Miscellaneous

SI Prefixes

Prefix	Symbol	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

Units

$$\text{Pa} = \text{N/m}^2$$

$$\text{W} = \text{J/s} = \text{N} \cdot \text{m/s}$$

$$\text{J} = \text{N} \cdot \text{m}$$

$$\text{N} = \text{kg} \cdot \text{m/s}^2$$

$$\text{V} = \text{J/C}$$

Unit Conversions

$$1 \text{ cal} = 4.184 \text{ J}$$

$$°\text{F} = \frac{9}{5}°\text{C} + 32$$

$$\text{K} = °\text{C} + 273.15$$

Vectors

A vector on a Cartesian chart

Figure 1: A vector on a Cartesian chart

$$\vec{r} = (r, \theta)$$

$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2}$$

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right)$$

$$r_x = r\cos\theta$$

$$r_y = r\sin\theta$$

$\vec{r}$	vector
r	magnitude (length) of $\vec{r}$
θ	direction (angle relative to the horizontal)
r_x	x-component of $\vec{r}$
r_y	y-component of $\vec{r}$

- A vector in general is a quantity that is made up of 2 scalar quantities, magnitude and direction.
- In physics, vectors are represented by arrows. The length of the arrow represents the magnitude of the vector and the direction of the arrow represents the direction of the vector.
- The vectors $\vec{r}$ and $-\vec{r}$ have the same magnitude but opposite directions.

Unit Vectors

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{r} = r_x\hat{i} \implies 1 \text{ dimension}$$

$$\vec{r} = r_x\hat{i} + r_y\hat{j} \implies 2 \text{ dimensions}$$

$$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k} \implies 3 \text{ dimensions}$$

$\hat{r}$	unit vector of $\vec{r}$
$\vec{r}$	vector
$\lvert\vec{r}\rvert$	magnitude of $\vec{r}$
r_x	x-component of $\vec{r}$
r_y	y-component of $\vec{r}$
$\hat{i}$	unit vector in the x direction
$\hat{j}$	unit vector in the y direction
$\hat{k}$	unit vector in the z direction

- A unit vector of a vector $\vec{r}$ is a vector in the same direction as $\vec{r}$ with a magnitude of 1.
- In the equations, $\hat{i}$ and $\hat{j}$ give a direction to r_x and r_y transforming them into vectors.

Vector Arithmetic

Scalar Multiplication

$$a\vec{r} = (ar, \theta)$$

a	scalar
$\vec{r}$	vector
r	magnitude of $\vec{r}$ (scalar)
θ	direction of $a\vec{r}$ (relative to the horizontal)

Scalar multiplication of a vector $\vec{r}$ by a scalar a is a vector in the same direction as $\vec{r}$ with a magnitude of ar.

Addition & Subtraction

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$

$\vec{A}, \vec{B}$	vectors
A_x, B_x	x-components of $\vec{A}, \vec{B}$
A_y, B_y	y-components of $\vec{A}, \vec{B}$
$\hat{i}, \hat{j}$	unit vector in the x, y direction (*See*: **Unit Vectors**)

On a graph, if you connect the vectors $\vec{A}$ and $\vec{B}$ head to tail, the vector from the tail of $\vec{A}$ to the head of $\vec{B}$ is the sum of $\vec{A}$ and $\vec{B}$.

Dot Product

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\theta = 0° \implies \vec{A} \parallel \vec{B} \implies \vec{A} \cdot \vec{B} = AB$$

$$\theta = 90° \implies \vec{A} \perp \vec{B} \implies \vec{A} \cdot \vec{B} = 0$$

$$\theta = 180° \implies \vec{A} \parallel \vec{B} \text{ (anti-parallel)} \implies \vec{A} \cdot \vec{B} = -AB$$

$\vec{A}, \vec{B}$	vectors
A, B	magnitudes of $\vec{A}, \vec{B}$
θ	angle between $\vec{A}, \vec{B}$
$\hat{i}, \hat{j}$	unit vector in the x, y direction (*See*: **Unit Vectors**)

The dot product of two vectors $\vec{A}$ and $\vec{B}$ is a scalar.

Cross Product

$$\vec{A} \times \vec{B} = AB\sin\theta\ \hat{n}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

$\vec{A}, \vec{B}$	vectors to be multiplied
A, B	magnitudes of the vectors
θ	angle between the vectors
$\hat{n}$	unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$

For direction of $\hat{n}$ use RHR (*See*: **Right Hand Rule**).

RHR Diagram

Figure 2: RHR Diagram

Right Hand Rule

- Direction of rotation is determined by the position of the thumb
  - If the thumb points in the direction of the axis of rotation (**typically upward**), then the direction is positive
  - If the thumb points in the opposite direction of the axis of rotation (**typically downward**), then the direction is negative

Gradient (Vector)

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \text{ (Cartesian)}$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \text{ (Spherical)}$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z} \text{ (Cylindrical)}$$

- **Note**: ∂ is a partial derivative
  - $\frac{\partial}{\partial x}$ means take the derivative with respect to x while holding all other variables constant.

7

Integration

$$\int_{x_1}^{x_2} x^n dx = \frac{x^{n+1}}{n+1}\Big|_{x_1}^{x_2} = \frac{x_2^{n+1}}{n+1} - \frac{x_1^{n+1}}{n+1}$$

x_1	lower limit of integration
x_2	upper limit of integration
n	power of x

Capacitors

For charging/discharging, *see*: **Capacitor Charging in Circuits**

Created by 2 plates of area A, equal and opposite charge Q separated by a distance d

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$$

$$\vec{E}_{\text{one plate}} = \frac{\sigma}{2\epsilon_0}\hat{n}$$

$$\sigma = \frac{Q}{A}$$

$\vec{E}$	Electric field between plates
σ	Charge density on each plate
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2$)
$\hat{n}$	Unit vector perpendicular to the plates (from positive to negative plate)
Q	Charge on each plate
A	Area of each plate

Capacitance

SI unit: F (Farads)

$$C = \frac{Q}{V}$$

$$C = \frac{\epsilon_0 A}{d}$$

C	Capacitance
Q	Charge (C)
V	Potential difference (*See*: **Electric Potential**)
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2$)
A	Area of each plate
d	Distance between plates

Energy of a Capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

$$U = \int u_E dV = \frac{1}{2}\epsilon_0 \int E^2 dV$$

$$U = \frac{1}{2}\epsilon_0 E^2 \times \text{Volume between plates}$$

$$V = -Ed = -\frac{\sigma}{\epsilon_0}d$$

U	Energy of a capacitor
C	Capacitance (*See*: **Capacitance**)
V	Potential difference (*See*: **Electric Potential**)
u_E	Energy density of electric field
E	Electric field between plates (*See*: **Electric Field**)
V	Potential difference (*See*: **Electric Potential**)
σ	Charge density on each plate (*See*: **Capacitors**)
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2$)

Capacitors in Series/Parallel

See: **Series vs Parallel** for differences between series and parallel

$$\frac{1}{C_{\text{eq}}} = \sum_i \frac{1}{C_i} \text{ (Series)}$$

$$C_{\text{eq}} = \sum_i C_i \text{ (Parallel)}$$

C_{eq}: capacitance of capacitor equivalent to the series/parallel combination

Dielectrics

Dielectric: Insulating material between the plates of a capacitor

$$C = \kappa C_0$$

$$\epsilon = \kappa\epsilon_0$$

C	Capacitance
κ	Dielectric constant (*See*: **Table 5**)
C_0	Capacitance without dielectric
ϵ	Permittivity of dielectric material (*See*: **Table 5**)
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2$)

Table 5: Dielectric Constants

Material	κ
Air	1.0006
Aluminum oxide	8.4
Glass (Pyrex)	5.6
Paper	3.5
Plexiglas	3.4
Polyethylene	2.3
Polystyrene	2.6
Quartz	3.8
Tantalum Oxide	26
Teflon	2.1
Water	80

Conductive slab between plates

Inserting a conductive slab between the plates of a capacitor increases the capacitance the same as if the slab and the area filled by the slab were removed from the capacitor

Current and Resistance

- Current: I
  - flow of charge
  - **SI unit**: A (Amps)
- Resistance: R
  - opposition to current
  - **SI unit**: Ω (Ohms)

$$I = \frac{\Delta Q}{\Delta t}$$

$$J = \frac{I}{A}$$

I	Current
Q	Charge
t	Time
J	Current density
A	Area

Ohm's Law

$$V = IR$$

$$J = \sigma E = \frac{E}{\rho}$$

$$\sigma = \frac{1}{\rho}$$

$$R \equiv \frac{\rho L}{A}$$

V	Potential difference (*See*: **Electric Potential**)
I	Current
R	Resistance
J	Current density
σ	Conductivity
ρ	Resistivity (*See*: **Table 6**)
E	Electric field
L	Length (of wire)
A	Cross-sectional area (of wire)

Table 6: Resistivity

Material	Resistivity: ρ ($\Omega\cdot$m)
Metals	
Aluminum	2.65×10^{-8}
Copper	1.68×10^{-8}
Gold	2.24×10^{-8}
Iron	9.71×10^{-8}
Mercury	9.84×10^{-7}
Silver	1.59×10^{-8}
Solutions	
1-molar $CuSO_4$	3.9×10^{-4}
1-molar HCL	1.7×10^{-2}
1-molar NaCl	1.4×10^{-4}
H_2O	2.6×10^{5}
Human Blood	0.70
Seawater	0.22
Semiconductors	
Geranium	0.5
Silicon	3×10^{3}
Insulators	
Ceramic	$10^{11} - 10^{14}$
Glass	$10^{10} - 10^{14}$
Polystyrene	$10^{15} - 10^{17}$
Rubber	$10^{13} - 10^{16}$
Wood (dry)	$10^{8} - 10^{14}$

Drift Velocity

$$I = nqA\vec{v}_d e$$

$$J = nq\vec{v}_d e$$

$$n = \frac{\rho}{m}$$

I	Current
n	Number of charge carriers (atoms) per unit volume
q	Charge of each atom ($-e = -1.6 \times 10^{-19}$C)
A	Cross-sectional area
J	Current density
$\vec{v}_d$	Drift velocity
e	Elementary charge (1.6×10^{-19}C)
ρ	mass density
m	Mass of each atom

Power

$$P = IV = I^2R = \frac{V^2}{R}$$

P	Power (W)
I	Current (A)
V	Potential (V) (*See*: **Electric Potential**)
R	Resistance (Ω)

Circuits

Current is the same everywhere in a series circuit

Circuit Simulator

Voltmeter vs Ammeter

- Voltmeter: parallel
  - measures voltage across 2 terminals
  - ideally has infinite resistance
- Ammeter: series
  - measures current through itself
  - ideally has zero resistance

Series vs Parallel

Series: same current, different voltage Parallel: same voltage, different current

Resistors in Parallel

Voltage is the same across each resistor

Current is split between each resistor

Equivalent resistance is less than the smallest resistor

$$I_p = \sum_i I_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$I_1 = I\frac{R_2}{R_1 + R_2} \text{ (2 resistors)}$$

$$I_2 = I\frac{R_1}{R_1 + R_2} \text{ (2 resistors)}$$

Resistors in Series

Current is the same across each resistor

Voltage is split between each resistor

Equivalent resistance is the sum of all resistors

$$V_s = \sum_i V_i$$

$$R_s = \sum_i R_i$$

$$V_1 = V\frac{R_1}{R_1 + R_2} \text{ (2 resistors)}$$

$$V_2 = V\frac{R_2}{R_1 + R_2} \text{ (2 resistors)}$$

I	Current (*See*: **Current and Resistance**)
R	Resistance (*See*: **Current and Resistance**)
V	Potential difference (*See*: **Electric Potential**)
I_p	Total current in parallel circuit
I_i, I_1, I_2	Current in each resistor in parallel circuit
R_p, R_s	Total resistance in parallel and series circuit (respectively)
R_i, R_1, R_2	Resistance in each resistor in the circuit
V_s	Total voltage in series circuit
V_i, V_1, V_2	Voltage in each resistor in series circuit

Kirchoff's Voltage Law

The sum of all voltages in a closed loop is zero

$$\sum_i V_i = 0 \text{ (closed loop)}$$

- Draw loops & assign current (I) to each.
- Find $\sum_i V_i$ for each loop: Start with battery EMF/voltage (ε), then go around loop subtract voltage drops
- Voltage drops across resistor $ = IR$
  - $I = \sum I_i$ (each loop)

Kirchoff's Current Law

The sum of all currents entering a node is zero

$$\sum_i I_i = 0 \text{ (node)}$$

- Choose nodes & assign voltage (V) to each.
- Assign current (I) to each branch entering each node
- Find $\sum_i I_i$ for each node: Incoming current is positive, outgoing current is negative

Capacitor Charging in Circuits

For in-depth capacitor mechanics, *see*: **Capacitors**

Initial State ($t = 0$)

Capacitor acts as a short circuit (no voltage)

Plates have 0 charge

$$q = 0 \implies E = 0 \implies V_c = 0 \text{ (min)}$$

$$I = \frac{\varepsilon}{R} \text{ (max)}$$

Intermediate State ($0 < t < \infty$)

Capacitor charges/discharges

Exponential growth/decay

$$q = Q(1 - e^{-t/z})$$

$$I = \frac{\varepsilon}{R}e^{-t/z}$$

$$V_c = \varepsilon(1 - e^{-t/z})$$

$$z = RC \text{ (time constant)}$$

Final State ($t \to \infty$)

Enough time for capacitor to sufficiently charge

Capacitor is effectively an open circuit (no current)

$$q = Q = CV \implies E = \frac{\sigma}{\epsilon_0} \implies V_c = \varepsilon \text{ (max)}$$

$$I = 0 \text{ (min)}$$

q	Charge (*See*: **Capacitors**)
Q	Maximum charge (*See*: **Capacitors**)
C	Capacitance (*See*: **Capacitors**)
E	Electric field (*See*: **Capacitors**)
V_c	Voltage across capacitor (*See*: **Capacitors**)
I	Current (*See*: **Current and Resistance**)
R	Resistance (*See*: **Current and Resistance**)
ε	Electromotive force (EMF, same as voltage) (*See*: **Electric Potential**)
σ	Surface charge density (*See*: **Capacitors**)
ϵ_0	Permittivity of free space (*See*: **Capacitors**)
t	Time since circuit closed

Magnetic Field

SI Unit: Tesla (T)

Magnetic field can be generated by current

Direction of current determines the direction of the magnetic field via the right-hand rule

ε	Electromotive force (EMF, same as voltage) (*See*: **Electric Potential**)
σ	Surface charge density (*See*: **Capacitors**)
ϵ_0	Permittivity of free space (*See*: **Capacitors**)
t	Time since circuit closed

Magnetic Field

SI Unit: Tesla (T)

Magnetic field can be generated by current

Direction of current determines the direction of the magnetic field via the right-hand rule

Biot-Savart Law

Magnetic field generated by a current-carrying wire

$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2}$$

$\vec{B}$	Magnetic field
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
I	Current in wire
$d\vec{l}$	Differential length of wire
$\hat{r}$	Unit vector from wire to point of interest
r	Distance from wire to point of interest

Use right-hand rule to determine direction of magnetic field

Magnetic field lines are circles around the wire (azimuthal)

Follows inverse square law

Ampere's Law

- **Requirements**:
  - $\vec{B}$ is constant along the amperian loop
  - $\vec{B}$ is tangential to the amperian loop
- For a straight wire, amperian loop is a centered around the wire

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encircled}}$$

$$B \cdot L = \mu_0 I_{\text{encircled}}$$

$\vec{B}$	Magnetic field
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
I	Current in wire
$d\vec{l}$	Differential length of wire
r	Distance from wire to point of interest
L	Length of amperian loop $= 2\pi r$

Gauss's Law for Magnetism

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$\vec{B}$	Magnetic field
$d\vec{A}$	Differential area element

Magnetic Field of Simple Current Distributions

Straight Wire

$$B = \frac{\mu_0 I}{2\pi r}$$

B	Magnetic field at point of interest
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
I	Current in wire
r	Distance from wire to point of interest

Loop of Wire

$$\vec{B} = \frac{\mu_0 I r_w^2}{2r}\hat{i}$$

$$x = 0 \implies \vec{B} = \frac{\mu_0 I}{2r_w}\hat{i} \text{ (center)}$$

$$x \gg r_w \implies \vec{B} = \frac{\mu_0 I r_w^2}{2x^3}\hat{i} \text{ (far-field)}$$

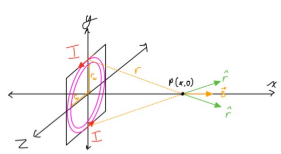

Figure 13: Diagram of loop

$$r = \sqrt{r_w^2 + x^2}$$

B	Magnetic field
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
I	Current in wire
r_w	Radius of wire
r	Distance from wire to point of interest
x	Distance from center of loop to point of interest

Sheet (Plane) of Current

$$B = \frac{1}{2}\mu_0 J_s$$

B	Magnetic field
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
J_s	Surface current density

Solenoid (Coil)

$$B = \mu_0 n I$$

B	Magnetic field
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
n	Number of turns per unit length
I	Current in wire

Force of a Magnetic Field on a Moving Charge

$$\vec{F} = q\vec{v} \times \vec{B}$$

$\vec{F}$	Force on the particle (charge)
q	Charge of the particle
$\vec{v}$	Velocity of the particle

Cyclotron Motion

$$f = \frac{qB}{2\pi m}$$

f	Cyclotron frequency (period of rotation)
q	Charge of the particle
B	Magnetic field
m	Mass of the particle

Note: $\times$ is the cross product (*See*: **Cross Product**)

Magnetic Force on a Straight Wire

$$\vec{F} = I\vec{l} \times \vec{B} \text{ (from uniform field)}$$

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ (from parallel wire)}$$

$\vec{F}$	Force on the particle (charge)
I	Current in wire
$\vec{l}$	Length of wire
$\vec{B}$	Magnetic field
d	Distance between wires
μ_0	Permeability of free space $= 4\pi \times 10^{-7} \frac{Tm}{A}$
I_1, I_2	Current in each wire

Force is attractive if currents are in the same direction, repulsive if in opposite directions

Note: $\times$ is the cross product (*See*: **Cross Product**)