8th Grade Class Notes

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Contents

1 Measures of Central Tendency and Spread

When analyzing a set of data, there are different ways to summarize both the center of the data and the spread of the data. Two common ways to describe the center of a dataset are the *median* and the *mean*.

Similarly, two ways to describe how spread out the data are include the *interquartile range* (IQR) and the *standard deviation*.

Each of these measures has its strengths, particularly when it comes to handling *outliers*—data points that are significantly higher or lower than the others.

1.1 Median and Interquartile Range

The median is the middle value in a set of data when the numbers are arranged in order. It splits the data into two equal halves: half of the values are below the median, and half are above.

The interquartile range (IQR) measures the spread of the middle 50% of the data. It is calculated as the difference between the third quartile (the value below which 75% of the data falls) and the first quartile (the value below which 25% of the data falls).

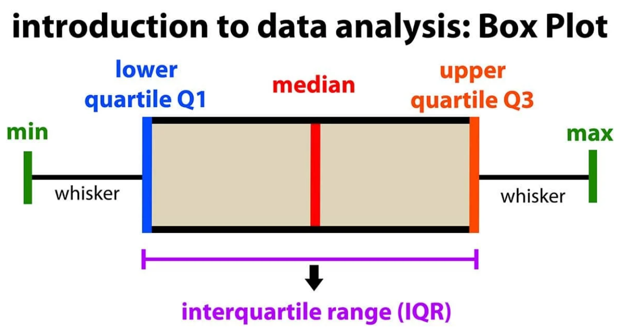


Figure 1: A box-and-whisker plot showing the distribution of data, highlighting the median, quartiles, and outliers.

- The median and IQR are most useful when the data contains outliers. This is because the median is not affected by extremely large or small values, while the mean can be pulled significantly in one direction by outliers.
- For example, consider the incomes of a small group of people. If one person in the group is a millionaire, the *mean* income would be much higher than most people's actual incomes, making it a misleading measure of the "typical" income. In this case, the *median* income would give a better idea of what most people in the group earn.

Example:

- Data set: $\{10, 12, 15, 18, 100\}$ (an outlier at 100).
- **Median**: The median is 15, which is a good representation of the middle of the data.
- **IQR**: The IQR tells us how spread out the middle 50% of the data is, ignoring the outlier at 100.

1.2 Mean and Standard Deviation

The mean is the average of the values in a data set. It is found by adding all the values together and dividing by the number of values. The standard deviation is a measure of how spread out the numbers are from the mean. A small standard deviation means that the values are close to the mean, while a large standard deviation indicates that the values are more spread out.

- The mean and standard deviation are best used when the data is *symmetric* and does not contain outliers. Because the mean is sensitive to extremely large or small values, outliers can skew the result and give a misleading picture of the data.

- When the data is fairly uniform, the mean provides a good measure of central tendency, and the standard deviation gives a clear indication of how much the data values vary around the mean.

Example:

- Data set: $\{10, 12, 15, 18, 20\}$.

- **Mean**: The mean is 15, which accurately reflects the central tendency of the data since there are no outliers.

- **Standard Deviation**
  - The standard deviation, σ (lowercase sigma), gives a sense of how closely clustered the data points are around the mean. In this population, $\sigma = 3.69$.
  - If the data set were $\{1, 8, 15, 22, 29\}$, the mean would still be 15, but $\sigma = 9.90$

1.3 Choosing Between Median and Mean

The key difference between using the median or the mean depends on whether your data contains *outliers*.

- If your data has extreme values (outliers), the *median* and *IQR* will give you a better understanding of the typical values in your data set.

- If your data does not have outliers and is relatively symmetric, the *mean* and *standard deviation* provide a more precise description of both the center and the spread of the data.

In summary:

- Use the *median* and *IQR* when there are *outliers* or a *skewed distribution*.

- Use the *mean* and *standard deviation* when the data is *symmetrically distributed* and *outlier-free*.

2 Plugging in Numbers for Equations

You may be asked to solve an equation when given a value for the variable. For example:

$$\begin{aligned}
d &= 3l + 2 && \text{Original equation} \\
d &= 3(4) + 2 && \text{Substitute } l = 4 \\
d &= 12 + 2 && \text{Multiply 3 by 4} \\
d &= 14 && \text{Simplify}
\end{aligned}$$

You might also be asked whether an ordered pair satisfies an equation. To solve this, remember that an ordered pair represents the x and y values, in that order. For example, given the equation: $y = 2x - 5$, you can determine if the ordered pair $(3, 1)$ satisfies the equation by substituting $x = 3$ and $y = 1$:

$$
\begin{aligned}
y &= 2x - 5 && \text{Original equation} \\
1 &= 2(3) - 5 && \text{Substitute } x = 3 \text{ and } y = 1 \\
1 &= 6 - 5 && \text{Multiply 2 by 3} \\
1 &= 1 && \text{Simplify, the equation is true}
\end{aligned}
$$

Since the equation is true when substituting the values, the ordered pair $(3, 1)$ does satisfy the equation. If the equation were false after substituting the values, then the ordered pair would not satisfy the equation.

3 Graphing

When asked to graph a linear equation on the coordinate plane, there are several approaches. First, we can make a table by choosing values for x and then solving for y. From this, we can make a table of x and y values, plot those points on the coordinate plane, and connect the points to form a line.

3.1 Creating a Table from an Equation

Let's take the equation $y = 2x + 1$. We can create a table by selecting values for x and calculating the corresponding y values.

x	y
-2	-3
0	1
2	5

Now we can plot the points $(-2, -3)$, $(0, 1)$, and $(2, 5)$ on the coordinate plane. By connecting these points, we form the line representing the equation $y = 2x + 1$.

3.2 Graphing Using Point-Slope Form

You can also take advantage of an equation in point-slope form by remembering how to find the slope and y-intercept. Recall that *slope* is $\frac{\text{rise}}{\text{run}}$, meaning how much the line goes up (or down) for every unit it moves horizontally. The slope, denoted as m, tells you how steep the line is, and the y-intercept, denoted as b, is the point where the line crosses the y-axis.

The general form of a linear equation is:

$$y = mx + b$$

where:

- m is the slope (rise/run)
- b is the y-intercept

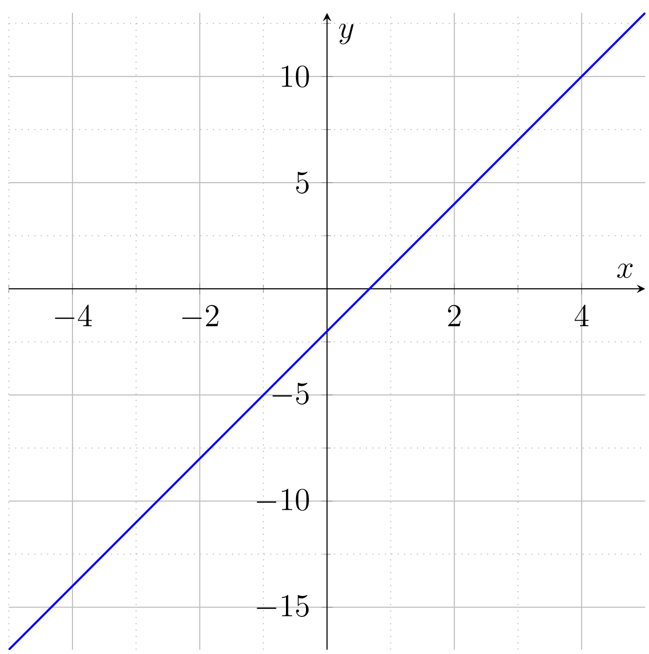
To graph an equation like $y = 3x - 2$, you can start by plotting the y-intercept, which is the point $(0, -2)$. From there, use the slope $m = 3$, which means for every 1 unit increase in x, y increases by 3 units (rise of 3 and run of 1). Plot a second point based on this slope, then draw a line through both points.

3.3 Graphing Using Slope and y-Intercept

Let's graph the equation $y = 3x - 2$. Here's how to do it:

1. The y-intercept is -2, so plot the point $(0, -2)$.
2. The slope is 3, which is $\frac{3}{1}$. This means from the y-intercept, move up 3 units and 1 unit to the right. Plot this second point at $(1, 1)$.
3. Now, connect the two points to form the line.

This is a quick and effective way to graph linear equations without needing to make a full table of values.



In this example, the line crosses the y-axis at -2 and follows a slope of 3.

4 Functions, Function Notation, and Sequences

A function is similar to the idea of x and y notation, where x represents an input and y represents the result or output. In basic algebra, you might have seen equations like

$$y = 2x + 3$$

This is a way of saying that for any value of x, we can find y by following this rule: multiply x by 2, then add 3.

Mathematicians use *function notation* to make this idea more general and easier to work with. Instead of writing $y = 2x + 3$, we can use function notation to write

$$f(x) = 2x + 3$$

Here, $f(x)$ is just another way of writing "the result when you plug x into the function." It's like a shortcut for showing how x changes into something else, depending on the rule of the function.

If you want to know what happens when $x = 4$, you would write

$$\begin{aligned}
f(x) &= 2x + 3 && \text{Original function}\\
f(4) &= 2(4) + 3 && \text{Substitute } 4 \text{ for } x\\
f(4) &= 8 + 3 && \text{Multiply } 2 \times 4\\
f(4) &= 11 && \text{Simplify the expression}
\end{aligned}$$

Function notation makes it easier to talk about different rules and to work with multiple functions at once, which is helpful when solving more complex problems.

A problem set might define several functions and ask you to solve for different numbers. This can be confusing if they choose different functions just to throw you off.

A problem set might define several functions and ask you to solve for different values. This can sometimes be tricky if different functions are used, but it's important to focus on the specific rule for each function and apply it carefully.

Given $f(n) = 8n - 3$ and $g(n) = 3n - 10$, solve $f(1)$, $g(0)$, $f(5)$.

$$\begin{aligned}
f(1) &= 8(1) - 3 && \text{Multiply } 8 \times 1\\
f(1) &= 8 - 3 && \text{Simplify the multiplication}\\
f(1) &= 5 && \text{Simplify the subtraction}
\end{aligned}$$

$$\begin{aligned}
g(0) &= 3(0) - 10 && \text{Multiply } 3 \times 0\\
g(0) &= 0 - 10 && \text{Simplify the multiplication}\\
g(0) &= -10 && \text{Simplify the subtraction}
\end{aligned}$$

$$
\begin{aligned}
f(5) &= 8(5) - 3 && \text{Multiply } 8 \times 5 \\
f(5) &= 40 - 3 && \text{Simplify the multiplication} \\
f(5) &= 37 && \text{Simplify the subtraction}
\end{aligned}
$$

4.1 Sequences

You may be given a sequence where you are asked to recognize a pattern, or given a description of a pattern. A sequence is just an ordered list of numbers, and often there is a specific rule that determines what comes next. Recognizing the pattern can help you figure out future numbers in the sequence or fill in missing terms. It can also be useful to reduce this pattern to an equation, which allows you to find any term in the sequence without having to write out the whole list.

For example, consider this situation:

> Sarah counted the number of tiles she placed on a wall each day. On the first day, she placed 3 tiles. On the second day, she placed 7 tiles. On the third day, she placed 11 tiles, and so on. Each day, the number of tiles she placed increased by 4 compared to the previous day.

If we look closely, we can see a pattern: each day Sarah places 4 more tiles than the previous day. This forms a sequence: 3, 7, 11, 15, ...

To express this in an equation, we notice that each number in the sequence is part of a linear pattern. On the first day, she places 3 tiles, on the second day, she places 7, and on the third day, she places 11. The difference between each number is 4, so this is an arithmetic sequence.

We can write the rule for the number of tiles Sarah places on day n as:

$$a_n = 3 + 4(n - 1)$$

Here, a_n represents the number of tiles placed on day n, and n is the day number. The number 3 is the starting number (the number of tiles placed on the first day), and 4 is the amount by which the number of tiles increases each day. Simplifying the equation:

$$a_n = 4n - 1$$

Now, you can use this equation to find the number of tiles Sarah will place on any given day without having to continue the sequence manually.

5 Functions vs. Relations

> A relation is a function if each x-value is paired with exactly one y-value.

Think of a function as a machine that accepts a number and always spits out the same result—given that number.

They test this three different ways:

- Giving a set of ordered pairs.
- Graphing.
- Showing a map from Domain to Range.

But the concept is the same. Always ask yourself: *Are there any xs with more than one y?*

5.1 As ordered pairs

An ordered pair is written in the form (x, y), where x is the input, and y is the output. If no x-value is repeated with different y-values, then the relation is a function. For example:

$$\{(1, 2), (3, 4), (5, 6), (7, 8)\}$$

This is a function because each x-value corresponds to exactly one y-value.
However, consider this set:

$$\{(2, 3), (2, 4), (5, 6), (7, 8)\}$$

This is not a function because the x-value 2 is paired with both 3 and 4, violating the rule that each x-value can only be paired with one y-value.

5.2 As a graph

Graphing is another way to test if a relation is a function. You can use the vertical line test: if any vertical line drawn on the graph crosses the relation more than once, the relation is not a function.

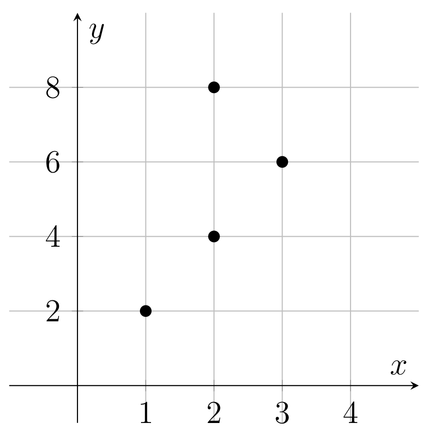


Figure 2: Graph of relation $\{(1, 2), (2, 4), (3, 6), (2, 8)\}$ fails the vertical line test because $x = 2$ maps to two different y-values.

5.3 As a mapping of domain to range

A function can also be represented by mapping the Domain (the set of all possible x-values) to the Range (the set of all possible y-values). For a relation to be a function, each element in the Domain must map to exactly one element in the Range.

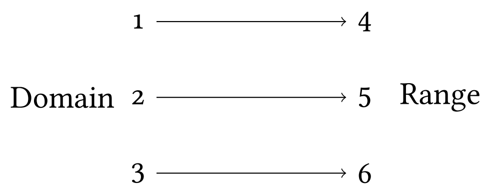


Figure 3: Mapping from Domain $\{1, 2, 3\}$ to Range $\{4, 5, 6\}$ shows a function. Each x-value in the Domain maps to exactly one y-value in the Range.

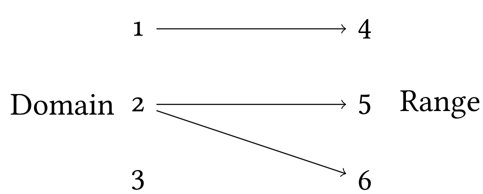


Figure 4: Mapping from Domain $\{1, 2, 3\}$ to Range $\{4, 5, 6\}$ is not a function because 2 in the Domain maps to two different elements in the Range (5 and 6).