Physics 1 Reference

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Contents

SI Prefixes

Prefix	Symbol	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

Kinematics (Motion)

Displacement

$$\Delta x = x_f - x_i$$

x_f	final position
x_i	initial position

Displacement is a **vector quantity** (See: Vectors).

Average Velocity

$$\bar{v} = \frac{\Delta \vec{x}}{\Delta t}$$

Δx	Displacement (over the time period) (See: Displacement)
Δt	Change in time (See: Change in Time)

Velocity is a **vector quantity**, it measures direction and magnitude.

Average Acceleration

$$\bar{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Δv	Change in velocity (over the time period) (See: Change in Velocity)
Δt	Change in time (See: Change in Time)

Acceleration is a **vector quantity**, it measures direction and magnitude.
Note: Direction of acceleration is not always the same as direction of velocity. (See: Circular Motion).

Instantaneous Velocity

$$v = \frac{d\vec{x}}{dt}$$

v	velocity
x	position
t	time

Can ususally be found using equations for constant acceleration (See: Constant Acceleration).

Linear Motion with Constant Acceleration

$$v_f = v_i + a\Delta t$$

$$\Delta x = \frac{1}{2}\left(v_i + v_f\right)\Delta t$$

$$\Delta x = v_i\Delta t - \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

v_f	final velocity
v_i	initial velocity
a	acceleration
Δt	Change in time (See: Change in Time)
Δx	Displacement (over the time period) (See: Displacement)

The above equations only work when acceleration is constant

Projectile Motion

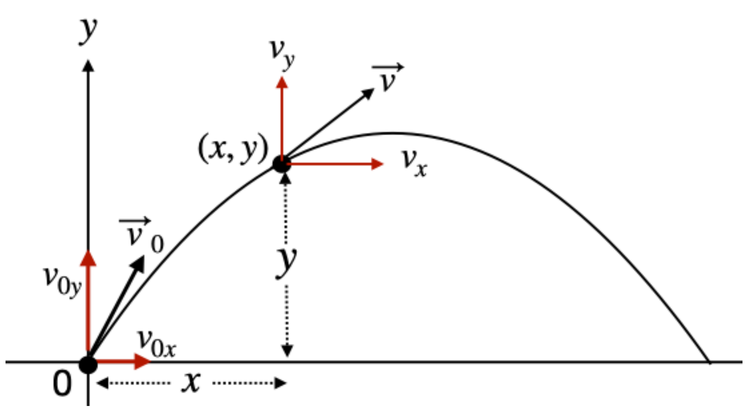

Figure 1: Graph of an object in projectile motion

$$x = v_{0x}t = v_0 t\cos\theta$$

$$y = v_{0y}t - \frac{1}{2}gt^2 = v_0 t\sin\theta - \frac{1}{2}gt^2$$

$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$

$$v_x = v_0 \cos\theta$$

$$v_y = v_0 \sin\theta - gt$$

$$t_{\text{projectile hits ground}} = \frac{v_0^2}{g} \sin 2\theta$$

x	horizontal position
y	vertical position
v_{0x}	initial horizontal velocity
v_{0y}	initial vertical velocity
v_0	initial velocity
g	acceleration due to gravity (9.8 m/s^2)
t	time

The equations are all derived from the equations of motion with constant acceleration assuming the following:

- $a_x = 0$
- $a_y = -g$

They are derived and used by breaking the motion into x (horizontal) and y (vertical) components, solving each as if they were a 1D motion problem, and then combining the results with vector addition (See: Addition / Subtraction).

Free Fall

$$v_f = v_i - gt$$

$$y = v_i t - \frac{1}{2} g t^2$$

$$v_f^2 = v_i^2 - 2gh$$

$$t_{\text{object hits ground}} = \sqrt{\frac{2h}{g}}$$

$$\vec{a} = -g\hat{j} \implies a = g$$

v_f	final velocity
v_i	initial velocity
g	acceleration due to gravity
y	vertical position (at time t)
h	height (initial y position)
t	time
$\vec{a}$	acceleration

Apparent Weight

If apparent weight is am and mass is m then there is downward acceleration of $(1-a)g$.

Circular Motion

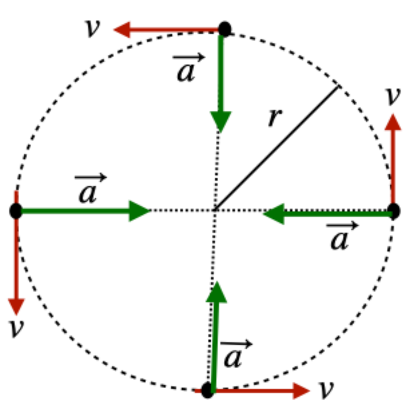

Figure 2: Circular motion at different parts of the circle

$$\vec{a} = -\omega^2 \vec{r}$$

$$T = \frac{2\pi r}{v}$$

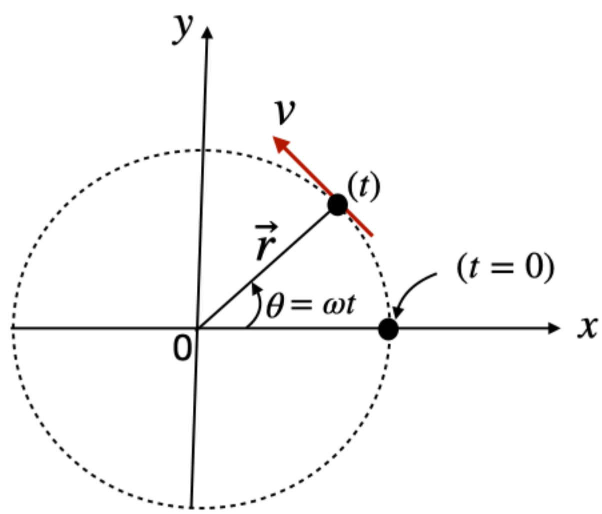

Figure 3: Circular motion at angle θ

$$\vec{v} = -\omega r \sin\theta \hat{i} + \omega r \cos\theta \hat{j}$$

$$|\vec{v}| = \omega r$$

$$\vec{r} = r\cos\theta \hat{i} + r\sin\theta \hat{j}$$

$$\theta = 0 \implies \vec{v} = \omega r \hat{j}$$

$$\theta = \frac{\pi}{2} \implies \vec{v} = -\omega r \hat{i}$$

$$\vec{F}_{\text{net}} = \frac{mv^2}{r}$$

$\vec{a}$	acceleration
$\vec{r}$	position
v	velocity
r	radius
T	period (time it takes to go full circle)
ω	constant

θ	angle (relative to the horizontal)
$\hat{i}$	unit vector in the x direction
$\hat{j}$	unit vector in the y direction

Note: acceleration points in the direction of $-\vec{r}$ (See: Vectors). Since the magnitude of velocity is constant, the acceleration is only affected by change of direction.

Forces

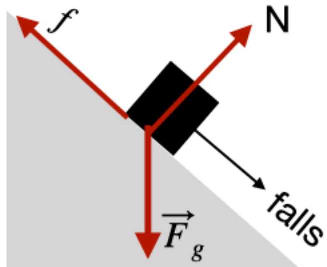

Figure 4: Forces acting on an object

A force is a **vector quantity** that causes an object to accelerate

Conservative Forces

$$\oint \vec{F} \cdot d\vec{r} = 0 \implies W_{A \to B} + W_{B \to A} = 0$$

- A force is **conservative** if the work done by the force on an object moving between two points is independent of the path taken by the object
- If an object moves in a round trip (i.e. it ends up at the same point it started) then the net work done by a conservative force is zero

Gravitational Force (weight)

$$\vec{F}_g = mg$$

$\vec{F}_g$	weight
m	mass
g	acceleration due to gravity, ($9.8 \frac{m}{s^2}$ on Earth's surface)

- Always vertically downward
- Gravitational force is **conservative**

Spring Force

$$F_s = -kx$$

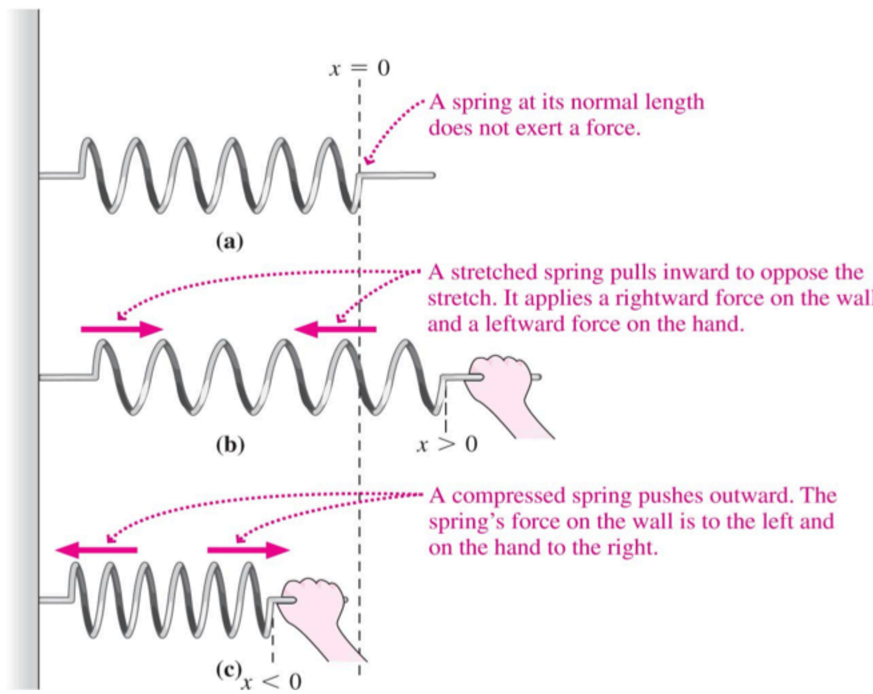

Figure 5: Spring Diagram

F_s	spring force
k	spring constant (how stiff the spring is)
x	displacement from equilibrium position

- *Ideal* spring force is **conservative**
  - An *ideal* spring is massless and frictionless and doesn't lose energy to heat
  - Springs we deal with are *ideal* unless otherwise stated

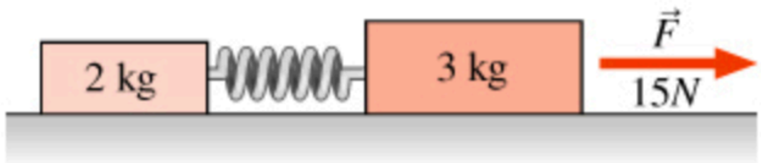

Figure 6: Spring Example

$$\frac{F_s}{m_a} = \frac{F - F_s}{m_b}$$

F_s	spring force
F	net force exerted on the system
m_a	mass of object A
m_b	mass of object B

Non-Conservative Forces

- A force is **non-conservative** if the work done by the force on an object moving between two points is dependent of the path taken by the object
- If an object moves in a round trip (i.e. it ends up at the same point it started) then the net work done by a non-conservative force is not zero

Friction

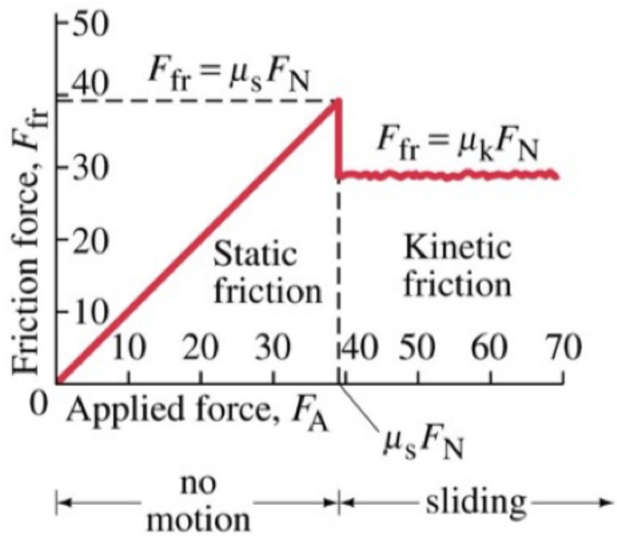

Figure 7: Static and Kinetic Friction

F_{fr}	friction force
μ_s	coefficient of static friction
μ_k	coefficient of kinetic friction
F_N	normal force

- Always parallel to the surface
- Frictional force is **non-conservative**
- On a **frictionless** surface, $f = 0$

Static Friction

$$\vec{f}_s \leq \mu_s N$$

f_s	static friction force
μ_s	coefficient of static friction
$\vec{N}$	normal force

Kinetic Friction

$$\vec{f} = \mu_k \vec{N}$$

$\vec{f}$	friction force
μ_k	coefficient of kinetic friction

$\vec{N}$	normal force

Normal Force

$$\vec{N} = mg\cos\theta$$

$\vec{N}$	normal force
m	mass
g	acceleration due to gravity, ($9.8\frac{m}{s^2}$ on Earth's surface)
θ	angle of incline (relative to the horizontal)

- Represents the force of the surface pushing against the object
- Always perpendicular to the surface
- Normal force is **non-conservative**
- If the object is on an incline, the normal force is less than the weight†.
- If the object is on a horizontal surface, the normal force is equal to the weight†.
- † (See: Gravitational Force).

Tension

- Represents the force exerted by a string or rope on an object
- Always parallel to the rope
- Equal on both sides of the rope
- Tension is **non-conservative**

Example

$$\vec{F}_x = T\cos\theta = Ma$$

$$\vec{F}_y = N + T\sin\theta - Mg = 0$$

$\vec{F}_x$	net force in the x direction
$\vec{F}_y$	net force in the y direction
T	tension (force from string)
θ	angle of incline (relative to the horizontal)
M	mass of object
a	acceleration
N	normal force
g	acceleration due to gravity, ($9.8\frac{m}{s^2}$ on Earth's surface)

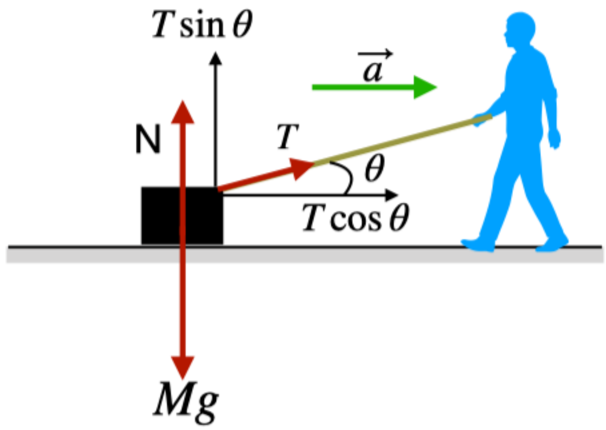

Figure 8: Forces example with tension

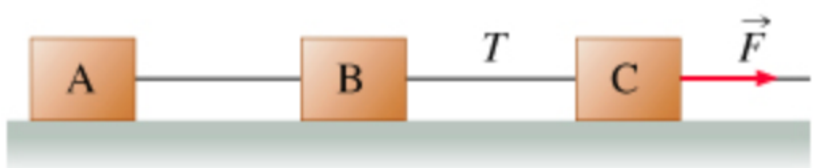

Figure 9: String Example

String Example

$$\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C$$

$$\vec{F} = (m_A + m_B + m_C)a$$

$$\vec{T} = \vec{F} - \vec{F}_{\mathrm{C}}$$

$\vec{F}$	force (in diagram)
$\vec{T}$	tension
$\vec{F}_A$	net force exerted on object A
$\vec{F}_B$	net force exerted on object B
$\vec{F}_C$	net force exerted on object C
m_A	mass of object A
m_B	mass of object B
m_C	mass of object C
a	acceleration

Newton's Second Law

$$F_{\mathrm{Net}} = ma$$

$$\vec{p} = m\vec{v}$$

F_{Net}	the vector sum of all forces acting on the object (See: Addition / Subtraction)
m	mass
a	acceleration
$\vec{p}$	momentum
$\vec{v}$	velocity

The equation implies Newton's First Law: $F_{\mathrm{Net}} = 0 \implies a = 0 \implies$ the object is at rest or moving at a constant velocity

Energy

$$E = K + U$$

E	energy
K	kinetic energy
U	potential energy

SI unit: Joule (J)

Kinetic Energy

$$K = \frac{1}{2}mv^2$$

K	kinetic energy
m	mass
v	velocity

Potential Energy

$$\Delta U_{A\to B} = -\int_A^B \vec{F} \cdot \vec{dr} = -W_{A\to B}$$

- Potential energy is the energy of an object due to its position
- Potential energy is always relative to some reference point
- Equations can also be used to determine work done by a force.

Gravitational Potential Energy

$$U_g = mgh$$

U_g	gravitational potential energy
m	mass
g	acceleration due to gravity
h	height

Stores work done against gravity

Elastic Potential Energy

$$U_s = \frac{1}{2}kx^2$$

U_s	elastic potential energy
k	spring constant
x	displacement from equilibrium

Stores work done in stretching or compressing a spring

Work

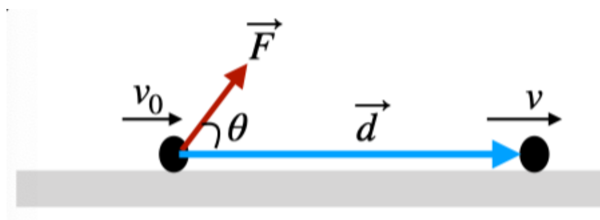

Figure 10: Work of a force $\vec{F}$

$$W = \int_A^B \vec{F} \cdot \vec{dr}$$

$$W = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

$$W = W_c + W_{nc}$$

$$W_{\text{Net}} = \sum W$$

W	work done by force $\vec{F}$
$\vec{F}$	force
$\vec{d}$	displacement
W_c	work done by conservative forces
W_{nc}	work done by non-conservative forces
d	magnitude of displacement
W_{Net}	net work done by all forces on the object

Work-Kinetic Energy Theorem

$$W = \Delta K = K_f - K_i$$

W	work done by force $\vec{F}$
ΔK	change in kinetic energy
K_f	final kinetic energy
K_i	initial kinetic energy

- Work is a scalar quantity
- SI unit: Joule (J)

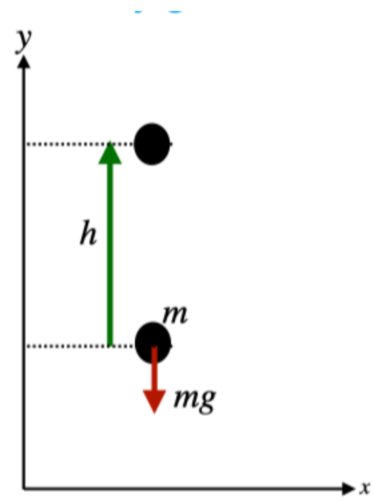

Figure 11: Work done by gravity in moving an object upward

Work done by Gravity Example

$$W_g = -mgh$$

W_g	work done by gravity
m	mass
g	acceleration due to gravity
h	height

$$W_g = mgh$$

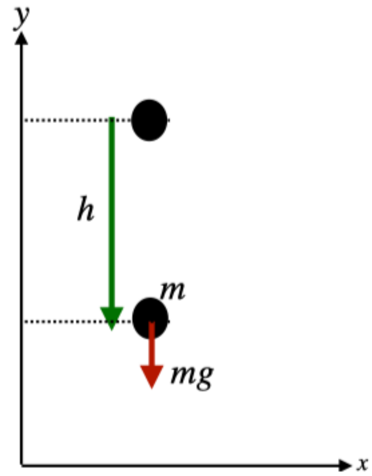

Figure 12: Work done by gravity in moving an object down

W_g	work done by gravity
m	mass
g	acceleration due to gravity
h	height

Work done by a Variable Force

$$W = \int_{x_i}^{x_f} F(x)dx$$

W	work done by force $F(x)$
x_i	initial position
x_f	final position
$F(x)$	force at a given position x

$F(x)$ is a function of position. The force depends on the position of the object

Work done by a Spring

$$W = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} -kxdx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 = \frac{1}{2}kx^2$$

$$F(x) = kx$$

W	work done by force $F(x)$
x_i	initial position
x_f	final position
$F(x)$	force at a given position x
k	spring constant

Mechanical Energy

$$E_m = K + U$$

E	mechanical energy
K	kinetic energy
U	potential energy

SI unit: Joule (J)

Conservation of Mechanical Energy

$$W_{nc} = 0 \implies E_{m_i} = E_{m_f}$$

W_{nc}	work done by non-conservative forces
E_{m_i}	mechanical energy at initial position
E_{m_f}	mechanical energy at final position

- Derived from the work-kinetic energy theorem
- $W_{nc} = 0$ means that there are no non-conservative forces acting on the object
- The only non-conserative force is friction
  - Tension cancels itself out when considering the entire system
  - Normal force is perpendicular to the displacement, so work done by normal force is 0

Power

$$P = \frac{W}{\Delta t}$$

$$P = \vec{F} \cdot \vec{v} \text{ (if force is constant)}$$

P	power
W	work
Δt	change in time
$\vec{F}$	force
$\vec{v}$	velocity

- SI unit: Watt (W)
- 1 W = 1 J/s

Momentum

$$\vec{p} = m\vec{v}$$

$\vec{p}$	momentum
m	mass
$\vec{v}$	velocity

Newton's Law of Universal Gravitation

$$F_g = G\frac{m_1 m_2}{r^2}$$

F_g	force of gravity
G	gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2
m_1	mass of object 1
m_2	mass of object 2
r	distance between the two objects

- Strictly applied to point masses (particles)
- Always attractive
- If one of the masses is the Earth (or similar object), then r is the distance from the center of the Earth to the object

Gravitational Potential Energy

$$U_g = -G\frac{m_1 m_2}{r}$$

U_g	gravitational potential energy
G	gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2
m_1	mass of object 1
m_2	mass of object 2
r	distance between the two objects

Circular Orbits

$$v = \sqrt{\frac{GM}{r}}$$

$$T = \frac{2\pi r}{v}$$

v	tangential velocity of the object
G	gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2
M	mass of the object being orbited
r	distance between the two objects
T	period of the orbit

The object is orbiting the other object

Gravitational Field

$$\vec{E} = \frac{\vec{F}}{m}$$

$\vec{E}$	gravitational field
$\vec{F}$	gravitational force
m	mass of the object

Escape Velocity

$$v_e = \sqrt{\frac{2GM}{r}}$$

v_e	escape velocity
G	gravitational constant $G = 6.67 \times 10^{-11}\ \mathrm{Nm}^2/\mathrm{kg}^2$
M	mass of the planet
r	radius of planet

Center of Mass

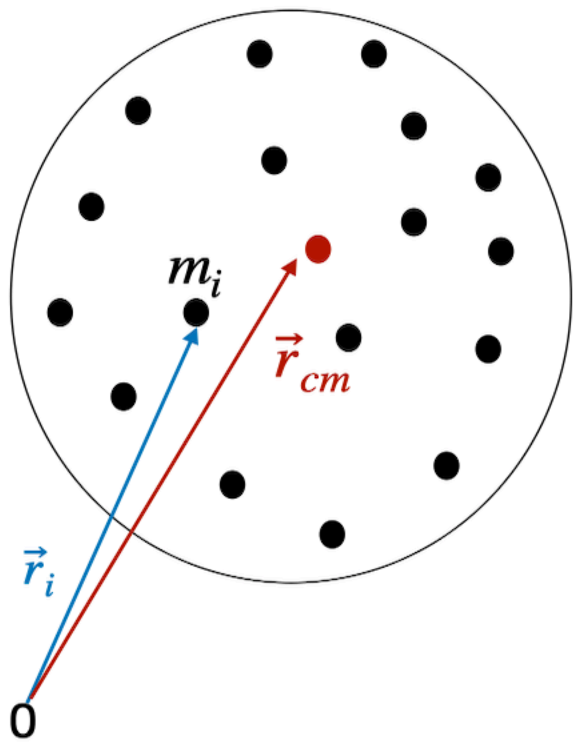

Figure 13: A multi-particle system with reference point 0

$$\vec{F}_{net} = M\frac{d^2\vec{r}_{cm}}{dt^2} = M\frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm}$$

$\vec{F}$	net force on the system
M	total mass of the system
$\vec{r}$	position of a particle
$\vec{v}$	velocity of a particle
$\vec{a}$	acceleration of a particle
$\vec{r}_{cm}$	position of the center of mass of the system
$\vec{v}_{cm}$	velocity of the center of mass of the system
$\vec{a}_{cm}$	acceleration of the center of mass of the system

The center of mass is the point where the net force on the system acts

Discrete distribution of mass

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{M}$$

$$x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$$

$$y_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{M}$$

$$z_{cm} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$

r_{cm}	position of the center of mass of the system
x_{cm}	x-coordinate of the center of mass of the system
y_{cm}	y-coordinate of the center of mass of the system
z_{cm}	z-coordinate of the center of mass of the system
x_i	x-coordinate of the ith particle
y_i	y-coordinate of the ith particle
z_i	z-coordinate of the ith particle

Continuous distribution of mass

$$\vec{r}_{cm} = \lim_{\Delta m \to 0} \frac{\sum_{i=1}^{n} \Delta m_i \vec{r}_i}{M} = \frac{\int \vec{r} dm}{M}$$

$$x_{cm} = \frac{\int x dm}{M}$$

$$y_{cm} = \frac{\int y dm}{M}$$

$$z_{cm} = \frac{\int z dm}{M}$$

r_{cm}	position of the center of mass of the system
x_{cm}	x-coordinate of the center of mass of the system
y_{cm}	y-coordinate of the center of mass of the system
z_{cm}	z-coordinate of the center of mass of the system
x	x-coordinate of a particle
y	y-coordinate of a particle

z	z-coordinate of a particle
dm	mass of a particle

If the mass is distributed uniformly, then $dm = \rho dV$, where ρ is the density of the object and dV is the volume of the object.

Total Momentum

$$\vec{p}_{tot} = \sum_{i=1}^{n} \vec{p}_i = M\vec{v}_{cm}$$

$\vec{p}$	momentum of a particle
$\vec{p}_{tot}$	total momentum of the system
M	total mass of the system

Conservation of Linear Momentum

$$\vec{F}_{net} = 0 \implies \vec{p}_{tot} = \text{constant}$$

If the net external force on a system ($\vec{F}_{net}$) is zero, then $\vec{p}_{tot}$ is constant and the center of mass of the system moves with constant velocity.

Total Kinetic Energy

$$K_{tot} = \sum_{i=1}^{n} K_i = \frac{1}{2}Mv_{cm}^2$$

K	kinetic energy of a particle
K_{tot}	total kinetic energy of the system
M	total mass of the system

Collisions

$$\int_{t_i}^{t_f} \vec{F}(t)dt \approx \vec{F}_{avg}\Delta t$$

t_i	start time of the collision
t_f	end time of the collision
$\vec{F}$	force on a particle
$\vec{F}_{avg}$	average force on a particle
Δt	change in time (See: Change in Time)

Elastic Collisions

Elastic collisions are collisions where the total kinetic energy of the system is conserved.

Inelastic Collisions

Inelastic collisions are collisions where the total kinetic energy of the system is not conserved.

Rotational Motion

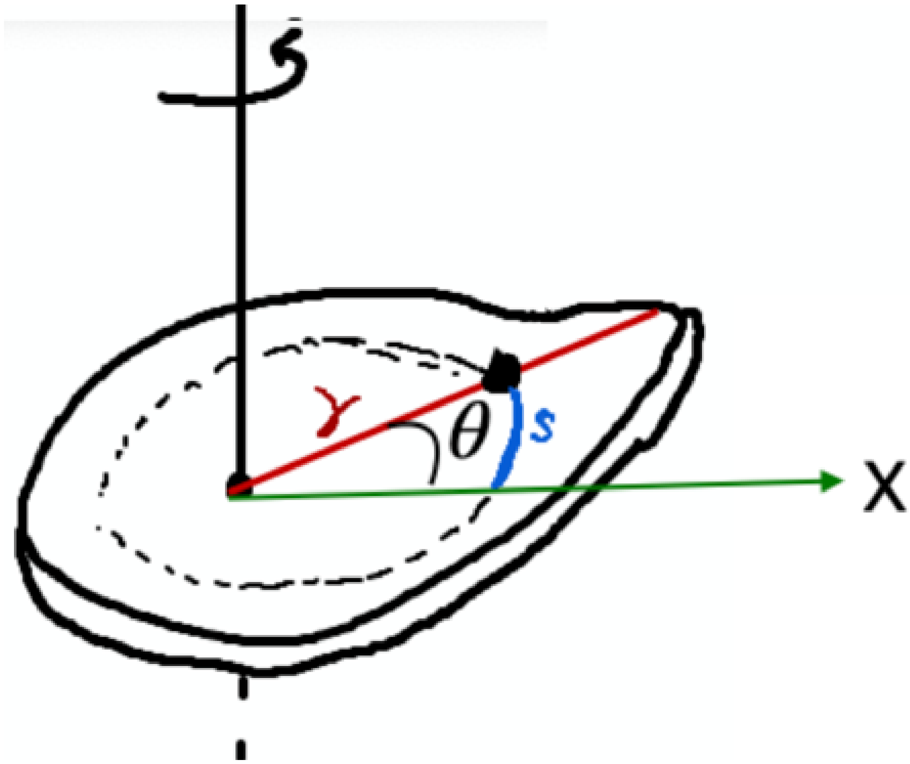

Figure 14: Rotational Motion about an axis

$$s = r\theta$$

$$v_t = r\omega$$

$$a_t = r\alpha$$

$$a_r = \frac{v_t^2}{r} = r\omega^2$$

θ	angular displacement of a particle (See: Angular Displacement)
s	arc length of a particle
r	radius of a particle (from the axis of rotation)

v_t	tangential velocity of a particle
ω	angular velocity of a particle (See: Angular Velocity)
a_t	tangential acceleration of a particle
α	angular acceleration of a particle (See: Angular Acceleration)
a_r	radial (centripetal) acceleration of a particle

Right Hand Rule

- Direction of rotation is determined by the position of the thumb
  - If the thumb points in the direction of the axis of rotation (**typically upward**), then the direction is positive
  - If the thumb points in the opposite direction of the axis of rotation (**typically downward**), then the direction is negative

Angular Displacement

$$\Delta\theta = \theta_f - \theta_i$$

$\Delta\theta$	change in angular displacement
θ_f	final angular displacement
θ_i	initial angular displacement

For Direction (See: Right Hand Rule)

Angular Velocity

$$1\ rpm = \frac{2\pi}{60}\ rad/s$$

$$1\ rev/s = 2\pi\ rad/s$$

For Direction (See: Right Hand Rule) Measured in Radians per Second (rad/s), or Revolutions per Minute/Second (rpm & rev/s)

Average Angular Velocity

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular Velocity

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} = \frac{d\vec{\theta}}{dt}$$

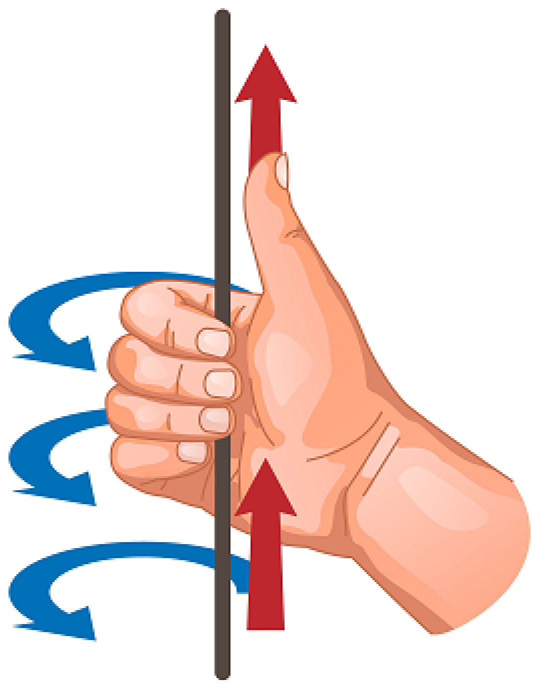

Figure 15: RHR Diagram

ω	angular velocity
$\Delta\theta$	change in angular displacement
Δt	change in time (See: Change in Time)
$\frac{d\theta}{dt}$	derivative of angular displacement with respect to time

Angular Acceleration

Average Angular Acceleration

$$\bar{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t}$$

Instantaneous Angular Acceleration

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

$\bar{\alpha}$	average angular acceleration
$\vec{\alpha}$	angular acceleration
$\Delta\vec{\omega}$	change in angular velocity
Δt	change in time (See: Change in Time)
$\frac{d\omega}{dt}$	derivative of angular velocity with respect to time

Rotational Motion with Constant Angular Acceleration

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

ω_f	final angular velocity
ω_i	initial angular velocity
α	angular acceleration
t	time
θ_f	final angular displacement

θ_i	initial angular displacement

Equations parralel the equations of motion for linear motion (See: Linear Motion with Constant Acceleration)

Inertia

$$\mathrm{I} = mr^2 \implies \text{single-particle system}$$

$$\mathrm{I} = \sum_{i=1}^{n} m_i r_i^2 \implies \text{multi-particle discrete systems}$$

$$\mathrm{I} = \int r^2 dm \implies \text{multi-particle continuous systems}$$

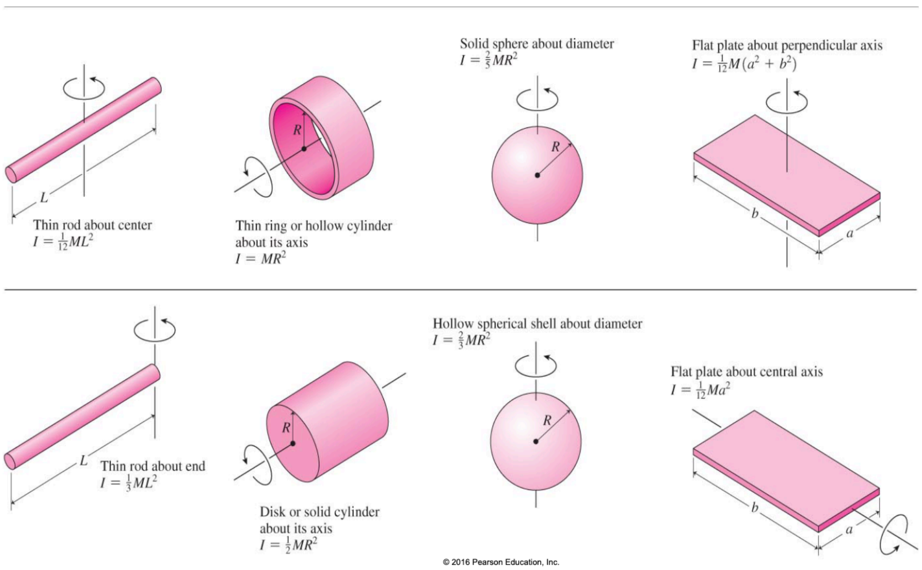

Figure 16: Inertia of different objects

I	moment of inertia
m	mass
n	number of particles
r_i	distance from the axis of rotation to the ith particle
r	distance from the axis of rotation to the point of application of the force

dm	mass element

- Inertia in rotational motion parrallels mass in linear motion, and is a measure of the resistance of an object to changes in its rotational motion. (higher inertia = harder to rotate)
- **Note**: Inertia doesn't have a direction, but it does depend on the axis of rotation

Parallel Axis Theorem

$$\mathrm{I} = \mathrm{I}_{cm} + md^2$$

I	moment of inertia about an axis parallel to I_{cm}
I_{cm}	moment of inertia about the center of mass
m	total mass of the system
d	distance from the center of mass to the axis of rotation

Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF_{\perp} = rF\sin\theta$$

$$\tau = \mathrm{I}\alpha$$

$$\tau = \frac{d\vec{L}}{dt}$$

τ	torque
r	distance from the axis of rotation to the point of application of the force
F	magnitude of the force
θ	angle between the force and the lever arm (radius)
I	moment of inertia (See: Inertia)
α	angular acceleration (See: Angular Acceleration)
$\vec{L}$	angular momentum (See: Angular Momentum)
$\frac{d\vec{L}}{dt}$	derivative of angular momentum with respect to time

- For Direction (See: Right Hand Rule)
- $\vec{r} \times \vec{F}$ is the cross product of $\vec{r}$ and $\vec{F}$ (See: Cross Product)
- Torque is the rotational equivalent of force

Rotational Kinetic Energy

$$K = \frac{1}{2}\mathrm{I}\omega^2$$

K	rotational kinetic energy
I	moment of inertia (See: Inertia)
ω	angular velocity (See: Angular Velocity)

Rotational kinetic energy is the energy of an object due to its rotational motion

Rolling Motion

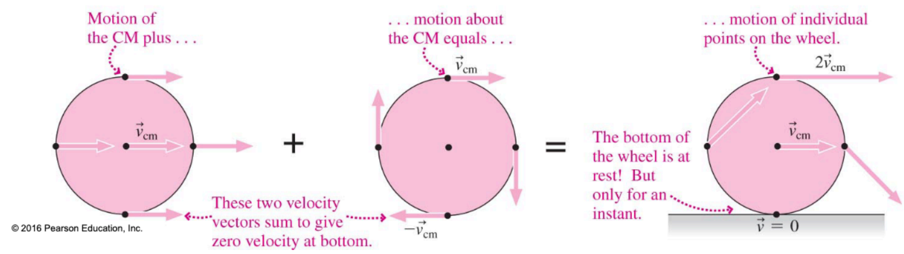

Figure 17: Rolling motion

$$v_{cm} = \omega R$$

$$a_{cm} = \alpha R$$

$$\omega = \frac{2\pi}{\Delta t}$$

v_{cm}	velocity of the center of mass
ω	angular velocity (See: Angular Velocity)
R	radius of the object
Δt	time it takes for the object to make one full rotation (See: Change in Time)

Rolling motion is a combination of translational and rotational motion

Kinetic Energy of Rolling Motion

$$K = K_{trans} + K_{rot}\frac{1}{2}mv_{cm}^2 + \frac{1}{2}\mathrm{I}\omega^2$$

K	kinetic energy
K_{trans}	translational kinetic energy
K_{rot}	rotational kinetic energy
m	mass of the system
v_{cm}	velocity of the center of mass
I	moment of inertia (See: Inertia)
ω	angular velocity (See: Angular Velocity)

The kinetic energy of an object rolling motion is the sum of the translational and rotational kinetic energies

Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = \mathrm{I}\vec{\omega}$$

$$\vec{L} = m(\vec{r} \times \vec{v})$$

$$L = mrv\sin\theta$$

$\vec{L}$	angular momentum
$\vec{r}$	distance from the axis of rotation to the point of application of the force
$\vec{p}$	linear momentum
I	moment of inertia (See: Inertia)
ω	angular velocity (See: Angular Velocity)
m	mass
$\vec{v}$	velocity
θ	angle between the velocity and the lever arm (radius)

For Direction (See: Right Hand Rule)

Conservation of Angular Momentum

$$\tau_{\text{Net Ext}} = 0 \implies \vec{L}_i = \vec{L}_f$$

$\tau_{\text{Net Ext}}$	net external torque
$\vec{L}_i$	initial angular momentum
$\vec{L}_f$	final angular momentum

Angular momentum is conserved when the net external torque is zero

Static Equilibrium

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\tau_{any} = 0$$

$\sum F$	sum of the forces
τ_{any}	torque at any axis of rotation
F_x	x-component of the force
F_y	y-component of the force
τ	torque

An object is in static equilibrium when it is at rest and the net force and the torque acting at any axis is zero

Simple Harmonic Motion

Harmonic Motion of a Horizontal Spring

$$\frac{md^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\omega^2 m$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

$$x = A\cos(\omega t + \phi)$$

$$v = -A\omega\sin(\omega t + \phi)$$

$$x(t = 0) = A \implies \phi = 0$$

$$x(t = 0) = 0 \implies \phi = \frac{\pi}{2}$$

m	mass of the object
x	displacement from equilibrium
k	spring constant
ω	angular frequency (See: Angular Frequency)
T	period (the time it takes for the spring to complete one full oscillation)
A	amplitude (maximum displacement from equilibrium)
ϕ	phase constant

The displacement of an object attached to a spring is a sinusoidal function of time

Harmonic Motion of a Vertical Spring

$$\frac{md^2y}{dt^2} = mg - k(\Delta l + y) = mg - k\Delta l - ky$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

m	mass of the object
y	displacement from equilibrium
k	spring constant
ω	angular frequency (See: Angular Frequency)
T	period (the time it takes for the spring to complete one full oscillation)
Δl	change in length of the spring when the object is attached to it

Simple Pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

T	period (the time it takes for the spring to complete one full oscillation)
l	length of the pendulum
g	acceleration due to gravity

A simple pendulum is a point mass suspended from a massless string

Physical Pendulum

$$T = 2\pi\sqrt{\frac{\mathrm{I}}{mgh}}$$

T	period (the time it takes for the spring to complete one full oscillation)
I	moment of inertia about the pivot point

m	mass of the object
g	acceleration due to gravity
h	distance from the pivot point to the center of mass

A physical pendulum is a rigid body suspended from a pivot point

Linked Equations

Vectors

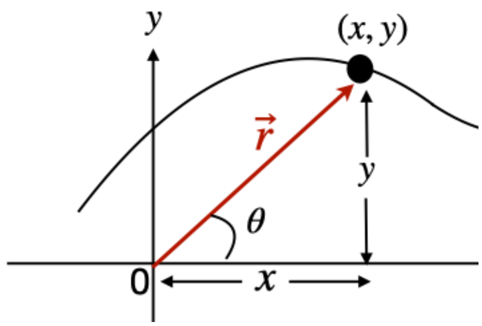

Figure 18: A vector on a Cartesian chart

$$\vec{r} = (r, \theta)$$

$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2}$$

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right)$$

$$r_x = r\cos\theta$$

$$r_y = r\sin\theta$$

$\vec{r}$	vector
r	magnitude (length) of $\vec{r}$
θ	direction (angle relative to the horizontal)
r_x	x-component of $\vec{r}$
r_y	y-component of $\vec{r}$

- A vector in general is a quantity that is made up of 2 scalar quantities, magnitude and direction.
- In physics, vectors are represented by arrows. The length of the arrow represents the magnitude of the vector and the direction of the arrow represents the direction of the vector.
- The vectors $\vec{r}$ and $-\vec{r}$ have the same magnitude but opposite directions.

Unit Vectors

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{r} = r_x\hat{i} \implies 1 \text{ dimension}$$

$$\vec{r} = r_x\hat{i} + r_y\hat{j} \implies 2 \text{ dimensions}$$

$$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k} \implies 3 \text{ dimensions}$$

$\hat{r}$	unit vector of $\vec{r}$
$\vec{r}$	vector
$\lvert\vec{r}\rvert$	magnitude of $\vec{r}$
r_x	x-component of $\vec{r}$
r_y	y-component of $\vec{r}$
$\hat{i}$	unit vector in the x direction
$\hat{j}$	unit vector in the y direction
$\hat{k}$	unit vector in the z direction

- A unit vector of a vector $\vec{r}$ is a vector in the same direction as $\vec{r}$ with a magnitude of 1.
- In the equations, $\hat{i}$ and $\hat{j}$ give a direction to r_x and r_y transforming them into vectors.

Vector Arithmetic

Scalar Multiplication

$$a\vec{r} = (ar, \theta)$$

a	scalar
$\vec{r}$	vector
r	magnitude of $\vec{r}$ (scalar)
θ	direction of $a\vec{r}$ (relative to the horizontal)

Scalar multiplication of a vector $\vec{r}$ by a scalar a is a vector in the same direction as $\vec{r}$ with a magnitude of ar.

Addition & Subtraction

$$\vec{A}+\vec{B}=(A_x+B_x)\hat{\imath}+(A_y+B_y)\hat{\jmath}$$

$$\vec{A}-\vec{B}=(A_x-B_x)\hat{\imath}+(A_y-B_y)\hat{\jmath}$$

$\vec{A}$, $\vec{B}$	vectors
A_x, B_x	x-components of $\vec{A}$, $\vec{B}$
A_y, B_y	y-components of $\vec{A}$, $\vec{B}$
$\hat{\imath}$, $\hat{\jmath}$	unit vector in the x, y direction (See: Unit Vectors)

On a graph, if you connect the vectors $\vec{A}$ and $\vec{B}$ head to tail, the vector from the tail of $\vec{A}$ to the head of $\vec{B}$ is the sum of $\vec{A}$ and $\vec{B}$.

Dot Product

$$\vec{A}\cdot\vec{B}=AB\cos\theta$$

$$\theta=0° \implies \vec{A}\parallel\vec{B} \implies \vec{A}\cdot\vec{B}=AB$$

$$\theta=90° \implies \vec{A}\perp\vec{B} \implies \vec{A}\cdot\vec{B}=0$$

$$\theta=180° \implies \vec{A}\parallel\vec{B}\ \text{(anti-parallel)} \implies \vec{A}\cdot\vec{B}=-AB$$

$\vec{A}$, $\vec{B}$	vectors
A, B	magnitudes of $\vec{A}$, $\vec{B}$
θ	angle between $\vec{A}$, $\vec{B}$
$\hat{\imath}$, $\hat{\jmath}$	unit vector in the x, y direction (See: Unit Vectors)

The dot product of two vectors $\vec{A}$ and $\vec{B}$ is a scalar.

Integration

$$\int_{x_1}^{x_2} x^n dx = \frac{x^{n+1}}{n+1}\bigg|_{x_1}^{x_2} = \frac{x_2^{n+1}}{n+1}-\frac{x_1^{n+1}}{n+1}$$

x_1	lower limit of integration
x_2	upper limit of integration

n	power of x

Change in Time

$$\Delta t = t_f - t_i$$

t_f	final time
t_i	initial time

Usually, Δt will just be given as an amount of time, it is sometimes written as t.

Change in Velocity

$$\Delta v = v_f - v_i$$

v_f	final velocity
v_i	initial velocity

Note: Δv is a vector quantity, it uses vector subtraction (See: Addition / Subtraction).

Cross Product

$$\vec{A} \times \vec{B} = AB \sin\theta \; \hat{n}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

$\vec{A}, \vec{B}$	vectors to be multiplied
A, B	magnitudes of the vectors
θ	angle between the vectors
$\hat{n}$	unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$

For direction of $\hat{n}$ use RHR (See: Right Hand Rule)