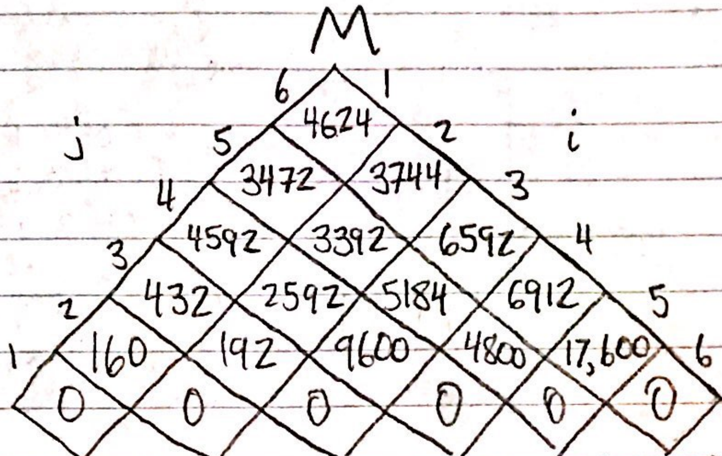
Ethan Dowalter

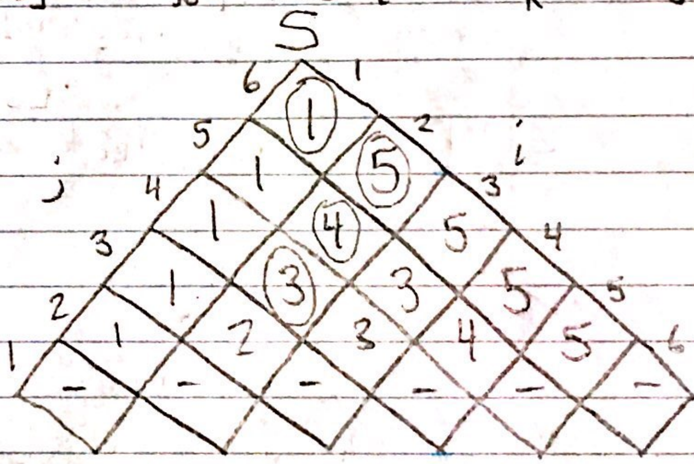
CS 4040 - HW 4

1)

Matrix	Rows	Cols
A	10	2
B	2	8
C	8	12
D	12	100
E	100	4
F	4	44



$$M[i,j] = \min_{i \le K < j} \{ M[i,K] + M[K+1,j] + rows[A_i]cols[A_K]cols[A_j] \}$$



$M[1,2] = 10 \cdot 2 \cdot 8 = 160$
$\Rightarrow S[1,2] = 1$
$M[2,3] = 2 \cdot 8 \cdot 12 = 192$
$\Rightarrow S[2,3] = 2$
$M[3,4] = 8 \cdot 12 \cdot 100 = 9600$
$\Rightarrow S[3,4] = 3$
$M[4,5] = 12 \cdot 100 \cdot 4 = 4800$
$\Rightarrow S[4,5] = 4$
$M[5,6] = 100 \cdot 4 \cdot 44 = 17{,}600$
$\Rightarrow S[5,6] = 5$

$$M[1,3] = \min \left\{ \begin{array}{l} 0 + 192 + 10 \cdot 2 \cdot 12 = 432 \\ 160 + 0 + 10 \cdot 8 \cdot 12 = 1120 \end{array} \right\} = 432 \Rightarrow S[1,3] = 1$$

$$M[2,4] = \min \left\{ \begin{array}{l} 0 + 9600 + 2 \cdot 8 \cdot 100 = 11{,}200 \\ 192 + 0 + 2 \cdot 12 \cdot 100 = 2592 \end{array} \right\} = 2592 \Rightarrow S[2,4] = 3$$

$$M[3,5] = \min \left\{ \begin{array}{l} 0 + 4800 + 8 \cdot 12 \cdot 4 = 5184 \\ 9600 + 0 + 8 \cdot 100 \cdot 4 = 12{,}800 \end{array} \right\} = 5184 \Rightarrow S[3,5] = 3$$

$$M[4,6] = \min \left\{ \begin{array}{l} 0 + 17{,}600 + 12 \cdot 100 \cdot 44 = 70{,}400 \\ 4800 + 0 + 12 \cdot 4 \cdot 44 = 6912 \end{array} \right\} = 6912 \Rightarrow S[4,6] = 5$$

$$M[1,4] = \min \left\{ \begin{array}{l} 0 + 2592 + 10 \cdot 2 \cdot 100 = 4592 \\ 160 + 9600 + 10 \cdot 8 \cdot 100 = 17{,}760 \\ 432 + 0 + 10 \cdot 12 \cdot 100 = 12{,}432 \end{array} \right\} = 4592 \Rightarrow S[1,4] = 1$$

$$M[2,5] = \min \left\{ \begin{array}{l} 0 + 5184 + 2 \cdot 8 \cdot 4 = 5248 \\ 192 + 4800 + 2 \cdot 12 \cdot 4 = 5088 \\ 2592 + 0 + 2 \cdot 100 \cdot 4 = 3392 \end{array} \right\} = 3392 \Rightarrow S[2,5] = 4$$

$$M[3,6] = \min \left\{ \begin{array}{l} 0 + 6912 + 8 \cdot 12 \cdot 44 = 11{,}136 \\ 9600 + 17{,}600 + 8 \cdot 100 \cdot 44 = 62{,}400 \\ 5184 + 0 + 8 \cdot 4 \cdot 44 = 6592 \end{array} \right\} = 6592 \Rightarrow S[3,6] = 5$$

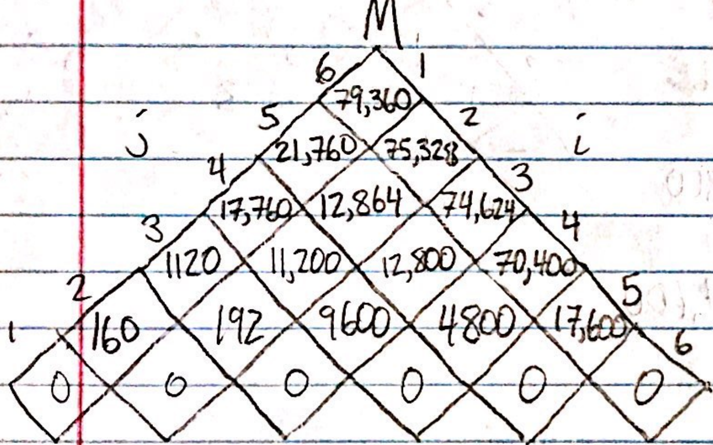
$$M[1,5] = \min \begin{Bmatrix} 0 + 3392 + 10 \cdot 2 \cdot 4 = 3472 \\ 160 + 5184 + 10 \cdot 8 \cdot 4 = 5664 \\ 432 + 4800 + 10 \cdot 12 \cdot 4 = 5712 \\ 4592 + 0 + 10 \cdot 100 \cdot 4 = 8592 \end{Bmatrix} = 3472 \Rightarrow S[1,5] = 1$$

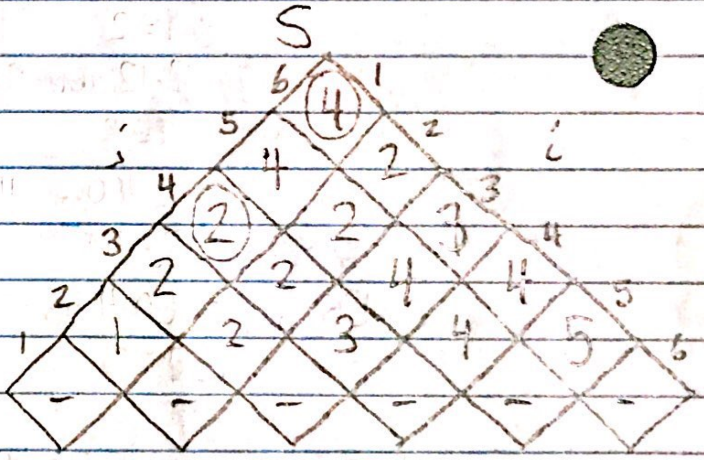
$$M[2,6] = \min \begin{Bmatrix} 0 + 6592 + 2 \cdot 8 \cdot 44 = 7296 \\ 192 + 6912 + 2 \cdot 12 \cdot 44 = 8160 \\ 2592 + 17{,}600 + 2 \cdot 100 \cdot 44 = 28{,}992 \\ 3392 + 0 + 2 \cdot 4 \cdot 44 = 3744 \end{Bmatrix} = 3744 \Rightarrow S[2,6] = 5$$

$$M[1,6] = \min \begin{Bmatrix} 0 + 3744 + 10 \cdot 2 \cdot 44 = 4624 \\ 160 + 6592 + 10 \cdot 8 \cdot 44 = 10{,}272 \\ 432 + 6912 + 10 \cdot 12 \cdot 44 = 12{,}624 \\ 4592 + 17{,}600 + 10 \cdot 100 \cdot 44 = 66{,}192 \\ 3472 + 0 + 10 \cdot 4 \cdot 44 = 5232 \end{Bmatrix} = 4624 \Rightarrow S[1,6] = 1$$

Therefore, the optimal parenthization is: $\boxed{A((((B \cdot C)D)E)F)}$ which gives $\boxed{4624}$ total multiplications.

2) $M[i,j] = \max\limits_{i \le k < j} \{ M[i,k] + M[k+1,j] + rows[A_i] cols[A_k] cols[A_j] \}$





$M[1,2] = 160 \Rightarrow S[1,2] = 1$

$M[2,3] = 192 \Rightarrow S[2,3] = 2$

$M[3,4] = 9600 \Rightarrow S[3,4] = 3$

$M[4,5] = 4800 \Rightarrow S[4,5] = 4$

$M[5,6] = 17{,}600 \Rightarrow S[5,6] = 5$

$M[1,3] = \max \begin{Bmatrix} 432 \\ 1120 \end{Bmatrix} = 1120 \Rightarrow S[1,3] = 2$

$M[2,4] = \max \begin{Bmatrix} 11{,}200 \\ 2592 \end{Bmatrix} = 11{,}200 \Rightarrow S[3,4] = 2$

$M[3,5] = \max \begin{Bmatrix} 5184 \\ 12{,}800 \end{Bmatrix} = 12{,}800 \Rightarrow S[3,5] = 4$

$M[4,6] = \max \begin{Bmatrix} 70{,}400 \\ 6912 \end{Bmatrix} = 70400 \Rightarrow S[4,6] = 4$

$$M[1,4] = \max \begin{Bmatrix} 0 + 11{,}200 + 10 \cdot 2 \cdot 100 = 13{,}200 \\ 160 + 9600 + 10 \cdot 8 \cdot 100 = 17{,}760 \\ 1120 + 0 + 10 \cdot 12 \cdot 100 = 13{,}120 \end{Bmatrix} = 17{,}760 \Rightarrow S[1,4] = 2$$

$$M[2,5] = \max \begin{Bmatrix} 0 + 12{,}800 + 2 \cdot 8 \cdot 4 = 12{,}864 \\ 192 + 4800 + 2 \cdot 12 \cdot 4 = 5088 \\ 11{,}200 + 0 + 2 \cdot 100 \cdot 4 = 12{,}000 \end{Bmatrix} = 12{,}864 \Rightarrow S[2,5] = 2$$

$$M[3,6] = \max \begin{Bmatrix} 0 + 70{,}400 + 8 \cdot 12 \cdot 44 = 74{,}624 \\ 9600 + 17{,}600 + 8 \cdot 100 \cdot 44 = 62{,}400 \\ 12{,}800 + 0 + 8 \cdot 4 \cdot 44 = 14{,}208 \end{Bmatrix} = 74{,}624 \Rightarrow S[3,6] = 3$$

Ethan Dowalter

CS 4040 - HW 4 (cont.)

cont. 2)

$$M[1,5] = \max \begin{cases} 0 + 12{,}864 + 10 \cdot 2 \cdot 4 = 12{,}944 \\ 160 + 12{,}800 + 10 \cdot 8 \cdot 4 = 13{,}280 \\ 1120 + 4800 + 10 \cdot 12 \cdot 4 = 6400 \\ 17{,}760 + 0 + 10 \cdot 100 \cdot 4 = 21{,}760 \end{cases} = 21{,}760 \Rightarrow S[1,5] = 4$$

$$M[2,6] = \max \begin{cases} 0 + 74{,}624 + 2 \cdot 8 \cdot 44 = 75{,}328 \\ 192 + 70{,}400 + 2 \cdot 12 \cdot 44 = 71{,}456 \\ 11{,}200 + 17{,}600 + 2 \cdot 100 \cdot 44 = 37{,}600 \\ 12{,}864 + 0 + 2 \cdot 4 \cdot 44 = 13{,}216 \end{cases} = 75{,}328 \Rightarrow S[2,6] = 2$$

$$M[1,6] = \max \begin{cases} 0 + 75{,}328 + 10 \cdot 2 \cdot 44 = 76{,}208 \\ 160 + 74{,}624 + 10 \cdot 8 \cdot 44 = 78{,}304 \\ 1120 + 70{,}400 + 10 \cdot 12 \cdot 44 = 76{,}800 \\ 17{,}760 + 17{,}600 + 10 \cdot 100 \cdot 44 = 79{,}360 \\ 21{,}760 + 0 + 10 \cdot 4 \cdot 44 = 23{,}520 \end{cases} = 79{,}360 \Rightarrow S[1,6] = 4$$

Therefore, the optimal parenthization is: $\boxed{((A \cdot B)(C \cdot D))(E \cdot F)}$ which gives $\boxed{79{,}360}$ total multiplications

3) One modification which would allow QuickSort to run in $O(n \log_2 n)$ time even in the worst case would be to intelligently select the pivot each time by implementing the Select algorithm we covered in class which finds the median of an array in $O(n)$ time. Doing this guarantees that the array gets split exactly (or as close to exactly as possible) in half, which means that the maximum depth of the recursion is $\log_2 n$, and at each step, $O(n)$ amount of work is being done. Thus, the time complexity of this modified QuickSort would be $O(n \log_2 n)$ in the best, worst, and average case.

4)

M	i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
j		A=	C	G	C	C	G	A	T	G	T	C	C	G	A	T	C	C
0	B =	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	G	0	0	↖1	←1	←1	↖1	←1	←1	↖1	←1	←1	←1	↖1	←1	←1	←1	←1
2	G	0	0	↖1	←1	←1	↖2	←2	←2	↖2	←2	←2	←2	↖2	←2	←2	←2	←2
3	C	0	↖1	←1	↖2	↖2	←2	←2	←2	←2	←2	↖3	↖3	←3	←3	←3	↖3	↖3
4	C	0	↖1	←1	↖2	↖3	←3	←3	←3	←3	←3	↖3	↖4	←4	←4	←4	↖4	↖4
5	C	0	↖1	←1	↖2	↖3	←3	←3	←3	←3	←3	↖4	↖4	←4	←4	←4	↖5	↖5
6	T	0	↑1	←1	↑2	↑3	←3	←3	↖4	←4	↖4	←4	←4	←4	←4	↖5	←5	←5
7	T	0	↑1	←1	↑2	↑3	←3	←3	↖4	←4	↖5	←5	←5	←5	←5	↖5	←5	←5
8	T	0	↑1	←1	↑2	↑3	←3	←3	↖4	←4	↖5	←5	←5	←5	←5	↖6	←6	←6
9	A	0	↑1	←1	↑2	↑3	←3	↖4	←4	←4	↑5	←5	←5	←5	↖6	←6	←6	←6
10	A	0	↑1	←1	↑2	↑3	←3	↖4	←4	←4	↑5	←5	←5	←5	↖6	←6	←6	←6
11	G	0	↑1	↖2	←2	↑3	↖4	←4	←4	↖5	←5	←5	←5	↖6	←6	←6	←6	←6
12	T	0	↑1	↑2	←2	↑3	↑4	←4	↖5	←5	↖6	←6	←6	←6	←6	↖7	←7	←7
13	C	0	↖1	↑2	↖3	↖3	↑4	←4	↑5	←5	↑6	↖7	↖7	←7	←7	←7	↖8	↖8
14	A	0	↑1	↑2	↑3	←3	↑4	↖5	←5	←5	↑6	↑7	←7	←7	↖8	←8	←8	←8
15	G	0	↑1	↖2	↑3	←3	↖4	↑5	←5	↖6	←6	↑7	←7	↖8	←8	←8	←8	←8
16	C	0	↖1	↑2	↖3	↖4	←4	↑5	←5	↑6	←6	↖7	↖8	←8	←8	←8	↖9	↖9
17	A	0	↑1	↑2	↑3	↑4	←4	↖5	←5	↑6	←6	↑7	↑8	←8	↖9	←9	←9	←9

$$M[i,j] = \begin{cases} 0 & \text{if } i=0 \text{ or } j=0 \\ M[i-1,j-1]+1 & \text{if } i,j>0 \text{ and } x_i = y_j \\ \max(M[i-1,j], M[i,j-1]) & \text{else} \end{cases}$$

To break ties, pick $M[i-1,j]$ when $M[i-1,j] = M[i,j-1]$

So, one longest common subsequence between A and B is: $\boxed{\text{"CCCAGTCCA"}}$ which has length $\boxed{9}$

Scanned with CamScanner

Ethan Dowalter

CS 4040 - HW 4 (cont.)

5)

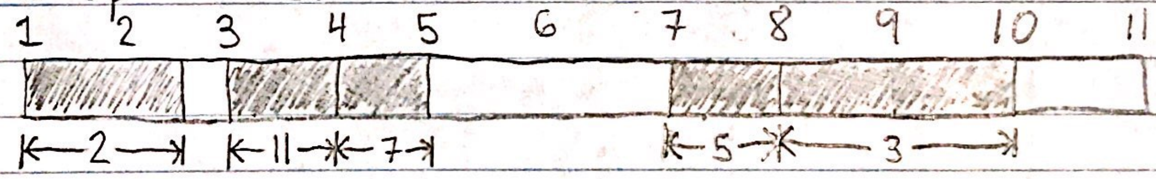
Activity #	s_i	f_i
1	2	3
2	1	2.5
3	8	10
4	1	5
5	7	8
6	2	5
7	4	5
8	8	11
9	2	6
10	5	9
11	3	4

→ $\boxed{\text{Sort by } f_i}$ →

Activity #	s_i	f_i
2	1	2.5
1	2	3
11	3	4
4	1	5
6	2	5
7	4	5
9	2	6
5	7	8
10	5	9
3	8	10
8	8	11

Then, pick the activities in the order in which they appear in the sorted list, making sure that $s_{i+1} \geq f_i$.

One optimal solution would be:



Activity #

$A_1 = \#2 \Rightarrow s_1 = 1, f_1 = 2.5$ $\qquad f_1 = 2.5$

$A_2 \neq \#1 \Rightarrow s_2 = 2, f_2 = 3$ $\qquad s_2 \ngeq f_1$

$A_2 = \#11 \Rightarrow s_2 = 3, f_2 = 4$ $\qquad s_2 \geq f_1$ ✓ $\qquad f_2 = 4$

$A_3 = \#4 \Rightarrow s_3 = 1, f_3 = 5$ $\qquad s_3 \ngeq f_2$

$A_3 = \#6 \Rightarrow s_3 = 2, f_3 = 5$ $\qquad s_3 \ngeq f_2$

$A_3 = \#7 \Rightarrow s_3 = 4, f_3 = 5$ $\qquad s_3 \geq f_2$ ✓ $\qquad f_3 = 5$

$A_4 = \#9 \Rightarrow s_4 = 2, f_4 = 6$ $\qquad s_4 \ngeq f_3$

$A_4 = \#5 \Rightarrow s_4 = 7, f_4 = 8$ $\qquad s_4 \geq f_3$ ✓ $\qquad f_4 = 8$

$A_5 = \#10 \Rightarrow s_5 = 5, f_5 = 9$ $\qquad s_5 \ngeq f_4$

$A_5 = \#3 \Rightarrow s_5 = 8, f_5 = 10$ $\qquad s_5 \geq f_4$ ✓ $\qquad f_5 = 10$

$A_6 = \#8 \Rightarrow s_6 = 8, f_6 = 11$ $\qquad s_6 \ngeq f_5$