Miscellaneous

SI Prefixes

Prefix	Symbol	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

Units

$$\text{Pa} = \text{N/m}^2$$

$$\text{W} = \text{J/s} = \text{N}\cdot\text{m/s}$$

$$\text{J} = \text{N}\cdot\text{m}$$

$$\text{N} = \text{kg}\cdot\text{m/s}^2$$

$$\text{V} = \text{J/C}$$

Unit Conversions

$$1\ \text{cal} = 4.184\ \text{J}$$

$$°\text{F} = \frac{9}{5}°\text{C} + 32$$

$$\text{K} = °\text{C} + 273.15$$

Vectors

A vector on a Cartesian chart

Figure 1: A vector on a Cartesian chart

$$\vec{r} = (r, \theta)$$

$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2}$$

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right)$$

$$r_x = r\cos\theta$$

$$r_y = r\sin\theta$$

$\vec{r}$	vector
r	magnitude (length) of $\vec{r}$
θ	direction (angle relative to the horizontal)
r_x	x-component of $\vec{r}$
r_y	y-component of $\vec{r}$

- A vector in general is a quantity that is made up of 2 scalar quantities, magnitude and direction.
- In physics, vectors are represented by arrows. The length of the arrow represents the magnitude of the vector and the direction of the arrow represents the direction of the vector.
- The vectors $\vec{r}$ and $-\vec{r}$ have the same magnitude but opposite directions.

Unit Vectors

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{r} = r_x\hat{i} \implies 1 \text{ dimension}$$

$$\vec{r} = r_x\hat{i} + r_y\hat{j} \implies 2 \text{ dimensions}$$

$$\vec{r} = r_x\hat{i} + r_y\hat{j} + r_z\hat{k} \implies 3 \text{ dimensions}$$

$\hat{r}$	unit vector of $\vec{r}$
$\vec{r}$	vector
$\lvert\vec{r}\rvert$	magnitude of $\vec{r}$
r_x	x-component of $\vec{r}$
r_y	y-component of $\vec{r}$
$\hat{i}$	unit vector in the x direction
$\hat{j}$	unit vector in the y direction
$\hat{k}$	unit vector in the z direction

- A unit vector of a vector $\vec{r}$ is a vector in the same direction as $\vec{r}$ with a magnitude of 1.
- In the equations, $\hat{i}$ and $\hat{j}$ give a direction to r_x and r_y transforming them into vectors.

Vector Arithmetic

Scalar Multiplication

$$a\vec{r} = (ar, \theta)$$

a	scalar
$\vec{r}$	vector
r	magnitude of $\vec{r}$ (scalar)
θ	direction of $a\vec{r}$ (relative to the horizontal)

Scalar multiplication of a vector $\vec{r}$ by a scalar a is a vector in the same direction as $\vec{r}$ with a magnitude of ar.

Addition & Subtraction

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$

$\vec{A}, \vec{B}$	vectors
A_x, B_x	x-components of $\vec{A}, \vec{B}$
A_y, B_y	y-components of $\vec{A}, \vec{B}$
$\hat{i}, \hat{j}$	unit vector in the x, y direction (*See*: **Unit Vectors**)

On a graph, if you connect the vectors $\vec{A}$ and $\vec{B}$ head to tail, the vector from the tail of $\vec{A}$ to the head of $\vec{B}$ is the sum of $\vec{A}$ and $\vec{B}$.

Dot Product

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\theta = 0° \implies \vec{A} \parallel \vec{B} \implies \vec{A}\cdot\vec{B} = AB$$

$$\theta = 90° \implies \vec{A} \perp \vec{B} \implies \vec{A}\cdot\vec{B} = 0$$

$$\theta = 180° \implies \vec{A} \parallel \vec{B} \text{ (anti-parallel)} \implies \vec{A}\cdot\vec{B} = -AB$$

$\vec{A}, \vec{B}$	vectors
A, B	magnitudes of $\vec{A}, \vec{B}$
θ	angle between $\vec{A}, \vec{B}$
$\hat{i}, \hat{j}$	unit vector in the x, y direction (*See*: **Unit Vectors**)

The dot product of two vectors $\vec{A}$ and $\vec{B}$ is a scalar.

Cross Product

$$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n}$$

$$\hat{j}\times\hat{i} = -\hat{k}$$

$$\hat{k}\times\hat{j} = -\hat{i}$$

$$\hat{i}\times\hat{k} = -\hat{j}$$

$\vec{A}, \vec{B}$	vectors to be multiplied
A, B	magnitudes of the vectors
θ	angle between the vectors
$\hat{n}$	unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B}$

For direction of $\hat{n}$ use RHR (*See*: **Right Hand Rule**).

RHR Diagram

Figure 2: RHR Diagram

Right Hand Rule

- Direction of rotation is determined by the position of the thumb
  - If the thumb points in the direction of the axis of rotation (**typically upward**), then the direction is positive
  - If the thumb points in the opposite direction of the axis of rotation (**typically downward**), then the direction is negative

Gradient (Vector)

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \text{ (Cartesian)}$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \text{ (Spherical)}$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z} \text{ (Cylindrical)}$$

- **Note**: ∂ is a partial derivative
  - $\frac{\partial}{\partial x}$ means take the derivative with respect to x while holding all other variables constant.

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Integration

$$\int_{x_1}^{x_2} x^n dx = \frac{x^{n+1}}{n+1}\Big|_{x_1}^{x_2} = \frac{x_2^{n+1}}{n+1} - \frac{x_1^{n+1}}{n+1}$$

x_1	lower limit of integration
x_2	upper limit of integration
n	power of x

Induction

Changing magnetic field induces an electric field

Faraday's Law

$$\varepsilon_{\text{induced}} = -\frac{d\phi_B}{dt}$$

Recall: $\varepsilon = \oint \vec{E}\cdot d\vec{l}$ (*See*: **Electric Potential**)

$\varepsilon_{\text{induced}}$	Induced electromotive force (EMF, same as voltage) (*See*: **Electric Potential**)
$\frac{d\phi_B}{dt}$	Rate of change of magnetic flux (*See*: **Magnetic Flux**)

- RHR for direction of induced current and magnetic field
  - Determining $\vec{E}$ and I around a loop
    - Increasing $\vec{B}$: use right hand rule on $-\vec{B}$
    - Decreasing $\vec{B}$: use right hand rule on $\vec{B}$
- ϕ_B is maximum when $\vec{B} \perp d\vec{A}$
- ϕ_B is 0 (minimum) when $\vec{B} \parallel d\vec{A}$

Magnetic Energy Density TODO: verify link > Compare to electric energy density (*See*: **Capacitors**) > > Represents stored energy in magnetic field

$$u_B = \frac{B^2}{2\mu_0}$$

u_B	Magnetic energy density
B	Magnetic field (*See*: **Magnetic Field**)
μ_0	Permeability of free space ($\mu_0 = 4\pi\times10^{-7}\frac{Tm}{A}$)

Inductors

Solenids encircle their own magnetic flux giving *self-inductance*

$$L \equiv \frac{\Phi}{I} = \frac{NBA}{I} = \mu_0 N^2 \frac{A}{L}$$

$$\Phi = N\phi$$

$$B = \mu_0 n I$$

$$\text{AC} \implies I(t) = I_0\cos\omega t$$

L	Inductance
Φ	Magnetic flux
I	Current
N	Number of turns
ϕ	Magnetic flux per turn
B	Magnetic field (*See*: **Magnetic Field**)
μ_0	Permeability of free space ($\mu_0 = 4\pi\times10^{-7}\frac{Tm}{A}$)
n	Number of turns per unit length $= \frac{N}{L}$
A	Cross-sectional area of one loop $A = \pi r^2$

Inductors in Circuits

$$L = \frac{\phi_B}{I} \text{ (self-inductance)}$$

$$I = \frac{\varepsilon_0}{R}(1 - e^{-Rt/L})$$

$$\varepsilon_{\text{induced}} = -L\frac{dI}{dt}$$

L	Inductance
ϕ_B	Magnetic flux
I	Current
$\varepsilon_{\text{induced}}$	Induced electromotive force (EMF, same as voltage) (*See*: **Electric Potential**)
R	Resistance (*See*: **Current and Resistance**)
t	Time

Inductor Charging in Circuits Initial State ($t = 0$)

Inductor is effectively an open circuit (no current)

$$I = 0 \text{ (min)} \qquad V = \varepsilon \text{ (max)}$$

Intermediate State ($0 < t < \infty$)

Inductor charges/discharges

Exponential growth/decay

$$I_L(t) = \frac{\varepsilon}{R}(1 - e^{-t/\tau}) \qquad V_L(t) = \varepsilon e^{-t/\tau}$$

$$\tau = \frac{L}{R} \text{ (time constant)}$$

Final State ($t \to \infty$)

Enough time for Inductor to sufficiently charge

Inductor acts as a short circuit (no voltage)

$$I = \frac{\varepsilon}{R} \text{ (max)} \qquad V = 0 \text{ (min)}$$

I	Current (*See*: figure above)
V	Potential difference (*See*: **Electric Potential**)
ε	Electromotive force (EMF, same as voltage) (*See*: **Electric Potential**)
R	Resistance (*See*: figure above, and **Current and Resistance**)

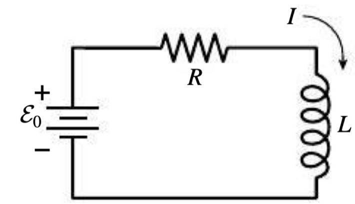


Figure 14: Inductor in a circuit

Energy Stored in an Inductor

$$U = \frac{1}{2}LI^2$$

U	Energy stored in an inductor
L	Inductance
I	Current

Maxwell's Equations

$$\oiint \vec{E}\cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0} \text{ (Gauss's Law)}$$

$$\oiint \vec{B}\cdot d\vec{A} = 0 \text{ (Gauss's Law for Magnetism)}$$

$$\oint_{\text{Amperian Loop}} \vec{B}\cdot d\vec{l} = \mu_0(I_{\text{encircled}} + I_{\text{displacement}}) \text{ (Modified Ampere's Law)}$$

$$\text{where } I_{\text{displacement}} = \epsilon_0\frac{d\Phi_E}{dt} = \epsilon_0\frac{d}{dt}\oiint\vec{E}\cdot d\vec{A} = \epsilon_0\oiint\frac{\partial\vec{E}}{\partial t}\cdot d\vec{A} \text{ (Displacement Current)}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt} \text{ (Faraday's Law)}$$

$\vec{E}$	Electric field
$\vec{B}$	Magnetic field
$d\vec{A}$	Differential area element

Electromagnetic Waves

- Electromagnetic waves are transverse waves
  - Electric and magnetic fields are perpendicular to eachother and the direction of propagation
- Electromagnetic waves are self-propagating

Speed of Electromagnetic Waves

$$-\frac{\partial B}{\partial x\partial t} = \frac{\partial^2 E}{\partial x^2} = \mu\epsilon\frac{\partial^2 E}{\partial t^2} \implies v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{n}$$

$$\text{vacuum} \implies \mu = mu_0 \quad \epsilon = \epsilon_0 \quad v = c$$

$$\text{material} \implies \mu = \mu_r\mu_0 \quad \epsilon = \epsilon_r\epsilon_0$$

$$E = vB$$

v	Speed of light (Electromagnetic waves)
c	Speed of light in a vacuum ($c = 3\times10^8\frac{m}{s}$)
μ	Permeability of material
μ_0	Permeability of free space ($\mu_0 = 4\pi\times10^{-7}\frac{Tm}{A}$)
ϵ	Permittivity of material
ϵ_0	Permittivity of free space ($\epsilon_0 = 8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2$)
E	Electric field
B	Magnetic field
n	Index of refraction

Einstein's Theory of Special Relativity c is the speed limit of the universe.

Frequency and Wavelength

$$v = \lambda f$$

v	Speed of light (Electromagnetic waves)
λ	Wavelength
f	Frequency

Intensity

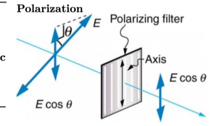
$$S \equiv \frac{P}{\text{Area}} = \frac{P}{4\pi r^2} = uv$$

$$u = u_E + u_B = \epsilon E^2 + \frac{B^2}{2\mu}$$

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu} \text{ (Poynting vector)}$$

S	Intensity of electromagnetic wave
P	Power
u	Energy density
v	Speed of electromagnetic wave
u_E, u_B	Energy density of electric and magnetic fields respectively (*See*: **Magnetic Energy Density** & **Capacitors**)
E, $\vec{E}$	Electric field
B, $\vec{B}$	Magnetic field
ϵ	Permittivity of material
μ	Permeability of material
r	Distance from source of electromagnetic wave

Polarization



$$S = S_0\cos^2\theta$$

$$E = E_0\cos\theta$$

S	Intensity of electromagnetic wave after passing through a polarizer
S_0	Intensity of electromagnetic wave before passing through a polarizer
θ	Angle between the polarizer and the direction of polarization
E	Electric field after passing through a polarizer
E_0	Electric field before passing through a polarizer

Radiation Pressure

SI Unit: Pa (Pascals)

$$P_{\text{rad}} = \frac{\bar{S}}{c}$$

P_{rad}	Radiation pressure
S	Intensity of electromagnetic wave
c	Speed of light in a vacuum ($c = 3\times10^8\frac{m}{s}$)

Optics

Law of Reflection

$$\theta_i = \theta_r$$

θ_i	Angle of incidence (angle from incoming wave to normal)
θ_r	Angle of reflection (angle from reflected wave to normal)

Law of Refraction (Snell's Law)

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

n_1, n_2	Index of refraction of material 1 and 2 respectively
θ_1	Angle of incidence (angle from incoming wave to normal)
θ_2	Angle of refraction (angle from refracted wave to normal)

Total Internal Reflection

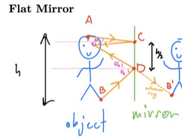
When $\theta_i > \theta_c$, the wave is reflected back into the material
Used in fiber optics

$$\sin\theta_c = \frac{n_2}{n_1}$$

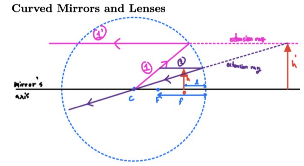
Dispersion Different colors refract at different angles

Image Formation

Flat Mirror



Curved Mirrors and Lenses



$$M \equiv \frac{h'}{h} = \frac{l'}{l}$$

$$\frac{1}{l} + \frac{1}{l'} = \frac{1}{f} \text{ (Thin Lens)}$$

$$\frac{n_1}{l} + \frac{n_2}{l'} = \frac{n_2 - n_1}{R} \text{ (Thick Lens, Type I)}$$

$$\frac{1}{l} + \frac{1}{l'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ (Thick Lens, Type II)}$$

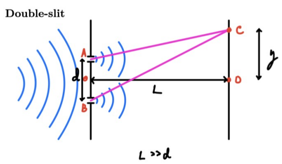
M	Magnification factor
o, i	object and image location respectively
h, h'	object and image height respectively
l, l'	object and image distance from mirror (center)respectively
F	focal point of mirror (point where parallel rays converge)
f	focal length (distance from mirror to focal point)
R_1, R_2	radii of curvature of mirror (positive for concave, negative for convex)
n_1, n_2	indices of refraction of outside and inside the lens respectively

- **Sign Convention**:
  - Mirrors
    - f is positive for concave mirrors, negative for convex mirrors
    - l is positive
    - l' is positive for real images, negative for virtual images
  - Lenses
    - f, R is positive for converging (convex) lenses, negative for diverging (concave) lenses
    - l is positive
    - l' is positive for image on the other side of the lens, negative for image on the same side of the lens

Interference & Diffusion

Diffraction is the spreading of light as it passes through a slit

Double-slit



$$\Delta\text{path} = BC - AC = \sqrt{L^2 + \left(y + \frac{d}{2}\right)^2} - \sqrt{L^2 + \left(y - \frac{d}{2}\right)^2}$$

$$\begin{aligned}\Delta\text{path} &= d\sin\theta \\ &= m\lambda \text{ (constructive interference)} \\ &= (2m+1)\frac{\lambda}{2} \text{ (destructive interference)}\end{aligned}$$

$$y_m = L\tan\theta_n$$

d	Distance between slits
L	Distance from slits to screen
y	Distance (height) from center of screen to point of interest
θ	Angle from center of slits to point of interest
m	Order of interference (Integer ≥ 0)

Thin-film/Radio wave interference

$$\begin{aligned}\Delta\text{path} &= 2d - \frac{\lambda}{2} \\ &= m\lambda \text{ (constructive interference)} \\ &= (2m+1)\frac{\lambda}{2} \text{ (destructive interference)}\end{aligned}$$

d	Thickness of film/distance between antennas
λ	Wavelength of light
m	Order of interference (Integer ≥ 0)