HW3 Camera Calibration

Yucong Chen 2019533079

Part A: Zhengyou Zhang's Method

Algorithm

Firstly find the corner points in images by OpenCV functions. I further added the subpixel optimization process to get precise corner locations.

Now we have the equation between world points and pixel coords:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & \gamma & u_0 & 0 \\ 0 & \beta & u_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

The objection is to find the intrinsics matrix K and extrinsics R and t.

From this, first of all I find the Homography matrix H by solving the linear optimization problem such that:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = H \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Then let $B = K^{-T}K^{-1}$, and use H to construct vectors v_{11}, v_{12}, v_{22}. After that we build a equation set:

$$\begin{pmatrix} v_{12}^T \\ (v_{11} - v_{12})^T \end{pmatrix} b = 0$$

Where b is flattened B. Then use SVD to solve it (constraint b's norm to be 1). So that we can have B.

Now we can get K from B. After that, from the first equation we can also have the corresponding R and t.

Results

Here is the result of the two cameras:

$$K_A = \begin{pmatrix} 1470.4 & 1.7 & 1229.1 \\ 0 & 1469.9 & 1033.7 \\ 0 & 0 & 1 \end{pmatrix}$$

$$K_B = \begin{pmatrix} 1461.7 & 3.3 & 1219.7 \\ 0 & 1459.7 & 1022.3 \\ 0 & 0 & 1 \end{pmatrix}$$

Reprojection Error ($l2$) of A: 0.106

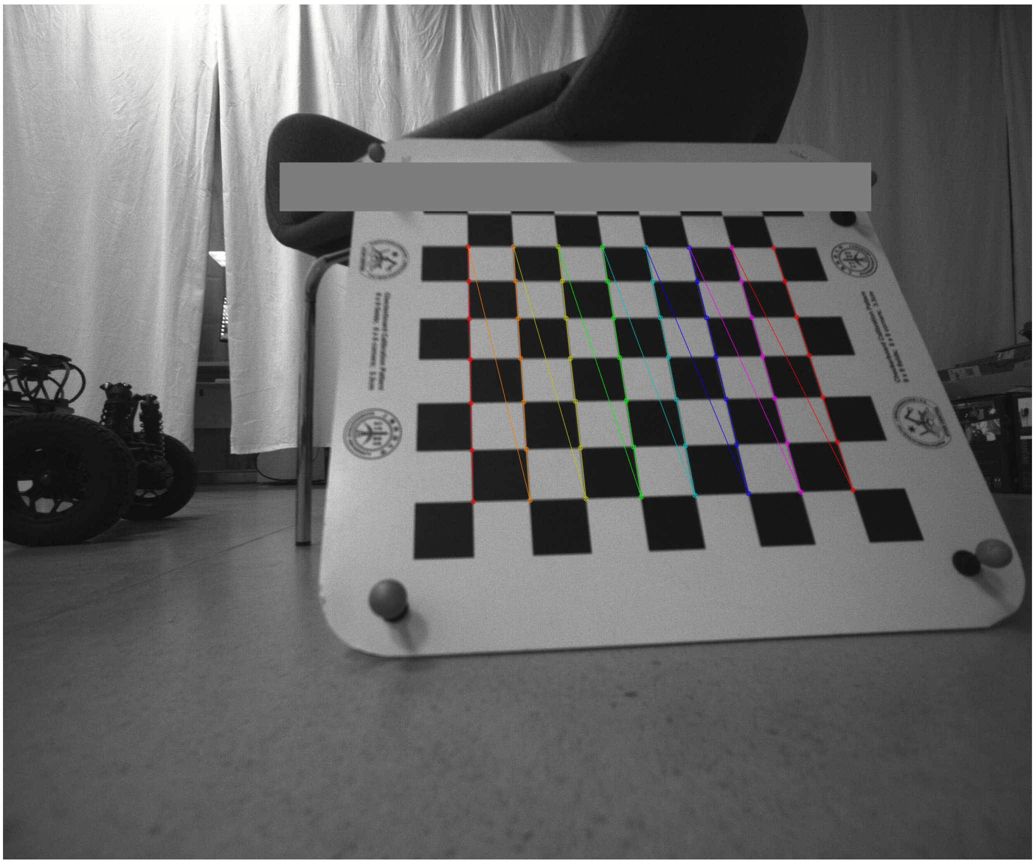
Reprojection Error ($l2$) of B: 0.105

And the extrinsics are saved in the `outputs/partA`

Part B: Hand-Eye Calibration

Algorithm (Bonus*)

Similar to PartA to find the corners:



From the Problem settings we get this formulation (All are $SE(3)$):

$$A_i A_j^{-1} Y = Y B_i B_j^{-1}$$

And now the formula can be written as:

$$\begin{pmatrix} R_A & t_A \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R_X & t_X \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} R_X & t_X \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R_B & t_B \\ 0 & 1 \end{pmatrix}$$

Expand and get:

$$\begin{cases} R_A R_X = R_X R_B \\ \\ R_A t_X + t_A = R_X t_B + t_X \end{cases}$$

Define the Kronecker Product operation as $\otimes$

From the first equation, we can get:

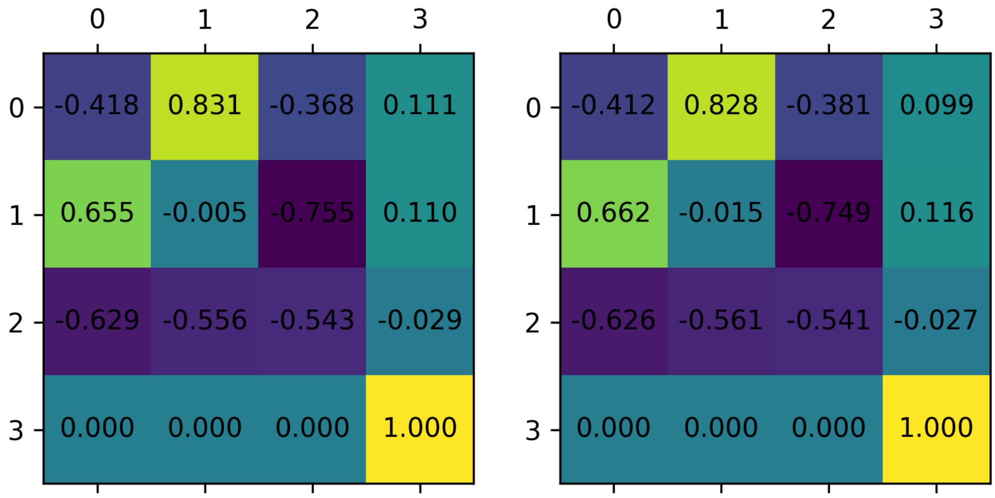
$$(R_A \otimes I - I \otimes R_B^T)\, vec(R_X) = 0$$

Solve it using SVD, then we get R_X and then we put it into the second equation, we now have t_X (least square)

$$Y = \begin{pmatrix} R_X & t_X \\ 0 & 1 \end{pmatrix}$$

Results

From Camera A (left) and Camera B (right), we can get Y as follows:



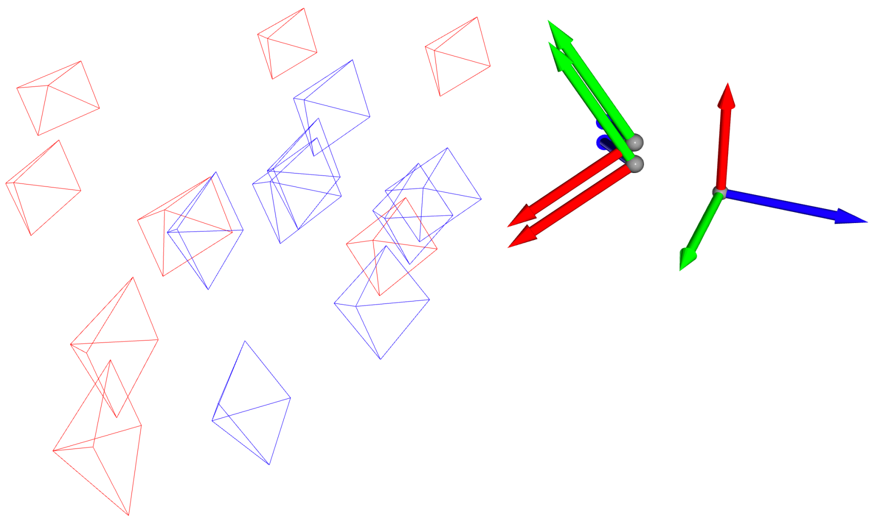
Reprojection Error ($l2$) of A: 0.068

Reprojection Error ($l2$) of B: 0.075

As we can see, the two matrices are similar to each other.

The difference is $||Y_A^{-1}Y_A - I||_2 = 0.0253$

And as the demonstration here:



Where read cameras are from A and blue cameras are from B. The right most coordinate frame is checkerboard frame while those two closely aligned frames are marker frames from Y_A and Y_B. We can see that they are very close to each other.