The Great Camp of American Consulting

Analyzing Mortgage Rates Through Examination of Current Market Data

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Abstract:

Objective:

When determining what the best options are for buying a house with the help of USBank, borrowers will struggle with what the best plan is for them. They are unaware of or need help differentiating between how the different plans can affect their finances and future. This consulting company's goal is to provide a breakdown of how USBanks different plans will cost a borrower in the long term through examples, charts/ graphs constructed through R code, and analysis of the four different terms: 30-years, 20-years, 15-years, and 10-years.

Goal:

We hope that, by USBank partnering up with The Great Camp of American Consulting, that USBank sees the benefits we provide and continues to work with us. With our analysis of the different mortgages, we hope to attract more borrowers for USBank. These borrowers would use this report to help them choose the best plan for them since this report provides a simple and easy to understand breakdown of the different plans.

Conclusion:

This report will hopefully satisfy the many borrowers of USBank by helping them understand how the different mortgage plans; 30-year term, 20-year term, 15-year term, and 10-year term, may affect them financially and their overall future in terms of costs. They will feel comfortable in choosing USBank as their loan provider and will trust that they will be fair and honest throughout the whole process. We hope that our analysis eases the pressure form borrowers as we know how stressful it is to buy a house for the first time. We hope that they are able to examine our analysis with easily understandable information and determine the best option for themselves.

Introduction and Data:

Problem Identification:

The Great Camp of American Consulting is working with USBank to provide them with the best mortgage loan document for borrowers to efficiently use. Through this, we hope to greatly increase the number of customers USBank has. We will display charts and graphs to help borrowers/readers examine mortgages and the many costs that are associated with them.

This report's objective is to be used as a tool for potential borrowers to help them navigate through USBank's different mortgage options. Many borrowers find it difficult to determine the advantages and disadvantages of different mortgage options and to have a tool that can ease some of the stress of picking one of these options. This report, thus, will help USBank attract more customers due to it simplifying this process.

In order to compare profits between different mortgage rates, we used the mortgage payment model described in Professor Samuel Shen's textbook, *Introduction to Modern Mathematical Modeling With R*. We also used a five step process, called the DAESI math modeling approach, to model the mortgages. This process is also described in Professor Samuel Shen's textbook. In order to compare mortgages, we will need to determine the monthly mortgage payments per the different terms. We will use the processes previously described to do so.

In order to find data on terms, rates, and APRs, we looked at the USBank website and gathered the appropriate data. We will assume a given principle of $200,000 with an APR of 2.817% and a 30-year fixed term. This will be compared with fixed terms of 20-years, 15-years, and 10-years with their appropriate APRs.

Math Modeling Method:

Since a principal loan was given, we will be using a mortgage payment model that was specified in Professor Samuel Shen's textbook. The model uses the following variables:

- P for principal interest/loan
- r for interest rate per month
- n for number of months in the term
- x for the monthly payment

We would use these values P, r, and n to solve for the monthly payment, x. It is important to note that interest is a compounding property and so interest on the principal loan is reviewed. If we construct a model for the first month, it would look like the following:

$$P_1 = P + Pr - x$$

$$P_1 = P(1+r) - x$$

Then if we look at the second month, we have:

$$P_2 = P_1(1+r) - x$$

$$P_2 = (P(1+r) - x)(1+r) - x$$

$$P_2 = P(1+r)^2 - (1+r)x - x$$

We then consider the model when an arbitrary-k value is used in place of number of months:

$$P_k = P_{k-1}(1+r) - x$$

Then we can substitute the equation above into the month before:

$$P_k = P(1+r)^k - (1+r)^{(k-1)}x - \ldots - (1+r)^2x - (1+r)x - x$$

$$P_k = P(1+r)^k - x[(1+r)^{k-1} + \ldots + (1+r)^2 + (1+r) + 1]$$

The above equation can also be rewritten using the following summation formula for a geometric series:

$$1 + a + a^2 + ... + a^{(k-1)} = \frac{1-a^k}{1-a}$$

After implementing the above formula into our equation for P_k we have the following:

$$P_k = P(1+r)^k - x\frac{1-(1+r)^k}{1-(1+r)}$$

$$P_k = P(1+r)^k + x[\frac{1-(1+r)^k}{r}]$$

This equation can then be rewritten to solve for x:

$$x = \frac{[-P*(1+r)^n*r]}{[1-(1+r)^n]}$$

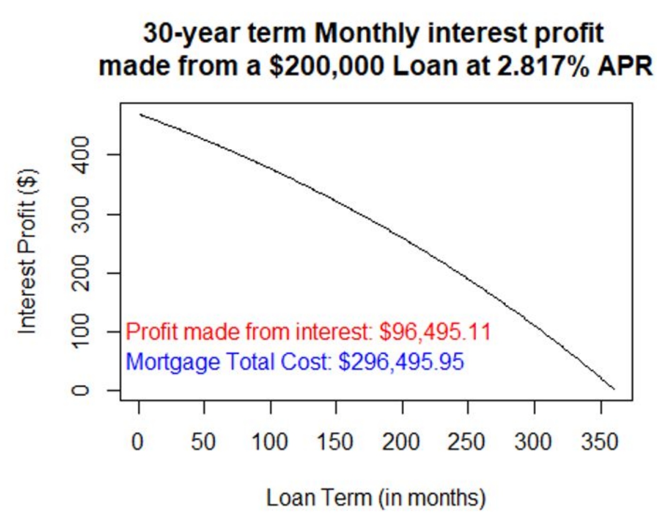
It is important to note that the equation above can only be used if P, r, and n is known.

Data Through Use of Math Modeling Method:

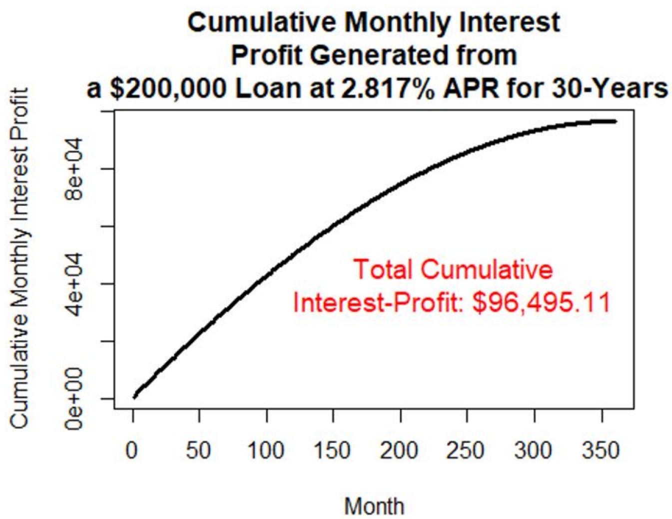
In order to find x, when given a 30-year term, \$200,000 principal interest (P), and a 2.817 % APR, we have to determine r. This can be determined by dividing 2.817% by 12 to get 0.23475%. Since there are 360 months in 30 years, n will equal 360. The calculation for x is below:

$$x = \frac{[-200{,}000(1+0.0023475)^{360}0.0023475]}{[1-(1+0.0023475)^{360}]} = \$823.60$$

Below is a graph of the monthly interest profit made over the course of 360 months in the 30-year term. This graph followed the same condition specified earlier, with the 2.817% APR and a principal interest of \$200,000.



From what we can see in the previous graph, the line goes down at a curve. This means that profits decrease at a faster rate as time goes on in the 30-year time span. The next graph shows the total amount of money made each month added to the months before it. This means that this graph represents the cumulative money made each month throughout the 30-year term:



The graph here is also at a curve which would thus mean that the profits being added decrease over time and so there is not a constant flow of profits each month. The fact that monthly profits decrease over time should stay constant no matter what term is chosen.

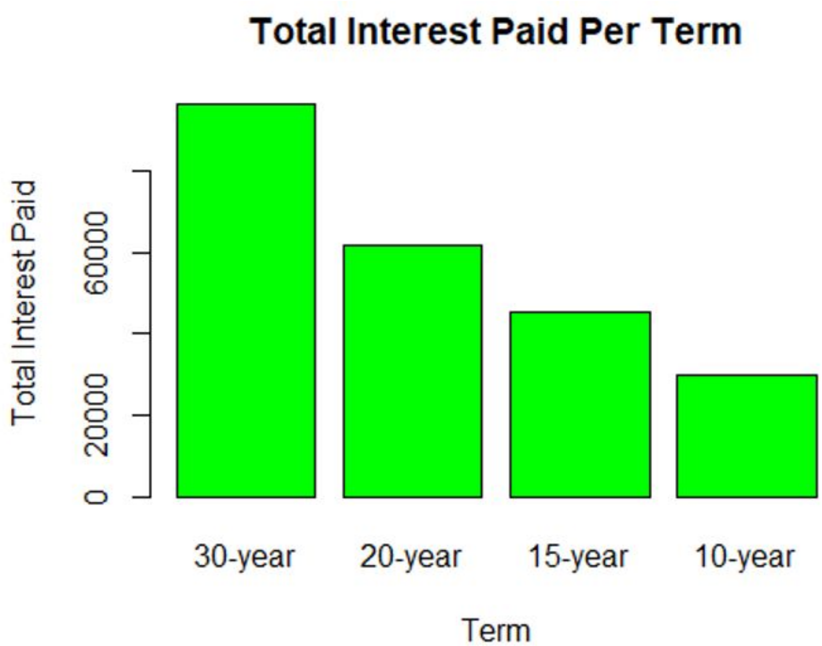
We then noted that a borrower's APR can vary depending on his or her circumstances and so we created a table that provides a sensitivity analysis of the 30-year, 20-year, 15-year, and 10-year terms below using a variety of different APR's. Principal interest/loan is still $200,000.

APR's	30-year term Monthly payments	20-year term Monthly payments	15-year term Monthly payments	10-year term Monthly payments
3.317%	$877.78	$1141.20	$1411.86	$1960.62
3.067%	$850.45	$1115.92	$1387.62	$1937.41
2.817%	$823.60	$1090.96	$1363.63	$1914.37
2.567%	$797.23	$1066.35	$1339.90	$1891.50

2.317%	$771.35	$1042.07	$1316.42	$1868.80

In the chart, the APR's are each separated by .25%. This sensitivity analysis shows that an increase in APR will cause the monthly payments to increase. It is also important to note that the shorter the term, the higher the monthly payments are too. Thus these two factors will grant USBank higher monthly profits if chosen.

We then needed to compare the total interest paid of all the four different terms to determine what the best term is for the right consumer. The bar graph below demonstrates the differences in total interest paid.



As seen in the bar graph above, if a borrower chose the 30-year term plan, they would end up paying the most amount of total interest. The graph shows that the shorter the term, the least amount of total interest is paid and total interest paid increases as the term increases. If we combine what we know from this graph to what we learned from the sensitivity analysis table, we can conclude that if a borrower chose the 30-year plan, they would end up paying more overall in interest but less in interest per month and if the borrower chose the 10-year plan, they would pay less in the overall interest than the other terms, but would be paying more in interest per month. Thus, the best term option for a borrow depends on what he or she wants. If they want to spend less overall money and are willing to pay more per month, then they would want to choose one of the lower term options, preferably either 15-year term or 10-year term. If they are willing to spend more overall money so they can spend less per month, then they would likely choose one of the higher term options, either 30-year term or 20-year term. These options depend on the financial situation and future of the borrowers and they would use these facts and analyses to determine what the best term option would be for them.

Conclusion:

This report will help borrowers gain confidence in their choice in terms of their mortgage plan since they analyzed the information given to them in this report. It provides them with the mathematics behind how each mortgage plan will affect them in the long run. It provides them with the benefits and consequences of each plan. If a borrower chooses a plan that lasts for a long time, then he or she will end up paying less monthly but more by the end of the term. If a borrower chooses a plan that lasts for a short time, then he or she will end up paying more monthly, but less overall. This is exemplified through easy to understand tables and graphs presented in the report.

This method does have some issues due to the mathematical explanation involving high level math, people who are not as skilled in this area may struggle to understand some of the modeling or mathematical conclusions in the report. If a borrower wants to understand how a certain to obtain a certain value through the calculations, he or she may find it difficult to do so since the math behind it may be hard to understand. For example, if a borrower knows their API and wants to use it in calculations, they may find that difficult. In place of that however, is tables and graphs to help borrowers predict any values they need. For instance, if a borrower knows their API, they can go to the sensitivity analysis table in the report and get a feeling on, generally, what their monthly interest will be by comparing it to the API's used in the table. Aside from that issue, we at The Great Camp of American Consulting hope to provide borrowers of USBank a great and easy to understand report that is well appreciated by them and USBank themselves.

References:

Current U.S. Bank mortgage rates. (2020, April 24). Retrieved December 03, 2020, from https://www.usbank.com/home-loans/mortgage/mortgage-rates.html

Shen, Samuel S.P. Introduction to Modern Mathematical Modeling with R. N.p.: WileyInterscience, 2019. Print.

Appendix:

```
P = 200000

r = 0.02817 / 12

n = 30 * 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.02567 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.02317 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.03067 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.03317/ 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)
```

```
X

P30<-200000

r30<-0.02817/12

k30 <- 360

MIR30 <- P30+r30*P30

MIP30 <- MIR30 - P30

MIP30

x30 <- (P30*(1+r30)^(k30)*r30)/((1+r30)^(k30)-1)

x30

monthlyloanprincipal30 <- (MIR30 - x30)

IRmat30 <- matrix(NA,nrow=k30,ncol=1)

for( i in 1:nrow(IRmat30) ){

  IRmat30[i,]<- MIP30

  P30 <- P30*(1+r30)-x30

  MIR30 <- P30+r30*P30

  MIP30 <- MIR30 - P30

}

totalprofit30<-colSums(IRmat30)

totalprofit30

plot.new()

plot(1:nrow(IRmat30),IRmat30[,1],type="l",main = "30-year term Monthly interest profit

\nmade from a $200,000 Loan at 2.817% APR",

     xlab="Loan Term (in months)",ylab="Interest Profit ($)", lwd=0.8)
```

```
text(130,100,"Profit made from interest: $96,495.11",col="red",cex=1)

text(120,50,"Mortgage Total Cost: $296,495.95",col="blue",cex=1)

CFIRmat30<-matrix(NA,nrow=k30,ncol=1)

for( j in 1:nrow(CFIRmat30) ){

  CFIRmat30[j,1]<- sum(IRmat30[1:j,1])

}

plot(1:nrow(CFIRmat30),CFIRmat30[,1],type="l",main = "Cumulative Monthly Interest

Profit Generated from

 a $200,000 Loan at 2.817% APR for 30-Years",

xlab="Month",ylab="Cumulative Monthly Interest Profit",

 lwd=3)

text(240,40000, "Total Cumulative

Interest-Profit: $96,495.11",col = "red",cex = 1.2)

n = 20 * 12

P = 200000

r = 0.02817 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.02567 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.02317 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)
```

X

r = 0.03067 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.03317/ 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

n = 15 * 12

P = 200000

r = 0.02817 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.02567 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.02317 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.03067 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.03317/ 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

```
X

n = 10 * 12

P = 200000

r = 0.02817 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.02567 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.02317 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.03067 / 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

r = 0.03317/ 12

X = (-P * (1+r)^n * r)/(1 - (1 + r)^n)

X

P30<-200000

r30<-0.02817/12

k30 <- 360

MIR30 <- P30+r30*P30

MIP30 <- MIR30 - P30
```

```
MIP30

x30 <- (P30*(1+r30)^(k30)*r30)/((1+r30)^(k30)-1)

x30

monthlyloanprincipal30 <- (MIR30 - x30)

IRmat30 <- matrix(NA,nrow=k30,ncol=1)

for( i in 1:nrow(IRmat30) ){

  IRmat30[i,]<- MIP30

  P30 <- P30*(1+r30)-x30

  MIR30 <- P30+r30*P30

  MIP30 <- MIR30 - P30

}

totalprofit30<-colSums(IRmat30)

totalprofit30

P20<-200000

r20<-0.02817/12

k20 <- 240

MIR20 <- P20+r20*P20

MIP20 <- MIR20 - P20

MIP20

x20 <- (P20*(1+r20)^(k20)*r20)/((1+r20)^(k20)-1)

x20

monthlyloanprincipal20 <- (MIR20 - x20)

IRmat20 <- matrix(NA,nrow=k20,ncol=1)
```

```
for( i in 1:nrow(IRmat20) ){
  IRmat20[i,]<- MIP20
  P20 <- P20*(1+r20)-x20
  MIR20 <- P20+r20*P20
  MIP20 <- MIR20 - P20
}
totalprofit20<-colSums(IRmat20)
totalprofit20
P15<-200000
r15<-0.02817/12
k15 <- 180
MIR15 <- P15+r15*P15
MIP15 <- MIR15 - P15
MIP15
x15 <- (P15*(1+r15)^(k15)*r15)/((1+r15)^(k15)-1)
x15
monthlyloanprincipal15 <- (MIR15 - x15)
IRmat15 <- matrix(NA,nrow=k15,ncol=1)
for( i in 1:nrow(IRmat15) ){
  IRmat15[i,]<- MIP15
  P15 <- P15*(1+r15)-x15
  MIR15 <- P15+r15*P15
  MIP15 <- MIR15 - P15
```

```
}
totalprofit15<-colSums(IRmat15)
totalprofit15
P10<-200000
r10<-0.02817/12
k10 <-120
MIR10 <- P10+r10*P10
MIP10 <- MIR10 - P10
MIP10
x10 <- (P10*(1+r10)^(k10)*r10)/((1+r10)^(k10)-1)
x10
monthlyloanprincipal10 <- (MIR10 - x10)
IRmat10 <- matrix(NA,nrow=k10,ncol=1)
for( i in 1:nrow(IRmat10) ){
  IRmat10[i,]<- MIP10
  P10 <- P10*(1+r10)-x10
  MIR10 <- P10+r10*P10
  MIP10 <- MIR10 - P10
}
totalprofit10<-colSums(IRmat10)
totalprofit10
totalprof <- c(totalprofit30, totalprofit20, totalprofit15, totalprofit10)
barplot(totalprof, main = "Total Interest Paid Per Term",
```

```
xlab = "Term", ylab = "Total Interest Paid",
names.arg = c("30-year", "20-year", "15-year", "10-year"),
col = "green")
```