Introducing features of agency into computational models of infectious disease

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Outline

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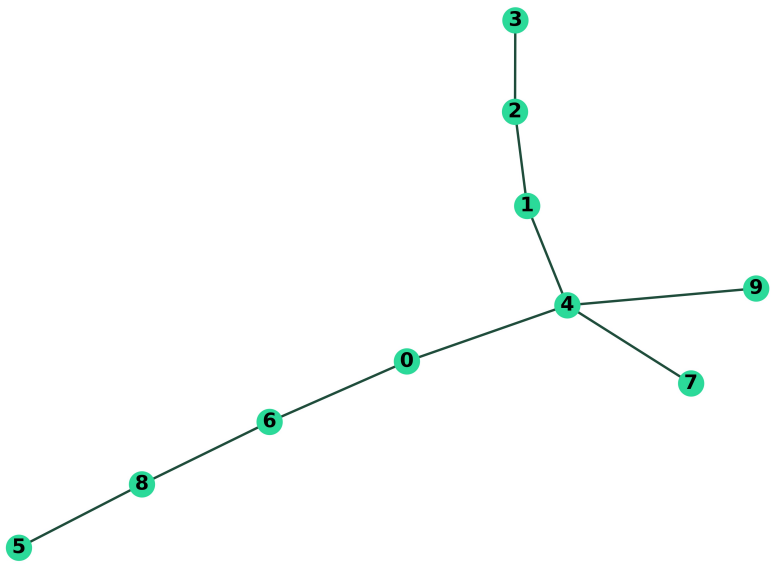
- Objects are called *vertices* or *nodes*
- Relations between objects are called *edges*.

Examples of Graphs



Example: Tree

A tree is a special type of graph where *any two vertices are connected by exactly one path.*

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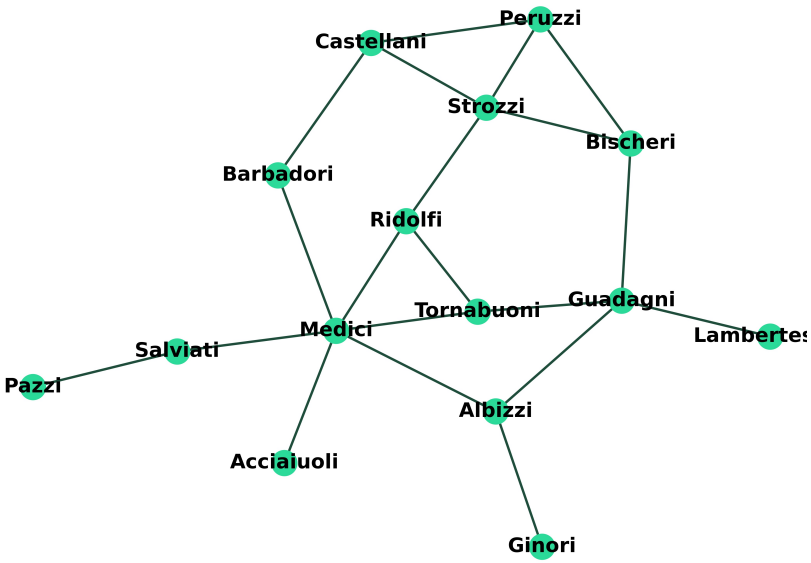

Example: Florentine Families Graph

Depicts the marital alliances between Renaissance Florentine families [4].

Using Graphs to Model Disease Spread

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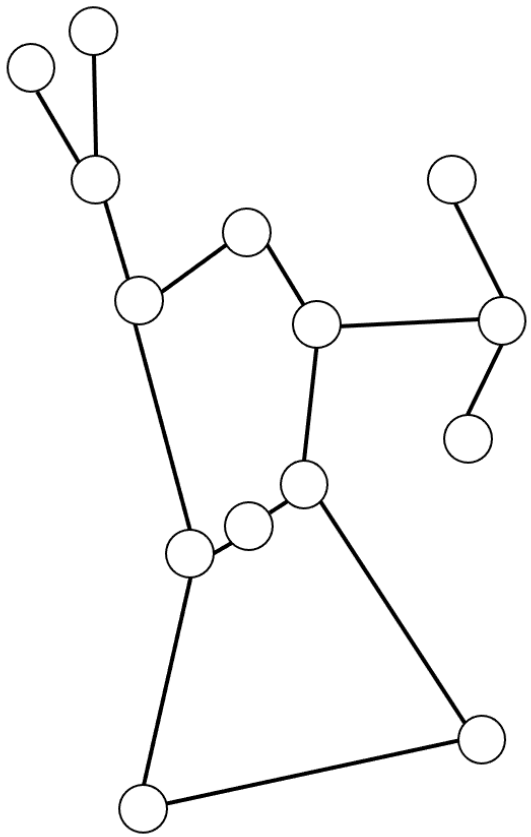
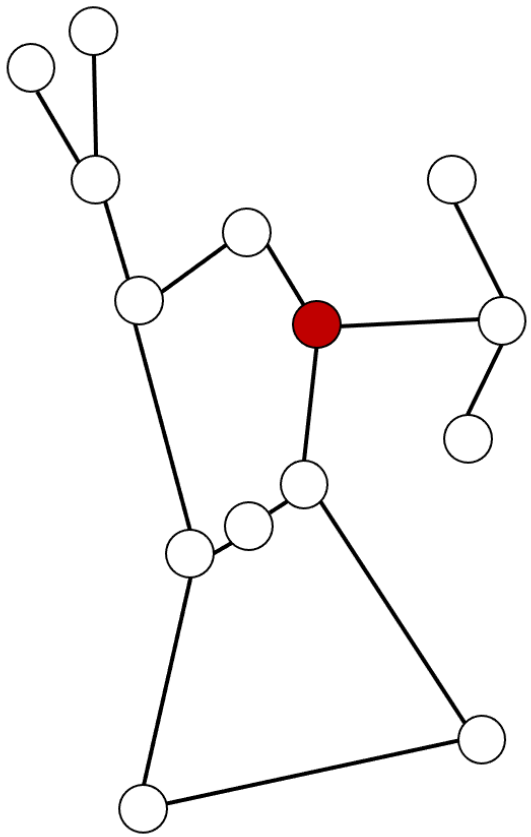
- At $t = 0$, a fire breaks out at some vertex in the graph.
- Firefighter then 'protects' some other vertex.
- Fire spreads to any adjacent vertices neither protected nor burnt.
- Firefighter protects another vertex, the fire spreads again and so on.

Example



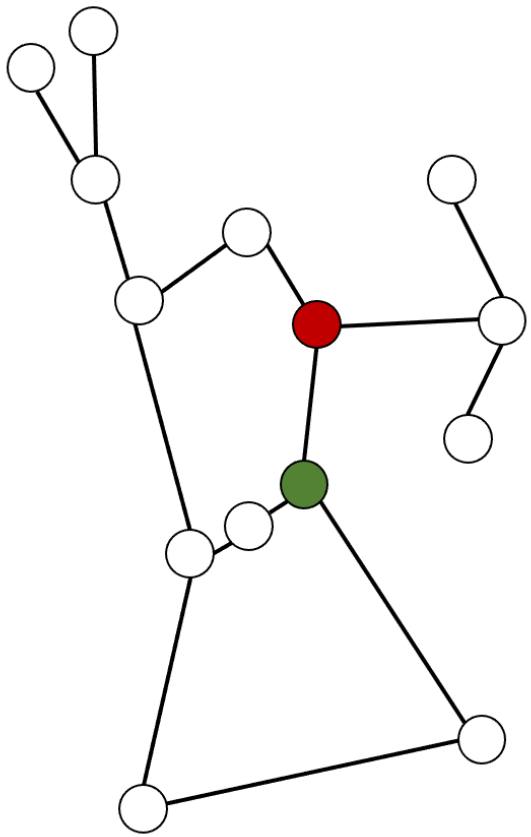
Original Graph

Example



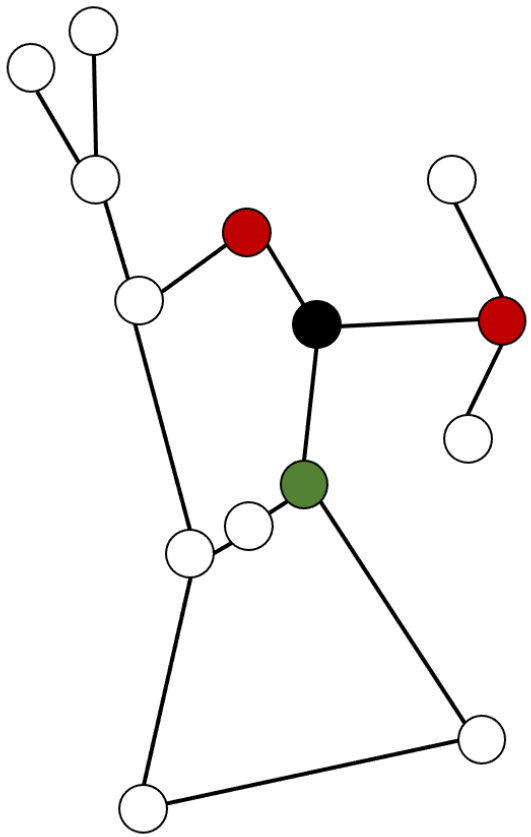
Outbreak

Example



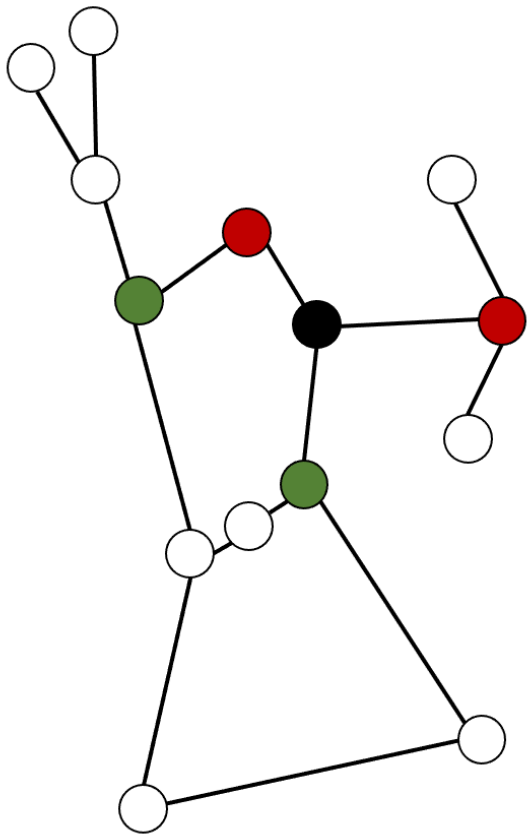
Defence

Example



Fire Spreads

Example



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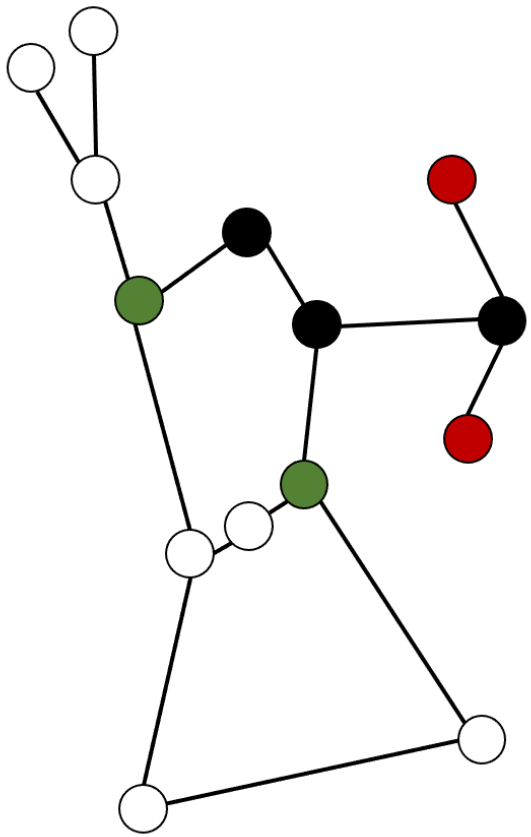

Fire Spreads

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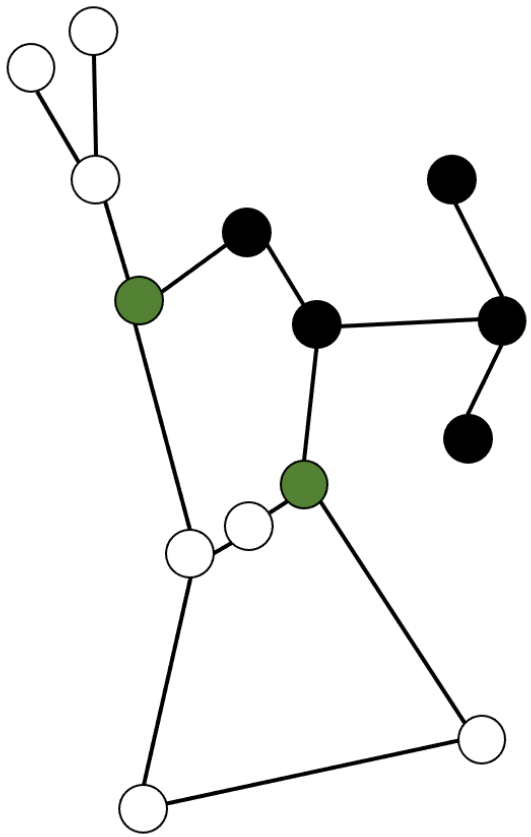

Fire contained

Firefighter as a Model for Disease Spread

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Limitations:

- Fairly rudimentary model for disease spread but already NP-hard.
- Defence and infection are discrete but epidemic propagation is a stochastic process.
- Only interventions in halting disease spread are *external*, no way for individuals to avoid contraction personally.

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If each vertex in the graph represents an individual agent, we can start to assign agency-related attributes. We are particularly interested in each agent's *internal inclination towards protection*. In context, this might involve:

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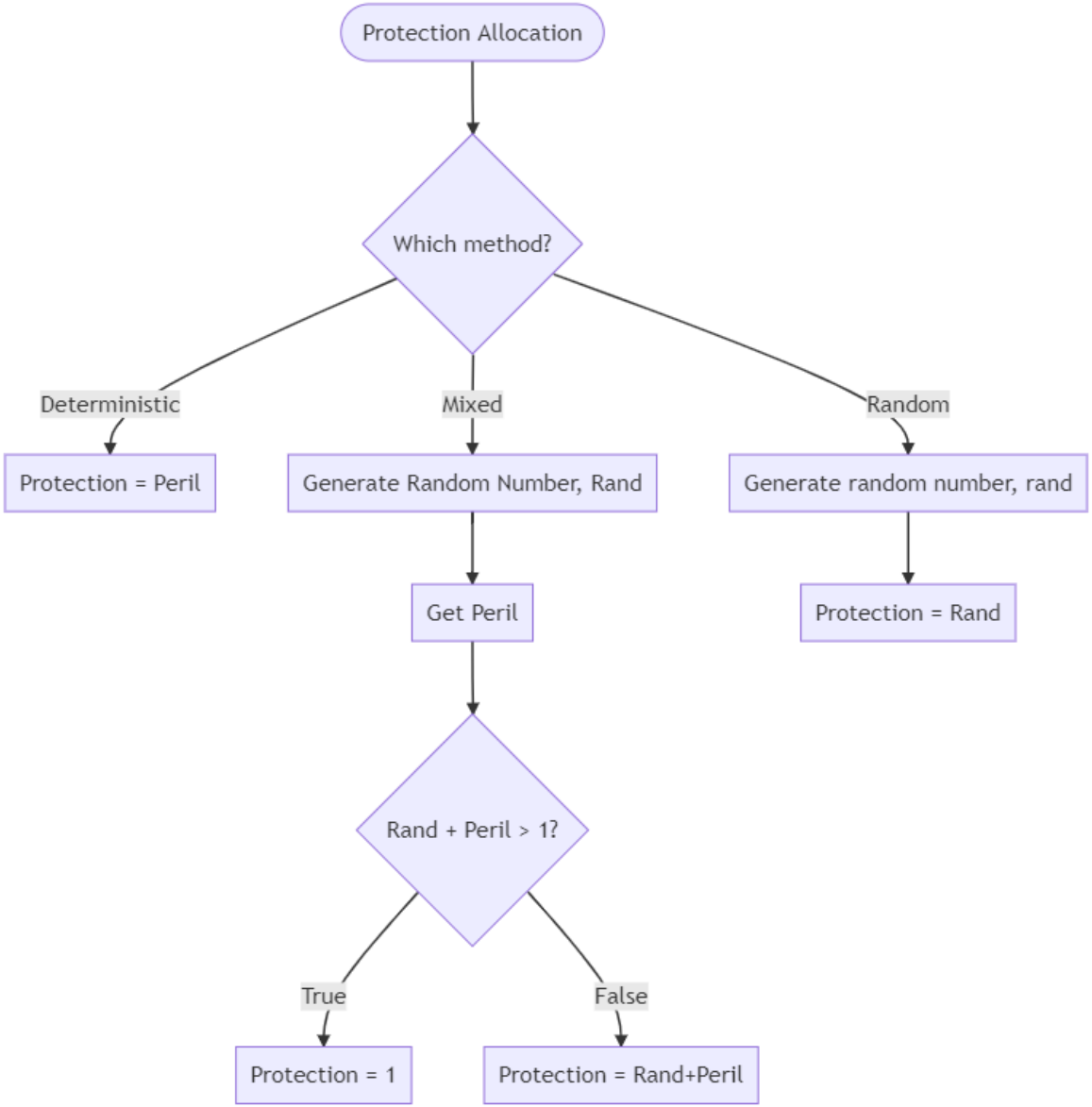
- Wearing PPE correctly
- Hand hygiene
- Strict physical distancing


Protection Allocation
Which method?
Deterministic
Mixed
Random
Protection = Peril
Generate Random Number, Rand
Generate random number, rand
Get Peril
Protection = Rand
Rand + Peril > 1?
True
False
Protection = 1
Protection = Rand+Peril

Defence Strategies

What do these amendments mean for how the game is played? In the usual formulation, general rule of thumb: *for sparse graphs, defend based on proximity to fire (breaking ties on degree); for dense graphs, defend based on degree (breaking ties on proximity)*.

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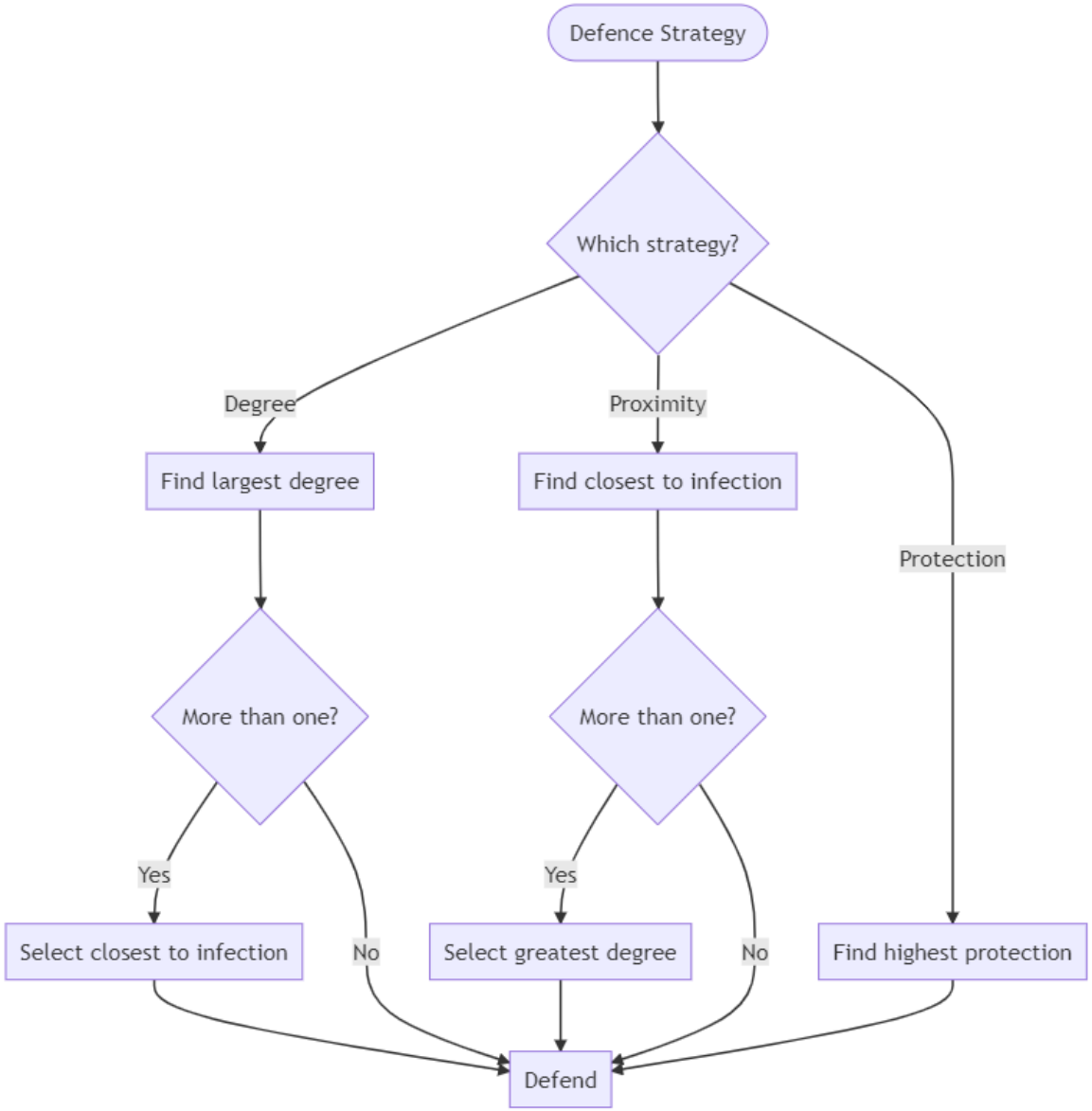
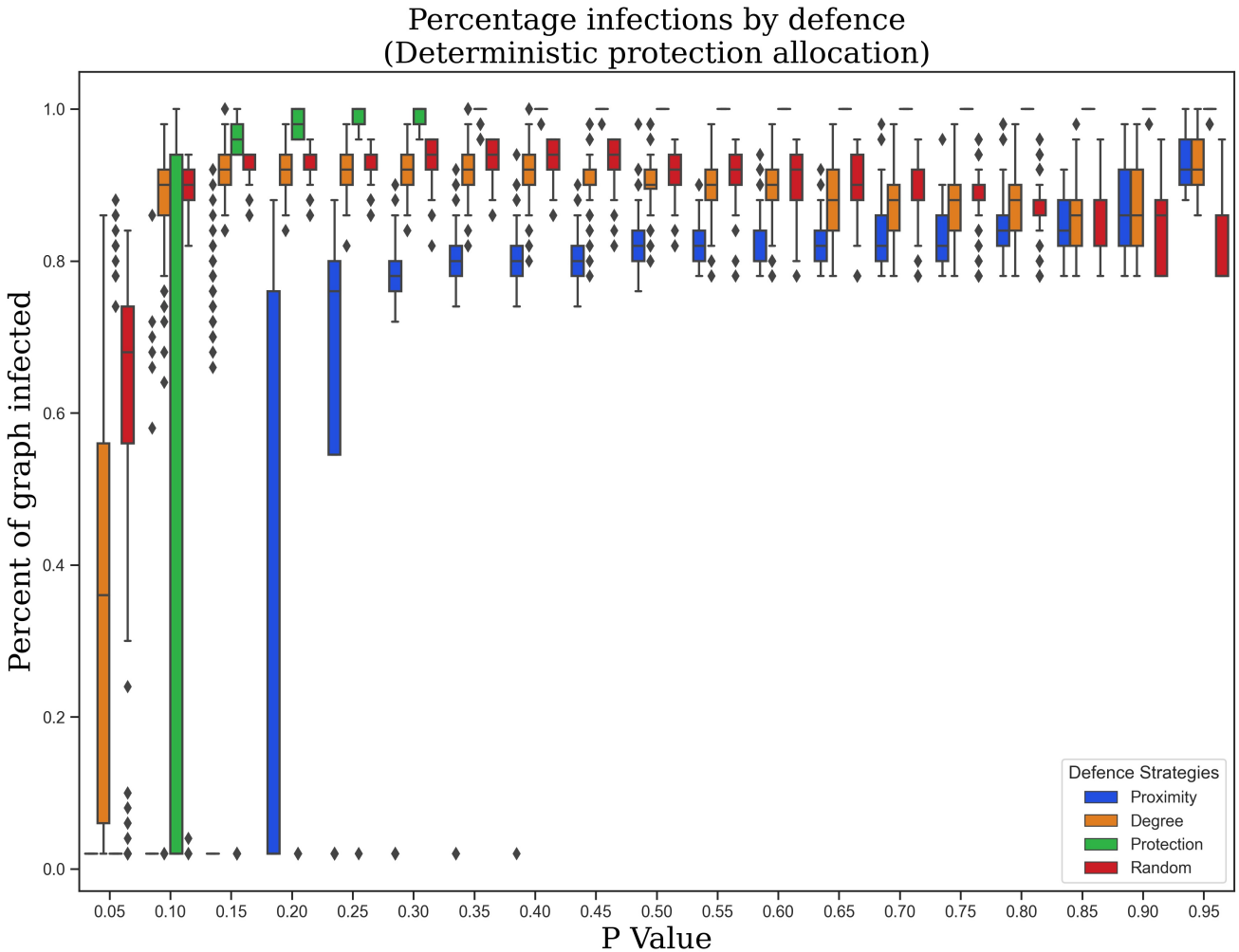
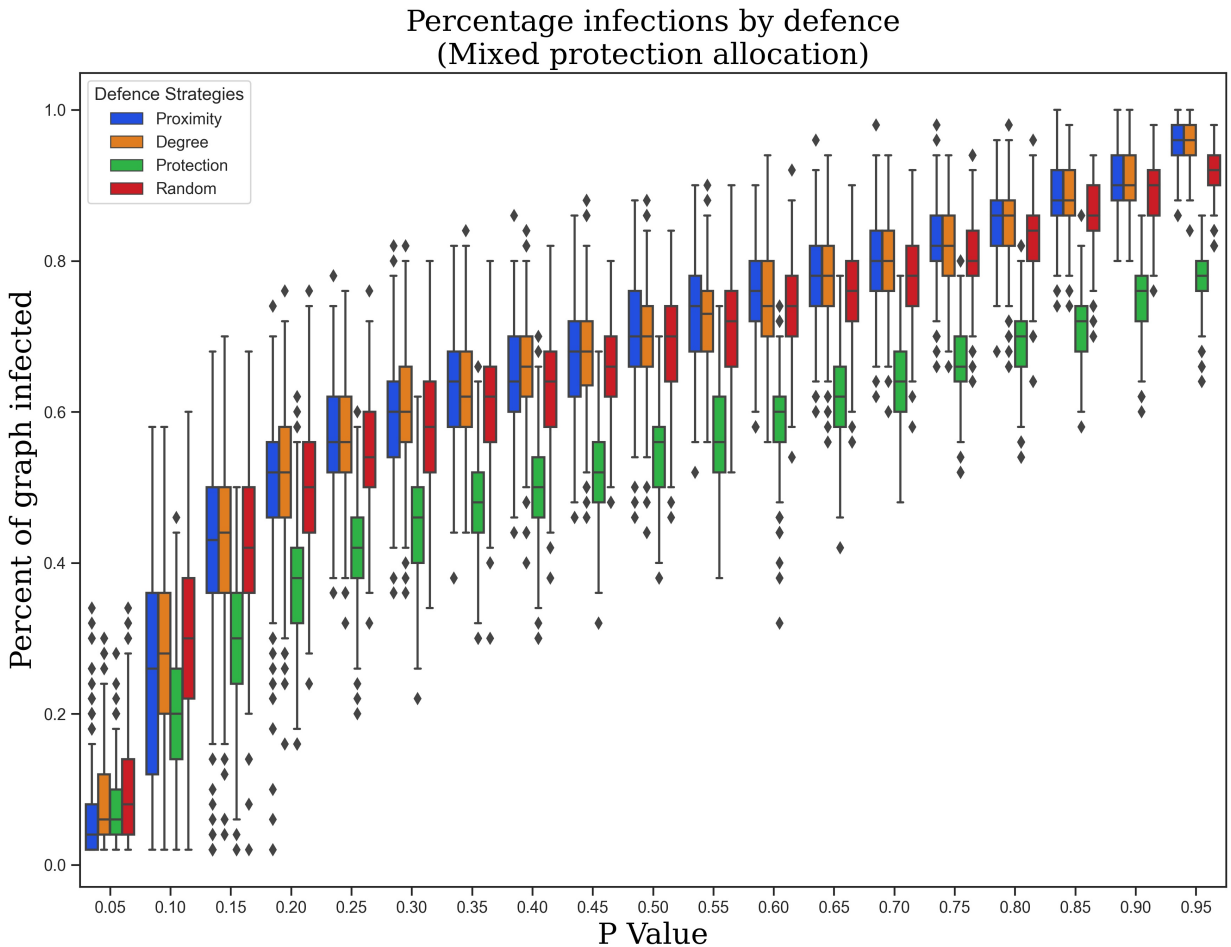
However, in our adjusted formulation we have more candidates for defence strategies. One such novel strategy is to defend based on highest agent protection rating.


Defence Strategy
Which strategy?
Degree
Proximity
Protection
Find largest degree
Find closest to infection
More than one?
More than one?
Yes
Yes
No
No
Select closest to infection
Select greatest degree
Find highest protection
Defend

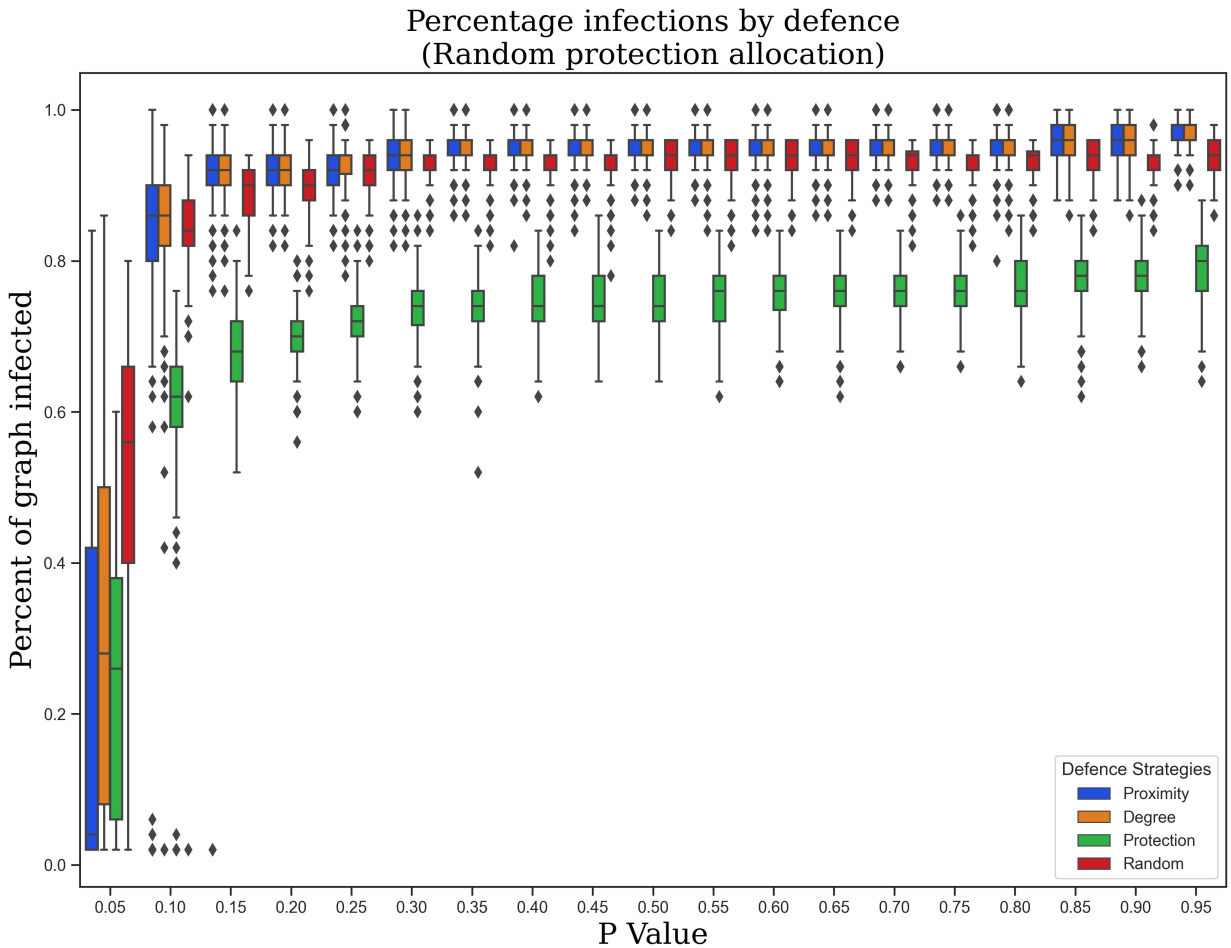
Deterministic Protection



Mixed Protection



Random Protection



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We discussed:

- Some fundamentals of Graph Theory, including FIREFIGHTER
- Using FIREFIGHTER as a model for disease spread
- Failings of this approach to disease modelling
- How agency can address these issues

References


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B. L. Hartnell, *Firefighter! an application of domination*, in 25th Manitoba Conference on Combinatorial Mathematics and Computing, University of Manitoba in Winnipeg, Canada, 1995.


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Questions?