Homework 1

1. (**20 points**) Please describe your math background in mathematical analysis and linear algebra. What is your expectation of this course? (you can use Chinese language for this problem).

2. (**20 points**) Eigenvalue. (a) Write down the definition of eigenvalue. (b) Given a non-square matrix, is it possible to find eigenvalues? Use proofs or counter-examples to illustrate your argument. (c) Describe the geometric perspective of eigenvalue. (d) Explain the motivation of introducing the concept of eigenvalue from both theoretic and practical perspectives.

3. (**20 points**) (a) Given a real number a, find the limit: $\lim\limits_{a \to 0^+} a^a$

   (b) Compute the integral $\int_{-\infty}^{\infty} e^{-x^2} \, dx$

4. (**20 points**) Given two positive real numbers θ and t, we define a function

$$\phi(t) = -\theta t + \ln(1 - \theta + \theta e^t).$$

   Please show that $\phi(t) \leq \frac{1}{8} t^2$.

5. (**20 points**) Given k skill levels, we define a reward function $H(\cdot) : \{1, \ldots, k\} \to R$. Then for skill levels $x, y \in \{1, \ldots, k\}$, we define a soft-max function

$$\pi(x) = \frac{e^{H(x)}}{\sum_{y=1}^{k} e^{H(y)}}.$$

   Please show the following result: for any skill level $a \in \{1, \ldots, k\}$, we have

$$\frac{\partial \pi(x)}{\partial H(a)} = \pi(x) \left(1_{\{x=a\}} - \pi(a)\right),$$

   where 1_A is an index function of events, being 1 when event A is true and being 0 otherwise.