Kinetics

Constant Acceleration

$$v_f = v_i + a\Delta t$$

$$\Delta x = \frac{1}{2}\left(v_i + v_f\right)\Delta t$$

$$\Delta x = v_i \Delta t - \frac{1}{2}a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

Projectile Motion

$$x = v_{0x}t = v_0 t\cos\theta$$

$$y = v_{0y}t - \frac{1}{2}gt^2 = v_0 t\sin\theta - \frac{1}{2}gt^2$$

$$v_{0x} = v_0\cos\theta$$

$$v_{0y} = v_0\sin\theta$$

$$v_x = v_0\cos\theta$$

$$v_y = v_0\sin\theta - gt$$

$$t_{\text{projectile hits ground}} = \frac{v_0^2}{g}\sin 2\theta$$

Free Fall

$$v_f = v_i - gt$$

$$y = v_i t - \frac{1}{2}gt^2$$

$$v_f^2 = v_i^2 - 2gh$$

$$t_{\text{object hits ground}} = \sqrt{\frac{2h}{g}}$$

$$\vec{a} = -g\hat{j} \implies a = g$$

Circular Motion

$$\vec{a} = -\omega^2\vec{r}$$

$$T = \frac{2\pi r}{v}$$

$$\vec{v} = -\omega r\sin\theta\hat{i} + \omega r\cos\theta\hat{j}$$

$$|\vec{v}| = \omega r$$

$$\vec{r} = r\cos\theta\hat{i} + r\sin\theta\hat{j}$$

$$\theta = 0 \iff \vec{v} = \omega r\hat{j}$$

$$\theta = \frac{\pi}{2} \iff \vec{v} = -\omega r\hat{i}$$

$$\vec{F}_{\text{net}} = \frac{mv^2}{r}$$

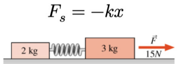
Translational Dynamics

Gravitational Force (weight)

$$\vec{F}_g = mg$$

Spring Force

$$F_s = -kx$$



$$\frac{F_s}{m_a} = \frac{F - F_s}{m_b}$$

Static Friction

$$\vec{f}_s \leq \mu_s N$$

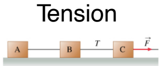
Kinetic Friction

$$\vec{f} = \mu_k\vec{N}$$

Normal Force

$$\vec{N} = mg\cos\theta$$

Tension



$$\vec{T} = \vec{F} - \vec{F}_B$$

Newtons Law

$$F_{\text{Net}} = ma$$

$$\vec{p} = m\vec{v}$$

Kinetic Energy

$$K = \frac{1}{2}mv^2$$

Potential Energy

$$\Delta U_{A\to B} = -\int_A^B \vec{F}\cdot d\vec{r} = -W_{A\to B}$$

Gravity $\quad U_g = mgh$

Spring $\quad U_s = \frac{1}{2}kx^2$

Work

$$W = \int_A^B \vec{F}\cdot d\vec{r}$$

$$W = \vec{F}\cdot\vec{d}$$

$$W = \Delta K = K_f - K_i$$

$$W_g = -mgh$$

$$W = \int_{x_i}^{x_f} F(x)dx$$

Power

$$P = \frac{W}{\Delta t}$$

$$P = \vec{F}\cdot\vec{v} \text{ (if force is constant)}$$

Universal Gravity

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$U_g = -G\frac{m_1 m_2}{r}$$

Circular Orbits

$$v = \sqrt{\frac{GM}{r}}$$

$$T = \frac{2\pi r}{v}$$

Gravitational Field

$$\vec{E} = \frac{\vec{F}}{m}$$

Escape Velocity

$$v_e = \sqrt{\frac{2GM}{r}}$$

Center of Mass

$$\vec{r}_{cm} = \frac{\sum_{i=1}^n m_i\vec{r}_i}{M}$$

$$\vec{r}_{cm} = \lim_{\Delta m\to 0}\frac{\sum_{i=1}^n \Delta m_i\vec{r}_i}{M} = \frac{\int \vec{r}dm}{M}$$

$$\vec{p}_{tot} = \sum_{i=1}^n \vec{p}_i = M\vec{v}_{cm}$$

$$\vec{F}_{net} = 0 \implies \vec{p}_{tot} = \text{constant}$$

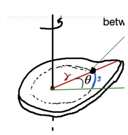
$$K_{tot} = \sum_{i=1}^n K_i = \frac{1}{2}Mv_{cm}^2$$

Collisions

$$\int_{t_i}^{t_f}\vec{F}(t)dt \approx \vec{F}_{avg}\Delta t$$

Elastic collisions are collisions where the total kinetic energy of the system is conserved.

Rotational Motion



$$s = r\theta$$

$$v_t = r\omega$$

$$a_t = r\alpha$$

$$a_r = \frac{v_t^2}{r} = r\omega^2$$

$$1\ rpm = \frac{2\pi}{60}\ rad/s$$

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$\vec{\omega} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\theta}}{\Delta t} = \frac{d\vec{\theta}}{dt}$$

$$\bar{\alpha} = \frac{\Delta\vec{\omega}}{\Delta t}$$

$$\vec{\alpha} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

Constant Acceleration

$$\omega_f = \omega_i + \alpha t$$

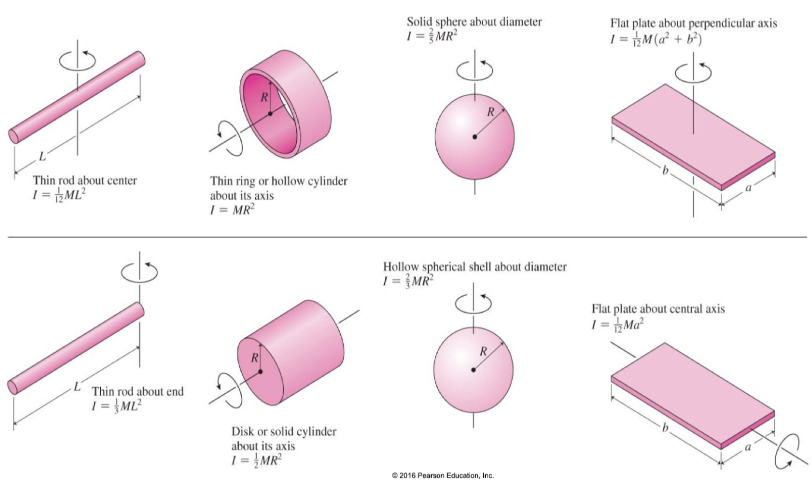
$$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

Inertia

$$\mathrm{I} = mr^2$$



Parallel Axis Theorem

$$\mathrm{I} = \mathrm{I}_{cm} + md^2$$

Torque

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$\tau = rF_\perp = rF\sin\theta$$

$$\tau = \mathrm{I}\alpha$$

$$\tau = \frac{d\vec{L}}{dt}$$

Kinetic Energy

$$K = \frac{1}{2}\mathrm{I}\omega^2$$

Rolling Motion

$$v_{cm} = \omega R$$

$$a_{cm} = \alpha R$$

$$\omega = \frac{2\pi}{\Delta t}$$

$$K = K_{trans} + K_{rot}\frac{1}{2}mv_{cm}^2 + \frac{1}{2}\mathrm{I}\omega^2$$

Angular Momentum

$$\vec{L} = \vec{r}\times\vec{p}$$

$$\vec{L} = \mathrm{I}\vec{\omega}$$

$$\vec{L} = m(\vec{r}\times\vec{v})$$

$$L = mrv\sin\theta$$

$$\tau_{\text{Net Ext}} = 0 \implies \vec{L}_i = \vec{L}_f$$

Static Equillibrium

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\tau_{any} = 0$$

Simple Harmonic Motion

Horizontal Spring

$$\frac{md^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\omega^2 m$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

$$x = A\cos(\omega t + \phi)$$

$$v = -A\omega\sin(\omega t + \phi)$$

$$x(t=0) = A \implies \phi = 0$$

$$x(t=0) = 0 \implies \phi = \frac{\pi}{2}$$

Vertical Spring

$$\frac{md^2y}{dt^2} = mg - k(\Delta l + y) = mg - k\Delta l - ky$$

$$\frac{d^2y}{dt^2} = -\omega^2 y$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega}$$

Simple Pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Physical Pendulum

$$T = 2\pi\sqrt{\frac{\mathrm{I}}{mgh}}$$

Vectors

$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2}$$

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right)$$

$$r_x = r\cos\theta$$

$$r_y = r\sin\theta$$

Vector Addition

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Dot Product

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

Cross Product

$$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n}$$

$$\hat{j}\times\hat{i} = -\hat{k}$$

$$\hat{k}\times\hat{j} = -\hat{i}$$

$$\hat{i}\times\hat{k} = -\hat{j}$$