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Research Statement
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Overview

This note is intended as a summary of the research accomplishments I have developed throughout my academic career up to this point. I will start with a very general overview of what motivates my work and the general themes in my research. Then, in the following section, I will give a more detailed description of specific research highlights.

Very broadly my work can be characterized as the study of statistical problems which have some nontrivial component of spatial dimension. I am particularly interested in statistical problems where the answers or methods depend on the dimension of the ambient space $\mathbb{R}^d$. A cogent example of this phenomenon, described in detail below, concerns the equivalence or orthogonality of a large class of Gaussian random field models with separate variance and spatial scale parameters. The conclusion, very briefly, is that under fixed domain asymptotics: when $d = 1, 2, 3$ it is impossible to consistently distinguish a change in pointwise variance with a change in spatial scale; but when $d > 4$ the opposite is true so that variance and scale are consistently distinguishable ($d = 4$ is still open). Problems like these, where spatial dimension play a fundamental role in the statistical conclusions, are what motivate a large part of my work.

A second characterizing feature of my research is the use of smooth invertible transformations—or deformations—in solutions to statistical problems. Deformations can provide an elegant tool for such things as developing nonstationary random field models, incorporating prior morphological shape information in multivariate density estimates, and for computational approximations to spatial prediction, to name a few. In my work with cosmologist Lloyd Knox we analyze and estimate a physical deformation arising in cosmology which is, not only observable, but also characterizes the density fluctuations of dark matter. The main challenge of working with deformations is their non-linear nature: adding two deformations does not necessarily result in another deformation. This makes it difficult to construct flexible classes of deformations, find estimates and quantify statistical uncertainty. Indeed, a large part of my work is dedicated to overcoming these difficulties and exploring solutions which will make deformations a flexible and powerful tool for the statistician practitioner.

One last feature of my work, which is worth stating in the overview, is that I have made a point of researching both applied and theoretical problems in statistics. My hope is that, taken together, they give me a broader perspective on statistical inference in general. The majority of my applied work is focused on using deformation estimates to measure the density fluctuations of dark matter from cosmic microwave background observations. In contrast, my theoretical work has been primarily dedicated to proving fixed domain asymptotic results for random fields, especially in the context of nonstationarity induced by a spatially varying local scale. Both the applied and theoretic problems have been rewarding and interesting. Indeed, I plan to continue working on both types of problems: applied work to connect with the scientific community while also using theoretical statistical arguments to gain a deeper understanding of the statistical problems at hand.

Research Highlights

Nonstationary models for random fields. Recognizing the importance of nonstationary random fields due to the recent abundance of large data sets from remote sensing and satellite imagery, Michael Stein (University of Chicago) and I started studying deformed isotropic random fields as a flexible class of nonstationary models. At that time, methods for estimating deformed isotropic random fields had been developed only when observing the deformed field at sparse spatial locations with independent replicates. The ubiquity of high resolution imagery led us to study a different observation scenario under which one observes a single realization of the deformed field on a dense grid. Our research led to the the paper [11] in which we develop a complete methodological package—from model assumptions to algorithmic recovery of the deformation—for the class of nonstationary processes obtained by deforming isotropic Gaussian random fields in two dimensions. A key contribution of this research is the use of quasiconformal theory as a tool, in two dimensions, for modeling and estimating deformations. Two other notable contributions are the careful analysis of the smoothing problem for locally estimated parameters and the development of an algorithm for recovery of the invertible transformation from the estimated parameters.

Soon after the paper with Michael Stein, I began investigating consistency questions related to the deformed random field model. The natural regime to study these questions is fixed domain asymptotics where one observes a fixed realization of the deformed random field on a grid where the grid spacing approaches zero. At that time it was not clear whether one could control the propagation of error through the nonlinear transform that relates the estimable parameters with the invertible deformation. In collaboration with Sourav Chatterjee (Courant Institute, NYU) we developed chaining arguments to control the supremum of the error field for kernel smoothed quadratic variation estimators. Using these techniques, along with some surprisingly useful results about Bergman spaces and projections, we established strong consistency results under mild assumptions on the isotropic random field and the deformation in two dimensions (see [5]).

Equivalence and orthogonality of Gaussian random field models. The problem of estimating a scale and sheer of the local coordinates of a deformed isotropic field leads naturally to questions relating to estimating the effects of an unknown spatial scale and an unknown amplitude change in a single realization of a Gaussian random field. In particular, let $Z(x)$ be a stationary random field on $x \in \mathbb{R}^d$. The main question is: can one consistently tell the difference between a realization of

$Z(\alpha x)$ and a realization of $\sigma Z(x)$. This question was motivated by the seminal work of Hao Zhang [17] who showed, perhaps surprisingly, that when $d \leq 3$, Z is a Gaussian Matérn random field, and $\alpha^{2\nu} = \sigma^2$ (in this case ν is a particular smoothness parameter), then it is impossible to distinguish between these two models when both fields are observed everywhere, without noise, on a bounded spatial region in $\mathbb{R}^d$. As an extension of this result I was able to establish in [1] the opposite is true when $d > 4$: one can consistently estimate α and σ^2 under fixed domain asymptotics for Gaussian Matérn random fields ($d = 4$ is still open). The paper utilizes quadratic variations and a careful study of rates to (in certain cases) get an estimate of the coefficient on the second principle irregular term which then allows one to separate the effects of α and σ^2.

Estimating weak lensing of the Cosmic microwave background. The majority of my applied work in the field of cosmology has been dedicated to estimating the gravitational distortion of cosmic microwave background (CMB) radiation as it passes through dark matter. The CMB is a remnant of the big bang and fills the Universe with an almost-uniform radiation. Recent observations can map the microscope variations from uniformity in the CMB. However, the observations do not precisely map the CMB, but a slight distortion due to the bending of the CMB light from intervening matter. This is call gravitational lensing. Estimating this lensing is important for a number of reasons including, but not limited to, understanding cosmic structure, constraining cosmological parameters and detecting gravity waves. My work in this area has resulted in two papers [9], [10] published in Physical Review D, two conference papers [2],[13] and a technical report currently under review [4].

The results found in [9] and [13] develop a new local likelihood estimate of weak lensing and was done under collaboration with Lloyd Knox (Professor of Physics, UC Davis) and Alexander van Engelen (Postdoc, Department of Physics, Stony Brook). Our method avoids the computational difficulties associated with a full scale likelihood approach and circumvents the typical Taylor truncation bias which corrupts most other estimates of lensing. This approach estimates the local curvature of the gravitational potential on sliding local neighborhoods of the observed CMB and polarization fields. A low pass filter of the true gravitational potential is then constructed by stitching together local curvature estimates. One of the main advantages of this approach is that it can easily handle point source foregrounds, masking, nonstationary noise and nonstationary beams.

The results found in [10] and [4] study the state-of-the-art quadratic estimator of weak lensing, initially developed by Hu and Okamoto [15], [16]. This estimator is interesting from two perspectives. First, from a statistical standpoint the estimator is not completely understood. The quadratic estimator works by estimating the correlation across the Fourier frequencies of the lensed CMB, due to the nonstationary of the lensing effect. This is a subtle operation and leads to unexpected statistical behavior in the estimator. Secondly, from a scientific perspective, this estimator is the state-of-the-art for reconstructing the weak lensing distortion of the CMB. This makes it an indispensable tool for probing the nature of dark matter. It is these two perspectives, statistical and scientific, that make it an interesting object of study.

Our most recent findings have resulted in a manuscript titled: "Decomposing CMB lensing power with simulation" [4]. The quadratic estimator of weak lensing works in part through a delicate cancelation of terms in a Taylor expansion of the lensing effect on the CMB. In our manuscript we present two simulation based approaches for exploring the nature of this cancelation for both the CMB intensity and the polarization fields. One of the highlights of this paper is a detailed analysis of a recent proposal for a bias-reduced modified quadratic estimator. We find that the modified quadratic estimate does reduce estimation bias. However, this is accomplished by effectively increasing the magnitude of a first and second order bias to the point of cancelation when the correct model for the spectral density of the gravitational potential is used to generate the lensed weights. This is different behavior than was previously expected and can effect future experimental results. We demonstrate, that in future ACTpol/SPTpol experiments the bias in the EB estimator can be effectively ignored. For the TE and the EE estimators, however, the bias does contribute significantly to projected error bars and may need to be corrected to give the estimator inferential power beyond a nominal fiducial uncertainty.

In the paper [10] we study the advantages obtained by relaxing the first order unbiasedness constraint used to derive the quadratic estimator. Our new estimate requires the user to propose a fiducial model for the spectral density of the unknown lensing potential but the resulting estimator is developed to be robust to misspecification of this model. The role of the fiducial spectral density is to give the estimator superior statistical performance in a neighborhood of the fiducial mode while controlling the statistical errors when the fiducial spectral density is drastically wrong. One of the biggest advantages of our new estimate is the Bayesian underpinnings of the estimator which allow construction of estimates and uncertainty quantification for nonlinear functionals of the gravitational potential.

Transformation methods for nonparametric density estimation. After my exposure to the elegance of using smooth invertible transformations for developing flexible, physically realistic models of nonstationary random fields I began investigating their use in other areas of statistics. Marc Coram (Stanford) and I started using these transformations for nonparametric and semi-parametric density estimation problems. We began by seeing whether estimating an invertible transformation to a target density could be realistically done and whether these methods could solve some curse of dimensionality issues in high-dimensional density estimation.

One of the major developments of the past year has been the proof of a spline-type characterization of an infinite dimensional optimization problem for estimating a transformation or deformation of $\mathbb{R}^d$ which pushes forward an unknown sampling distribution to some known target probability measure. The main challenge of working with deformations is their non-linear nature: adding two deformations does not necessarily result in another deformation. This makes it difficult to construct flexible classes of deformations, find estimates and quantify statistical uncertainty. In the report [7] we circumvent these challenges by adapting the powerful tools developed by Grenander, Miller, Younes, Trouve and co-authors in the image processing and computational anatomy literature to generate estimates of the deformation with all the required properties: nonparametric flexibility, smoothness, invertability and computational tractability. We establish the existence of a penalized maximum likelihood estimate of a deformation which has a finite dimensional characterization similar to those results found in the spline literature. This finite dimensional characterization is a key component of the numerical computation of these estimates which are nominally defined as an infinite dimensional minimizer of a penalized likelihood. The spline-type representation establishes that the initial velocity field, in a geodesic dynamic flow representation of the deformation model, must be a member of a known finite dimensional subset of a reproducing kernel Hilbert vector space. Our results are derived utilizing an Euler-Lagrange characterization of the PMLE which also establishes a surprising connection to a generalization of Stein's lemma for characterizing the normal distribution.

One of the applications of a warping characterization of a probability density function is the ability to generate nonstationary covariance tapers. The paper [8] explores this application for the problem of spatial kriging. This work has been done in collaboration with a number of researchers at a variety of institutions: Raphael Huser (Ph. D. student, Switzerland); Douglas Nychka (National Center for Atmospheric Research); Marc Coram (Department of Health Research and Policy, Stanford). In [8], we show how to generate nonstationary covariance tapers such that the taper neighborhoods can depend on observation density: larger neighborhoods for sparsely observed areas; smaller neighborhood for densely observed areas. This ensures that tapering neighborhoods do not have too many points to cause computational problems but simultaneously have enough local points for accurate prediction.

Local likelihood estimation for nonstationary random fields. In the paper [12], Michael Stein and I develop a weighted local likelihood estimate for the parameters that govern the local spatial dependency of a locally stationary random field. The advantage of this local likelihood estimate is that it smoothly downweights the influence of faraway observations, works for irregular sampling locations, and when designed appropriately, can trade bias and variance for reducing estimation error.

The motivation for this paper arose from the problem of estimating the local distortion of a warped isotropic random field [11]. In that paper we simply divided the observation locations into local neighborhoods and fitted a linear distortion of the isotropic random field on each neighborhood. Typically two problems arise with this approach. First, the range of validity of a stationary approximation can be too small to contain enough local data to estimate it. Second, it can produce non-smooth local parameter estimates, which is undesirable in many cases. There do exist alternative weighted local likelihood techniques, but unfortunately they are not applicable to random fields. These alternative techniques utilize the independence structure of the data to decompose the log-likelihood as a sum, the summands of which are downweighted as a function of some spatial covariate. In the random field case, however, there is no independence and no such decomposition of the log-likelihood. In [12] we present an exposition, through computation, simulation and some theory, of our version of local likelihood estimation. The local likelihood is generated by re-weighting terms in a telescoping sum of the incremental changes in a stationary likelihood when adding observations by their distance to a local neighborhood midpoint. A large portion of the paper is devoted to the discussion of different ways of constructing and estimating the weights used in our local likelihood that downweight the influence of distant observations of the random field.

Cloud height estimation from satellite imagery. Clouds play a major role in determining the Earth's energy budget. As a result, monitoring and characterizing the distribution of clouds becomes important in global studies of climate. In collaboration with Bin Yu (UC Berkeley) and the MISR team at the Jet Propulsion Laboratory we developed a new stereo matching algorithm for cloud height estimation using multi-angle cameras provided by the MISR instrument on the Terra satellite. By viewing the multi-angle cloud images as discrete sub-samples of a continuous random random field, one can view cloud-top height estimation as a statistical parameter estimation problem. Under this paradigm new tools become available for recovering the height from the MISR images and, in some cases, improve sensitivity and allow fine tuning for different cloud ensembles. Our work on the new height estimator resulted in a sub-contract award for two months of research funded by the Jet Propulsion Laboratory. The main focus of this project was to use the special nature of our new estimator to recover the heights of a two layer cloud ensemble: an optically thin high cloud layer and a bottom, optically thick and textured cloud. Our results are reported in the the paper [14].

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