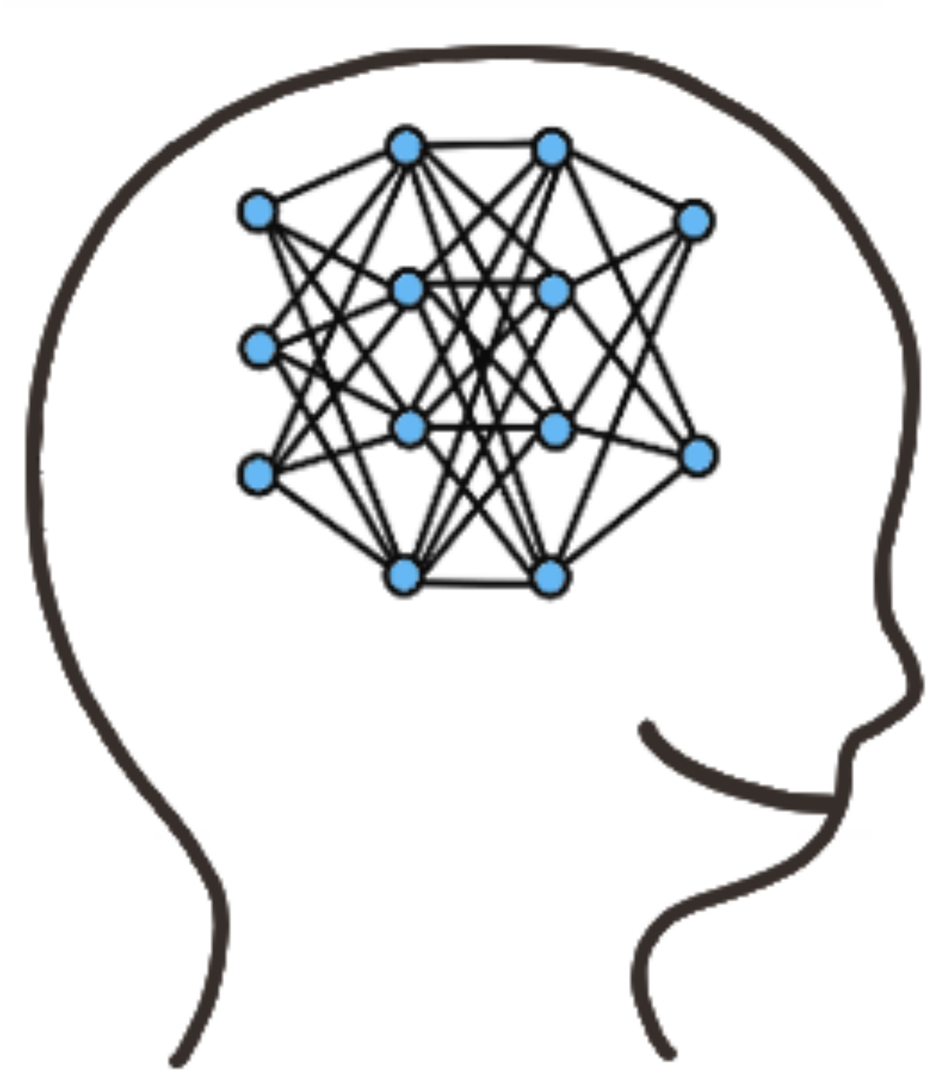


線性迴歸與正規化

Linear Regression & Regularization

Overfitting 處理

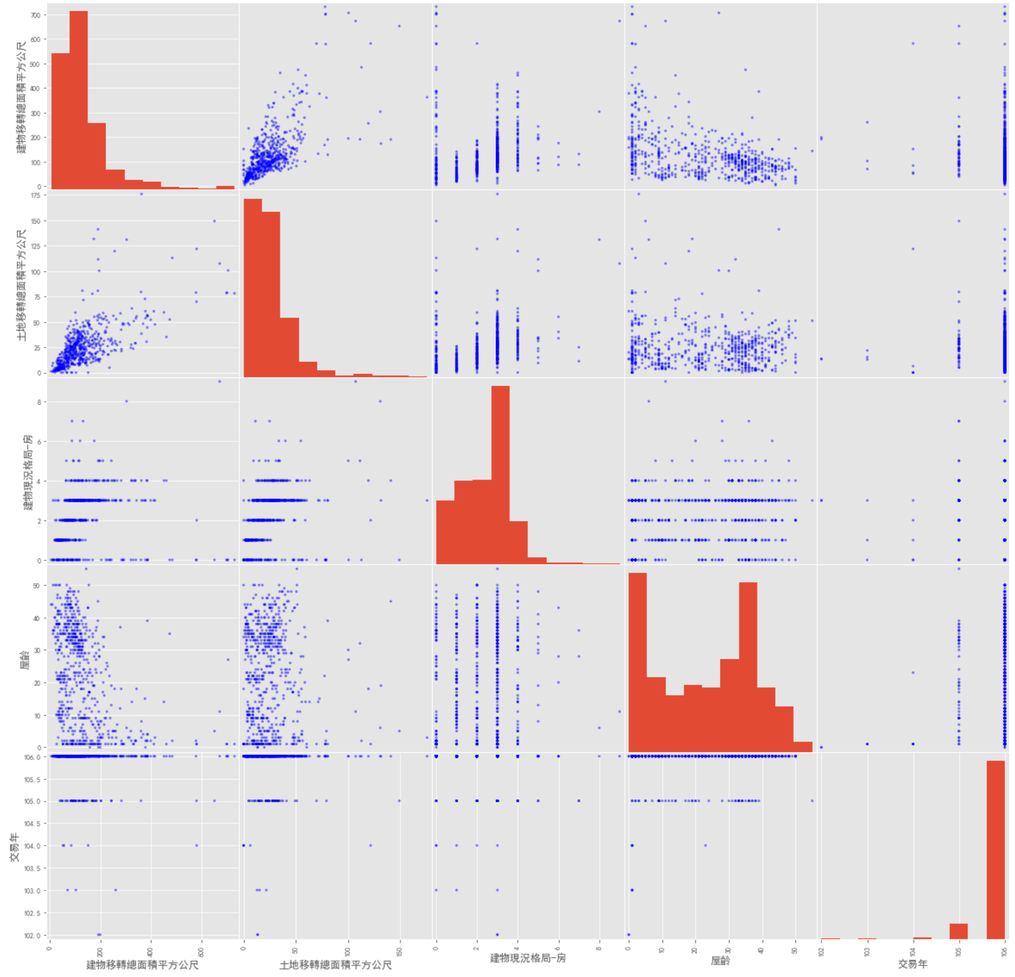
- 減少特徵數量：
  - 手動挑選特徵(利用domain knowledge)
  - 降維、特徵提取(Feature extraction)
  - 特徵重要性計算、特徵選取(Feature Selection)
- 增加資料量
- 正規化：降低權重過高的情況

Notes

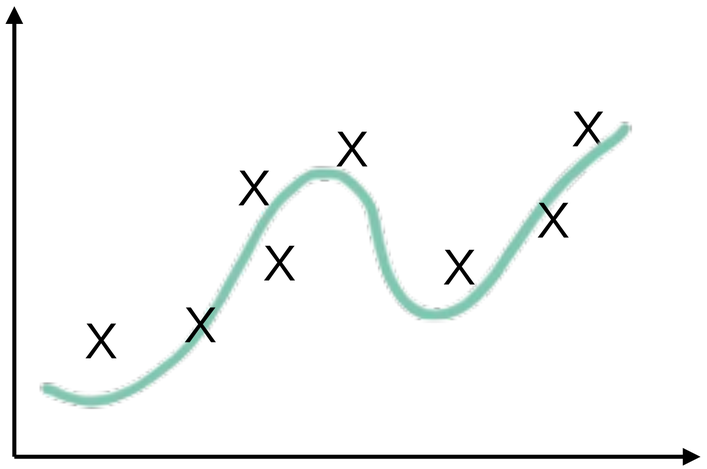
- 於決策樹會再教大家如何做特徵選取(Feature Selection)
- 於非監督式學習的章節會再教大家如何降維與特徵提取(Feature extraction)

檢查特徵存在的線性關係

- 相關性分析
- 散佈圖



正規化（Regularization）



$$y = w_0 + w_1 x_1 + w_2 x_1^{\,2} + w_3 x_1^{\,3} + w_4 x_1^{\,4}$$

$$J(w) = \frac{1}{2m}\sum_{i=1}^{m}\left(\hat{y}^{(i)} - y^{(i)}\right)^2 + 1000 w_4 x_1^{\,4}$$

懲罰項(penalty)

$$w_4 \approx 0$$

- Regularization

$$J(w) = \frac{1}{2m}\sum_{i=1}^{m}\left(\hat{y}^{(i)} - y^{(i)}\right)^2 + \alpha\sum_{j=1}^{n} w^2$$

Notes

- 此方法又稱為權重衰減(Weight Decay)
- 限制weight的增長

L1, L2 正規化

- L2 Regularization

$$J(w)=\frac{1}{2m}\sum_{i=1}^{m}\left(\hat{y}^{(i)}-y^{(i)}\right)^2+\alpha\sum_{j=1}^{n}w^2$$

Notes

- alpha 越大，正規化懲罰越大，無限大時w=o
- alpha 越小，正規化懲罰越小，alpha=o時，等於無正規化的線性迴歸

- L1 Regularization

$$J(w)=\frac{1}{2m}\sum_{i=1}^{m}\left(\hat{y}^{(i)}-y^{(i)}\right)^2+\alpha\sum_{j=1}^{n}|w|$$

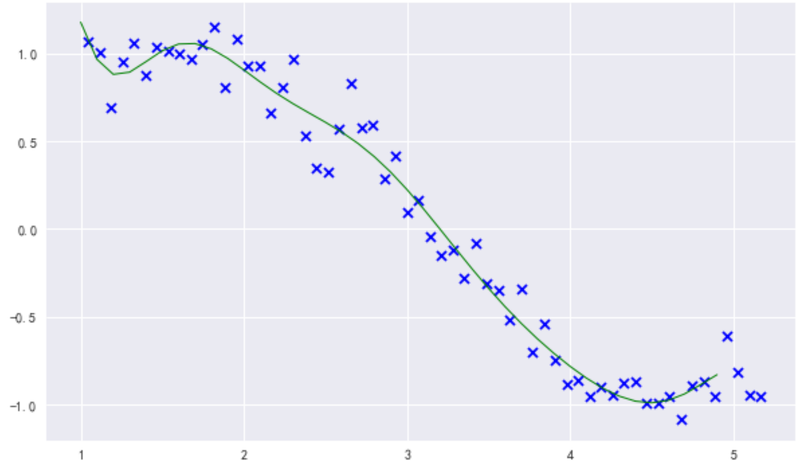
含正規化的迴歸

- Linear Regression with L2 Regularization
  - 脊迴歸 (Ridge Regression)
- Linear Regression with L1 Regularization
  - 最小絕對值收斂和選擇算子、套索算法 (least absolute shrinkage and selection operator, LASSO)
- Linear Regression with both
  - 彈性網 (Elastic Net)

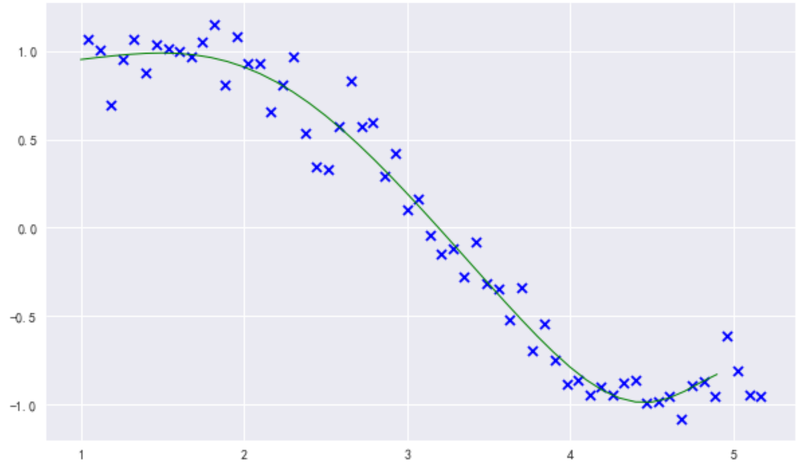
Notes

- Ridge Regression
  - from sklearn.linear_model import Ridge
- LASSO
  - from sklearn.linear_model import Lasso
- ElasticNet
  - from sklearn.linear_model import ElasticNet

Linear Regression with Regularization



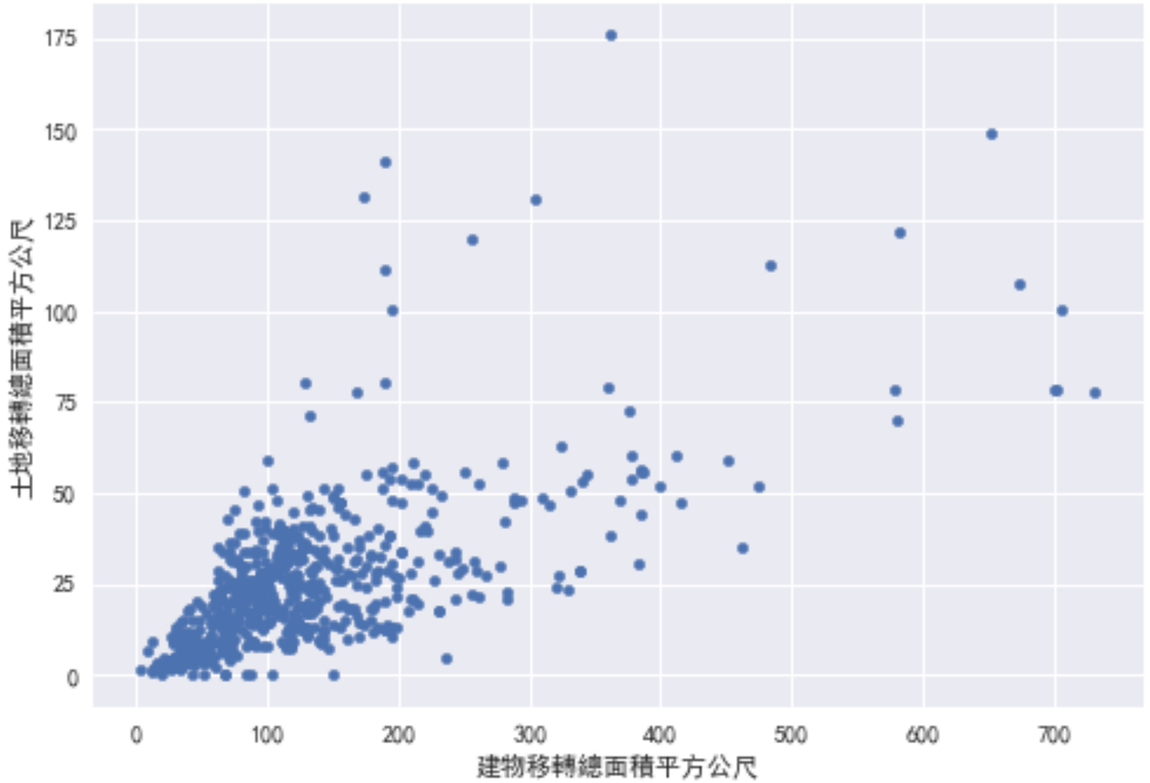

```
degree=12
Linear Regression
```




```
degree=12
Ridge Regression
(alpha=1)
```

共線性（Collinearity）

- 特徵之間存在線性相關：共線性(Collinearity)




```
Linear Regression weights:
建物移轉面積：2191.4998606
土地移轉面積：-275.7035364
屋齡：-118.04770571
```

土地移轉面積越大，售價越低？

Handling Collinearity with Ridge

- Experiment Results:

```
Linear Regression weights:
建物移轉面積：2191.4998606
土地移轉面積：-275.7035364
屋齡：-118.04770571
R Square: 0.727309001534
```

```
Ridge Regression weights: (alpha=100)
建物移轉面積：1506.46178566
土地移轉面積：137.74155002
屋齡：-308.67028393
R Square: 0.660919727018
```

Notes

- **解決共線性問題，只是使權重值具解釋性，但準確度不一定會提升**