CS 4040 - HW 1 Ethan Dowalter

1) i) a) The code is correct

b) $O(n)$

c) $O(\log_2 n)$

d) $O(n)$

e) $O(\log_2 n)$

ii) a) The code is correct.

b) $O(n)$

c) $O(\log_3 n)$

d) $O(\log_3 n)$

e) $O(\log_3 n)$

iii) a) The code is correct.

b) $O(n)$

c) $O(n)$

d) $O(1)$

e) $O(1)$

iv) a) The code is incorrect. Counterexample: int a[2]={3,3} int b[2]={3,5} will return True when a's and b's elements are not the same.

b) $O(n^2)$

c) $O(1)$

d) $O(n)$

e) $O(1)$

2) a) Prove $2n \in O(6n^2+n+42)$

Def. of Big Oh says:

$O(g(n))$ is the set of all functions $f(n)$ such that there exist positive constants c and n_0 such that:

$$0 \le f(n) \le cg(n) \quad \forall n \ge n_0$$

$$0 \le 2n \le c(6n^2+n+42) \qquad \text{substitution}$$

$$0 \le 2 \le c(6n+1+\tfrac{42}{n}) \qquad \text{algebra } (n>0)$$

$(2 \le 49)$

The above is true given $c = n_0 = 1$ and $\forall n \ge 1$, thus

$$2n \in O(6n^2+n+42) \checkmark$$

(because $\lim_{n\to\infty}(6n+1+\frac{42}{n}) = \infty$)

b) Disprove $2n^2 \in o(3n^2+12n)$

Def. of Little oh says:

$$o(g(n)) = \left\{ f(n) \;\middle|\; \begin{array}{l} \forall c>0,\ \exists n_0>0 \text{ s.t.} \\ 0 \le f(n) < cg(n) \quad \forall n \ge n_0 \end{array} \right\}$$

$$0 \le 2n^2 < (3n^2+12n)c \qquad \text{substitution}$$

$$0 \le 2 < c(3+\tfrac{12}{n}) \qquad \text{algebra } (n>0)$$

$$0 \le n < \frac{12}{\frac{2}{c}-3} \qquad \text{algebra } (c>0)$$

Which is impossible because $\lim_{c\to\infty}\left(\frac{12}{\frac{2}{c}-3}\right) = -4$, but $n>0$, so this inequality cannot hold true for all c.

Thus,

$$2n^2 \notin o(3n^2+12)$$

CS 4040 - HW 1 (cont.)

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2) c) Prove $3n^2+42 \in \Omega(4n+1)$

Def. of Big Omega says:

$\Omega(g(n))$ is the set of all functions $f(n)$ such that there exist positive constants c and n_0 such that:

$$0 \le c\,g(n) \le f(n) \quad \forall n \ge n_0$$

$$0 \le c(4n+1) \le 3n^2+42 \quad \text{substitution}$$

$$0 \le c\left(4+\frac{1}{n}\right) \le 3n+\frac{42}{n} \quad \text{algebra } (n>0)$$

$$0 \le c \le \frac{3n+\frac{42}{n}}{4+\frac{1}{n}}$$

Which is true, given $c = n_0 = 1$ $(1 \le 9)$ $\forall n > 1$

$\left(\lim_{n\to\infty} \frac{3n+\frac{42}{n}}{4+\frac{1}{n}} = \infty\right)$, thus

$$3n^2+42 \in \Omega(4n+1) \checkmark$$

3) a) True

b) True

c) True

d) False

e) True

f) True

g) False

h) True

i) True

j) False