HW2 Kalman filter

General Algorithm

Problem Formulation

$x_k = f(x_{k-1}, u_k, w_k)$

$z_k = h(x_k, v_k)$

Here w_k and v_k are the process and observation noises which are both assumed to be zero mean multivariate Gaussian noises with covariance Q_k and R_k respectively. u_k is the control vector.

We define the Jacobian matrix: $F_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1|k-1}, u_k}$, $H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k-1|k-1}}$, $L_k = \left.\frac{\partial f}{\partial \omega}\right|_{\hat{x}_{k-1|k-1}, u_k}$ and $M_k = \left.\frac{\partial h}{\partial v}\right|_{\hat{x}_{k|k-1}}$

Predic

Predicted state estimate: $\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_k)$

Predicted covariance estimate: $P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + L_k Q_k L_k^T$

Update

Innovation or measurement residual: $\tilde{y}_k = z_k - h(\hat{x}_{k|k-1})$

Innovation (or residual) covariance: $S_k = H_k P_{k|k-1} H_k^T + M_k R_k M_k^T$

Near-optimal Kalman gain: $K_k = P_{k|k-1} H_k^T S_k^{-1}$

Updated state estimate: $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k$

Updated covariance estimate: $P_{k|k} = (I - K_k H_k) P_{k|k-1}$

PartA

In this setting, we have (under noise-free notations):

$x_k = (\mathrm{x}_k, \mathrm{y}_k, \theta)$ at time k, $u_k = (\delta_1, \delta_t, \delta_2)$

$f(x_{k-1}, u_k)[0] = x_{k-1}[0] + u_k[1] \cdot \cos(u[0] + x_{k-1}[2])$

$f(x_{k-1}, u_k)[1] = x_{k-1}[1] + u_k[1] \cdot \sin(u[0] + x_{k-1}[2])$

$f(x_{k-1}, u_k)[2] = x_{k-1}[2] + u_k[1] + u_k[2]$

$h(x_k) = x_k$

And the Jacobians then would be:

$$F = \begin{pmatrix} 1 & 0 & -u_k[1] \cdot \sin\left(u[0] + x_{k-1}[2]\right) \\ 0 & 1 & u_k[1] \cdot \cos\left(u[0] + x_{k-1}[2]\right) \\ 0 & 0 & 1 \end{pmatrix}$$

$$H = I$$

$$L = \begin{pmatrix} -u_k[1] \cdot \sin\left(u[0] + x_{k-1}[2]\right) & \cos\left(u[0] + x_{k-1}[2]\right) & 0 \\ u_k[1] \cdot \cos\left(u[0] + x_{k-1}[2]\right) & \sin\left(u[0] + x_{k-1}[2]\right) & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

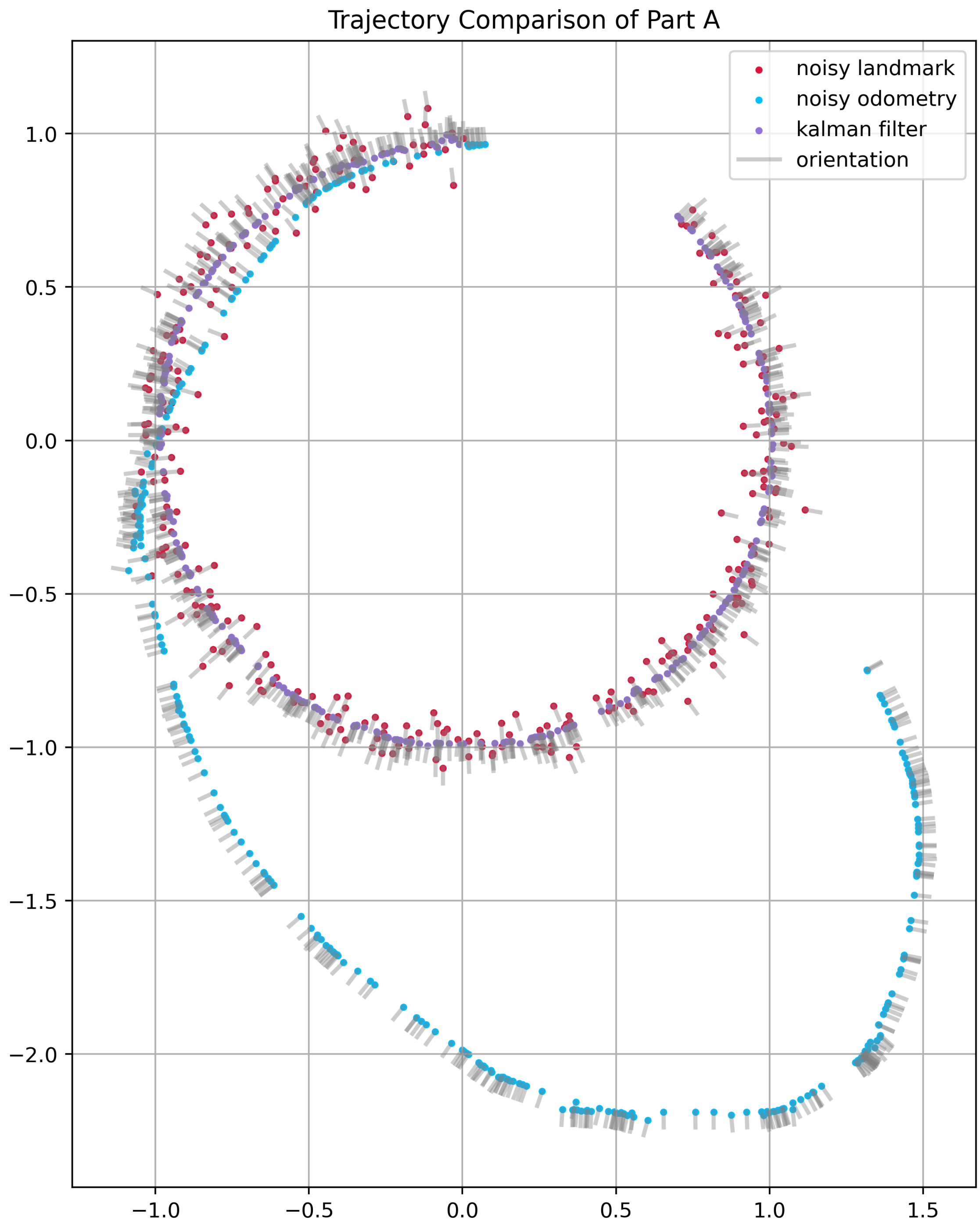
$$M = I$$

$$Q = \begin{pmatrix} 0.02^2 & 0 & 0 \\ 0 & 0.05^2 & 0 \\ 0 & 0 & 0.02^2 \end{pmatrix}$$

$$R = \begin{pmatrix} 0.05^2 & 0 & 0 \\ 0 & 0.05^2 & 0 \\ 0 & 0 & 0.03^2 \end{pmatrix}$$

Initialize P with R,

Run the EKF algorithm, we got the result as follows:



We can see that my algorithm converges to a smooth Standard-shaped circle.

PartB

In this setting, we have (under noise-free notations):

$x_k = (\mathrm{x}_k, \mathrm{y}_k, \theta, v_x, v_y, \omega)$ at time k,

$f(x_{k-1})[0] = x_{k-1}[0] + x_{k-1}[3]$

$f(x_{k-1})[1] = x_{k-1}[1] + x_{k-1}[4]$

$f(x_{k-1})[2] = x_{k-1}[2] + x_{k-1}[3]$

$f(x_{k-1})[3] = x_{k-1}[3]$

$f(x_{k-1})[4] = x_{k-1}[4]$

$f(x_{k-1})[5] = x_{k-1}[5]$

$h(x_{k-1})[0] = x_{k-1}[0] + x_{k-1}[3]$

$h(x_{k-1})[1] = x_{k-1}[1] + x_{k-1}[4]$

$h(x_{k-1})[2] = x_{k-1}[2] + x_{k-1}[3]$

And the Jacobians then would be:

$$F = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$L = I$

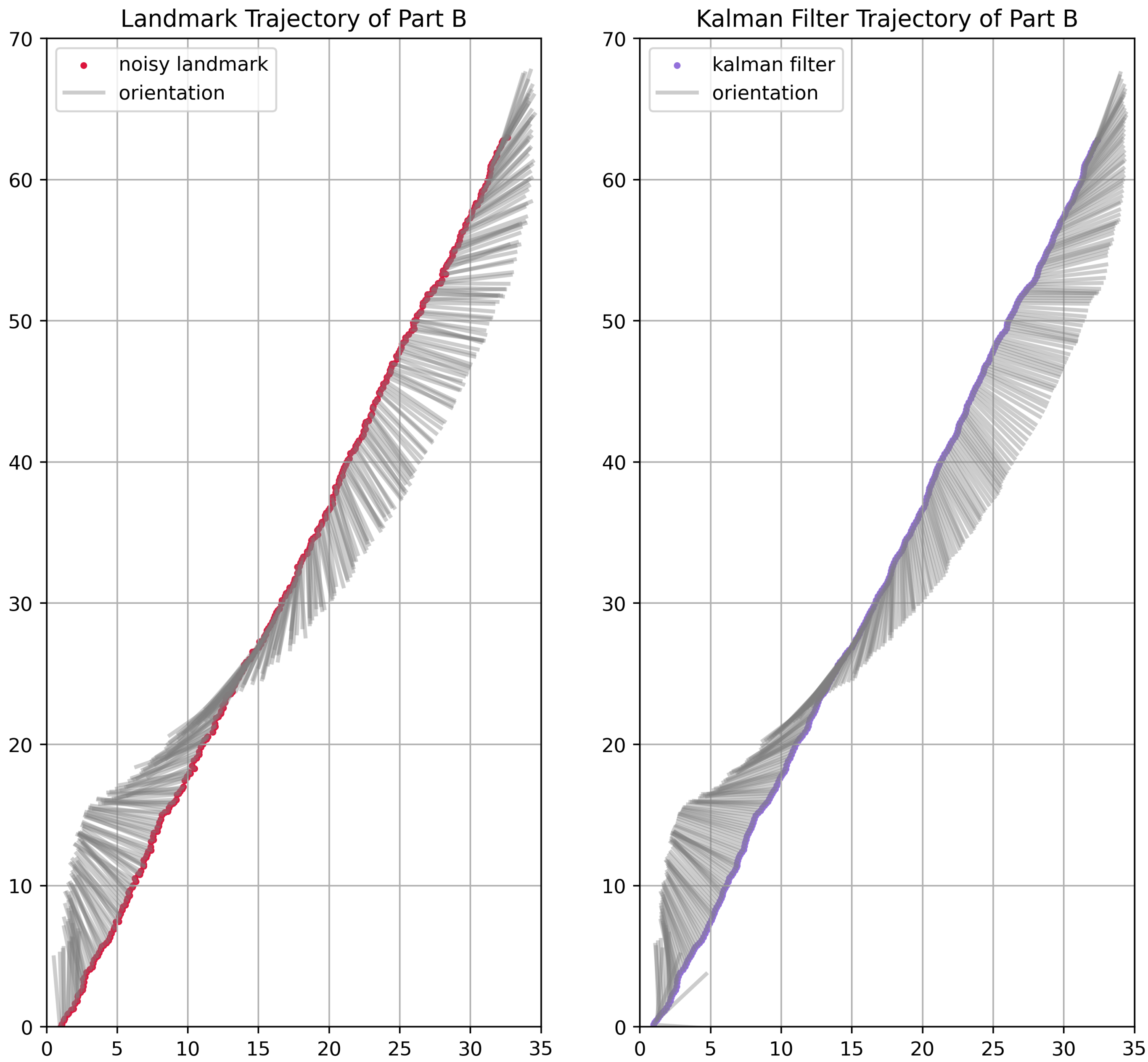
$M = I$

$$Q = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.05^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.05^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.02^2 \end{pmatrix}$$

$$R = \begin{pmatrix} 0.05^2 & 0 & 0 \\ 0 & 0.05^2 & 0 \\ 0 & 0 & 0.05^2 \end{pmatrix}$$

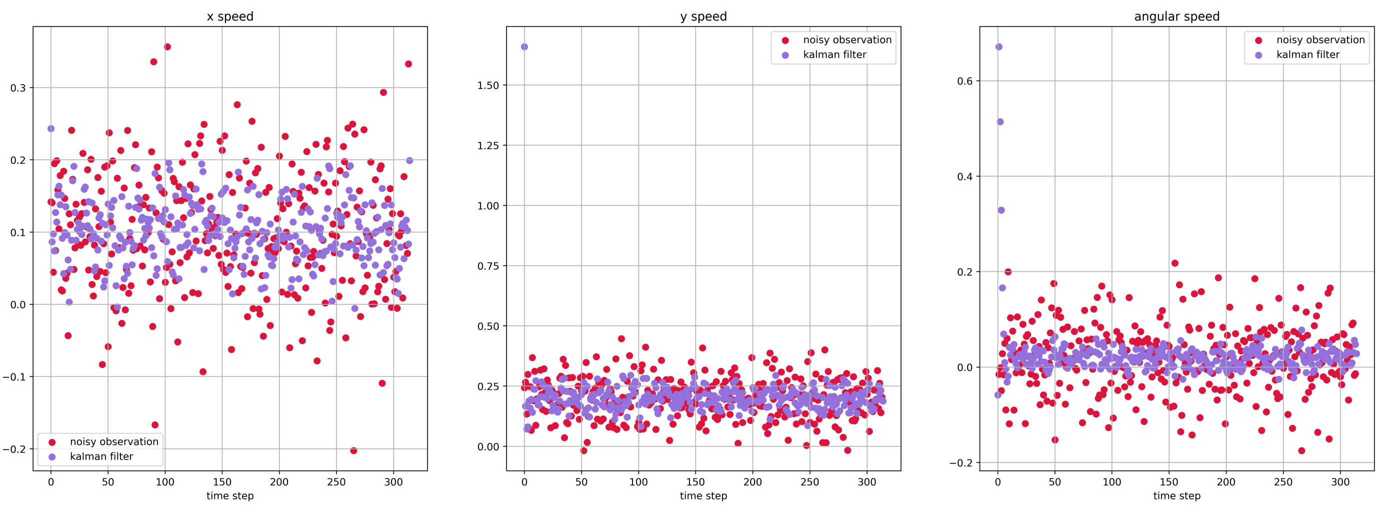
Initialize P with R and Q,

Run the EKF algorithm, we got the result as follows:



We can see that my algorithm converges to a smooth trajectory.

And we can see the velocity:



As we can see, KF canceled most of the observation noise and tried to maintain the original variety of velocity at the same time.