Thursday, October 5, 2023 12:08 PM

D.8)
c)

$t_p = \frac{\pi}{w_d}$ $t_r = \frac{1}{2} t_p = \frac{1}{2} \frac{\pi}{w_n\sqrt{1-\zeta^2}}$ if $\zeta = 0.707$, $t_r = \frac{2.2}{w_n}$

$t_r = 2$
$\zeta = 0.7$

Transfer function $m = 5$ $b = 0.5$ $k = 3$

$$\frac{1/m}{s^2 + b/m\, s + k/m} \leftarrow \frac{b_0}{s^2 + a_1 s + a_0}$$

$$\frac{0.2}{s^2 + .1s + .6}$$

For the mass spring, do the following:

(a) Suppose that the design requirements are that the rise time is $t_r \approx 2$ seconds, with a damping ratio of $\zeta = 0.7$. Find the desired closed loop characteristic polynomial $\Delta_{cl}^d(s)$, and the associated pole locations. Find the proportional and derivative gains k_P and k_D to achieve these specifications, and modify the simulation from HW D.7 to verify that the step response satisfies the requirements.

(b) Suppose that the size of the input force is limited to $F_{max} = 6$ N. Modify the simulation to include a saturation block on the force F. Using the rise time t_r as a tuning parameter, tune the PD control gains so that the input just saturates when a step of size of 1 meter is placed on z^r. Plot the step response showing that saturation does not occur for a 1 meter step input for the new control gains.

$$Y(s) = \frac{b_0 k_P}{s^2 + (a_1 + b_0 k_D)s + (a_0 + b_0 k_P)} Y_r(s)$$

$$Y(s) = \frac{1/m\, k_P}{s^2 + (b/m + 1/m\, k_D)s + (k/m + 1/m\, k_P)}$$

$w_n = \frac{\pi}{t_r\sqrt{1-\zeta^2}} \cdot \frac{1}{2} = 1.1$ $\Delta_{cl}^d = s^2 + 2\zeta w_n s + w_n^2 = s^2 + 1.54s + 1.21$ $\Delta_{cl} = s^2 + (0.1 + 0.2k_D)s + (0.6 + 0.2k_P)$

$1.54 = 0.1 + 0.2k_D \rightarrow \boxed{7.2 = k_D}$
$1.21 = 0.6 + 0.2k_P \rightarrow \boxed{3.05 = k_P}$

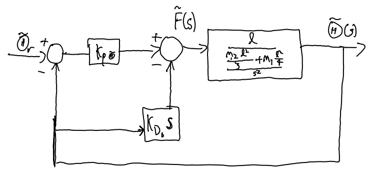
$$\frac{-1.54 \pm \sqrt{1.54^2 - 4(1.21)}}{2}$$

$$\boxed{\begin{aligned} p_1 &= -0.77 + .7856j \\ p_2 &= -0.77 - .7856j \end{aligned}}$$

E.8
a)

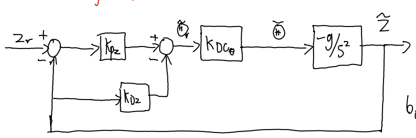
$\tilde{\Theta}(s) = \frac{\frac{l}{\frac{m_2 l^2}{3} + m_1 z_e^2}}{s^2} \cdot F(s)$, $\tilde{Z}(s) = -g/s^2\, \Theta(s)$

Inner loop:



How does PD change Z_r to Θ_r

Outer loop:



$l = 0.5$ $m_1 = 0.35$ $m_2 = 2$ $g = 9.8$

$$b_0 = \frac{l}{\frac{m_2 l^2}{3} + m_1 \frac{l^2}{4}} = 2.652$$

b) $\zeta = 0.707$
$t_{r\theta} = 1$
$w_n = 2.2$

$\Delta_{cl}^d = s^2 + 2\zeta w_n s + w_n^2 = s^2 + 3.1108s + 4.84$ $\Delta_{cl} = s^2 + b_0 k_D s + b_0 k_P$

$3.1108 = 2.652 k_D$
$4.84 = 2.652 k_P$

$$\boxed{\begin{aligned} k_{D\theta} &= 1.173 \\ k_{P\theta} &= 1.825 \end{aligned}}$$

c)

K_{DC} is when $s = 0$

$$K_{DC} = \frac{b_0 k_P}{b_0 k_P} = 1$$

d)

$t_{rz} = 10$ $\zeta = 0.707$ $w_n = 2.2/t_{rz} =$

$\Delta_{cl}^d = s^2 + 2\zeta w_n s + w_n^2$
$= s^2 + 0.31108s + 0.0484$ $\Delta_{cl} = s^2 + b_0 k_D s + b_0 k_P$

$0.31108 = -9.8 k_D \rightarrow$
$0.0484 = -9.8 k_P \rightarrow$

$$\boxed{\begin{aligned} k_D &= -0.0317 \\ k_P &= -0.00494 \end{aligned}}$$

F.8

$\tilde{\Theta}(s) = \left(\frac{1}{J_c + 2m_r d^2} / s^2\right) \tilde{\tau}(s)$ $\tilde{Z}(s) = \frac{\left(\frac{-F_e}{m_c + 2m_r}\right)}{s^2 + \frac{\mu}{m_c + 2m_r} s} \tilde{\Theta}(s)$ $\Delta_{cl}^d = s^2 + 2\zeta w_n s + w_n^2$

$\hat{H}(s) = \frac{\frac{1}{m_c + 2m_r}}{s^2} \tilde{F}(s)$

Use the following physical parameters: $m_c = 1$ kg, $J_c = 0.0042$ kg m^2, $m_r = 0.25$ kg, $m_\ell = 0.25$ kg, $d = 0.3$ m, $\mu = 0.1$ kg/s, $g = 9.81$ m/s^2.

a) $t_{rh} = 8$
$\zeta_h = 0.707$

$w_n = 2.2/8 = 0.275$ $b_0 = \frac{1}{m_c + 2m_r} \Rightarrow \frac{1}{1 + 2 \cdot 0.25} = 0.667$

$\Delta_{cl}^d = s^2 + 0.3885s + 0.0756$ $\Delta_{cl} = s^2 + b_0 k_D + b_0 k_P$

$0.3885 = 0.667 k_{DH} \rightarrow$
$0.0756 = 0.667 k_{PH} \rightarrow$

$$\boxed{\begin{aligned} k_{DH} &= 0.582 \\ k_{PH} &= 0.1133 \end{aligned}}$$

b) $\zeta_\theta = 0.707$ $w_{n\theta} = 2.2/t_{r\theta} = 2.75$ $b_{0\theta} = \frac{1}{J_c + 2m_r d^2} = \frac{1}{J_c + 0.045}$
$t_{r\theta} = 0.8$

$\Delta_{cl}^d = s^2 + 3.88s + 7.56$ $\Delta_{cl} = s^2 + \frac{k_D s}{J_c + 0.045} + \frac{k_P}{J_c + 0.045}$

$$3.88 J_c + 0.1746 = \boxed{K_{D0} = 0.191}$$
$$7.56 J_c + 0.3402 = \boxed{K_{P0} = 0.372}$$

c) $K_{DC} = 1$

d) $t_{r_2} = 10(0.8) = 8$ $\quad w_n = 2.2/8 = 0.275$ $\quad b_0 = \cancel{\dfrac{(m_c + 2m_r)g}{m_c + 2m_r}} = g$ $\quad a_1 = \dfrac{\mu}{m_c + 2m_r} = 0.0667$

$\zeta = 0.707$

$$\Delta^d_{c_1} = s^2 + 0.3885 + 0.0756 \qquad \Delta_{c1} = s^2 + (a_1 + b_0 K_D)s + (a_0 + b_0 K_p)$$
$$= s^2 + (0.0667 + gK_D) + gK_p$$

$$0.3885 = 0.0667 + 9.8 K_D$$
$$0.0756 = 9.8 k_p$$

$$\boxed{\begin{aligned} K_{D2} &= 0.0328 \\ K_{p2} &= 0.0077 \end{aligned}}$$