Physics 2 Reference

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Contents

Miscellaneous

SI Prefixes

Prefix	Symbol	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}

Prefix	Symbol	Value
deka	da	10^1
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

Units

$$\mathrm{Pa} = \mathrm{N/m^2}$$

$$\mathrm{W} = \mathrm{J/s} = \mathrm{N \cdot m/s}$$

$$\mathrm{J} = \mathrm{N \cdot m}$$

$$\mathrm{N} = \mathrm{kg \cdot m/s^2}$$

$$\mathrm{V} = \mathrm{J/C}$$

Unit Conversions

$$1\ \mathrm{cal} = 4.184\ \mathrm{J}$$

$$^{\circ}\mathrm{F} = \frac{9}{5}{}^{\circ}\mathrm{C} + 32$$

$$\mathrm{K} = {}^{\circ}\mathrm{C} + 273.15$$

Vectors

A vector on a Cartesian chart

Figure 1: A vector on a Cartesian chart

$$\vec{r} = (r, \theta)$$

$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2}$$

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right)$$

$$r_x = r\cos\theta$$

$$r_y = r\sin\theta$$

$\vec{r}$	vector
r	magnitude (length) of $\vec{r}$
θ	direction (angle relative to the horizontal)
r_x	x-component of $\vec{r}$
r_y	y-component of $\vec{r}$

- A vector in general is a quantity that is made up of 2 scalar quantities, magnitude and direction.
- In physics, vectors are represented by arrows. The length of the arrow represents the magnitude of the vector and the direction of the arrow represents the direction of the vector.
- The vectors $\vec{r}$ and $-\vec{r}$ have the same magnitude but opposite directions.

Unit Vectors

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{r} = r_x\hat{i} \implies \text{1 dimension}$$

$$\vec{r} = r_x\hat{i} + r_y\hat{j} \implies \text{2 dimensions}$$

$$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k} \implies 3 \text{ dimensions}$$

$\hat{r}$	unit vector of $\vec{r}$
$\vec{r}$	vector
$\lvert\vec{r}\rvert$	magnitude of $\vec{r}$
r_x	x-component of $\vec{r}$
r_y	y-component of $\vec{r}$
$\hat{i}$	unit vector in the x direction
$\hat{j}$	unit vector in the y direction
$\hat{k}$	unit vector in the z direction

- A unit vector of a vector $\vec{r}$ is a vector in the same direction as $\vec{r}$ with a magnitude of 1.
- In the equations, $\hat{i}$ and $\hat{j}$ give a direction to r_x and r_y transforming them into vectors.

Vector Arithmetic

Scalar Multiplication

$$a\vec{r} = (ar, \theta)$$

a	scalar
$\vec{r}$	vector
r	magnitude of $\vec{r}$ (scalar)
θ	direction of $a\vec{r}$ (relative to the horizontal)

Scalar multiplication of a vector $\vec{r}$ by a scalar a is a vector in the same direction as $\vec{r}$ with a magnitude of ar.

Addition & Subtraction

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$

$\vec{A}$, $\vec{B}$	vectors
A_x, B_x	x-components of $\vec{A}$, $\vec{B}$
A_y, B_y	y-components of $\vec{A}$, $\vec{B}$
$\hat{i}$, $\hat{j}$	unit vector in the x, y direction (*See*: **Unit Vectors**)

On a graph, if you connect the vectors $\vec{A}$ and $\vec{B}$ head to tail, the vector from the tail of $\vec{A}$ to the head of $\vec{B}$ is the sum of $\vec{A}$ and $\vec{B}$.

Dot Product

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\theta = 0° \implies \vec{A} \parallel \vec{B} \implies \vec{A} \cdot \vec{B} = AB$$

$$\theta = 90° \implies \vec{A} \perp \vec{B} \implies \vec{A} \cdot \vec{B} = 0$$

$$\theta = 180° \implies \vec{A} \parallel \vec{B} \text{ (anti-parallel)} \implies \vec{A} \cdot \vec{B} = -AB$$

$\vec{A}$, $\vec{B}$	vectors
A, B	magnitudes of $\vec{A}$, $\vec{B}$
θ	angle between $\vec{A}$, $\vec{B}$
$\hat{\imath}$, $\hat{\jmath}$	unit vector in the x, y direction (*See*: **Unit Vectors**)

The dot product of two vectors $\vec{A}$ and $\vec{B}$ is a scalar.

Cross Product

$$\vec{A} \times \vec{B} = AB\sin\theta\ \hat{n}$$

$$\hat{\jmath} \times \hat{\imath} = -\hat{k}$$

$$\hat{k} \times \hat{\jmath} = -\hat{\imath}$$

$$\hat{\imath} \times \hat{k} = -\hat{\jmath}$$

$\vec{A}, \vec{B}$	vectors to be multiplied
A, B	magnitudes of the vectors
θ	angle between the vectors
$\hat{n}$	unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$

For direction of $\hat{n}$ use RHR (*See*: **Right Hand Rule**).

RHR Diagram

Figure 2: RHR Diagram

Right Hand Rule

- Direction of rotation is determined by the position of the thumb
  - If the thumb points in the direction of the axis of rotation (**typically upward**), then the direction is positive
  - If the thumb points in the opposite direction of the axis of rotation (**typically downward**), then the direction is negative

Gradient (Vector)

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \text{ (Cartesian)}$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \text{ (Spherical)}$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z} \text{ (Cylindrical)}$$

- **Note**: ∂ is a partial derivative
  - $\frac{\partial}{\partial x}$ means take the derivative with respect to x while holding all other variables constant.

Integration

$$\int_{x_1}^{x_2} x^n dx = \frac{x^{n+1}}{n+1}\Big|_{x_1}^{x_2} = \frac{x_2^{n+1}}{n+1} - \frac{x_1^{n+1}}{n+1}$$

x_1	lower limit of integration
x_2	upper limit of integration
n	power of x

Thermodynamics

Motion of heat

Temperature

SI unit: K (Kelvin)

Temperature $\neq$ Heat

Note: Temperature is a macroscopic property that is a consequence of microscopic motion.

Temperature Scales

See: **Unit Conversions**

Celsius (°C)	Water freezes at 0°C and boils at 100°C
Fahrenheit (°F)	Human body temperature is 98.6°F If it rises above 100°F it is considered a fever.
Kelvin (K)	Absolute zero is 0K

Types of Heat Transfer

- **Conduction**: Heat transfer through direct contact
  - *e.g.* Touching a hot stove
  - *Requires* free electrons (e.g. metals)
- **Convection**: Heat transfer through fluid motion
  - *e.g.* Boiling water, wind, convection oven/air fryer
  - *Requires* fluid motion (e.g. gas/air, liquid/water)
- **Radiation**: Heat transfer through electromagnetic waves
  - *e.g.* Sunlight, microwave, fire
  - *Requires* no medium

Heat Transfer Rate

SI unit: W (Watts)

Depends on the type of heat (*See*: **Types of Heat**)

$$H = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

ΔQ	Change in heat (*See*: **Heat Exchange**)
Δt	Time interval
$\frac{dQ}{dt}$	Derivative of heat with respect to time

Conduction

$$H = k\frac{A}{\Delta x}\Delta T$$

$$\mathcal{R} = \frac{\Delta x}{k} \implies H = \frac{A\Delta T}{\mathcal{R}}$$

$$R = \frac{\mathcal{R}}{A} \implies H = \frac{\Delta T}{R}$$

k	Thermal conductivity (*See*: **Table 1**)
A	Surface Area
Δx	Thickness of material
ΔT	Change in temperature ($T_f - T_i$)
$\mathcal{R}$	R-Factor (Resistance Factor)
R	Thermal Resistance

Table 1: Thermal Conductivity (k)

Material	k (W/m $\cdot$ K)	k (Btu $\cdot$ in/h $\cdot$ ft^2 $\cdot$ °F)
Air	0.026	0.18
Aluminum	237	1644
Concrete (varies)	1	7
Copper	401	2780
Fiberglass	0.042	0.29
Glass	$0.7 \to 0.9$	$5 \to 6$
Goose down	0.043	0.30
Helium	0.14	0.97
Iron	80.4	558
Steel	46	319
Styrofoam	0.029	0.20
Water	0.61	4.3
Wood (pine)	0.11	0.78

Radiation

$$P = e\sigma AT^4$$

P	Power (*See*: **Power (Physics 1 Reference)**)

e	Emissivity (between 0 and 1)
σ	Stefan-Boltzmann constant ($5.67 \times 10^{-8} \mathrm{W/m^2 \cdot K^4}$)
A	Surface area
T	Temperature of object (in Kelvin)

Heat Capacity

SI unit: J/K (Joules per Kelvin)

$$C = \frac{\Delta Q}{\Delta T}$$

ΔQ	Change in heat
ΔT	Change in temperature ($T_f - T_i$)

Specific Heat

SI unit: $\mathrm{J/kg \cdot K}$

Number of joules required to raise the temperature of one kilogram of a substance by one kelvin.

$$c = \frac{C}{m}$$

C	Heat capacity (*See*: **Heat Capacity**)
m	Mass

Table 2: Specific Heat (c)

Substance	c ($\mathrm{J/kg \cdot K}$)
Aluminum	900
Concrete	880
Copper	386
Iron	447
Glass	753
Mercury	140
Steel	502
Stone (granite)	840
Water	4184
Ice (−10°C)	2090

Substance	c (J/kg $\cdot$ K)
Wood	1400

Temperature range $0 \to 100$°C (unless otherwise specified)

Heat Exchange

SI unit: J (Joules)

$$\Delta Q = mc\Delta T$$

m	Mass
c	Specific heat (*See*: **Specific Heat**)
ΔT	Change in temperature $(T_f - T_i)$

Heat Balance

$$\Delta Q_1 + \Delta Q_2 = 0$$

ΔQ_1	Heat change in object 1 (*See*: **Heat Exchange**)
ΔQ_2	Heat change in object 2 (*See*: **Heat Exchange**)

Note: Objects 1 and 2 must be in thermal contact with each other, and thermally isolated from the rest of the universe.

Heat of Transformation

SI unit: J (Joules)

$$Q = mL$$

Q	Heat needed to vaporize/melt m kg of substance
m	Mass
L	Latent heat

Table 3: Latent Heat (L)

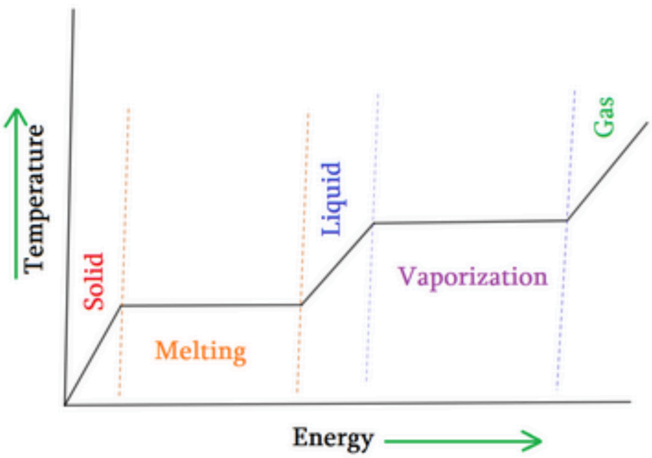


Figure 3: Figure 1: Temperature vs Energy. The long horizontal segments indicate much energy is being used to melt/vaporize

Substance	Melting Point (K)	L_f (kJ/kg)	Boiling Point (K)	L_v (kJ/kg)
Ethyl alcohol	159	109	351	879
Copper	1357	205	2840	4726
Lead	601	24.7	2013	858
Mercury	234	11.3	630	296
Oxygen	54.8	14.8	90.2	213
Sulfur	388	53.6	718	306
Water	273.15	334	373.15	2257
Uranium dioxide	3120	259	3815	1533

Ideal Gas Law

$$PV = kNT = nRT$$

P	Pressure
V	Volume
k	Boltzmann constant (1.38×10^{-23}J/K)
N	Number of molecules
n	Number of moles ($n = \frac{N}{N_A}$)
R	Ideal gas constant ($R = k \cdot N_A = 8.31$J/mol $\cdot$ K)
T	Temperature

N_A is Avogadro's number (6.02×10^{23})

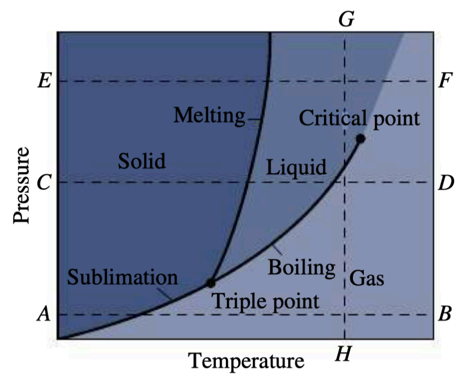


Figure 4: Figure 2: A phase diagram showing solid, liquid, and gas phases on a plot of pressure versus temperature.

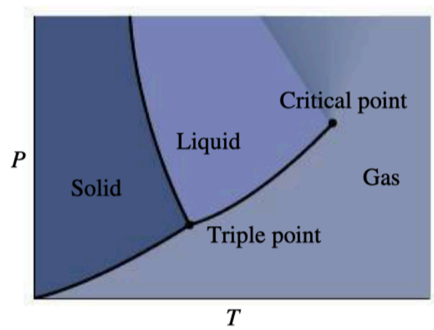


Figure 5: Figure 3: Phase diagram for water. Compare the solid–liquid boundary with that of Fig. 2

Thermal Expansion

SI unit: K^{-1} (Kelvin)

Measures how much an object expands when heated

Coefficient of *Linear* Expansion

$$\alpha = \frac{\Delta L/L}{\Delta T}$$

α	Coefficient of linear expansion (*See*: **Table 4**)
ΔL	Change in length
L	Original length
ΔT	Change in temperature ($T_f - T_i$)

Coefficient of *Volume* Expansion

$$\beta = \frac{\Delta V / V}{\Delta T}$$

β	Coefficient of volume expansion (*See*: **Table 4**)
ΔV	Change in volume
V	Original volume
ΔT	Change in temperature ($T_f - T_i$)

Table 4: α and β

Solids	α (K^{-1})	Liquids	β (K^{-1})
Aluminum	2.4×10^{-5}	Air	3.7×10^{-3}
Brass	1.9×10^{-5}	Ethyl alcohol	7.5×10^{-4}
Copper	1.7×10^{-5}	Gasoline	9.5×10^{-4}
Pyrex glass	3.2×10^{-6}	Mercury	1.8×10^{-4}
Ice	5.1×10^{-5}	Water ($1°C$)	-4.8×10^{-5}
Invar	0.9×10^{-6}	Water ($20°C$)	2.0×10^{-4}
Steel	1.2×10^{-5}	Waterm ($50°C$)	5.0×10^{-4}

$\approx$ Room temperatrue (unless otherwise specified)

Invar is designed to have a very low coefficient of linear expansion

Temperature and Molecular Kinetic Energy

$$\frac{1}{2}m\bar{v^2} = \frac{3}{2}kT$$

m	Mass
$\bar{v^2}$	Average squared velocity
k	Boltzmann constant (1.38×10^{-23}J/K)
T	Temperature

$\frac{1}{2}m\bar{v}^2$	Average kinetic energy of a molecule (*See*: **Kinetic Energy (Physics 1 Reference)**)

Internal Energy

SI unit: J (Joules)

First Law of Thermodynamics

$$\Delta E_{int} = Q + W$$

ΔE_{int}	Change in internal energy
Q	Heat transferred to the system (*See*: **Heat Exchange**)
W	Work done on the system (*See*: **Work (Physics 1 Reference)**)

- Positive Q means a net heat input to the system
- Negative Q means heat leaves the system
- Positive W means work is done on the system
- Negative W means the system does work on its surroundings

Work in terms of Pressure and Volume

SI unit: J (Joules)

$$W = -\int_{V_1}^{V_2} PAdx = -\int_{V_1}^{V_2} PdV$$

$$W = Q_h - Q_c$$

Note: Must be in a closed system.

W	Work (*See*: **Work (Physics 1 Reference)**)
P	Pressure
A	Area
V_1	Initial volume
V_2	Final volume
Q_h	Heat added to the system (*See*: **Heat Exchange**)
Q_c	Heat removed from the system (*See*: **Heat Exchange**)

Work done on the system is negative

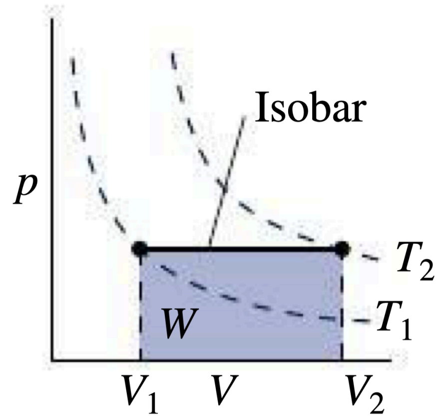


Figure 6: Isoparic P-V Diagram

Isobaric Process

Constant pressure

$$P = \text{constant}$$

$$Q = \Delta E_{int} - W = nC_p \Delta T$$

$$W = P\Delta V$$

$$\Delta S_{12} = nC_p \ln \frac{T_2}{T_1}$$

W	Work (*See*: **Work (Physics 1 Reference)**)
n	Number of moles
C_p, C_v	Molar specific heat at constant pressure/volume (*See*: **Degrees of Freedom and Specific Heat**)
ΔT	Change in temperature ($T_f - T_i$)
P	Pressure
ΔV	Change in volume $V_f - V_i$
R	Ideal gas constant ($R = k \cdot N_A = 8.31\text{J/mol} \cdot \text{K}$)
m	Mass
ΔS_{12}	Change in entropy from state 1 to state 2 (*See*: **Entropy**)
T_1, T_2	Temperatures at states 1 and 2

Isovolumic Process

Constant volume

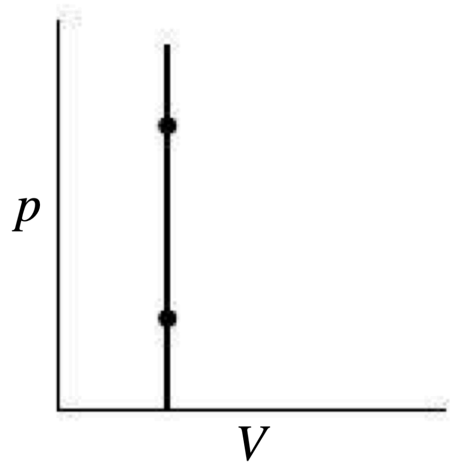


Figure 7: Isovolumic P-V Diagram

Same as isochoric & isometric process

$$V = \text{constant}$$

$$W = 0$$

$$Q = \Delta E_{int} = nC_v \Delta T$$

$$\Delta S_{12} = nC_v \ln \frac{T_2}{T_1}$$

W	Work (*See*: **Work (Physics 1 Reference)**)
ΔE_{int}	Change in internal energy (*See*: **Newtons First Law of Thermodynamics**)
Q	Heat (*See*: **Heat Exchange**)
n	Number of moles
C_v	Molar specific heat at constant volume (*See*: **Degrees of Freedom and Specific Heat**)
ΔT	Change in temperature $(T_f - T_i)$
ΔS_{12}	Change in entropy from state 1 to state 2 (*See*: **Entropy**)

T_1, T_2	Temperatures at states 1 and 2

Isothermal Process

Constant temperature

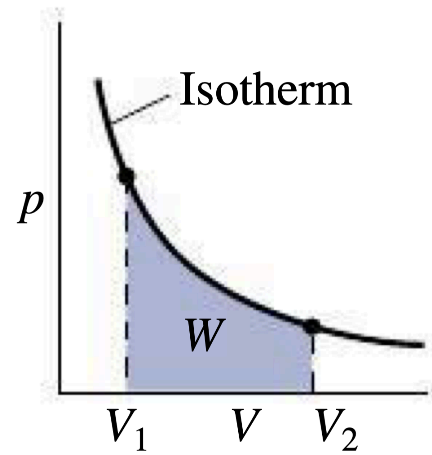


Figure 8: Isotherm P-V Diagram

$$T = \text{constant}$$

$$\Delta E_{int} = 0 \implies Q = -W$$

$$W = -nRT \ln \frac{V_2}{V_1}$$

$$PV = \text{constant}$$

$$\Delta S_{12} = \frac{\Delta Q_{12}}{T}$$

W	Work (*See*: **Work (Physics 1 Reference)**)
n	Number of moles
R	Ideal gas constant ($R = k \cdot N_A = 8.31\text{J/mol} \cdot \text{K}$)
T	Temperature

V_1	Initial volume
V_2	Final volume
ΔE_{int}	Change in internal energy (*See*: **First Law of Thermodynamics**)
Q	Heat (*See*: **Heat Exchange**)
ΔS_{12}	Change in entropy from state 1 to state 2 (*See*: **Entropy**)
ΔQ_{12}	Change in heat from state 1 to state 2 (*See*: **Heat Exchange**)

Adiabatic Process

No heat exchange

$$Q = 0 \text{ (No heat exchange)}$$

$$\Delta E_{int} = W = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$$

$$PV^{\gamma} = \text{constant}$$

$$TV^{\gamma-1} = \text{constant}$$

$$\gamma = \frac{C_p}{C_v}$$

$$\Delta S = 0$$

W	Work (*See*: **Work (Physics 1 Reference)**)
ΔE_{int}	Change in internal energy (*See*: **First Law of Thermodynamics**)
P_1	Initial pressure
P_2	Final pressure
V_1	Initial volume
V_2	Final volume
γ	Ratio of specific heats ($\gamma = \frac{C_p}{C_v}$)
T	Temperature
C_p, C_v	Molar specific heat at constant pressure/volume (*See*: **Degrees of Freedom and Specific Heat**)
ΔS	Change in entropy (*See*: **Entropy**)

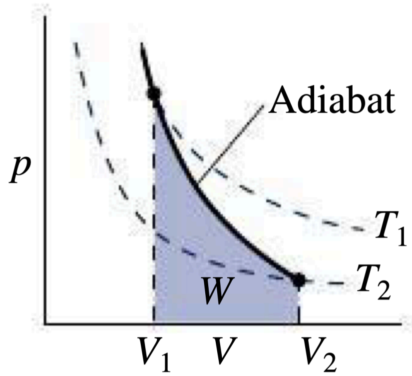


Figure 9: Adiabat P-V Diagram

Degrees of Freedom and Specific Heat

$$C_p = C_v + R$$

> **Note**: The number of **degrees of freedom** is the number of independent ways a molecule can store energy.

Monatomic Gas

$$C_v = \frac{3}{2}R$$

$$\gamma = \frac{5}{3}$$

Diatomic Gas

$$C_v = \frac{5}{2}R$$

$$\gamma = \frac{7}{5}$$

Triatomic Gas

$$C_v = \frac{7}{2}R$$

R	Ideal gas constant ($R = k \cdot N_A = 8.31\text{J/mol} \cdot \text{K}$)
C_v	Molar specific heat at constant volume
γ	Ratio of specific heats ($\gamma = \frac{C_p}{C_v}$), used in Adiabatic Process (*See*: **Adiabatic Process**)

Entropy

SI unit: J/K (Joules per Kelvin)

- Measure of disorder
- Systems tend to move towards a state of maximum entropy
- Higher entropy means more energy is unable to do work

$$\Delta S_{12} = \int_1^2 \frac{dQ}{T}$$

$$E_{\text{unavailable}} = T_{\text{min}} \Delta S$$

ΔS	Change in entropy from state 1 to state 2
ΔQ	Change in heat from state 1 to state 2 (*See*: **Heat Exchange**)
T	Temperature
$E_{\text{unavailible}}$	Energy that becomes unavailable
T_{min}	Minimum temperature

2nd Law of Thermodynamics

Kelvin–Planck statement It is impossible to construct a heat engine operating in a cycle that extracts heat from a reservoir and delivers an equal amount of work (has 100% efficiency).

Clausius statement It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body.

Heat Engines

Note: Heat engines convert heat into work

$$e = \frac{W}{Q_h} = 1 - \frac{|Q_c|}{|Q_h|}$$

e	Efficiency
W	Work done *by* the engine ($W = Q_h - Q_c$) (*See*: **Work (Physics 1 Reference)**)
Q_h	Heat added to the system (*See*: **Heat Exchange**)
Q_c	Heat removed from the system (*See*: **Heat Exchange**)

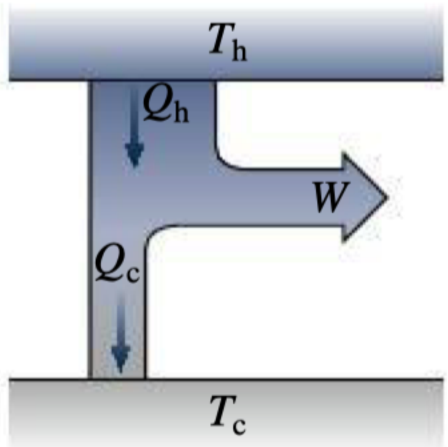


Figure 10: Energy-flow diagram for a heat engine

Carnot Engine

Note: The Carnot engine is the most efficient heat engine possible it has the **Maximum Efficiency** of any heat engine.

$$e = 1 - \frac{T_c}{T_h}$$

Coefficient of Performance

$$\text{COP} = \frac{\text{what we want}}{\text{what we put in}}$$

$$\text{COP}_{\text{refrigerator}} = \frac{T_c}{T_h - T_c} = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c}$$

$$\text{COP}_{\text{heat pump}} = \frac{T_h}{T_h - T_c} = \frac{Q_h}{W} = \frac{Q_h}{Q_h - Q_c}$$

COP	Coefficient of performance
T_c	Temperature of the cold reservoir
T_h	Temperature of the hot reservoir
Q_h	Heat added to the system (*See*: **Heat Exchange**)
Q_c	Heat removed from the system (*See*: **Heat Exchange**)

W	Work done *by* the engine ($W = Q_h - Q_c$) (*See*: **Work (Physics 1 Reference)**)

Electricity and Magnetism

Electric Field

SI unit: N/C (Newtons per Coulomb)

$$\vec{F} = q\vec{E}$$

$$e = -q_e = 1.6 \times 10^{-19}\text{C}$$

$$\vec{E} = \vec{\nabla}V$$

$\vec{F}$	Force exerted on q by $\vec{E}$
q	Charge
$\vec{E}$	Electric field
e	Smallest possible charge
q_e	Charge of an electron
$\vec{\nabla}$	Gradient operator (vector) (See: **Gradient (Vector)**)
V	Electric potential

Recall: Newton's Second Law: $\vec{F} = m\vec{a}$

Like charges repel, opposite charges attract.

Coulomb's Field (Electric Field of a Point Charge)

$$\vec{E} = \frac{kQ}{r^2}\hat{r}$$

$\vec{E}$	Electric field
k	Coulomb's constant ($k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9\text{N} \cdot \text{m}^2/\text{C}^2$)
Q	Charge
r	Distance from host charge
$\hat{r}$	Radial unit vector (pointing from host charge to test charge)

Can also be used for far-field approximation of a dipole or line of charge.

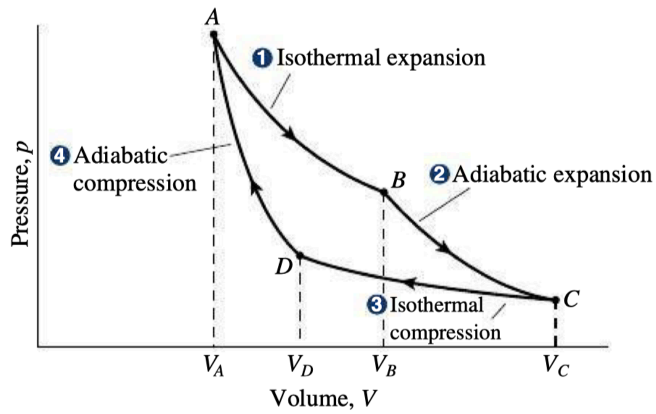
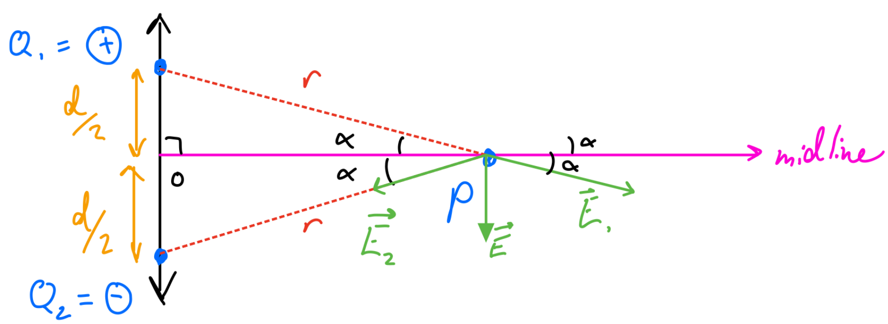


Figure 11: P-V Diagram for a Carnot Engine

Electric Field of a Dipole

Dipole: Two equal and opposite charges separated by a distance d



$$\vec{E} = -\frac{kp}{r^3}\hat{j}$$

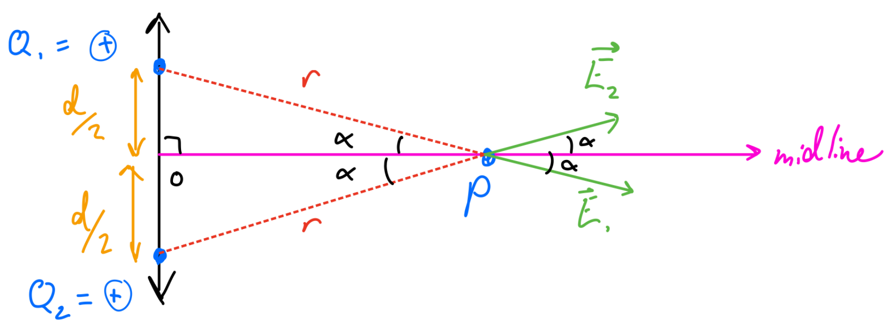
$$r = \sqrt{x^2 + \frac{d^2}{4}}$$

$\vec{E}$	Electric field
k	Coulomb's constant ($k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$)
p	Dipole moment ($p = Qd$)
r	Distance from each charge in dipole to P

x	Distance from dipole to P along the axis of the dipole
d	Distance between charges in the dipole
$\hat{j}$	Unit vector pointing from negative to positive charge
Q	Magnitude of each charge in the dipole

Electric Field of 2 Positive Charges

Charges have equal magnitude Q and are separated by a distance d



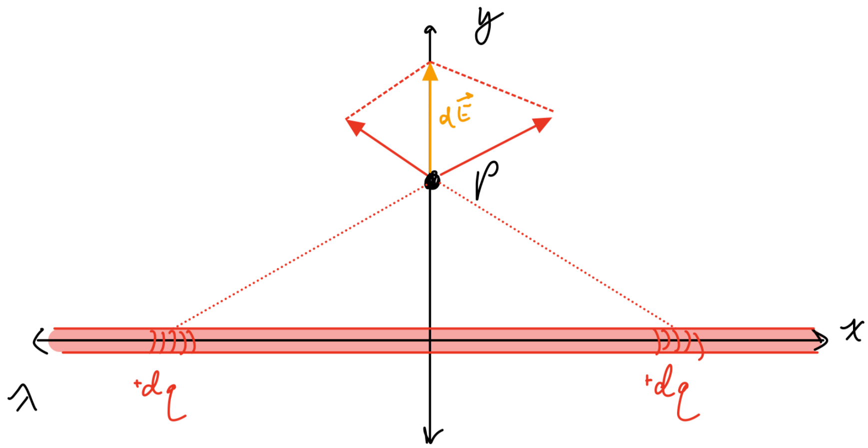
$$\vec{E} = \frac{2kQx}{r^3}\hat{i}$$

$$r = \sqrt{x^2 + \frac{d^2}{4}}$$

$\vec{E}$	Electric field at point $P = (x, 0)$
k	Coulomb's constant ($k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$)
Q	Magnitude of each charge
r	Distance from each charge to P
x	Distance from charges along the axis to P
d	Distance between charges

Electric Field of a Line of Charge

Line is infinite and has even charge density λ



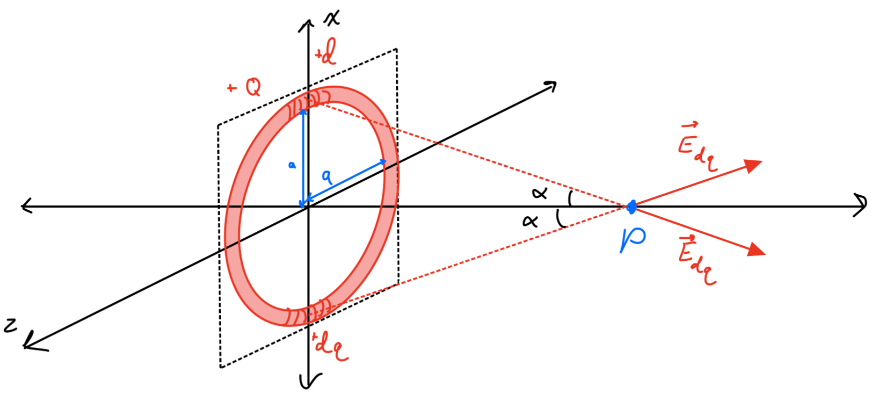
$$\vec{E} = \frac{2k\lambda}{y}\hat{i}$$

$$\lambda = \frac{dQ}{dx} = \frac{Q}{L} \text{ for line of length } L$$

$\vec{E}$	Electric field at point $P = (x, 0)$
k	Coulomb's constant ($k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$)
λ	Charge density
Q	Total charge
y	Distance from line to P

Electric Field of a Ring of Charge

Ring is finite and has even charge density λ



$$\vec{E} = \frac{kQx}{(x^2 + a^2)^{3/2}}\hat{i}$$

$\vec{E}$	Electric field at point $P = (x, 0)$
k	Coulomb's constant ($k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \mathrm{N \cdot m^2/C^2}$)
Q	Total charge
x	Distance from ring to P
a	Radius of ring

Gauss's Law

Electric Flux

Electric Flux is the amount of electric field passing through a surface (area).

$$\Phi = \oiint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$E \parallel A \implies \Phi = EA$$

Φ	Electric flux
$\oiint$	Surface integral
$\vec{E}$	Electric field
$d\vec{A}$	Differential area element
q_{enclosed}	Charge enclosed by the surface
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C^2/N \cdot m^2}$)

Electric Field Inside/Outside a Spherical Charge Distribution

$$r < a \implies E = \frac{kQr}{a^3} \text{ (inside)}$$

$$r > a \implies E = \frac{kQ}{r^2} \text{ (outside)}$$

E	Electric field
k	Coulomb's constant ($k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2$)
Q	Total charge
r	Distance from center of sphere to E
a	Radius of spherical charge distribution

Electric Potential (Voltage)

SI unit: V (Volts)

In charged conductors, the charge density is generally highest, and the field strongest, where a conductor curves sharply.

$$\Delta V_{AB} = \frac{\Delta U_{AB}}{q} - \int_A^B \vec{E} \cdot d\vec{r}$$

$$\Delta V_{AB} = -\vec{E}\Delta\vec{r} \text{ (in a uniform field)}$$

$$dV = -\vec{E} \cdot d\vec{r} = -Edr\cos\theta$$

ΔV	Change in electric potential (potential difference)
ΔU	Change in potential energy
q	Charge
$\vec{E}$	Electric field (*See*: **Electric Field**)
$d\vec{r}$	Differential displacement element

Potential of a Charge Distribution

$$V = k\sum_i \frac{q_i}{r_i} \text{ (Discrete)}$$

$$V = k\int \frac{dq}{r} \text{ (Continuous)}$$

V	Electric potential
k	Coulomb's constant ($k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$)
q	Charge
r	Distance from charge to V

Potential of a Dipole TODO: Image

TODO: verify & what is θ?

$$V = \frac{kp\cos\theta}{r^2}$$

V	Electric potential
k	Coulomb's constant ($k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$)
p	Dipole moment ($p = Qd$)
r	Distance from dipole to V
θ	Angle between dipole and V

Capacitors

For charging/discharging, *see*: **Capacitor Charging in Circuits**

Created by 2 plates of area A, equal and opposite charge Q separated by a distance d

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$$

$$\vec{E}_{\text{one plate}} = \frac{\sigma}{2\epsilon_0}\hat{n}$$

$$\sigma = \frac{Q}{A}$$

$\vec{E}$	Electric field between plates
σ	Charge density on each plate
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2$)
$\hat{n}$	Unit vector perpendicular to the plates (from positive to negative plate)
Q	Charge on each plate
A	Area of each plate

Capacitance

SI unit: F (Farads)

$$C = \frac{Q}{V}$$

$$C = \frac{\epsilon_0 A}{d}$$

C	Capacitance
Q	Charge (C)
V	Potential difference (*See*: **Electric Potential**)
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2$)
A	Area of each plate
d	Distance between plates

Energy of a Capacitor

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C}$$

$$U = \int u_E dV = \frac{1}{2} \epsilon_0 \int E^2 dV$$

$$U = \frac{1}{2} \epsilon_0 E^2 \times \text{Volume between plates}$$

$$V = -Ed = -\frac{\sigma}{\epsilon_0} d$$

U	Energy of a capacitor
C	Capacitance (*See*: **Capacitance**)
V	Potential difference (*See*: **Electric Potential**)
u_E	Energy density of electric field
E	Electric field between plates (*See*: **Electric Field**)
V	Potential difference (*See*: **Electric Potential**)
σ	Charge density on each plate (*See*: **Capacitors**)
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2$)

Capacitors in Series/Parallel

See: **Series vs Parallel** for differences between series and parallel

$$\frac{1}{C_{\text{eq}}} = \sum_i \frac{1}{C_i} \text{ (Series)}$$

$$C_{\text{eq}} = \sum_i C_i \text{ (Parallel)}$$

C_{eq}: capacitance of capacitor equivalent to the series/parallel combination

Dielectrics

Dielectric: Insulating material between the plates of a capacitor

$$C = \kappa C_0$$

$$\epsilon = \kappa \epsilon_0$$

C	Capacitance
κ	Dielectric constant (*See*: **Table 5**)
C_0	Capacitance without dielectric
ϵ	Permittivity of dielectric material (*See*: **Table 5**)
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2$)

Table 5: Dielectric Constants

Material	κ
Air	1.0006
Aluminum oxide	8.4
Glass (Pyrex)	5.6
Paper	3.5
Plexiglas	3.4
Polyethylene	2.3
Polystyrene	2.6
Quartz	3.8
Tantalum Oxide	26
Teflon	2.1
Water	80

Conductive slab between plates

Inserting a conductive slab between the plates of a capacitor increases the capacitance the same as if the slab and the area filled by the slab were removed from the capacitor

Current and Resistance

- Current: I
  - flow of charge
  - **SI unit**: A (Amps)
- Resistance: R
  - opposition to current
  - **SI unit**: Ω (Ohms)

$$I = \frac{\Delta Q}{\Delta t}$$

$$J = \frac{I}{A}$$

I	Current
Q	Charge
t	Time
J	Current density
A	Area

Ohm's Law

$$V = IR$$

$$J = \sigma E = \frac{E}{\rho}$$

$$\sigma = \frac{1}{\rho}$$

$$R \equiv \frac{\rho L}{A}$$

V	Potential difference (*See*: **Electric Potential**)
I	Current
R	Resistance
J	Current density

σ	Conductivity
ρ	Resistivity (*See*: **Table 6**)
E	Electric field
L	Length (of wire)
A	Cross-sectional area (of wire)

Table 6: Resistivity

Material	Resistivity: ρ ($\Omega{\cdot}$m)
Metals	
Aluminum	2.65×10^{-8}
Copper	1.68×10^{-8}
Gold	2.24×10^{-8}
Iron	9.71×10^{-8}
Mercury	9.84×10^{-7}
Silver	1.59×10^{-8}
Solutions	
1-molar $CuSO_4$	3.9×10^{-4}
1-molar HCL	1.7×10^{-2}
1-molar NaCl	1.4×10^{-4}
H_2O	2.6×10^{5}
Human Blood	0.70
Seawater	0.22
Semiconductors	
Geranium	0.5
Silicon	3×10^{3}
Insulators	
Ceramic	$10^{11} - 10^{14}$
Glass	$10^{10} - 10^{14}$
Polystyrene	$10^{15} - 10^{17}$
Rubber	$10^{13} - 10^{16}$
Wood (dry)	$10^{8} - 10^{14}$

Drift Velocity

$$I = nqA\vec{v}_d e$$

$$J = nq\vec{v}_d e$$

$$n = \frac{\rho}{m}$$

I	Current
n	Number of charge carriers (atoms) per unit volume
q	Charge of each atom ($-e = -1.6 \times 10^{-19}$C)
A	Cross-sectional area
J	Current density
$\vec{v}_d$	Drift velocity
e	Elementary charge (1.6×10^{-19}C)
ρ	mass density
m	Mass of each atom

Power

$$P = IV = I^2R = \frac{V^2}{R}$$

P	Power (W)
I	Current (A)
V	Potential (V) (*See*: **Electric Potential**)
R	Resistance (Ω)

Circuits

Current is the same everywhere in a series circuit

Circuit Simulator

Voltmeter vs Ammeter

- Voltmeter: parallel
  - measures voltage across 2 terminals
  - ideally has infinite resistance
- Ammeter: series
  - measures current through itself
  - ideally has zero resistance

Series vs Parallel

Series: same current, different voltage Parallel: same voltage, different current

Resistors in Parallel

Voltage is the same across each resistor

Current is split between each resistor

Equivalent resistance is less than the smallest resistor

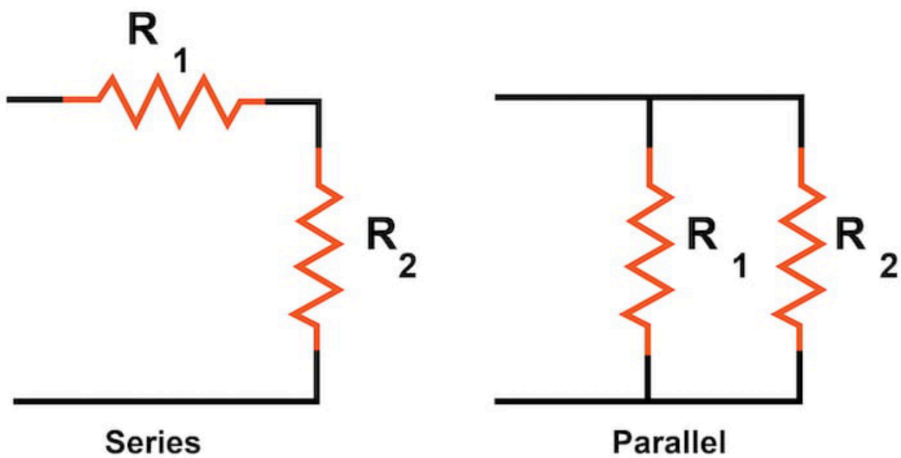


Figure 12: Example with resistors

$$I_p = \sum_i I_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$I_1 = I \frac{R_2}{R_1 + R_2} \text{ (2 resistors)}$$

$$I_2 = I \frac{R_1}{R_1 + R_2} \text{ (2 resistors)}$$

Resistors in Series

Current is the same across each resistor

Voltage is split between each resistor

Equivalent resistance is the sum of all resistors

$$V_s = \sum_i V_i$$

$$R_s = \sum_i R_i$$

$$V_1 = V \frac{R_1}{R_1 + R_2} \text{ (2 resistors)}$$

$$V_2 = V\frac{R_2}{R_1 + R_2} \text{ (2 resistors)}$$

I	Current (*See*: **Current and Resistance**)
R	Resistance (*See*: **Current and Resistance**)
V	Potential difference (*See*: **Electric Potential**)
I_p	Total current in parallel circuit
I_i, I_1, I_2	Current in each resistor in parallel circuit
R_p, R_s	Total resistance in parallel and series circuit (respectively)
R_i, R_1, R_2	Resistance in each resistor in the circuit
V_s	Total voltage in series circuit
V_i, V_1, V_2	Voltage in each resistor in series circuit

Kirchoff's Voltage Law

The sum of all voltages in a closed loop is zero

$$\sum_i V_i = 0 \text{ (closed loop)}$$

- Draw loops & assign current (I) to each.
- Find $\sum_i V_i$ for each loop: Start with battery EMF/voltage (ε), then go around loop subtract voltage drops
- Voltage drops across resistor $ = IR$
    - $I = \sum I_i$ (each loop)

Kirchoff's Current Law

The sum of all currents entering a node is zero

$$\sum_i I_{in} = \sum I_{out} \text{ (node)}$$

- Choose nodes & assign voltage (V) to each.
- Assign current (I) to each branch entering each node
- Find $\sum_i I_{in}$ incoming current
- Find $\sum_i I_{out}$ outgoing current is negative

Capacitor Charging in Circuits

For in-depth capacitor mechanics, *see*: **Capacitors**

Initial State $(t = 0)$

Capacitor acts as a short circuit (no voltage)

Plates have 0 charge

$$q = 0 \implies E = 0 \implies V_c = 0 \text{ (min)}$$

$$I = \frac{\varepsilon}{R} \text{ (max)}$$

Intermediate State $(0 < t < \infty)$

Capacitor charges/discharges

Exponential growth/decay

$$q = Q(1 - e^{-t/z})$$

$$I = \frac{\varepsilon}{R} e^{-t/z}$$

$$V_c = \varepsilon(1 - e^{-t/z})$$

$$z = RC \text{ (time constant)}$$

Final State $(t \to \infty)$

Enough time for capacitor to sufficiently charge

Capacitor is effectively an open circuit (no current)

$$q = Q = CV \implies E = \frac{\sigma}{\epsilon_0} \implies V_c = \varepsilon \text{ (max)}$$

$$I = 0 \text{ (min)}$$

---

q	Charge (*See*: **Capacitors**)
Q	Maximum charge (*See*: **Capacitors**)
C	Capacitance (*See*: **Capacitors**)
E	Electric field (*See*: **Capacitors**)
V_c	Voltage across capacitor (*See*: **Capacitors**)
I	Current (*See*: **Current and Resistance**)
R	Resistance (*See*: **Current and Resistance**)

ε	Electromotive force (EMF, same as voltage) (*See*: **Electric Potential**)
σ	Surface charge density (*See*: **Capacitors**)
ϵ_0	Permittivity of free space (*See*: **Capacitors**)
t	Time since circuit closed

Magnetic Field

SI Unit: Tesla (T)

Magnetic field can be generated by current

Direction of current determines the direction of the magnetic field via the right-hand rule

Biot-Savart Law

Magnetic field generated by a current-carrying wire

$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2}$$

$\vec{B}$	Magnetic field
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
I	Current in wire
$d\vec{l}$	Differential length of wire
$\hat{r}$	Unit vector from wire to point of interest
r	Distance from wire to point of interest

Use right-hand rule to determine direction of magnetic field

Magnetic field lines are circles around the wire (azimuthal)

Follows inverse square law

Ampere's Law

- **Requirements**:
  - $\vec{B}$ is constant along the amperian loop
  - $\vec{B}$ is tangential to the amperian loop
- For a straight wire, amperian loop is a centered around the wire

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encircled}}$$

$$B \cdot L = \mu_0 I_{\text{encircled}}$$

$\vec{B}$	Magnetic field
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
I	Current in wire
$d\vec{l}$	Differential length of wire
r	Distance from wire to point of interest
L	Length of amperian loop $= 2\pi r$

Gauss's Law for Magnetism

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$\vec{B}$	Magnetic field
$d\vec{A}$	Differential area element

Magnetic Field of Simple Current Distributions

Straight Wire

$$B = \frac{\mu_0 I}{2\pi r}$$

B	Magnetic field at point of interest
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
I	Current in wire
r	Distance from wire to point of interest

Loop of Wire

$$\vec{B} = \frac{\mu_0 I r_w^2}{2r} \hat{i}$$

$$x = 0 \implies \vec{B} = \frac{\mu_0 I}{2r_w} \hat{i} \text{ (center)}$$

$$x \gg r_w \implies \vec{B} = \frac{\mu_0 I r_w^2}{2x^3} \hat{i} \text{ (far-field)}$$

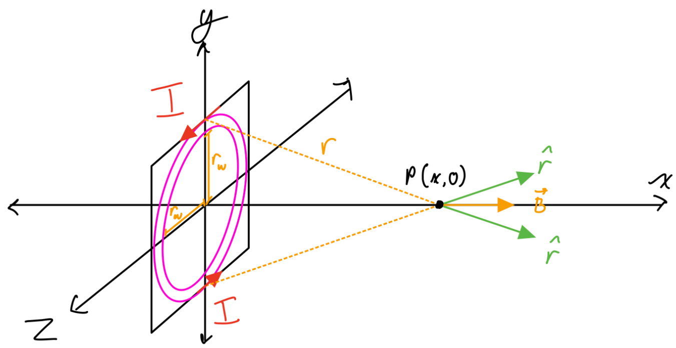


Figure 13: Diagram of loop

$$r = \sqrt{r_w^2 + x^2}$$

B	Magnetic field
μ_0	Permeability of free space $= 4\pi \times 10^{-7}\frac{Tm}{A}$
I	Current in wire
r_w	Radius of wire
r	Distance from wire to point of interest
x	Distance from center of loop to point of interest

Sheet (Plane) of Current

$$B = \frac{1}{2}\mu_0 J_s$$

B	Magnetic field
μ_0	Permeability of free space $= 4\pi \times 10^{-7}\frac{Tm}{A}$
J_s	Surface current density

Solenoid (Coil)

$$B = \mu_0 n I$$

B	Magnetic field
μ_0	Permeability of free space $= 4\pi \times 10^{-7}\frac{Tm}{A}$
n	Number of turns per unit length
I	Current in wire

Force of a Magnetic Field on a Moving Charge

$$\vec{F} = q\vec{v} \times \vec{B}$$

$\vec{F}$	Force on the particle (charge)
q	Charge of the particle
$\vec{v}$	Velocity of the particle

Cyclotron Motion

$$f = \frac{qB}{2\pi m}$$

f	Cyclotron frequency (period of rotation)
q	Charge of the particle
B	Magnetic field
m	Mass of the particle

Note: $\times$ is the cross product (*See*: **Cross Product**)

Magnetic Force on a Straight Wire

$$\vec{F} = I\vec{l} \times \vec{B} \text{ (from uniform field)}$$

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ (from parallel wire)}$$

$\vec{F}$	Force on the particle (charge)
I	Current in wire
$\vec{l}$	Length of wire
$\vec{B}$	Magnetic field
d	Distance between wires
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
I_1, I_2	Current in each wire

Force is attractive if currents are in the same direction, repulsive if in opposite directions

Note: $\times$ is the cross product (*See*: **Cross Product**)

Magnetic Flux

Magnetic flux is the amount of magnetic field passing through a surface (area).

$$\Phi = \oiint \vec{B} \cdot d\vec{A}$$

$$(\vec{B} \perp d\vec{A}) \ \wedge \ (B \text{ is constant}) \implies \Phi = BA$$

Φ	Magnetic flux
$\oiint$	Surface integral
$\vec{B}$	Magnetic field
$d\vec{A}$	Differential area element

Induction

Changing magnetic field induces an electric field

Faraday's Law

$$\varepsilon_{\text{induced}} = -\frac{d\phi_B}{dt}$$

Recall: $\varepsilon = \oint \vec{E} \cdot d\vec{l}$ (*See*: **Electric Potential**)

$\varepsilon_{\text{induced}}$	Induced electromotive force (EMF, same as voltage) (*See*: **Electric Potential**)
$\frac{d\phi_B}{dt}$	Rate of change of magnetic flux (*See*: **Magnetic Flux**)

- RHR for direction of induced current and magnetic field
  - Determining $\vec{E}$ and I around a loop
    - Increasing $\vec{B}$: use right hand rule on $-\vec{B}$
    - Decreasing $\vec{B}$: use right hand rule on $\vec{B}$
- ϕ_B is maximum when $\vec{B} \perp d\vec{A}$
- ϕ_B is 0 (minimum) when $\vec{B} \parallel d\vec{A}$

Magnetic Energy Density TODO: verify link > Compare to electric energy density (*See*: **Capacitors**) > > Represents stored energy in magnetic field

$$u_B = \frac{B^2}{2\mu_0}$$

u_B	Magnetic energy density
B	Magnetic field (*See*: **Magnetic Field**)
μ_0	Permeability of free space ($\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$)

Inductors

Solenids encircle their own magnetic flux giving *self-inductance*

$$L \equiv \frac{\Phi}{I} = \frac{NBA}{I} = \mu_0 N^2 \frac{A}{L}$$

$$\Phi = N\phi$$

$$B = \mu_0 n I$$

$$\text{AC} \implies I(t) = I_0 \cos \omega t$$

L	Inductance
Φ	Magnetic flux
I	Current
N	Number of turns
ϕ	Magnetic flux per turn
B	Magnetic field (*See*: **Magnetic Field**)
μ_0	Permeability of free space ($\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$)
n	Number of turns per unit length $= \frac{N}{L}$
A	Cross-sectional area of one loop $A = \pi r^2$

Inductors in Circuits

$$L = \frac{\phi_B}{I} \text{ (self-inductance)}$$

$$I = \frac{\varepsilon_0}{R}(1 - e^{-Rt/L})$$

$$\varepsilon_{\text{induced}} = -L \frac{dI}{dt}$$

L	Inductance

ϕ_B	Magnetic flux
I	Current
$\varepsilon_{\text{induced}}$	Induced electromotive force (EMF, same as voltage) (*See*: **Electric Potential**)
R	Resistance (*See*: **Current and Resistance**)
t	Time

Inductor Charging in Circuits Initial State ($t = 0$)

Inductor is effectively an open circuit (no current)

$$I = 0 \text{ (min)} \qquad V = \varepsilon \text{ (max)}$$

Intermediate State ($0 < t < \infty$)

Inductor charges/discharges

Exponential growth/decay

$$I_L(t) = \tfrac{\varepsilon}{R}(1 - e^{-t/\tau}) \qquad V_L(t) = \varepsilon e^{-t/\tau}$$

$$\tau = \frac{L}{R} \text{ (time constant)}$$

Final State ($t \to \infty$)

Enough time for Inductor to sufficiently charge

Inductor acts as a short circuit (no voltage)

$$I = \tfrac{\varepsilon}{R} \text{ (max)} \qquad V = 0 \text{ (min)}$$

I	Current (*See*: figure above)
V	Potential difference (*See*: **Electric Potential**)
ε	Electromotive force (EMF, same as voltage) (*See*: **Electric Potential**)
R	Resistance (*See*: figure above, and **Current and Resistance**)

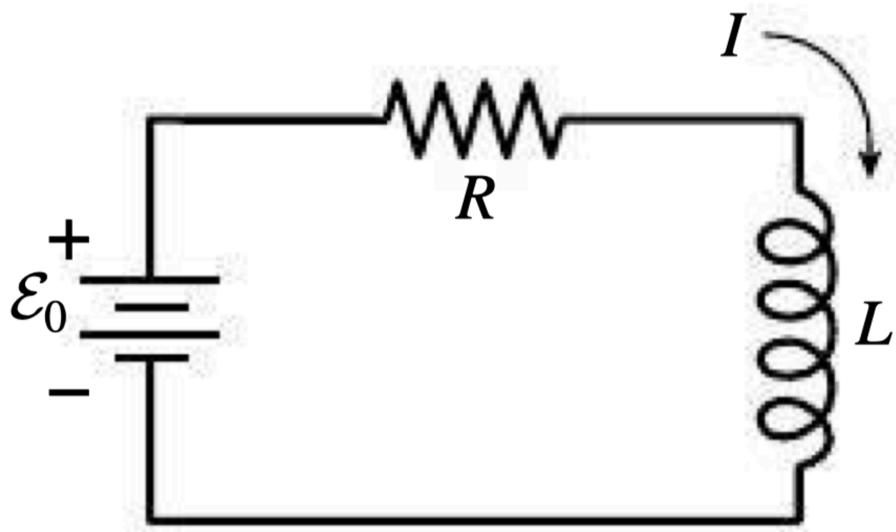


Figure 14: Inductor in a circuit

Energy Stored in an Inductor

$$U = \frac{1}{2}LI^2$$

U	Energy stored in an inductor
L	Inductance
I	Current

Maxwell's Equations

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0} \text{ (Gauss's Law)}$$

$$\oiint \vec{B} \cdot d\vec{A} = 0 \text{ (Gauss's Law for Magnetism)}$$

$$\oint_{\text{Amperian Loop}} \vec{B} \cdot d\vec{l} = \mu_0(I_{\text{encircled}} + I_{\text{displacement}}) \text{ (Modified Ampere's Law)}$$

where $I_{\text{displacement}} = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt} \oiint \vec{E} \cdot d\vec{A} = \epsilon_0 \oiint \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$ (Displacement Current)

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt} \text{ (Faraday's Law)}$$

$\vec{E}$	Electric field
$\vec{B}$	Magnetic field
$d\vec{A}$	Differential area element

Electromagnetic Waves

- Electromagnetic waves are transverse waves
  - Electric and magnetic fields are perpendicular to eachother and the direction of propagation
- Electromagnetic waves are self-propagating

Speed of Electromagnetic Waves

$$-\frac{\partial B}{\partial x \partial t} = \frac{\partial^2 E}{\partial x^2} = \mu\epsilon\frac{\partial^2 E}{\partial t^2} \implies v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{n}$$

$$\begin{aligned} \text{vacuum} &\implies \quad \mu = mu_0 \quad \epsilon = \epsilon_0 \qquad v = c \\ \text{material} &\implies \quad \mu = \mu_r\mu_0 \quad \epsilon = \epsilon_r\epsilon_0 \end{aligned}$$

$$E = vB$$

v	Speed of light (Electromagnetic waves)
c	Speed of light in a vacuum ($c = 3 \times 10^8 \frac{m}{s}$)
μ	Permeability of material
μ_0	Permeability of free space ($\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$)
ϵ	Permittivity of material
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2$)
E	Electric field
B	Magnetic field
n	Index of refraction

Einstein's Theory of Special Relativity c is the speed limit of the universe.

Frequency and Wavelength

$$v = \lambda f$$

v	Speed of light (Electromagnetic waves)
λ	Wavelength
f	Frequency

Intensity

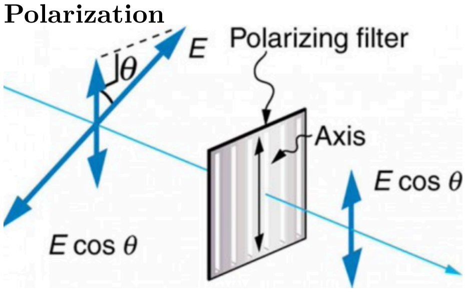
$$S \equiv \frac{P}{\text{Area}} = \frac{P}{4\pi r^2} = uv$$

$$u = u_E + u_B = \epsilon E^2 + \frac{B^2}{2\mu}$$

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu} \text{ (Poynting vector)}$$

S	Intensity of electromagnetic wave
P	Power
u	Energy density
v	Speed of electromagnetic wave
u_E, u_B	Energy density of electric and magnetic fields respectively (*See*: **Magnetic Energy Density** & **Capacitors**)
E, $\vec{E}$	Electric field
B, $\vec{B}$	Magnetic field
ϵ	Permittivity of material
μ	Permeability of material
r	Distance from source of electromagnetic wave

Polarization



$$S = S_0 \cos^2 \theta$$

$$E = E_0 \cos\theta$$

S	Intensity of electromagnetic wave after passing through a polarizer
S_0	Intensity of electromagnetic wave before passing through a polarizer
θ	Angle between the polarizer and the direction of polarization
E	Electric field after passing through a polarizer
E_0	Electric field before passing through a polarizer

Radiation Pressure

SI Unit: Pa (Pascals)

$$P_{\text{rad}} = \frac{\bar{S}}{c}$$

P_{rad}	Radiation pressure
S	Intensity of electromagnetic wave
c	Speed of light in a vacuum ($c = 3 \times 10^8 \frac{m}{s}$)

Optics

Law of Reflection

$$\theta_i = \theta_r$$

θ_i	Angle of incidence (angle from incoming wave to normal)
θ_r	Angle of reflection (angle from reflected wave to normal)

Law of Refraction (Snell's Law)

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

n_1, n_2	Index of refraction of material 1 and 2 respectively
θ_1	Angle of incidence (angle from incoming wave to normal)
θ_2	Angle of refraction (angle from refracted wave to normal)

Total Internal Reflection

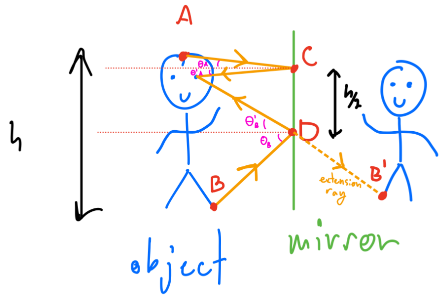
When $\theta_i > \theta_c$, the wave is reflected back into the material
Used in fiber optics

$$\sin\theta_c = \frac{n_2}{n_1}$$

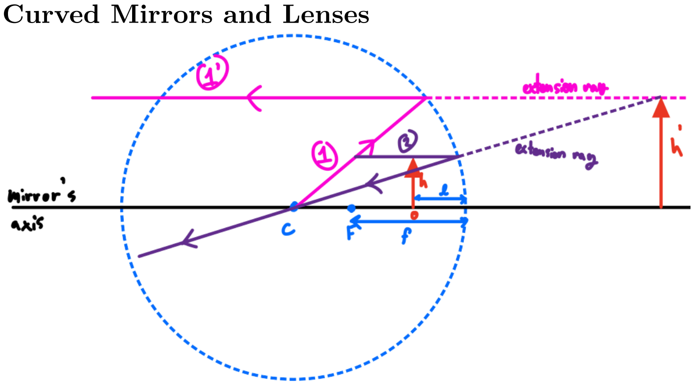
Dispersion Different colors refract at different angles

Image Formation

Flat Mirror



Curved Mirrors and Lenses



$$M \equiv \frac{h'}{h} = \frac{l'}{l}$$

$$\frac{1}{l} + \frac{1}{l'} = \frac{1}{f} \text{ (Thin Lens)}$$

$$\frac{n_1}{l} + \frac{n_2}{l'} = \frac{n_2 - n_1}{R} \text{ (Thick Lens, Type I)}$$

$$\frac{1}{l} + \frac{1}{l'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ (Thick Lens, Type II)}$$

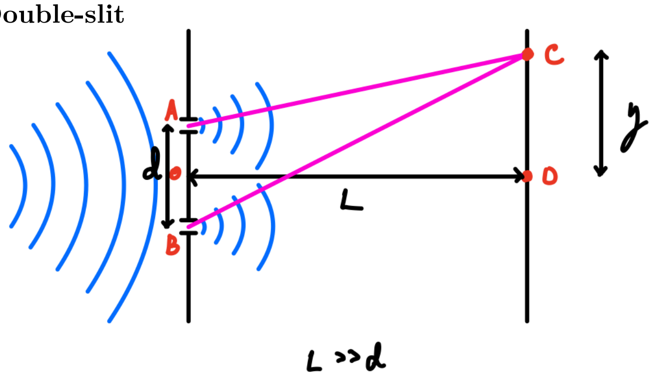
M	Magnification factor
o, i	object and image location respectively
h, h'	object and image height respectively
l, l'	object and image distance from mirror (center)respectively
F	focal point of mirror (point where parallel rays converge)
f	focal length (distance from mirror to focal point)
R_1, R_2	radii of curvature of mirror (positive for concave, negative for convex)
n_1, n_2	indices of refraction of outside and inside the lens respectively

- **Sign Convention**:
  - Mirrors
    - f is positive for concave mirrors, negative for convex mirrors
    - l is positive
    - l' is positive for real images, negative for virtual images
  - Lenses
    - f, R is positive for converging (convex) lenses, negative for diverging (concave) lenses
    - l is positive
    - l' is positive for image on the other side of the lens, negative for image on the same side of the lens

Interference & Diffusion

Diffraction is the spreading of light as it passes through a slit

Double-slit



$$\Delta\text{path} = BC - AC = \sqrt{L^2 + \left(y + \frac{d}{2}\right)^2} - \sqrt{L^2 + \left(y - \frac{d}{2}\right)^2}$$

$$\begin{aligned}\Delta\text{path} &= d\sin\theta \\ &= m\lambda \text{ (constructive interference)} \\ &= (2m+1)\frac{\lambda}{2} \text{ (destructive interference)}\end{aligned}$$

$$y_m = L\tan\theta_n$$

d	Distance between slits
L	Distance from slits to screen
y	Distance (height) from center of screen to point of interest
θ	Angle from center of slits to point of interest
m	Order of interference (Integer ≥ 0)

Thin-film/Radio wave interference

$$\begin{aligned}\Delta\text{path} &= 2d - \frac{\lambda}{2} \\ &= m\lambda \text{ (constructive interference)} \\ &= (2m+1)\frac{\lambda}{2} \text{ (destructive interference)}\end{aligned}$$

d	Thickness of film/distance between antennas
λ	Wavelength of light
m	Order of interference (Integer ≥ 0)