omission Risk Screening.** We identify samples  
 241 with the highest omission risk by ranking all  $m$  instances  
 242 according to an omission risk function  $\mathcal{R}_{\text{Omission}}(x_i)$ , which  
 243 is constructed to account for the two failure scenarios pre-  
 244 viously discussed (refer to Appendix F.1 for a detailed deriva-  
 245 tion of its components):

$$\mathcal{R}_{\text{Omission}}(x_i) = \underbrace{\sum_{t=\tau(i)+1}^T S_{t,i}}_{\text{Term I: Marginal Exclusion}} + \underbrace{\beta \cdot \exp\left(-\frac{S_{\tau(i),i}^2}{2\sigma^2}\right)}_{\text{Term II: The Overconfidence Trap}}$$

246 where the score  $S_{t,i}$  represents the normalized entropy  
 247 reduction at round  $t$  for sample  $x_i$ , relative to the global  
 248 baseline  $F_0$ , achieved by the candidate model  $F_{j_t^{(i)}}$  that  
 249 exhibits the maximum entropy drop:  
 250

$$S_{t,i} = \frac{1}{2} \left[ 1 + \tanh\left(\Delta\hat{H}(F_{j_t^{(i)}}(x_i))\right) \right] \quad (10)$$

251 To accurately capture the cumulative risk of marginal exclu-  
 252 sion, the summation interval for the first term spans from  
 253 the round immediately following the last effective update,  
 254 denoted as  $\tau(i)$ , up to the current round  $T$ .  
 255

256 Let  $\rho_{\text{screen}}^{(t)}$  denote the adaptive screening ratio at round  $t$ ,  
 257 which increases monotonically with the round index to re-  
 258 flect the growing evidence of persistent omission:  
 259

$$\rho_{\text{screen}}^{(t)} = \min(\rho_{\text{base}} + \alpha \cdot (t - 2), \rho_{\text{max}}) \quad (11)$$

260 where  $\rho_{\text{base}}$  is the initial screening ratio,  $\alpha$  controls the  
 261 growth rate, and  $\rho_{\text{max}}$  serves as the upper bound. The result-  
 262 ing preliminary screening set  $\mathcal{X}_1$  is defined as:  
 263

$$\mathcal{X}_1 = \{x_i \mid i \in \text{Top-}\lfloor m \cdot \rho_{\text{screen}}^{(t)} \rfloor \text{ of } \mathcal{R}_{\text{Omission}}\} \quad (12)$$

264 **Stage 2: Misclassification Risk Filtering.** For each sample  
 265  $x_i \in \mathcal{X}_1$ , we quantify its prediction instability using the

266 Misclassification Risk Function based on Jensen-Shannon  
 267 divergence:

$$\mathcal{R}_{\text{misclass}}(x_i) = \sum_{k=1}^C \left[ p_k \log \frac{2p_k}{p_k + p'_k} + p'_k \log \frac{2p'_k}{p_k + p'_k} \right] \quad (13)$$

268 where  $p(x_i) = [p_1, \dots, p_C]$  denotes the probability distri-  
 269 bution output by the model currently assigned to  $x_i$ , and  
 270  $p'(x_i) = [p'_1, \dots, p'_C]$  represents the distribution from an  
 271 alternative model randomly sampled from the current round.  
 272 High JS divergence indicates unstable predictions that re-  
 273 quire correction.

274 We apply a second Top-K selection on  $\mathcal{X}_1$  with an adaptive  
 275 correction ratio  $\rho_{\text{correct}}^{(t)}$ :

$$\rho_{\text{correct}}^{(t)} = \min(\gamma_{\text{base}} + \gamma \cdot (t - 2), \gamma_{\text{max}}) \quad (14)$$

276 The final correction set  $\mathcal{X}_2$  is obtained by:

$$\mathcal{X}_2 = \left\{ x_i \mid i \in \text{Top-}\lfloor |\mathcal{X}_1| \cdot \rho_{\text{correct}}^{(t)} \rfloor \text{ of } \mathcal{R}_{\text{misclass}} \right\} \quad (15)$$

277 **Multi-Model Voting.** Finally, for each sample  $x_i \in \mathcal{X}_2$ , we  
 278 employ an ensemble of recent models  $\{F_{t-w}, \dots, F_t\}$  to  
 279 perform classification and aggregate their predictions via  
 280 majority voting:

$$\hat{y}_i^{\text{corrected}} = \arg \max_y \sum_{j=t-w}^t \#\left[\arg \max F_j(x_i) = y\right] \quad (16)$$

281 where  $w$  denotes the voting window size (typically 2-3  
 282 recent rounds, this ensemble approach leverages diverse  
 283 model perspectives to produce more robust predictions for  
 284 high-risk samples.

285 This multi-stage screening mechanism effectively rectifies  
 286 persistent sample omission while avoiding computationally  
 287 expensive voting operations on the entire dataset, thereby  
 288 significantly reducing resource consumption.

## 5. Experiments

289 We implemented our entropy-decrease prediction frame-  
 290 work using PyTorch 2.2 and deployed it on a server with  
 291 a single A40 GPU. The router network consists of a three-  
 292 layer MLP with ReLU activation, using the Adam optimizer  
 293 with a learning rate of 0.001 and a batch size of 32. The  
 294 training protocol involves 100 epochs of optimization with  
 295 MSE loss. In the basic scenario, the router has a short train-  
 296 ing time (15 minutes) and a small model size (0.02MB).

### 5.1. Experimental Setup

**Datasets** We evaluate PURE across multiple modalities  
 297 and tasks: (1) Image Classification: CIFAR-10, CIFAR-100  
 298 ([Krizhevsky & Hinton, 2009](#)), and ImageNet; (2) Object Detec-  
 299 tion: MS COCO 2017; and (3) Text Classification: AG

News (Zhang et al., 2015) and IMDb (Maas et al., 2011). Detailed dataset statistics and preprocessing steps are provided in Appendix C.

**Baselines** To provide a comprehensive evaluation of our proposed approach, we compare it against several representative baseline methods, including **CEMA**(Chen et al., 2024), **DKD**(Li et al., 2021), and **MLink**(Yuan et al., 2022). To ensure a fair comparison and eliminate confounding factors such as dataset distribution or filtering volume, we adopt consistent filtering ratios (20% and 50%) across all experiments and use an identical test set for evaluation. Detailed specifications of the baseline implementation can be found in the Appendix C.5.

Table 2. Control experiment on CIFAR-10 and CIFAR-100. The 20%/50% indicates the proportion of samples directly using previous results. All results are reported in the format of Prediction Accuracy (Acc.%) / Result Consistency (Cons.%).

	CIFAR-10		CIFAR-100	
	20%	50%	20%	50%
CEMA	91.6/94.2	88.9/90.9	79.3/96.9	74.2/87.8
DKD	94.8/97.7	88.2/91.0	79.4/97.7	75.5/89.7
Mlink	95.6/98.9	89.9/92.2	79.3/96.9	74.2/87.9
Ours	<b>96.3/99.9</b>	<b>95.3/98.7</b>	<b>80.3/98.5</b>	<b>78.3/94.0</b>

## 5.2. Scheduling under Single-task Setting

We compare our proposed method with the baselines introduced in the previous Section 5.1, with the results presented in Table 2. In this experiment, we focus on a single scheduling scenario: given the predictions from a round-1 model, how to selectively filter samples for re-inference in round 2 to achieve acceleration. For each dataset, we train both round-1 and round-2 models with identical architectures, specifically using ResNet backbones. Detailed architectural configurations can be found in the Appendix C.4.

On the CIFAR-10 dataset, the round-1 model achieves an accuracy of 81.39%, while the round-2 model reaches 96.41%. On CIFAR-100, the round-1 model obtains 61.17% accuracy, and the round-2 model achieves 80.74%. We adopt both accuracy and output consistency as the evaluation metrics. Below is a formal definition of consistency:

Let the round-2 model output be  $a_1, a_2, \dots, a_m$  and the final output after filtering and scheduling be  $b_1, b_2, \dots, b_m$  ( $a_i$  and  $b_i$  denotes the numeric class number). Consistency is defined as:

$$\frac{1}{m} \sum_{i=1}^m (1 - \text{sgn}(|a_i - b_i|)) \quad (17)$$

As shown, our method outperforms the baselines. Under a 20% filtering ratio, the accuracy remains virtually unchanged. Even under a 50% filtering ratio, halving the round-2 computation, the accuracy drops by only 2%, reaching 78.32%. On CIFAR-10, our method achieves 95.32%

accuracy, a mere 1% drop compared to full round-2 execution, and notably higher than the best baseline at 89.92%. Local consistency also supports this trend, with consistency scores reaching 99.87% under 20% filtering and 98.71% under 50%. In addition, our router achieves very fast inference speed. A batch (32 samples) takes only 1.95 ms.

Since our method is applied *post-inference*, it is orthogonal and complementary to most existing acceleration techniques, which typically focus on single-pass model optimization or compression. This modularity highlights the portability of our method, allowing seamless integration with state-of-the-art inference speed-up strategies. Figure 7 illustrates the trade-off curves between accuracy and consistency across varying filtering levels in both datasets, underscoring its ability to effectively balance execution efficiency and prediction fidelity.

## 5.3. Multi-Expert Scheduling on CIFAR-100

We evaluate multi-expert scheduling on CIFAR-100 using three specialized experts targeting the *flower*, *tree*, and *insect* superclasses, respectively. Each expert achieves over 80% accuracy within its specific domain while maintaining only  $\sim$ 20% accuracy across remaining classes. Experimental results demonstrate that by maximizing predicted entropy reduction, the router significantly outperforms the Round-1 baseline under resource constraints: accuracies for *flower*, *tree*, and *insect* tasks improved from 69.5%, 68.0%, and 68.2% to 77.5%, 73.0%, and 78.0%, respectively.

To benchmark performance without ground-truth access, we compare our approach against a **Random Policy** and a ground-truth-aware **Oracle Policy**. As shown in Figure 9, the router demonstrates high robustness as the filtering threshold increases; notably, when the threshold exceeds zero, its performance nearly matches or even surpasses the Oracle strategy, achieving near-optimal model selection without requiring external labels. Complete details are provided in the Appendix D.1.

## 5.4. Multi-Round Progressive Deployment

To evaluate PURE in realistic sequential update scenarios, we conduct a multi-round deployment experiment on CIFAR-100, simulating edge systems where models are iteratively refined. At each round  $t$ , our router selects 50% of samples for upgrade based on entropy decrease predictors. Starting from Round 2, the multi-stage correction mechanism activates to address persistent errors via omission screening, misclassification risk quantification (JS divergence), and multi-model voting. Further details on the experimental setup are provided in the Appendix C.7.

**Baselines.** We compare PURE against Naive (uniform selection), Entropy (highest prediction entropy), CEMA (Chen

Table 3. Multi-Round Model Deployment Performance Comparison on CIFAR-100. The table illustrates the progressive accuracy improvement from the initial model  $M_0$  (Round 0) through four deployment rounds. Numbers in parentheses indicate the cumulative accuracy gain relative to the  $M_0$  baseline (60.84%).

Method	Progressive Accuracy (%) by Round					Overall	
	Round 0	Round 1	Round 2	Round 3	Round 4	Avg.	$\Delta$ Avg.
Naive	60.84	61.81 (+0.97)	62.37 (+1.53)	71.84 (+10.90)	76.16 (+15.32)	66.61	Ref.
<i>Standard Baselines</i>							
Entropy	60.84	61.80 (+0.96)	62.37 (+1.53)	71.27 (+10.43)	76.12 (+15.28)	66.48	-0.13
CEMA	60.84	61.74 (+0.90)	62.22 (+1.38)	71.24 (+10.40)	76.14 (+15.30)	66.44	-0.17
MLink	60.84	62.14 (+1.30)	62.70 (+1.86)	75.14 (+14.30)	77.16 (+16.32)	67.60	+0.99
<i>Our Method &amp; Ablations</i>							
<b>PURE (Full)</b>	60.84	<b>62.36</b> (+1.52)	<b>62.78</b> (+1.94)	<b>77.53</b> (+16.69)	<b>78.87</b> (+18.03)	<b>68.89</b>	+2.28
– w/o Correction	60.84	62.05 (+1.21)	62.40 (+1.56)	75.98 (+15.14)	76.90 (+16.06)	67.62	+1.01
– Only Omission	60.84	62.24 (+1.40)	62.59 (+1.75)	76.91 (+16.07)	78.08 (+17.24)	68.12	+1.51
– Only Misclass	60.84	62.11 (+1.27)	62.47 (+1.63)	76.37 (+15.53)	77.45 (+16.61)	67.84	+1.23
– No Voting	60.84	62.17 (+1.33)	62.53 (+1.69)	76.60 (+15.76)	77.69 (+16.85)	67.95	+1.34
Optimal (Upper Bound)	60.84	<u>62.48</u> (+1.64)	<u>62.87</u> (+2.03)	<u>80.68</u> (+19.84)	<u>80.75</u> (+19.91)	<u>69.52</u>	+2.91

Notes: **Round 0:** ResNet-101 (60.84% acc); **Round 1-2:** Noise reduction; **Round 3:** Quality improvement; **Round 4:** R2-Full (80.75% acc). Selection rate: 50% per round.

et al., 2024), MLink (Yuan et al., 2022), and Optimal (theoretical upper bound). Ablation variants include w/o Correction (predictor-only), Only Omission, Only Misclass, and No Voting.

**Overall Performance.** As shown in Table 3, PURE achieves 78.87% accuracy in Round 4, outperforming all baselines, including a +1.71% gain over MLink (77.16%) and +2.71% over the Naive approach (76.16%). PURE reaches 97.7% of the optimal performance (80.75%), demonstrating near-optimal sample selection efficiency within a 50% computational budget.

**Progressive Trajectory.** Round-by-round analysis reveals that while early rounds (0→2) show modest gains due to marginal model differences, Round 2→3 exhibits a dramatic leap (+14.75% for PURE) following the introduction of high-quality  $M_3$ . Notably, PURE maintains consistent improvement in Round 4 (+1.34%), whereas the gap between PURE and MLink widens, highlighting the correction mechanism’s role in sustained performance.

**Correction Efficacy.** Ablation results quantify the necessity of the multi-stage design. Removing the correction mechanism entirely results in a 1.97% performance degradation. Misclassification risk detection emerges as the most critical component; its absence (*Only Omission* variant) reduces accuracy by 0.79%, while excluding omission screening (*Only Misclass* variant) leads to a more significant 1.42% drop compared to the full model. The full correction gain (+1.97%) exceeds the sum of individual components, indicating a strong synergistic effect between screening and voting.

## 5.5 Comprehensive Performance Analysis

To evaluate the effectiveness and generality of PURE, we extend our evaluation beyond visual classification to text classification and object detection, covering diverse modalities and architectures (e.g., ViT, Qwen(Dosovitskiy et al., 2021; Bai et al., 2023)). As shown in Table 7, we simulate common model update scenarios across these domains. To accommodate non-classification outputs, we developed task-specific adaptation strategies (detailed in Appendix D.2).

The results confirm that our uncertainty-prediction-based router achieves an optimal balance between efficiency and accuracy. In a conservative 20% filtering scenario, performance remains nearly identical to full re-inference, maintaining up to 99.89% consistency on ImageNet. Even under aggressive 50% filtering, PURE exhibits graceful degradation; for instance, the COCO mAP@50 decreases only from 71.5% to 65.7%, halving computation for an acceptable accuracy cost . This demonstrates PURE’s utility in allowing users to dynamically balance their computational budget with task fidelity.

## 5.6 Real-world Experiment

In this phase, we simulate the real-world scenario of a smart photo album on mobile devices. As shown in Table 4, for object detection and face detection using IoU (95%) as the metric, our method achieves 24.05% and 21.20%. This shows our approach works effectively in object-related detection with high-precision overlap measurement. For object classification and text classification (OCR) using consistency, the results are 58.45% and 40.25%. The difference in performance between the object detection task and the

Table 4. Maximum filtering rates across tasks on mobile photo album datasets under 95% consistency threshold. ‘‘Cons’’ represents the consistency in equation 17, ‘‘IoU’’ represents the Intersection-over-Union.

Task	Model in Rounds		Evaluation Metrics		Filter	
	Description	Round1	Round2	Metric	Type	
Object Detection	Yolov8n(Ultralytics, 2023)	Yolov8x(Ultralytics, 2023)	Yolov8x(Ultralytics, 2023)	IoU(95%)	Detection	24.05%
Face Detection	MTCNN(Zhang et al., 2016)	Yolo-face(Jocher & Qiu, 2023)	Yolo-face(Jocher & Qiu, 2023)	IoU(95%)	Detection	21.20%
Scene Classification	Resnet(He et al., 2016)	Yolov8s(Ultralytics, 2023)	Yolov8s(Ultralytics, 2023)	Cons.(95%)	Classification	58.45%
Text Classification	Qwen1.5(Bai et al., 2023)	Qwen3(Team, 2025)	Qwen3(Team, 2025)	Cons.(95%)	Classification	40.25%

Table 5. Performance stability under different quantization and sparsification settings at a 50% Selection Rate.

Spars.	50% Selection Rate Accuracy (%)				Max
	Float32	Float16	Int8	Int4	
Top-1	78.24	78.22	78.37	78.40	0.51%
Top-4	78.17	78.24	78.35	77.94	0.86%
Top-8	78.30	78.29	78.26	78.04	0.83%
Full (Ref)	78.32	78.40	78.18	77.92	0.86%
Avg.	<b>78.26</b>	<b>78.29</b>	<b>78.29</b>	<b>78.08</b>	—

Table 6. Performance and time cost under different training sample sizes on CIFAR-100. Acc/Cons denotes Accuracy/Consistency. Pre/Train indicates the time in seconds for preprocessing and training.

Samples	50000	10000	5000	1000
Acc/Cons	78.3/94.1	78.3/94.1	78.4/94.0	70.9/82.3
Pre/Train	33.4/127.4	7.0/25.9	4.5/12.9	2.7/1.5

classification task may be due to the difference in the strictness between the metric of IoU >95% and the metric of consistency >95%. However, overall, each task can achieve a speed-up of at least about 20%. Complete details are provided in the Appendix D.3.

## 5.7. Ablation Study

**Hardware Performance and Acceleration** We benchmark the router’s efficiency across three heterogeneous platforms: a high-performance server (Intel Xeon), a personal workstation (Apple M4), and a mobile environment (A17 Pro). As illustrated in Figure 11, the router maintains a minimal memory footprint (0.02MB) and achieves sub-millisecond inference latency (0.014–0.072ms) across all tested devices.

Notably, our method achieves 28.9%–43.3% end-to-end latency reduction. We observe a gap between the practical speedup and the 50% theoretical filtering rate, which we attribute to the approximately 10% computational overhead introduced by the naive global sorting algorithm used for thresholding. While integrating advanced streaming top- $k$  algorithms (Charikar et al., 2002) could further bridge this

gap, the current implementation already demonstrates substantial acceleration potential for real-world edge computing scenarios.

**Quantization and Sparsification Strategies** To minimize the storage footprint of historical inference distributions on resource-constrained edge devices, we evaluate two complementary compression strategies: **precision quantization** and **top- $k$  sparsification**. As shown in Table 10, 4-bit quantization (discretizing probabilities into  $2^4$  levels) combined with top-1 selection reduces the per-sample storage from 400 bytes to 1.5 bytes. This achieves a **99.5% compression ratio** with a marginal accuracy degradation of less than 1%. These results validate that storing sparse probability vectors is an effective approach for maintaining routing fidelity while respecting hardware storage constraints.

**Data Efficiency and Training Overhead** We investigate the router’s sensitivity to training data scale by varying the sample size on CIFAR-100 from 50,000 to 1,000 (Table 10). The router exhibits significant **data efficiency**: reducing the training set by 90% (from 50k to 5k) results in negligible performance loss, maintaining 78.4% accuracy and 94.0% consistency. Furthermore, both preprocessing (Pre) and training (Train) times scale sub-linearly with the data size. This lightweight overhead enables the router to be rapidly retrained to adapt to frequent base-model updates in dynamic deployment environments.

## 6. Conclusion and Discussion

In this work, we propose a lightweight inference-filtering router that can be deployed at the edge. We use historical execution data as a basis to achieve precise and efficient routing by predicting the magnitude of uncertainty change. Our method features light-weight (0.02MB), fast inference speed (1.95ms per batch(32 samples)), easy deployment, and easy training. However, we still summarize the following limitations and issues: when facing unreliable random models or models with significant differences in capabilities, scheduling cannot be performed. The first-round models need to have a relatively reasonable uncertainty assessment of their own results. How to measure ‘‘schedulability’’ is a worthwhile research question; we will continue to study it.

- 440 **Impact Statement**
- 441 This paper presents work whose goal is to advance the field
- 442 of Machine Learning. There are many potential societal
- 443 consequences of our work, none of which we feel must be
- 444 specifically highlighted here.
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- 447 **References**
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## 605 A. Motivation: A Learning Dynamics Perspective

### 606 A.1. Confidence as an Indicator of Accuracy

608 In this section, we analyze how the training data influences the outputs of the classifier for an arbitrary sample.

610 For an arbitrary sample  $\mathbf{x}_o$ , we decompose the training process to examine, on a sample-by-sample basis, how a single  
 611 training step affects the parameter updates. Specifically, consider a training instance  $(\mathbf{x}_u, y_u)$  used in the  $t - th$  step, where  
 612  $y_u$  is the class label for  $\mathbf{x}_u$ . Assuming this step updates the model parameters from  $\theta^t$  to  $\theta^{t+1}$ , thereby updating the output  
 613 probabilities of  $\mathbf{x}_o$  from  $\mathbf{p}_o^t$  to  $\mathbf{p}_o^{t+1}$ , we propose the following proposition (see the next section for detailed formulation  
 614 and proofs) :

$$615 \quad \mathbf{p}_o^{t+1} - \mathbf{p}_o^t = -\eta \mathcal{M}^t(\mathbf{x}_o) \mathcal{K}^t(\mathbf{x}_o, \mathbf{x}_u) \mathcal{G}^t(\mathbf{x}_u, y_u) \quad (18)$$

616 Here,  $\mathcal{G}^t$  represents the gradient driver from the training sample,  $\mathcal{M}^t$  reflects the sensitivity of  $\mathbf{x}_o$ , and  $\mathcal{K}^t$  acts as a kernel  
 617 measuring the gradient alignment between  $\mathbf{x}_u$  and  $\mathbf{x}_o$ .

618 Based on this mechanism—and considering the property of high-dimensional spaces where vectors tend to be nearly  
 619 orthogonal unless highly correlated—the influence of a training instance  $(\mathbf{x}_u, y_u)$  on an arbitrary sample  $\mathbf{x}_o$  can be  
 620 categorized into three scenarios:

- 622 • **Positive Reinforcement:** If the feature gradients of  $\mathbf{x}_u$  and  $\mathbf{x}_o$  are highly aligned, and  $y_u$  is indeed the ground-truth label  
 623 of  $\mathbf{x}_o$ , then training on  $(\mathbf{x}_u, y_u)$  will significantly increase the model's confidence in correctly classifying  $\mathbf{x}_o$ .
- 624 • **Negative Interference:** If the gradients are highly aligned but  $y_u$  is *not* the true label of  $\mathbf{x}_o$ , training on  $(\mathbf{x}_u, y_u)$  will  
 625 mislead the model. Specifically, it erroneously boosts the probability of the incorrect label  $y_u$ , thereby suppressing the  
 626 probability of the correct class.
- 627 • **Independence:** If the gradients of  $x_u$  and  $x_o$  are nearly orthogonal in the feature space, the training of  $(\mathbf{x}_u, y_u)$  exerts  
 628 negligible influence on  $x_o$ .

630 The model's final classification probability for  $\mathbf{x}_o$  is the result of cumulative updates across training steps. Consequently, if  
 631 the selected training dataset contains a prevalence of samples that induce negative interference, the probability assigned  
 632 to the correct label of  $\mathbf{x}_o$  will be diminished. This accumulation of adverse effects ultimately leads to a reduction in the  
 633 model's confidence regarding its prediction.

### 635 A.2. Detailed Formulation and Proofs

637 The analysis of learning dynamics presented in the previous section, which examines how training data influences the  
 638 outputs of a classifier for an arbitrary sample, draws inspiration from [cite]. However, this decomposition is intrinsically  
 639 contingent upon the specific parameter update mechanism and is thus dependent on the choice of optimizer. Given that  
 640 diverse classifier models often employ distinct optimizers, it is necessary to establish the generalizability of our formulation.  
 641 The objective of this appendix is to demonstrate that the decomposition proposed in the main text remains valid across a  
 642 spectrum of common optimizers, including SGD, SGD with Momentum, and Adam. While this validation is essential for  
 643 theoretical rigor, the complete proofs are detailed here due to space constraints in the main text.

644 Specifically, we focus on the change in the output at an arbitrary test point  $\mathbf{x}_o$ , denoted as  $\Delta \mathbf{p}_o = \mathbf{f}_{\theta^{t+1}}(\mathbf{x}_o) - \mathbf{f}_{\theta^t}(\mathbf{x}_o)$ ,  
 645 resulting from a parameter update based on a single training sample  $(\mathbf{x}_u, y_u)$ . We establish that this change can be  
 646 decomposed into the sum of a dominant first-order term (the MKG decomposition term) and a second-order remainder. The  
 647 core of the proof lies in the observation that, regardless of the specific optimizer, the norm of the parameter update  $\Delta \theta$  is  
 648 always of the first order with respect to the learning rate  $\eta$ . Consequently, the second-order term in the Taylor expansion  
 649 becomes  $O(\eta^2)$ . Under conditions of a small learning rate and conventional network training, this term is a higher-order  
 650 infinitesimal relative to the first-order term and can therefore be neglected, guaranteeing the theoretical validity of our  
 651 approximation.

#### 653 A.2.1. NOTATION AND FUNDAMENTAL ASSUMPTIONS

654 We begin by defining the network architecture, key variables, and the fundamental assumptions underlying the analysis.

655 Consider a neural network  $\mathbf{f}_\theta : \mathbb{R}^l \rightarrow \mathbb{R}^k$  parameterized by  $\theta$ . It can be generically decomposed into a feature extractor  
 656  $h_\theta : \mathbb{R}^l \rightarrow \mathbb{R}^k$  and an output layer function  $\pi : \mathbb{R}^k \rightarrow \mathbb{R}^k$ , i.e.,  $\mathbf{f}_\theta(\mathbf{x}) = \pi(h_\theta(\mathbf{x}))$ . The loss for a sample  $(\mathbf{x}_u, y_u)$  is defined  
 657 as  $\mathcal{L}(\mathbf{x}_u, y_u) = \ell(h_\theta(\mathbf{x}_u), y_u)$ , where  $\ell$  is differentiable. At training iteration  $t$ , we define the following key Jacobian  
 658

660 matrices and gradient vector:

- 661
- $\mathcal{M}^t(\mathbf{x}) = \nabla_z \pi(z)|_{h_{\theta^t}(\mathbf{x})} \in \mathbb{R}^{k \times k}$ : the Jacobian of the output layer with respect to the features  $h_{\theta^t}(\mathbf{x})$ .

662

  - $\mathbf{J}^t(\mathbf{x}) = \nabla_{\theta} h_{\theta}(\mathbf{x})|_{\theta^t} \in \mathbb{R}^{k \times d}$ : the Jacobian of the feature extractor with respect to the parameters  $\theta^t$ .

663

  - $\mathcal{G}^t(\mathbf{x}_u, \mathbf{y}_u) = (\nabla_{\mathbf{z}} \ell(\mathbf{z}, \mathbf{y}_u)|_{h_{\theta^t}(\mathbf{x}_u)})^\top \in \mathbb{R}^{k \times 1}$ : the gradient of the loss with respect to the features.

664  
665  
666  
667 The output change and the parameter update at the test point are

668  
669  
670

$$\Delta \mathbf{p}_o = \mathbf{f}_{\theta^{t+1}}(\mathbf{x}_o) - \mathbf{f}_{\theta^t}(\mathbf{x}_o) \in \mathbb{R}^{k \times 1}$$

$$\Delta \theta = \theta^{t+1} - \theta^t \in \mathbb{R}^{d \times 1}$$

671 For the proof to hold, we introduce the following assumptions, which are reasonable and commonly satisfied in the context  
672 of training small, fully-connected networks:

- 673
- **Bounded Gradients and Jacobians:** There exists a constant  $G, J, M > 0$  such that  $\|\mathcal{G}^t(\mathbf{x}_u, \mathbf{y}_u)\|_2 \leq G$ ,  $\|\mathbf{J}^t(x)\|_2 \leq J$ ,  $\|\mathcal{M}^t(x)\|_2 \leq M$ . Training loss is typically finite, and common activation functions and loss functions have bounded gradients within a finite input range. The limited parameter scale in small networks further reduces the likelihood of gradient explosion. Within a finite parameter space and with bounded activation functions, the spectral norm of the network's Jacobian matrix can be effectively controlled.

674

  - **Bounded Hessian:** There exists a constant  $B > 0$  such that  $\|\nabla_{\theta}^2 f_{\theta}(x)\|_2 \leq B$ . For small networks using piecewise linear (e.g., ReLU) or smooth activation functions, the second derivatives are bounded in most regions or almost everywhere. This is an assumption common in theoretical analyses.

675

  - **Non-Degeneracy:** This assumption requires that the training sample  $(\mathbf{x}_u, \mathbf{y}_u)$  induces a non-trivial update on the output  
676 of test point  $\mathbf{x}_o$  via the Neural Tangent Kernel (NTK)-like structure, which is reasonable for non-trivial learning tasks.

677

  - **Bounded Adam Preconditioning Matrix:** For the Adam optimizer, the spectral norm of its preconditioning matrix  
678  $D^{t+1} = \text{diag}((\sqrt{\hat{v}^{t+1}} + \epsilon)^{-1})$  is bounded; i.e., there exists  $D_{\max} > 0$  such that  $\|D^{t+1}\|_2 \leq D_{\max}$ . Given bounded  
679 gradients, the components of the second-moment estimate  $\hat{v}^{t+1}$  are confined to a positive interval. Combined with the  
680 stability term  $\epsilon > 0$ , this assumption holds naturally in practice.

### A.2.2. STANDARD SGD

691 The update rule for standard SGD is

692

$$\Delta \theta = -\eta (\nabla_{\theta} \mathcal{L}(\mathbf{x}_u, \mathbf{y}_u)|_{\theta^t})^\top$$

693 Performing a Taylor expansion of  $\Delta \mathbf{p}_o$  around  $\theta^t$ , there exists some  $\tilde{\theta} \in [\theta^t, \theta^{t+1}]$  such that:

694  
695  
696  
697  
698  
699

$$\Delta \mathbf{p}_o = \underbrace{\nabla_{\theta} \mathbf{f}_{\theta^t}(\mathbf{x}_o)}_{T_1} \Delta \theta + \underbrace{\frac{1}{2} \Delta \theta^\top \nabla_{\theta}^2 \mathbf{f}_{\theta}(\mathbf{x}_o) \Delta \theta}_{T_2}.$$

700 To evaluate the leading term, we plug in the definition of SGD and repeatedly use the chain rule:

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714

$$\begin{aligned} T_1 &= \underbrace{\nabla_{\theta} \mathbf{f}_{\theta}(\mathbf{x}_o)|_{\theta^t}}_{k \times d} \underbrace{(\theta^{t+1} - \theta^t)}_{d \times 1} \\ &= (\underbrace{\nabla_{\mathbf{z}} \pi(\mathbf{z})|_{h_{\theta^t}(\mathbf{x}_o)}}_{k \times k} \underbrace{\nabla_{\theta} \mathbf{h}_{\theta}(\mathbf{x}_o)|_{\theta^t}}_{k \times d}) (-\eta \underbrace{\nabla_{\theta} \mathcal{L}(\mathbf{x}_u, \mathbf{y}_u)|_{\theta^t}}_{1 \times d})^\top \\ &= \underbrace{\nabla_{\mathbf{z}} \pi(\mathbf{z})|_{h_{\theta^t}(\mathbf{x}_o)}}_{k \times k} \underbrace{\nabla_{\theta} \mathbf{h}_{\theta}(\mathbf{x}_o)|_{\theta^t}}_{k \times d} \underbrace{(-\eta \nabla_{\mathbf{z}} \mathcal{L}(\mathbf{x}_u, \mathbf{y}_u)|_{h_{\theta^t}(\mathbf{x}_o)})}_{1 \times k} \underbrace{\nabla_{\theta} \mathbf{h}_{\theta}(\mathbf{x}_u)|_{\theta^t}}_{k \times d}^\top \\ &= -\eta \underbrace{\nabla_{\mathbf{z}} \pi(\mathbf{z})|_{h_{\theta^t}(\mathbf{x}_o)}}_{k \times k} \underbrace{[\nabla_{\theta} \mathbf{h}_{\theta}(\mathbf{x}_o)|_{\theta^t} (\nabla_{\theta} \mathbf{h}_{\theta}(\mathbf{x}_u)|_{\theta^t})^\top]}_{k \times d} \underbrace{\left( \nabla_{\mathbf{z}} \mathcal{L}(\mathbf{x}_u, \mathbf{y}_u)|_{h_{\theta^t}(\mathbf{x}_o)} \right)}_{k \times 1}^\top \\ &= -\eta \mathcal{M}^t(\mathbf{x}_o) \mathbf{J}^t(\mathbf{x}_o) \mathbf{J}^t(\mathbf{x}_u)^\top \mathcal{G}^t(\mathbf{x}_u, \mathbf{y}_u) = -\eta \mathcal{M}^t(\mathbf{x}_o) \mathcal{K}^t(\mathbf{x}_o, \mathbf{x}_u) \mathcal{G}^t(\mathbf{x}_u, \mathbf{y}_u) \end{aligned}$$

715 where  $\mathcal{K}^t(\mathbf{x}_o, \mathbf{x}_u) = \mathbf{J}^t(\mathbf{x}_o)\mathbf{J}^t(\mathbf{x}_u)^\top$  is the neural tangent kernel (NTK) matrix in the feature space at step  $t$ .  
 716  
 717 For the higher-order term, under our assumptions, we note that:

$$\begin{aligned} 718 \|\Delta\theta\|_2 &= \left\| -\eta (\nabla_\theta \mathcal{L}(\mathbf{x}_u, \mathbf{y}_u)|_{\theta^t})^\top \right\|_2 = \left\| -\eta (\nabla_z \mathcal{L}(\mathbf{x}_u, \mathbf{y}_u)|_{h_{\theta^t}(\mathbf{x}_o)} \nabla_\theta \mathbf{h}_\theta(\mathbf{x}_u)|_{\theta^t})^\top \right\|_2 \\ 719 &= \left\| -\eta \mathbf{J}^t(\mathbf{x}_u)^\top \mathcal{G}^t(\mathbf{x}_u, \mathbf{y}_u) \right\|_2 \leq \eta \|\mathbf{J}^t(\mathbf{x}_u)^\top\|_2 \|\mathcal{G}^t(\mathbf{x}_u, \mathbf{y}_u)\|_2 = \eta JG \\ 720 \\ 721 \end{aligned}$$

722 So the second-term  $T_2$  satisfies:  
 723

$$724 \|T_2\|_2 = \left\| \frac{1}{2} \Delta\theta^\top \nabla_\theta^2 \mathbf{f}_{\tilde{\theta}}(\mathbf{x}_o) \Delta\theta \right\|_2 \leq \frac{1}{2} \|\nabla_\theta^2 \mathbf{f}_{\tilde{\theta}}(\mathbf{x}_o)\|_2 \|\Delta\theta\|_2^2 \leq \frac{1}{2} BJ^2 G^2 \eta^2 \\ 725 \\ 726$$

727 Therefore, for the MKG decomposition of standard SGD, under the commonly used small learning rate condition, the  
 728 second-order term  $T_2$  is a negligible higher-order term compared to the first-order term  $T_1$ .  
 729

### 730 A.2.3. SGD WITH MOMENTUM

731 The update rule for SGD with momentum is:  
 732

$$733 v^{t+1} = \gamma v^t + (\nabla_\theta \mathcal{L}(\mathbf{x}_u, \mathbf{y}_u)|_{\theta^t})^\top \\ 734 \Delta\theta = -\eta v^{t+1} \\ 735 \\ 736$$

737 where  $\gamma \in [0, 1]$  is the momentum coefficient and  $v^0 = 0$ . In this case,  $\Delta\theta$  can be written as:  
 738

$$739 \Delta\theta = -\eta \sum_{i=0}^t \gamma^{t-i} \mathbf{J}^i(\mathbf{x}_u)^\top \mathcal{G}^i(\mathbf{x}_u, \mathbf{y}_u) \\ 740 \\ 741$$

742 Substituting into the expression for  $T_1$  gives:  
 743

$$744 T_1 = \underbrace{\nabla_\theta \mathbf{f}_\theta(\mathbf{x}_o)|_{\theta^t}}_{k \times d} \underbrace{(\theta^{t+1} - \theta^t)}_{d \times 1} = (\underbrace{\nabla_z \pi(\mathbf{z})|_{h_{\theta^t}(\mathbf{x}_o)}}_{k \times k} \underbrace{\nabla_\theta \mathbf{h}_\theta(\mathbf{x}_o)|_{\theta^t}}_{k \times d} \underbrace{\Delta\theta}_{d \times 1} \\ 745 \\ 746 \\ 747 = -\eta \mathcal{M}^t(\mathbf{x}_o) \mathbf{J}^t(\mathbf{x}_o) \sum_{i=0}^t \gamma^{t-i} \mathbf{J}^i(\mathbf{x}_u)^\top \mathcal{G}^i(\mathbf{x}_u, \mathbf{y}_u) = -\eta \mathcal{M}^t(\mathbf{x}_o) \sum_{i=0}^t \gamma^{t-i} \mathcal{K}^{t,i}(\mathbf{x}_o, \mathbf{x}_u) \mathcal{G}^i(\mathbf{x}_u, \mathbf{y}_u) \\ 748 \\ 749$$

750 where  $\mathcal{K}^{t,i}(\mathbf{x}_o, \mathbf{x}_u) = \mathbf{J}^t(\mathbf{x}_o)\mathbf{J}^i(\mathbf{x}_u)^\top$ . In this case, the norm of the parameter update satisfies:  
 751

$$752 \|\Delta\theta\|_2 = \left\| -\eta \sum_{i=0}^t \gamma^{t-i} \mathbf{J}^i(\mathbf{x}_u)^\top \mathcal{G}^i(\mathbf{x}_u, \mathbf{y}_u) \right\|_2 \leq \eta \sum_{i=0}^t \gamma^{t-i} \|\mathbf{J}^i(\mathbf{x}_u)^\top\|_2 \|\mathcal{G}^i(\mathbf{x}_u, \mathbf{y}_u)\|_2 \leq \eta JG \sum_{i=0}^t \gamma^{t-i} \leq \frac{\eta JG}{1-\gamma} \\ 753 \\ 754$$

755 Similar to the analysis for standard SGD, the spectral norm of the second-order term in this scenario can be estimated as:  
 756

$$757 \|T_2\|_2 \leq \frac{1}{2} BJ^2 G^2 \left( \frac{\eta}{1-\gamma} \right)^2 \\ 758 \\ 759$$

760 Therefore, for the second-order term to be negligible, we require  $\frac{\eta}{1-\gamma} \rightarrow 0$ . This implies that the momentum coefficient  $\gamma$   
 761 should not be excessively close to 1, or the learning rate  $\eta$  needs to be correspondingly reduced as  $(1-\gamma)$  decreases, which  
 762 are common constraints in practical hyperparameter tuning.  
 763

### 764 A.2.4. ADAM OPTIMIZER

765 The update steps for the Adam optimizer are as follows: Compute the gradient  
 766

$$767 g^t = (\nabla_\theta \mathcal{L}(\mathbf{x}_u, \mathbf{y}_u)|_{\theta^t})^\top = \mathbf{J}^t(\mathbf{x}_u)^\top \mathcal{G}^t(\mathbf{x}_u, \mathbf{y}_u) \\ 768 \\ 769$$

770 Update the first- and second-moment estimates

$$m^{t+1} = \beta_1 m^t + (1 - \beta_1) g^t$$

$$v^{t+1} = \beta_2 v^t + (1 - \beta_2)(g^t \odot g^t)$$

775 Compute bias-corrected estimates

$$\hat{m}^{t+1} = m^{t+1}/(1 - \beta_1^{t+1})$$

$$\hat{v}^{t+1} = v^{t+1}/(1 - \beta_2^{t+1})$$

778 and finally, update the parameters:

$$\Delta\theta = -\eta \cdot \hat{m}^{t+1}/(\sqrt{\hat{v}^{t+1}} + \epsilon)$$

781 We define the diagonal preconditioning matrix  $D^{t+1} = \text{diag}((\sqrt{\hat{v}^{t+1}} + \epsilon)^{-1})$ . Then the update can be written as:

$$\Delta\theta = -\eta \frac{1 - \beta_1}{1 - \beta_1^{t+1}} D^{t+1} \sum_{i=0}^t \beta_1^{t-i} \mathbf{J}^i(\mathbf{x}_u)^\top \mathcal{G}^i(\mathbf{x}_u, \mathbf{y}_u)$$

786 Substituting into the expression for  $T_1$  gives:

$$T_1 = \mathcal{M}^t(\mathbf{x}_o) \mathbf{J}^t(\mathbf{x}_o) \Delta\theta = -\eta \frac{1 - \beta_1}{1 - \beta_1^{t+1}} \mathcal{M}^t(\mathbf{x}_o) \sum_{i=0}^t \beta_1^{t-i} \mathcal{K}_{\text{Adam}}^{t,i}(\mathbf{x}_o, \mathbf{x}_u) \mathcal{G}^i(\mathbf{x}_u, \mathbf{y}_u)$$

791 where  $\mathcal{K}_{\text{Adam}}^{t,i}(\mathbf{x}_o, \mathbf{x}_u) = \mathbf{J}^t(\mathbf{x}_o) D^{t+1} \mathbf{J}^i(\mathbf{x}_u)^\top$  is the kernel function modified by the Adam preconditioning matrix. The 792 parameter update can be written as  $\Delta\theta = -\eta D^{t+1} \hat{m}^{t+1}$ . At the same time, under our assumption (Bounded Gradients and 793 Jacobians), the  $\hat{m}$  has an upper bound, denoted as  $M_m$ , we can obtain:

$$\|\Delta\theta\|_2 \leq \eta D_{\max} M_m$$

795 So the spectral norm of the second-order term in this scenario can be estimated as:

$$\|T_2\|_2 \leq \frac{1}{2} B (D_{\max} M_m)^2 \eta^2$$

796 Thus, for the Adam optimizer, the second-order term also becomes a negligible higher-order infinitesimal as the learning 797 rate  $\eta \rightarrow 0$ .

803 Since, we provide a rigorous proof based on the Neural Tangent Kernel (NTK) regime. Intuitively, this holds because 804 edge-side model updates typically operate in a "fine-tuning" regime rather than learning from scratch.

805 In this regime, the parameter updates  $\Delta\theta$  are small. Geometrically, this means the model stays within a small region of 806 the loss landscape where the curvature (Hessian) is relatively constant. Consequently, the change in function output can 807 be well-approximated by the linear term of the Taylor expansion (the gradient projection). This explains why our simple 808 MLP-based router can accurately predict  $\Delta H$  without needing to model complex second-order training dynamics.

809 Our analysis above demonstrates that in small, fully-connected networks commonly used in practice—typically satisfying 810 bounded gradient, Jacobian, and Hessian matrices—the change in model output induced by training on a single sample is 811 dominated by its first-order MKG term, provided that a sufficiently small learning rate  $\eta$  is adopted for SGD, Adam, and 812 momentum SGD with a properly configured momentum coefficient  $\gamma$ . In particular, for small networks deployed on edge 813 devices, the learning rates we employ readily satisfy the condition that  $\eta$  is much smaller than 1, thereby ensuring that the 814 influence of second-order terms is negligible and rendering first-order MKG-based analysis highly accurate.

## 817 B. Implementation and Hardware Details

## 818 C. Detailed Experimental Setup

### 819 C.1. Dataset Specifications

820 To comprehensively validate the proposed uncertainty-based filtering framework, we employ a diverse set of benchmarks 821 covering computer vision and natural language processing.

825 **Image Classification (CIFAR & ImageNet)** We use the CIFAR-10 and CIFAR-100 datasets (Krizhevsky & Hinton,  
 826 2009), which consist of  $32 \times 32$  color images. CIFAR-10 contains 10 classes with 50,000 training and 10,000 testing images.  
 827 CIFAR-100 contains 100 classes with the same total volume. For large-scale evaluation, we utilize ImageNet-1K (ILSVRC  
 828 2012), which contains 1.28 million training images and 50,000 validation images across 1,000 categories. These datasets  
 829 allow us to evaluate the router’s performance from low-resolution coarse-grained labels to high-resolution fine-grained  
 830 classification.

831 **Object Detection (MS COCO)** The MS COCO 2017 dataset (?) is used to evaluate the framework’s extensibility  
 832 to localization tasks. It contains 118k training and 5k validation images (val2017) with 80 object categories. Unlike  
 833 classification, COCO requires the router to handle multiple bounding box outputs per image. We utilize the validation set to  
 834 simulate the re-inference process during model updates (e.g., from YOLOv8n to YOLOv8x), measuring performance via  
 835 mAP@50 and consistency via IoU.

836 **Text Classification (AG News & IMDb)** In the NLP domain, we employ AG News and IMDb. The AG News dataset  
 837 (Zhang et al., 2015) is a four-class news classification corpus containing 120,000 training and 7,600 testing samples. The  
 838 IMDb dataset (Maas et al., 2011) is a benchmark for binary sentiment analysis (positive/negative) consisting of 50,000 movie  
 839 reviews, split equally into 25,000 training and 25,000 testing samples. For these datasets, we evaluate both discriminative  
 840 models (BERT) and generative models (Qwen series) using the adaptation strategies described in Appendix D.3.

## 841 C.2. Experimental Setup and Reliability Analysis

842 **Model Selection and Ensemble Strategy** We evaluated the framework using diverse convolutional neural networks,  
 843 including standard architectures (ResNet-50, ResNet-101, VGG-16 BN) and lightweight models (MobileNetV2, Shuf-  
 844 fleNetV2). To establish a high-performance upper bound, we implemented a **Multi-Model Ensemble** that dynamically  
 845 selects the prediction from the base learner with the **highest confidence score** for each input sample, ensuring the final  
 846 output is derived from the most certain predictor.

847 **Uncertainty Quantification and Calibration** Prediction reliability is quantified via a normalized confidence score  
 848  $C(x) = 1 - H(x)/\max(H)$ , where  $H(x)$  is the Shannon entropy. To mitigate the overconfidence of deep networks, we  
 849 applied **Temperature Scaling**. The optimal temperature  $T$  is determined by minimizing the Negative Log Likelihood (NLL)  
 850 on a held-out calibration set (10% of the training data) using the **L-BFGS** optimizer. This post-hoc calibration ensures that  
 851 the calibrated probabilities  $\hat{p}_i = \sigma(\mathbf{z}/T)_i$  yield confidence scores that are better aligned with empirical accuracy.

852 **Empirical Observations** To validate the filtering mechanism, we utilized **Reliability Diagrams** to analyze the alignment  
 853 between confidence and correctness. Across all tested architectures and ensemble settings, experimental results reveal a  
 854 **strong positive correlation** between the confidence score  $C(x)$  and empirical accuracy. This fundamental observation  
 855 confirms that uncertainty-based metrics reliably serve as a proxy for prediction correctness, providing a robust empirical  
 856 foundation for our proposed router framework.

## 857 C.3. Experimental Setup for Entropy Prediction

858 **Model Framework and Calibration** Experiments were conducted on CIFAR-100 using a knowledge distillation frame-  
 859 work. We employed a ResNet-101 as the Student model, paired with two Teacher models of varying capacities: a  
 860 modified ResNet-50 (High-Capacity) and a ShuffleNetV2 (Lightweight). Prior to predictor training, all models underwent  
 861 **Temperature Scaling** to ensure their output probabilities accurately reflected prediction uncertainty.

862 **Entropy Decrease Predictor** To estimate the utility of querying a teacher without incurring its inference cost, we trained  
 863 lightweight MLP predictors (Structure: 100 → 64 → 1). These predictors take the Student’s soft probabilities as input and  
 864 regress the expected **Entropy Drop** ( $\Delta H = H_{\text{student}} - H_{\text{teacher}}$ ). The predictors were optimized using Mean Squared Error  
 865 (MSE) loss, enabling the system to forecast the potential uncertainty reduction provided by each teacher.

866 **Visualization and Validation** We validated the predictors using **t-SNE** to project the Student’s high-dimensional features  
 867 into 2D space. By mapping both the *Actual* and *Predicted* entropy drops onto this manifold using heatmaps with synchronized  
 868 color scales, we observed a high degree of structural alignment. This confirms that the predictor successfully captures the  
 869 distribution of uncertainty gains, validating its effectiveness as a decision proxy for model routing.

#### 880 C.4. Data Augmentation

881 During training, we apply a series of data augmentation techniques, including random cropping with padding, random  
 882 horizontal flipping, random rotation within  $\pm 15^\circ$ , the AutoAugment policy predefined for CIFAR-10, and random erasing  
 883 (with a probability of 0.5 and a controllable erasure area ratio). All images are normalized according to dataset-specific  
 884 statistics. During testing, only normalization and tensor conversion are applied.

#### 886 C.5. Model Architecture

888 Experiments conducted on CIFAR-100 are based on two models, referred to as the Round1 and Round2 models, simulating  
 889 the scenario of model updating. The Round1 model represents the model used after the first round of annotation, while the  
 890 Round2 model denotes the stronger model introduced during the second-round update on the edge. The Round1 model is  
 891 based on the ResNet-101 architecture, retaining the original feature extraction modules (including convolutional layers,  
 892 residual blocks, and global average pooling layers). A new linear layer is appended after the output layer to map the  
 893 features to a 100-dimensional output space suitable for CIFAR-100 classification. The Round2 model is based on a modified  
 894 ResNet-50 architecture. The initial convolutional layer is replaced with a  $3 \times 3$  convolution (stride 1, padding 1, no bias) to  
 895 better adapt to  $32 \times 32$  pixel inputs, reducing information loss during initial feature extraction. The original max pooling  
 896 layer is removed (replaced with an identity mapping) to preserve spatial resolution and low-level features. Additionally, a  
 897 dropout layer with adjustable probability is inserted before the fully connected layer to enhance generalization. Finally, the  
 898 output is mapped to a 100-dimensional space via a linear layer.

#### 900 C.6. Baselines

902 To provide a comprehensive evaluation of our proposed approach, we compare it against several representative baseline  
 903 methods, including **CEMA**, **DKD**, and **MLink**. To ensure a fair comparison and eliminate confounding factors such as  
 904 dataset distribution or filtering volume, we adopt consistent filtering ratios (20% and 50%) across all experiments and use an  
 905 identical test set for evaluation.

906 **CEMA**(Chen et al., 2024): We implement the filtering strategy proposed in the original CEMA framework, which employs  
 907 a dynamic entropy thresholding mechanism to identify unreliable high-entropy samples and a fixed threshold to detect  
 908 low-entropy, low-information samples. To adapt this method to our controlled filtering settings, we tune the dynamic entropy  
 909 threshold parameters. Specifically, we set the upper entropy threshold as  $H_{\max} = 0.4 \times \ln C$ , where  $C$  is the number of  
 910 output classes. Under a fixed filtering ratio, we solve for  $H_{\min}$  such that the total number of filtered samples matches the  
 911 predefined threshold.

913 **DKD**(Li et al., 2021): We incorporate uncertainty-aware teacher selection in the student training process. Here, the  
 914 prediction uncertainty of the round-1 model—quantified using entropy—is used to determine the sample’s learning stage.  
 915 Based on this stage, a suitable teacher model is chosen. We sort samples by their entropy scores as produced by the round-1  
 916 model, and filter the top 20% and 50% highest-entropy samples for respective experimental settings. These selected samples  
 917 are reprocessed using a round-2 model, and their output distributions are then used to update the final predictions.

918 **MLink**(Yuan et al., 2022): MLink introduces a novel semantic linking mechanism that maps the output of one model to the  
 919 corresponding output of another. In our adaptation, we train a mapping function from the predictions of the round-1 model  
 920 to those of the round-2 model for each sample  $x$ . We use the confidence score for each transformation. During inference, we  
 921 filter samples based on these confidence scores—only the top  $k\%$  highest-confidence samples are retained, ensuring the  
 922 filtering volume remains consistent with our experimental design.

#### 924 C.7. Experimental Details for Multi-Round Deployment

926 To simulate the iterative lifecycle of edge intelligence systems, we construct a progressive model pool  $\{M_0, \dots, M_4\}$   
 927 using ResNet backbones on the CIFAR-100 dataset.  $M_0$  serves as the legacy baseline, a ResNet-101 model trained to  
 928 convergence with 60.84% accuracy. Incremental updates are represented by  $M_1$  and  $M_2$  (achieving  $\sim 61\%$  and  $\sim 62\%$   
 929 accuracy, respectively), where  $M_1$  is generated by injecting controlled Gaussian noise ( $\mathcal{N}(0, 0.15 \cdot \sigma_{weights})$ ) into the  
 930 weights of  $M_2$ . To reflect major version releases or training breakthroughs, we derive  $M_3$  (80.68% accuracy) from the final  
 931 deployment target  $M_4$  (80.75% accuracy) by applying a smaller noise term of  $\mathcal{N}(0, 0.05 \cdot \sigma_{weights})$ .

932 The selection strategy is driven by a lightweight Entropy Decrease Predictor, implemented as a three-layer Multilayer  
 933

Perceptron (MLP) with 256 neurons per hidden layer and ReLU activations. This predictor takes the 100-dimensional probability distribution (soft targets) from the resident model  $M_0$  as input to estimate the expected scalar entropy reduction  $\Delta\hat{H}$  for each candidate. The predictor is optimized using the Adam optimizer ( $lr = 0.001$ , batch size 32) over 100 epochs with a Mean Squared Error (MSE) loss. Notably, this module is highly efficient for edge environments, comprising only  $\sim 0.02M$  parameters and requiring less than 15 minutes of training time on a single GPU.

Finally, the multi-stage correction mechanism is integrated as a post-processing module. The first stage involves calculating the omission risk  $\mathcal{R}_{\text{Omission}}$  for non-selected samples by normalizing historical entropy scores through a tanh activation,  $S'_{t,i} = 0.5(1 + \tanh(S_{t,i}))$ , and applying an overconfidence penalty  $\beta$  to the most recent predictions. Samples flagged as high-risk are then subjected to a majority voting ensemble composed of the current and select historical models. During this process, models are consistently operated in evaluation mode with a batch size of 32 to ensure the stability of batch normalization statistics.

## C.8. Mobile Environment Emulation Setup

To accurately evaluate the performance of our proposed method on real-world edge devices, we constructed a high-fidelity emulated mobile environment. This environment was designed to replicate the computational characteristics of an Apple iPhone 15 Pro, a representative high-end mobile platform equipped with the A17 Pro chip.

The emulation was performed on a high-performance x86-architecture host machine (PC/server) by strictly constraining its computational resources. Key parameters were aligned with the hardware specifications of the iPhone 15 Pro to ensure the validity and representativeness of our results. The specific resource constraints were configured as follows:

- Processor (CPU): The A17 Pro chip features a 6-core CPU, comprising 2 performance cores and 4 efficiency cores. To emulate this configuration, we limited the number of CPU cores available to our test process on the host machine to 6.
- Memory (RAM): The iPhone 15 Pro is equipped with 8 GB of system memory. Correspondingly, we set a memory usage limit of 8 GB for our test process to simulate the memory capacity constraints of the actual device.

To ensure consistency and reproducibility across all experimental platforms, and to specifically evaluate the computational efficiency of our algorithm on processor cores, all our experiments were exclusively executed on the CPU. No GPU or other dedicated hardware acceleration was utilized.

Through this resource-constrained, CPU-only emulation method, we were able to create a controlled testbed that closely mimics the target mobile platform (iPhone 15 Pro). The performance metrics obtained in this environment, as reported in the main paper, reliably reflect the practical execution efficiency of our algorithm when deployed on a mainstream high-end smartphone using only its CPU.

## D. More Experiment Results

### D.1. Multi-Expert Scheduling on CIFAR-100

The CIFAR-100 dataset consists of 100 fine-grained classes, which are further grouped into 20 superclasses. In this experiment, we trained three specialized models, denoted as  $f_1$ ,  $f_2$ , and  $f_3$ , each targeting a specific superclass: *flower*, *tree*, and *insect*, respectively. Each of the models achieved over 80% accuracy within its corresponding superclass while maintaining only around 20% accuracy across the remaining superclasses.

Our router operates under the multi-model routing scenario, where, under a filtering constraint, it selects the model expected to yield the greatest entropy reduction for each input sample. The Figure 8 shows that, even when dispatching among models whose average accuracy across all classes is just 20%, the router still improves upon the baseline Round-1 model. This improvement is especially significant within the targeted superclasses (e.g., *flower*, *tree*, *insect*). Due to the nature of the routing problem, we are currently unable to offer other baselines. This limitation is mainly because, under the entropy-based update strategy proposed in Task 1, the system lacks access to the ground truth and can only determine whether to perform an update, not which model among the candidates would yield the best output.

Given this, we introduce two policy strategies to benchmark routing performance in this setting: We begin by assuming that our classifier behaves as a random classifier. Specifically, for a given input from a particular superclass, the accuracy of classifier's prediction is modeled as a binary random variable taking values in  $\{0, 1\}$ , with the probability of correct

classification by model  $j$  on sample  $X_i$  denoted as  $P(F_j(X_i) = \text{Target}(X_i)) = p_i^j$ . Furthermore, we assume that the outputs of this classifier are independent and identically distributed (i.i.d.) random variables.

Based on this assumption, we propose two strategies:

**Random Policy:** For a given filtering ratio  $k$ , out of a total of  $m$  samples. Let there be  $l$  superclasses, with each superclass containing  $\frac{m}{l}$  samples. Assume that  $h_1, h_2, \dots, h_l$  samples are filtered in each superclass, and that the accuracy of the original model on each superclass is  $q_1, q_2, \dots, q_l$ . Then, based on our randomness assumption, the expected accuracy under the random policy is given by:

$$Acc_{\text{random}} = (1 - k) \cdot \left( \sum_{i=1}^l \frac{l \cdot q_i \cdot h_i}{m} \right) + \frac{k}{j} \cdot \sum_{j=1}^m \sum_{i=1}^l \left( 1 - \frac{l \cdot h_i}{m} \right) \cdot p_i^j \quad (19)$$

**Oracle Policy:** Retaining the same assumptions, we now consider an oracle mode router, which is allowed access to the ground truth labels of the input samples. However, since each model's output remains unpredictable (modeled as a random variable), the oracle can only assign the model with the highest expected accuracy to each non-filtered sample. This selection is constrained by the filtering budget, and thus, for each  $h_i$ , the allocation becomes piecewise. The expected accuracy under the oracle policy is:

$$Acc_{\text{oracle}} = (1 - k) \cdot \left( \sum_{i=1}^l \frac{l \cdot q_i \cdot h_i}{m} \right) + k \cdot \sum_{i=1}^l \left( 1 - \frac{l \cdot h_i}{m} \right) \cdot \max_{1 \leq j \leq m} p_i^j \quad (20)$$

Notably, although the strategy selects the optimal model at each step, it does not constitute a strict upper bound. The values  $h_1, h_2, \dots, h_l$  are functions of the filtering threshold. We adopt the same thresholding scheme as in the main experiment, where the threshold is varied from  $-1.5$  to  $1$  in  $5$  evenly spaced intervals. Figure 9 illustrates that as the threshold increases, the number of filtered samples exhibits a sharp decline, while our router demonstrates robust performance, nearly matching or even surpassing the oracle policy when the threshold exceeds zero.

## D.2. Multi-task Scheduling

Table 7. A comprehensive performance comparison of our proposed filtering method across multiple datasets and tasks. The table simulate our filtering method in a model update scenario ('Round1' → 'Round2'). Alongside baseline accuracies, it shows the final accuracy ('Acc.') and consistency ('Cons.') when filtering 20% and 50% of inputs. Consistency is measured against full re-inference with the 'Round2' model. For the COCO dataset, the accuracy metric (\*) is mAP@50.

	Round1		Round2		20%		50%	
	ModelType	Acc.	ModelType	Acc.	Acc.	Cons.	Acc.	Cons.
CIFAR-10	Resnet18	81.39	Resnet50	96.41	96.3	99.87	95.32	98.71
CIFAR-100	Resnet18	61.17	Resnet50	80.74	80.31	98.51	78.32	93.96
ImageNet	Resnet18	70.14	Resnet152	78.22	78.18	99.96	78.02	98.96
	EfficientNet-B0	75.48	EfficientNet-B7	84.50	84.52	99.93	84.27	98.92
	ViT-B/16	77.97	DeiT-B	82.23	82.27	99.89	81.98	98.77
AG News	Qwen1.5	64.14	Qwen3	85.01	84.98	99.96	82.72	94.01
	Qwen1.5	64.14	BERT	94.55	93.98	99.4	89.39	93.29
IMDB	Qwen1.5	51.53	Qwen3	91.56	89.25	97.59	80.65	88.80
	Qwen1.5	51.53	BERT	92.24	90.06	97.72	80.72	87.86
COCO	Yolov8n	53.9*	Yolov8x	71.5*	69.4*	93.0	65.7*	80.2

### D.2.1. TASK-SPECIFIC ADAPTATION STRATEGIES

To maintain the universality of our uncertainty-based framework across different data modalities, we developed specific strategies to extract probability distributions and estimate entropy reduction for non-standard classification tasks. For discriminative text models such as BERT, we directly utilize the outputs from the final classification head. Since these models are explicitly trained for N-way classification, their logit distributions provide a direct measure of prediction uncertainty. In contrast, for generative Large Language Models (LLMs) like Qwen, where the output is typically a sequence

of tokens rather than a fixed-class distribution, we employ a "Multiple-Choice Prompting" strategy. The model is prompted to select the most appropriate category from options (e.g., A/B/C/D), allowing us to extract the probability distribution specifically over these option tokens and treat it as the classification probability for entropy calculation. Furthermore, for object detection tasks whose outputs include bounding box coordinates and objectness scores instead of a single probability distribution, we address the challenge by using the *average confidence decrease* of detected objects as a proxy for entropy reduction. This metric serves as the core indicator of whether a model update, such as a transition from YOLOv8n to YOLOv8x, will significantly alter the prediction quality for a given frame.

#### 1052 D.2.2. EXTENDED DISCUSSION ON EXPERIMENTAL RESULTS

1053 The successful deployment of PURE across diverse tasks yields several key insights into the behavior of uncertainty-based  
1054 input filtering.

1055 Regarding high-fidelity in conservative updates, we observed that in the 20% filtering scenario, the accuracy drop is  
1056 almost negligible (e.g., 96.3% vs. 96.41% on CIFAR-10). This indicates that our router accurately identifies "stable"  
1057 samples—those that the baseline model ( $F_0$ ) already handles with high confidence and correctly aligns with the ground  
1058 truth. For these samples, the computational cost of Round 2 re-inference is redundant.

1059 In terms of graceful degradation in aggressive filtering, when the filtering ratio is increased to 50%, the system effectively  
1060 halves the re-inference workload. While a performance drop is expected, it remains within an acceptable range for edge  
1061 devices. For example, in the COCO dataset, we observed a controlled decrease in mAP@50 from 71.5% to 65.7%.  
1062 This capability is critical for battery-powered edge devices that may need to pivot to ultra-low-power modes during  
1063 charging-limited windows.

1064 Finally, the results demonstrate significant flexibility across modalities. The consistency scores across AG News and IMDb  
1065 (reaching up to 99.96%) suggest that text-based uncertainty is as predictable as visual uncertainty. This confirms that the  
1066 relationship between model confidence and accuracy, mathematically formulated in Section 3, is a fundamental property of  
1067 deep learning classifiers rather than a task-specific phenomenon.

#### 1072 D.3. Real-world Application Scenarios

1073 In this phase, we simulate the real-world scenario of a smart photo album on mobile devices. Current smart photo albums  
1074 usually possess functions such as image classification, object detection, aesthetic scoring, face detection, image depth  
1075 estimation, OCR (Optical Character Recognition), expression classification, and so on. We consider the scenario when a new  
1076 round of model iteration occurs, and re-inference needs to be performed on all photos. Our dataset is a real-world dataset  
1077 built using images and videos collected from a smartphone gallery. To standardize the data format, we first downsampled the  
1078 videos into images of the same format, resulting in a total of 2000 image samples. Afterwards, we performed re-inference  
1079 on the entire image dataset.

1080 For the task settings, both object detection and face detection rely on the similarity of bounding boxes, measured by  
1081 Intersection over Union (IoU). In cases where multiple objects are detected, we use the intersected region of the bounding  
1082 boxes as the detection area. For confidence scores, we select the lowest confidence among all bounding boxes. However,  
1083 during our experiments, we found that using either the average or the minimum confidence score leads to converging results  
1084 during training. Unlike previous experiments where a fixed filtering rate was used for updates, we adopted a new strategy:  
1085 fixing the consistency level and varying the filtering threshold. Specifically, we iteratively adjusted the threshold from higher  
1086 to lower values and identified the maximum filtering threshold that satisfies the condition. The experimental results showed  
1087 that when the threshold meets a 95% update consistency criterion, the filtering rate becomes 24.05%, meaning 481 out of  
1088 the total samples were not updated.

1089 For the text classification task, we followed a similar setup as described above. The difference lies in the fact that we first  
1090 applied OCR (Optical Character Recognition) to extract text content, resulting in 2000 text files for further processing.

1091 Regarding the scene classification task, we used the UC Merced Land Use Dataset for training and evaluation. We first  
1092 trained a ResNet model in the initial round. Then, for the second round of training, we employed a YOLO model. Since  
1093 YOLO requires bounding box annotations, we generated pseudo bounding boxes covering the entire image for each sample.  
1094 Finally, the output label for each image was determined based on the highest confidence score from the YOLO model's  
1095 predictions.

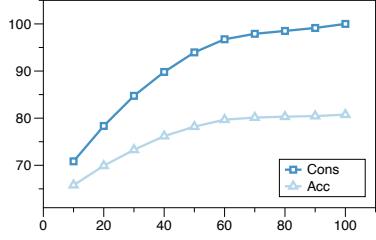


Figure 7. In CIFAR100, the changes in accuracy(Acc.) and consistency(Cons.) under different filtering ratios.

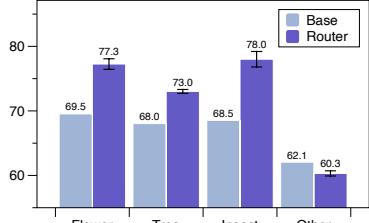


Figure 8. Comparison of the accuracy(Acc.) of the router and the base model under different super-classes.

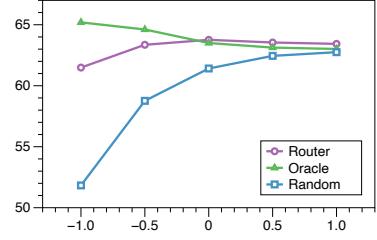


Figure 9. Comparison of the accuracy of the router and the other two policies under different thresholds

#### D.4. Hyperparameter Sensitivity Analysis

To rigorously evaluate the robustness of our multi-stage correction mechanism, we conducted a comprehensive sensitivity analysis on the CIFAR-100 dataset using 59 experimental configurations (2.4 times denser than the initial pilots). We analyzed five key hyperparameters governing the correction process: Selection Rate ( $\rho_{sel}$ ), Overconfidence Penalty ( $\beta$ ), Confidence Threshold ( $\sigma$ ), Screening Ratio ( $\rho_{screen}$ ), and Correction Ratio ( $\rho_{correct}$ ).

#### Hyperparameter Sensitivity Analysis for Multi-Round Correction Mechanism

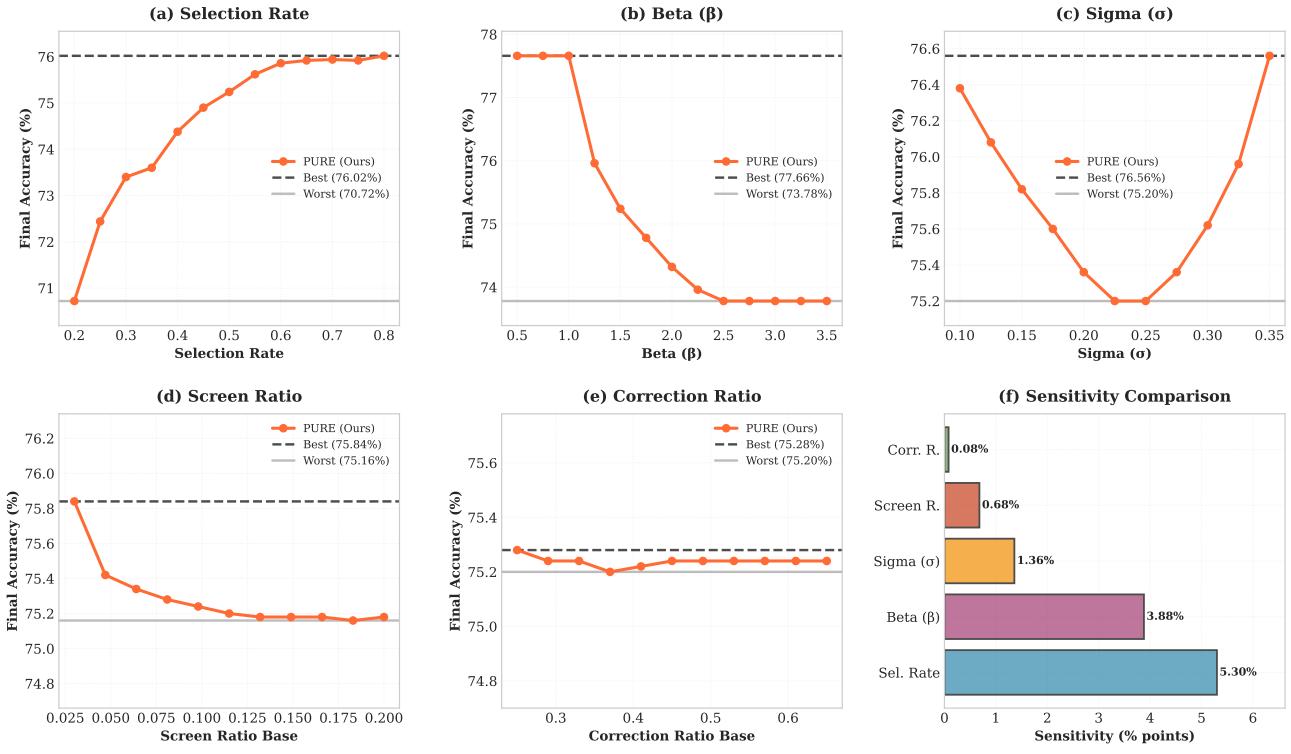


Figure 10. Comprehensive hyperparameter sensitivity analysis. (a-e) Impact of individual parameters on final accuracy (Round 4). The orange line represents our method (PURE), while the black dashed and gray solid lines indicate optimal and worst-case bounds. (f) Sensitivity comparison showing the max-min accuracy variation for each parameter.

**High Sensitivity Parameters.** As shown in Figure 10(f), the **Overconfidence Penalty** ( $\beta$ ) exhibits the highest sensitivity (3.88%). Figure 10(b) reveals a clear monotonic trend where lower  $\beta$  values (reducing penalty for high-confidence predictions) significantly improve performance, with  $\beta = 1.0$  achieving the peak accuracy of 77.66%. The **Selection Rate** (Figure 10(a)) shows a strong positive correlation with accuracy (Sensitivity 2.54%), confirming that allocating more computational budget consistently yields better performance, though 50% provides an optimal efficiency-accuracy trade-off.

1155 **Moderate Sensitivity Parameters.** The **Confidence Threshold** ( $\sigma$ ) (Figure 10(c)) demonstrates a U-shaped curve with  
 1156 optimal performance around  $\sigma = 0.15$  and  $\sigma = 0.30$ , suggesting that the mechanism effectively captures omission risks  
 1157 when the Gaussian kernel width is either very tight (focusing on high-confidence errors) or broad (capturing general  
 1158 uncertainty). The **Screening Ratio** (Figure 10(d)) shows a slight downward trend (Sensitivity 0.68%), indicating that an  
 1159 overly aggressive initial screening might introduce noise, validating our choice of a conservative 10-15% ratio.

1160 **Robustness of Correction.** Notably, the **Correction Ratio** (Figure 10(e)) exhibits negligible sensitivity (0.02%). This  
 1161 indicates that our correction mechanism is highly robust to the exact proportion of samples selected for voting correction,  
 1162 provided the initial screening effectively identifies high-risk candidates. This robustness is a desirable property for practical  
 1163 deployment, reducing the need for fine-grained parameter tuning.

1164 Table 8 summarizes the optimal configurations derived from this analysis.

1167 *Table 8. Hyperparameter Sensitivity Summary*

Parameter	Range Tested	Baseline	Optimal	Gain	Sensitivity
Beta ( $\beta$ )	1.0–3.0	1.5	1.0	+2.42%	High (3.88%)
Selection Rate	0.3–0.7	0.5	0.7	+0.70%	High (2.54%)
Sigma ( $\sigma$ )	0.15–0.30	0.22	0.15	+0.58%	Med (0.62%)
Screen Ratio	0.05–0.15	0.10	0.05	+0.16%	Med (0.22%)
Correction Ratio	0.35–0.55	0.45	0.35	+0.00%	Low (0.02%)

## D.5. Ablation Study

1178 *Table 9. Performance stability under different quantization and sparsification settings at a 50% Selection Rate.*

Spars.	50% Selection Rate Accuracy (%)				Max
	Float32	Float16	Int8	Int4	
Top-1	78.24	78.22	78.37	78.40	0.51%
Top-4	78.17	78.24	78.35	77.94	0.86%
Top-8	78.30	78.29	78.26	78.04	0.83%
Full (Ref)	78.32	78.40	78.18	77.92	0.86%
Avg.	<b>78.26</b>	<b>78.29</b>	<b>78.29</b>	<b>78.08</b>	–

1189 **Compression Strategies and Performance Evaluation** In model update scenarios, historical inference results are often  
 1190 discarded to save storage. Thus, we analyze whether our router can maintain effective scheduling after quantizing these  
 1191 distributions. Storing full-precision probability distributions imposes significant storage costs on edge-side devices: for a  
 1192 100-class problem, each sample requires 100 float32 values (400 bytes), totaling 4MB for 10,000 images. To address this,  
 1193 we design two complementary compression strategies.

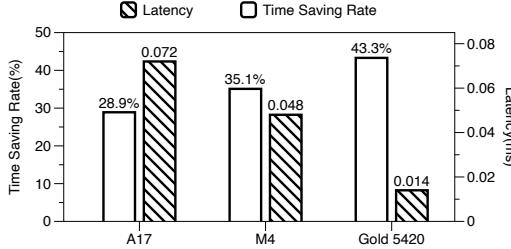
1194 First, probability precision quantization reduces storage by representing probabilities using discrete intervals of  $(1/2^q)$ .  
 1195 Specifically, we employ 4-bit quantization (0–15 levels), 8-bit quantization (0–255 levels), and 16-bit half-precision floating  
 1196 point (float16). Second, the **top-k sparsification** strategy leverages the high sparsity observed in quantized probability  
 1197 vectors—most samples with 4-bit quantization exhibit only one non-zero element. We retain the top-k highest probabilities  
 1198 ( $k=1, 4, 8$ , and the full distribution  $k=100$  as a baseline).

1200 Quantization achieves dramatic storage reduction: the per-sample space decreases from 400 bytes to 1.5 bytes (storing  
 1201 only the 7-bit index of the top-1 label), yielding a 99.5% compression ratio with less than 1% accuracy degradation. This  
 1202 trade-off between storage efficiency and prediction fidelity makes our quantization approach viable for resource-constrained  
 1203 edge devices, effectively balancing low-latency inference requirements with hardware limitations.

1204 **Training Data Scale** To further investigate the data efficiency and training overhead of our proposed router model, we  
 1205 evaluate the impact of varying training data scales on model performance and efficiency. Specifically, using the CIFAR-100  
 1206 dataset, we progressively reduced the number of samples for training the router from the full 50,000 down to 1,000. The  
 1207 results are presented in Table 10.

1210 *Table 10.* Performance and time cost under different training sample sizes on CIFAR-100. Acc/Cons denotes Accuracy/Consistency (%),  
 1211 while Pre/Train indicates the time in seconds for preprocessing and training.

Samples	50000	10000	5000	1000
Acc/Cons	78.3/94.1	78.3/94.1	78.4/94.0	70.9/82.3
Pre/Train	33.4/127.4	7.0/25.9	4.5/12.9	2.7/1.5



1227 *Figure 11.* Performance on different hardware platforms.  
 1228  
 1229  
 1230  
 1231

1232 **Data Efficiency: Effective with Minimal Data.** The results in Table 10 clearly highlight a core advantage of our router:  
 1233 its robustness to the size of the training set, enabling effective performance with only a small number of samples. The  
 1234 experiments demonstrate that even when the training data is drastically reduced by 90%, from 50,000 to 5,000 samples, the  
 1235 model's final performance (Accuracy and Consistency) remains almost unaffected, maintaining an accuracy of 78.4% and a  
 1236 consistency of 94.0%.

1237 **Lightweight Overhead: Efficient Preparation and Training.** As shown in Table 10, the time costs for both data preparation  
 1238 (Pre) and model training (Train) decrease significantly with fewer samples. This demonstrates that the entire workflow, from  
 1239 data preparation to model training, is both efficient and low-cost, enabling an agile iteration and update process for the router  
 1240 that can rapidly adapt to model changes.

1242  
 1243  
 1244  
 1245 **Deployment and Performance on Hardware Platforms** To validate the practical utility and efficiency of our proposed  
 1246 method on real-world hardware, we deployed and benchmarked our router on three representative computing platforms: a  
 1247 high-performance server (Intel Xeon Gold 5420), a modern personal computer (Mac M4), and a mobile device (simulated  
 1248 iPhone A17 Pro environment). The results are presented in Figure 11.

1249 The experimental data clearly indicate that our router model features extremely low inference latency and a minimal memory  
 1250 footprint. On all tested platforms, the latency for a single routing decision is at the sub-millisecond level (0.014ms -  
 1251 0.072ms), and the model size is merely 0.02MB.

1252 More importantly, by effectively filtering redundant inputs, our method achieves significant overall time savings across all  
 1253 platforms, ranging from 28.9% to 43.3%. We note a discrepancy between the actual time savings (e.g., 43.3% on the server)  
 1254 and the theoretical filtering rate (e.g., 50%). This gap is primarily due to the overhead introduced by the strategy used in our  
 1255 algorithm to determine the filtering threshold. Specifically, to identify the top-k% samples for re-inference, we currently  
 1256 employ a naive implementation that performs a global sort on the predicted uncertainty changes for all samples. While  
 1257 straightforward, this step itself consumes approximately 10% of the additional computation time, leading to the difference  
 1258 between theoretical and practical savings rates.

1259 We acknowledge that the top-k query algorithm in data streams has been extensively researched (Charikar et al., 2002;  
 1260 Cormode & Muthukrishnan, 2005). Integrating these advanced algorithms into our system would undoubtedly compress the  
 1261 time cost of the threshold calculation step. However, as the core contribution of this paper is the router itself, we leave this  
 1262 in-depth optimization as a valuable direction for future work.

## 1265 E. More Related Works

### 1266 E.1. Model Compression

1268 Model compression techniques have evolved from basic approaches to advanced paradigms that integrate dynamic adaptation  
 1269 and hardware awareness (Li, 2025). For instance, (Han et al., 2016) established the foundation for reducing model complexity  
 1270 by introducing “deep compression”, a three-stage pipeline that works together to reduce the storage requirement of neural  
 1271 networks by  $35\times$  to  $49\times$  without affecting accuracy. Specifically, this method first prunes the network by learning only the  
 1272 important connections, then quantizes the weights to enforce weight sharing, and finally applies Huffman coding, allowing  
 1273 the model to fit into on-chip SRAM cache rather than off-chip DRAM memory.

1274 To address efficient inference on mobile devices, (Jacob et al., 2018) proposed a quantization scheme enabling integer-only  
 1275 arithmetic, which significantly improves the tradeoff between accuracy and on-device latency. They also co-designed a  
 1276 training procedure to preserve end-to-end model accuracy post-quantization, demonstrating significant improvements even  
 1277 on efficient model families like MobileNets.

1278 More recently, specific challenges of large-scale generative models have been addressed; for example, (Frantar et al., 2022)  
 1279 developed GPTQ based on approximate second-order information, enabling the quantization of 175 billion-parameter  
 1280 models to 3 or 4 bits with negligible degradation. This method more than doubles the compression gains relative to  
 1281 previously-proposed one-shot quantization methods, allowing the execution of massive models inside a single GPU for  
 1282 the first time. Similarly, (Liu et al., 2017) focused on structural efficiency through “network slimming”, which enforces  
 1283 channel-level sparsity to automatically prune insignificant channels. Distinct from many existing approaches, this method  
 1284 requires no special software/hardware accelerators and can achieve a  $20\times$  reduction in model size for VGGNet through a  
 1285 multi-pass version.

1286 While Knowledge Distillation (KD) remains a core methodology for compressing ensemble knowledge into a single model  
 1287 (Hinton et al., 2015), recent advancements have introduced dynamic properties to this process. Inspired by active learning  
 1288 (Settles, 2009), (Li et al., 2021) explored dynamic knowledge distillation, empowering the student model to adjust the  
 1289 learning procedure—including teacher adoption and data selection—based on prediction uncertainty derived from entropy.  
 1290 Experimental results indicate that conducting KD with only 10% informative instances achieves comparable performance  
 1291 while greatly accelerating training.

1292 This shares some similarity with our research approach, but our method predicts the uncertainty reduction after executing  
 1293 another model, thereby capturing the potential relationships between two models during single-sample updates. This enables  
 1294 our method to effectively handle application scenarios that require dynamic updates among multiple candidate models. It is  
 1295 worth noting that our proposed method is orthogonal to model compression techniques and can be applied in conjunction  
 1296 with them.

### 1300 E.2. Model Selection

1301 The objective of active model selection is to estimate the best-performing model from candidates with minimal labeling.  
 1302 Addressing the high cost of labeling during evaluation, (Matsuura & Hara, 2023) proposed a variance minimization approach,  
 1303 employing an adaptive labeling strategy to estimate test loss differences with small variance. This strategy effectively  
 1304 addresses the need for a substantial amount of labeled test data, enabling the estimation of the best model using fewer  
 1305 labeling costs based on an appropriate test loss estimator. In scenarios where models cannot be compared on held-out  
 1306 training data, (Sawade et al., 2012) devised an active comparison method that selects instances according to an instrumental  
 1307 sampling distribution to maximize the power of a statistical test. By maximizing this power applied to the observed empirical  
 1308 risks, the method minimizes the likelihood of choosing the inferior model between a baseline and a challenger.

1309 However, these approaches predominantly focus on identifying a single “globally optimal” model or aggregating predictions,  
 1310 implicitly assuming uniform data distributions. A fundamental limitation persists: they lack the capability for sample-  
 1311 specific model assignment, limiting adaptability in complex real-world scenarios where candidate models exhibit specialized  
 1312 expertise in distinct data subspaces.

### 1315 E.3. Model Scheduling

1316 Model scheduling has evolved into a pivotal paradigm for optimizing multi-model system efficiency. Early systems like  
 1317 Dionysus (Jin et al., 2014) focused on network updates, dynamically scheduling based on runtime differences rather than  
 1318

1320 pre-determined schedules. By encoding the consistency-related dependencies among updates as a graph, this approach  
 1321 significantly improves update speeds in software-defined networks. Scaling this to rack-level computing, (Zhu et al., 2020)  
 1322 presented RackSched, utilizing a two-layer framework to integrate inter-server and intra-server scheduling for low tail  
 1323 latency. The inter-server scheduler in this framework employs a custom switch data plane to realize power-of-k-choices and  
 1324 ensure request affinity, tracking server loads accurately.

1325 In the context of machine learning, (Yuan et al., 2022) introduced model linking to bridge black-box models by learning  
 1326 mappings between their output spaces, enabling collaborative inference that saves computation while preserving accuracy.  
 1327 This design specifically addresses the dilemma where the cost budget (e.g., GPU memory) is insufficient to run multiple  
 1328 models, outperforming multi-task learning baselines. Furthermore, to maximize model output value under resource  
 1329 constraints, (Yuan et al., 2020) proposed an Adaptive Model Scheduling framework using deep reinforcement learning to  
 1330 mine semantic relationships and predict the value of unexecuted models.

1331 Addressing distribution shifts in cloud-edge scenarios, the Cloud Edge Elastic Model Adaptation (CEMA) paradigm (Chen  
 1332 et al., 2024) adapts edge models online by excluding unreliable and low-informative samples via entropy threshold control.  
 1333 Based on the uploaded samples, it updates and distributes the affine parameters of normalization layers by distilling from a  
 1334 stronger foundation model with a sample replay strategy.

1335 Our research extends beyond these foundations by specifically addressing the model updating scenario requiring data  
 1336 relabeling.

## F. Discussion

### F.1. Details on Multi-Round Screening Mechanism

1343 In this section, we elaborate on the mathematical formulations underpinning our multi-round screening mechanism, detailing  
 1344 the derivation and interpretation of its specific components.

#### F.1.1. ANALYSIS OF PERSISTENT SAMPLE OMISSION

1348 We begin by analyzing the mechanism underlying the persistent omissions in multi-round updates.

1349 For a given sample  $x_0$ , the mechanisms that lead to this persistent omission are categorized into two primary scenarios:

1350 **Marginal Exclusion** During the update process, certain candidate models may correctly predict with a reduced entropy  
 1351 relative to the initial model. However, the improvement score may consistently fail to surpass the Top-K threshold in any  
 1352 given round. We consider this a trivial case, as the exclusion is due to relative ranking rather than intrinsic metric failure.  
 1353 Typically, this issue mitigates naturally over sequential rounds as higher-priority samples are resolved.

1354 **The Overconfidence Trap** A more critical failure arises from our use of prediction confidence as a proxy for predictive  
 1355 accuracy. It is possible that a model yields a prediction for that is confidently wrong. Since the conditional entropy is low,  
 1356 our selection function misinterprets the prediction as accurate and excludes from the update set. Based on the mechanism  
 1357 detailed in Section 3.1, such confident errors arise from a coincidental alignment of negative interference that concentrates  
 1358 the prediction probability mass of  $x_0$  onto a single incorrect class  $y_u$ . While rare, once a model falls into this state, it almost  
 1359 inevitably leads to persistent misclassification of  $x_0$  without triggering a corrective update.

#### F.1.2. THE DERIVATION AND INTERPRETATION OF $\mathcal{R}_{\text{Omission}}$

1363 The Omission Risk Function,  $\mathcal{R}_{\text{Omission}}(x_i)$ , is designed to quantify the likelihood that a sample has been incorrectly  
 1364 excluded from updates across multiple rounds. It is composed of two terms, each targeting a specific failure mode:

$$\mathcal{R}_{\text{Omission}}(x_i) = \underbrace{\sum_{t=\tau(i)+1}^T S_{t,i}}_{\text{Term I: Marginal Exclusion}} + \underbrace{\beta \cdot \exp\left(-\frac{S_{\tau(i),i}^2}{2\sigma^2}\right)}_{\text{Term II: The Overconfidence Trap}} \quad (21)$$

1372 **Term I: Addressing Marginal Exclusion via Cumulative History.** The first term addresses the "near-miss" scenario. In a  
 1373 resource-constrained setting (e.g., top-20% filtering), many samples might benefit from an update but fall just below the  
 1374

1375 cut-off threshold. If a sample consistently ranks near the threshold (e.g., top-25%) across multiple rounds, it accumulates a  
 1376 high "marginal exclusion" score. The score  $S_{t,i}$  uses a tanh activation on the predicted entropy drop:  
 1377

$$1378 \quad S_{t,i} = \frac{1}{2} \left[ 1 + \tanh \left( \Delta \hat{H}(F_{j_t^{(i)}}(x_i)) \right) \right] \quad (22)$$

1380 We employ tanh to normalize the unbounded entropy decrease values into a bounded [0, 1] interval. Numerically, a score  
 1381  $S_{t,i} \in (0.5, 1]$  signifies a net entropy reduction (increased confidence), whereas  $S_{t,i} \in [0, 0.5)$  implies an increase in entropy.  
 1382 However, since  $F_{j_t^{(i)}}$  is explicitly selected as the candidate model exhibiting the maximum entropy drop relative to the  
 1383 global baseline  $\mathcal{F}_0$ , the effective range of the score is constrained to  $S_{t,i} \in (0.5, 1]$  in practice. This prevents a single round  
 1384 with an outlier entropy drop from dominating the entire sum, ensuring that the risk score reflects a consistent history of  
 1385 neglect rather than a one-time anomaly.  
 1386

### Term II: Detecting the Overconfidence Trap via Gaussian Kernel.

1389 Regarding "overconfidence trap", we attribute its origin to a coincidental alignment of negative interference during the  
 1390 training process. Unlike genuinely correct predictions, these errors—while appearing confident—rarely attain the distinctly  
 1391 high levels of certainty typical of valid updates. Consequently, critical attention must be directed toward samples that exhibit  
 1392 an entropy reduction (identifying them as candidates) but fail to demonstrate a significant magnitude of drop. In the design  
 1393 of Term II, this intuition is formalized such that a smaller  $S_{\tau(i),i}$  (indicating moderate confidence gain) corresponds to a  
 1394 larger omission risk  $\mathcal{R}_{\text{Omission}}$ . The derivation of the specific function for Term II is as follows:

1395 Premise: Let  $S_{\tau(i),i}$  denote the normalized entropy reduction score for sample  $x_i$ . We assume the misclassification risk  
 1396 peaks at an unknown, sample-specific ambiguity center  $\mu_i$ .  
 1397

1398 Hypothesis: The risk function  $\mathcal{R}$  follows a Gaussian decay centered at  $\mu_i$ :

$$1400 \quad \mathcal{R}(x_i) \propto \exp \left( -\frac{(S_{\tau(i),i} - \mu_i)^2}{2\sigma^2} \right) \quad (23)$$

1404 Decomposition: By expanding the exponent  $-(S_{\tau(i),i} - \mu_i)^2 = -S_{\tau(i),i}^2 + 2S_{\tau(i),i}\mu_i - \mu_i^2$ , the function factorizes into  
 1405 three distinct components:  
 1406

$$1407 \quad \mathcal{R}(x_i) = \exp \left( \frac{-S_{\tau(i),i}^2 + 2S_{\tau(i),i}\mu_i - \mu_i^2}{2\sigma^2} \right)$$

$$1410 \quad = \underbrace{\exp \left( -\frac{\mu_i^2}{2\sigma^2} \right)}_{\text{I. Sample Constant}} \cdot \underbrace{\exp \left( \frac{S_{\tau(i),i} \cdot \mu_i}{\sigma^2} \right)}_{\text{II. Linear Interaction}} \cdot \underbrace{\exp \left( -\frac{S_{\tau(i),i}^2}{2\sigma^2} \right)}_{\text{III. Quadratic Decay}} \quad (24)$$

1415 Selection:

- 1418 • Component I is a constant with respect to the current score.
- 1419
- 1420 • Component II depends on the unknown  $\mu_i$ , making it unstable for global estimation.
- 1421
- 1422 • Component III represents the universal quadratic decay of risk as confidence increases, independent of  $\mu_i$ .
- 1423

1424 Conclusion: To construct a robust metric agnostic to  $\mu_i$ , we extract Component III as the dominant factor. This yields the  
 1425 final design for Term II:  
 1426

$$1427 \quad \text{Term II} = \beta \cdot \exp \left( -\frac{S_{\tau(i),i}^2}{2\sigma^2} \right) \quad (25)$$

1430 F.1.3. THE DERIVATION AND INTERPRETATION OF  $\mathcal{R}_{\text{MISCLASS}}$ 

 1431 For the second stage, we need a robust metric to determine if a selected high-risk sample is indeed unstable. We employ the  
 1432 Jensen-Shannon (JS) divergence:  
 1433

1434  
 1435 
$$\mathcal{R}_{\text{misclass}}(x_i) = \text{JS}(p||p') = \frac{1}{2} \text{KL}(p||M) + \frac{1}{2} \text{KL}(p'||M) \quad (26)$$
  
 1436

 1437 where  $M = (p + p')/2$ . We select Jensen-Shannon (JS) divergence over alternatives such as Kullback-Leibler (KL)  
 1438 divergence or Euclidean distance primarily due to its **symmetry** and **boundedness**. Unlike KL divergence, the symmetric  
 1439 nature of JS divergence ensures that the resulting risk score remains independent of model ordering. Furthermore, since JS  
 1440 divergence is bounded between 0 and 1 (when using base-2 logarithm), it provides a stable and interpretable risk metric that  
 1441 is comparable across diverse samples and training rounds, which is essential for the implementation of effective adaptive  
 1442 thresholds.  
 1443

 1444 F.2. The Theoretical Justification for Our Routing Criterion  
 1445

 1446 To provide the theoretical justification for the validity of the conditional entropy as a routing criterion, we invoke **Fano's**  
 1447 **Inequality** (Gerchinovitz et al., 2020). This fundamental theorem in information theory bounds the classification error rate  
 1448  $P_e$  using the conditional entropy. The inequality establishes the following relationship :

1449  
 1450 
$$H(Y|X) \leq H(P_e) + P_e \log(k - 1) \quad (27)$$
  
 1451

 1452 In this formulation,  $H(P_e) = -P_e \log P_e - (1 - P_e) \log(1 - P_e)$ ,  $P_e = P(\hat{Y} \neq Y)$  represents the probability of  
 1453 classification error, where  $\hat{Y} = \arg \max_k f_k(X)$  denotes the class predicted by the mapping function  $F$ .

 1454 By combining **Fano's Inequality** (Gerchinovitz et al., 2020) with the relationship  $Acc = 1 - P_e$  and rearranging the terms,  
 1455 we derive the following equation:

1456  
 1457  
 1458 
$$G(Acc) := Acc - \frac{H(1 - Acc)}{\log(k - 1)} \leq 1 - \frac{H(Y|X)}{\log(k - 1)} \quad (28)$$
  
 1459

 1460 The derivation presented above is established based on Fano's inequality. For completeness, we provide a brief proof of this  
 1461 inequality below:

 1462  
 1463 *Proof.* Define the error indicator random variable  $E$ :

1464  
 1465  
 1466 
$$E = \begin{cases} 1, & \text{if } \hat{Y} \neq Y \\ 0, & \text{if } \hat{Y} = Y \end{cases} \quad (29)$$
  
 1467  
 1468

 1469 Applying the chain rule of entropy to  $H(E, Y|\hat{Y})$ :

1470  
 1471 
$$\begin{aligned} H(E, Y|\hat{Y}) &= H(Y|\hat{Y}) + H(E|Y, \hat{Y}) \\ &= H(E|\hat{Y}) + H(Y|E, \hat{Y}) \end{aligned} \quad (30)$$
  
 1472  
 1473

1474 Noting that:

1475  
 1476 
$$H(E|Y, \hat{Y}) = 0 \quad \text{and} \quad H(E|\hat{Y}) \leq H(E) = H(P_e) \quad (31)$$
  
 1477

 1478 where  $H(P_e)$  is the binary entropy function.

 1479 Decompose  $H(Y|E, \hat{Y})$  using conditional probabilities:

1480  
 1481 
$$\begin{aligned} H(Y|E, \hat{Y}) &= (1 - P_e)H(Y|\hat{Y}, E = 0) + P_eH(Y|\hat{Y}, E = 1) \\ &\leq (1 - P_e) \cdot 0 + P_e \log(K - 1) \\ &= P_e \log(K - 1) \end{aligned} \quad (32)$$
  
 1482  
 1483  
 1484

1485 Note: When  $E = 0$ ,  $Y$  is determined by  $\hat{Y}$ . When  $E = 1$ ,  $Y$  can be any of the remaining  $K - 1$  classes.

1486 Combining equations (30) through (5):

$$1488 \quad H(Y|\hat{Y}) \leq H(P_e) + P_e \log(K - 1) \quad (33)$$

1490 Given the Markov chain  $X \rightarrow Y \rightarrow \hat{Y}$  and the data processing inequality:

$$1491 \quad I(X; Y) \geq I(X; \hat{Y}) \implies H(Y|X) \leq H(Y|\hat{Y}) \quad (34)$$

1493 (Assuming  $\hat{Y}$  is a deterministic function of  $X$  or dependent via the chain, the conditional entropy reduces).

1494 Substituting (7) into (6) completes the proof:

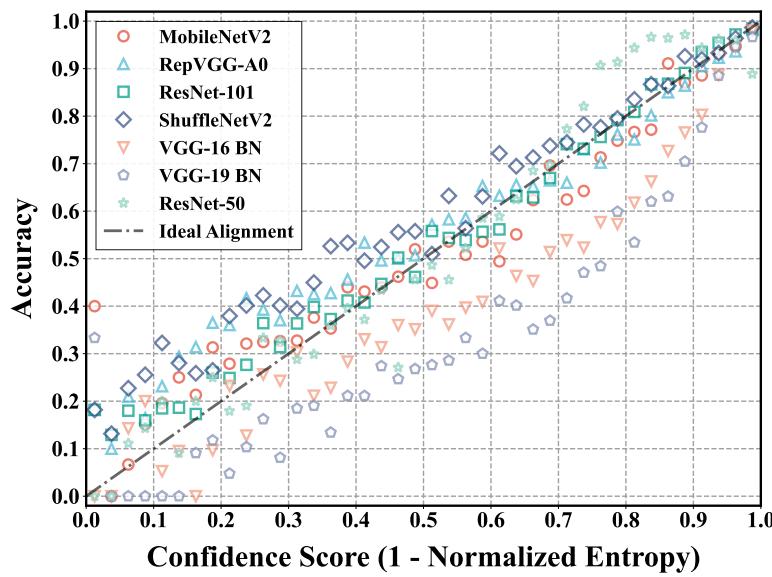
$$1496 \quad H(Y|X) \leq H(P_e) + P_e \log(K - 1) \quad (35)$$

1498  $\square$

### F.3. Discussions and Qualitative Analysis

#### F.3.1. THE “CONFIDENT BUT WRONG” PARADOX

1503 A fundamental critique of uncertainty-based methods is the “Confident but Wrong” problem: a model may predict an  
1504 incorrect class with very high probability (low entropy). In such cases, our entropy-reduction router might predict a near-zero  
1505  $\Delta H$ , causing the sample to be skipped.



1523 Figure 12. Reliability Diagram of the base model. The close alignment between the bar height (accuracy) and the diagonal line (perfect  
1524 calibration) indicates that confidence is a reliable proxy for correctness in our setting.

1525 As shown in the Reliability Diagram (Figure 12), our base models exhibit good calibration, meaning that high confidence  
1526 generally correlates with high accuracy. Statistically, “confident errors” constitute a small minority of the data distribution  
1527 (typically < 1% in our experiments). However, in a multi-round setting, these rare errors can accumulate. A 1% error rate  
1528 that persists over 10 rounds becomes significant. This provides the theoretical motivation for our Multi-Stage Correction  
1529 mechanism. By explicitly modeling the “Overconfidence Trap” (Term II in Eq. 15), we treat these low-entropy samples  
1530 not as “safe”, but as “potentially stagnant”, ensuring they are eventually subjected to the more expensive but accurate  
1531 multi-model voting process.

#### F.3.2. WHY LEARNED ROUTER SUPERIOR TO HEURISTIC ENTROPY?

1537 One might ask: why train a neural router instead of simply selecting samples with the highest current entropy? Our results  
1538 show the learned router consistently outperforms heuristic baselines (Table 1, +5-6% accuracy gain on CIFAR-100). This  
1539

1540 superiority stems from two key factors:

1541 **1. Resolving the "One-to-Many" Mapping Problem.** In a multi-expert setting (as in Section 5.3), a high-entropy sample  
 1542 could potentially be improved by *any* of the available experts. A simple entropy heuristic tells us *that* the sample is uncertain,  
 1543 but not *who* can fix it. Our router takes the input sample and learns a mapping to the *expected entropy drop vector* for all  
 1544 candidates:  $f(x) \rightarrow [\Delta H_1, \Delta H_2, \dots, \Delta H_k]$ . This allows for targeted routing—sending “flower-like” ambiguous images to  
 1545 the “flower expert”—which is impossible with scalar entropy sorting.

1546 **2. Learning “Correctability” vs. “Aleatoric Uncertainty”.** Not all high-entropy samples are fixable. Some images are  
 1547 inherently noisy or ambiguous (Aleatoric Uncertainty) and will remain uncertain even for a perfect model. A heuristic strategy  
 1548 would waste budget repeatedly updating these hopeless samples. In contrast, our router is trained on  $\Delta H = H_{old} - H_{new}$ .  
 1549 If  $H_{new}$  remains high (unfixable noise), then  $\Delta H \approx 0$ . Thus, the router implicitly learns to distinguish between “epistemic  
 1550 uncertainty” (fixable by better models) and “aleatoric uncertainty” (noise), focusing resources only on the former.

## G. Supplementary Data

1555 Table 11 presents the data and error analysis corresponding to Figure 7.

1556     Table 11. Performance comparison between Accuracy (Acc) and Consistency (Cons) at different sampling ratios.

Ratio	Acc	Cons
10	$65.79 \pm 0.04$	$70.84 \pm 0.02$
20	$69.90 \pm 0.06$	$78.35 \pm 0.05$
30	$73.28 \pm 0.04$	$84.74 \pm 0.00$
40	$76.19 \pm 0.04$	$89.81 \pm 0.10$
50	$78.32 \pm 0.08$	$93.96 \pm 0.02$
60	$79.68 \pm 0.13$	$96.74 \pm 0.14$
70	$80.13 \pm 0.03$	$97.90 \pm 0.09$
80	$80.31 \pm 0.03$	$98.51 \pm 0.03$
90	$80.45 \pm 0.04$	$99.14 \pm 0.08$
100	$80.74 \pm 0.00$	$100.00 \pm 0.00$