# 北京邮电大学 2022—2023 学年第二学期

# 《离散数学》期中考试试题

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| 考试注意事项 | 一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。<br>二、书本、参考资料、书包等物品一律放到考场指定位置。<br>三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。<br>四、学生必须将答题内容做在答题纸上，做在草稿纸上一律无效。 |

1. Construct the truth table for the following compound proposition.

$$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$$

[10 marks]

2. Show that $\neg(p \leftrightarrow q)$ and $(p \vee q) \wedge \neg (p \wedge q)$ are logically equivalent.

[10 marks]

3. Show that the argument with the following premises and conclusion is valid by using rules of inference.

Premises: $p \rightarrow q$, $r \rightarrow s$, $p \vee r$

Conclusion: $q \vee s$

[10 marks]

4. Let $A$, $B$ and $C$ be three sets. Show that

$$(A - B) - C = A - (B \cup C).$$

[10 marks]

5. Show that the set of real numbers between 0 and 1 is uncountable.

6. Please firstly use the Euclidean algorithm to find the gcd of 168 and 300, then write the gcd as a linear combination of 168 and 300.

[10 marks]

7. Prove that when $n \geq 2$, it holds that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n}$$

[10 marks]

8. Find the smallest non-negative integer satisfying the following three congruence.

$$x \equiv \begin{cases} 3 \pmod 7 \\ 3 \pmod 5 \\ 4 \pmod{12} \end{cases}$$

[10 marks]

9. 1) How many bit strings of length 10 either begin with three 1s or end with two 1s?

[5 marks]

2) Suppose that a cookie shop has three different kinds of cookies. How many different ways can eight cookies be chosen?

[5 marks]

10. Please use the principle of mathematical induction to prove the binomial theorem.

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \cdots + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n.$$

[10 marks]