Lecture 2

Announcements:

- My office hrs:
  Thurs 9am – 11am ~~MPS~~ MSB Room 4214
- Help sessions:
  Thurs 2pm – 4pm EPS Room 1317

Last lecture:

- Summation notation & preview of Box Models.

Today:

- Quick review of stat 13 probability rules.
- Using the probability mass function of a r.v. X for computing/updating probabilities associated with X.

Review Probability rules

- A probability model describes some game of chance ... like the toss of a 6-sided die or a random card picked from a deck.
- If you have a probability model you can compute the chance of any bet.
  - Bets are just sets of outcomes
    - If A is a bet then $P(A)$ denotes the chance or probability someone wins if betting on A for a single play of the game.

Example:

- Your a Bookie running a gambling parlor in your dorm.
- You allow students to bet on a single roll of a 6-sided die.
- Possible outcomes of the game
  $\{1, 2, 3, 4, 5, 6\}$
  - Let X denote the outcome of a single roll.
    This is a random variable.
- Here are some events that students can bet on:

$A =$ X is 2 $= \{2\}$

$B =$ X is even $= \{2, 4, 6\}$

$C =$ X is odd $= \{1, 3, 5\}$

$D = (X \leq 2) = \{1, 2\}$

$E =$ A or C happens $= \{2, 1, 3, 5\}$

$F =$ B and D happens simultaniously $= \{2\} = A$

$\vdots$

- As a student you want to compute $P(A), P(B), P(C), \ldots$ to decide what to bet on.
- Probability rules help you compute probabilities of complicated events/bets.

Review of Stat 13 probabilities

Rule 1: If all possible outcomes have the same chance of occuring then

$$P(A) = \frac{\#\text{ of outcomes in } A}{\text{total } \#\text{ of outcomes}}$$

Rule 2: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Rule 3: $$P(\underbrace{A|B}_{A \text{ given } B}) = \frac{P(A \text{ and } B)}{P(B)}$$

Rule 4: $P(A \text{ and } B) = P(A|B)P(B)$

Rule 5: $P(A) = 1 - P(A^c)$

The event A did not happen.

Example: Consider the events in the previous example.

$P(A) = P(X \text{ is } 2) = P(\{2\})$

all outcomes of X that correspond to a win for bet A

$$\overset{\text{Rule 1}}{=} \frac{\#\text{ of outcomes in } A}{\text{total } \#\text{ of possible outcomes}}$$

$$= \frac{1}{6}$$

$P(C) = P(X \text{ is odd}) = P(\{1,3,5\})$

$$\overset{\text{Rule 1}}{=} \frac{3}{6}$$

$P(E) = P(A \text{ or } C)$

$$\overset{\text{Rule 2}}{=} P(A) + P(C) - P(\underbrace{\overset{\{2\}}{A} \text{ and } \overset{\{1,3,5\}}{C}}_{\text{This can't happen}})$$

$$= \frac{1}{6} + \frac{3}{6} - 0$$

$$= \frac{4}{6}$$

$P(\underset{\{2\}}{A} \,/\, \underset{\{2,4,6\}}{B})$ = given that $X \in B$ what is the chance $X \in A$ also

$$\overset{\text{Rule 3}}{=} \frac{P(A \text{ and } B)}{P(B)}$$

$$= \frac{P(\{2\} \text{ and } \{2,4,6\})}{P(\{2,4,6\})}$$

$$= \frac{P(\{2\})}{P(\{2,4,6\})}$$

$$\overset{\text{Rule 1}}{=} \frac{(1/6)}{(3/6)} = \frac{1}{3}$$

Note: $\frac{1}{3}$ is different from $P(A) = 1/6$

In a way probabilities simply describe proportions of attributes accross a population.

i.e. if A, B, C are attributes for members of a population then

$P(A) \equiv$ proportion of the population with attribute A

$P(A^c) \equiv$ proportion of the population who do not have attribute A.

$P(A \cap B) \equiv$ proportion who have both A and B.

$P(A|B) \equiv$ among those with attribute B what is the proportion who also have attribute A.

Random variables

- A random variable (r.v.) X is just a number, the value of which is determined by a game of chance.

- r.v.s can be very complicated.
- We want a tool for working with them

Question:

- How can you tell when two r.v.s X, Y are the same in terms of their randomness?
- How can I visualize the randomness in a r.v. X?
- Is there such thing as a blueprint for a r.v. X I can store on my computer?

Answer:

- Every discrete r.v. X has associated with it a function $p_X(x)$ called the probability mass function or PMF for short
- The PMF completely characterizes the r.v. X and provides a blueprint for X that can be used to make copies and visualization.

Here is the definition of $p_X(x)$:

$$p_X(x) = P(X = x)$$

output this probability

input a fix number x

= the probability that you win if you bet X will be exactly x.

e.g. Roll a 6-sided die. Let X be the number showing.

Find the PMF $p_X(x)$.

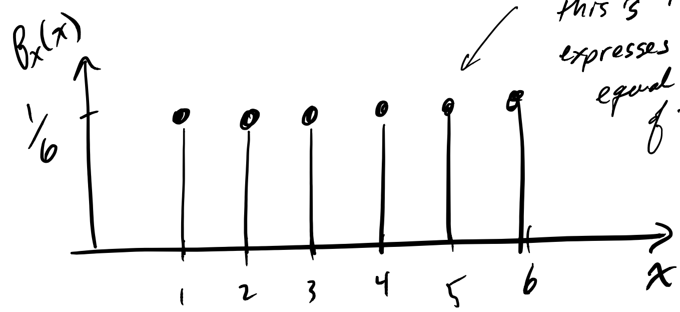
Convenient table format

x	$p_X(x)$	
1	1/6	$\leftarrow p_X(1) = P(X=1)$
2	1/6	$\leftarrow p_X(2) = P(X=2)$
3	1/6	⋮
4	1/6	
5	1/6	
6	1/6	
⋮	0	
x	0	
⋮	⋮	

possible values of X

any other number other than 1, 2, .., 6

Visualize $p_X(x)$



The fact that this is "flat" expresses the equal chance of the dice.

To find the probability of some event involving X just add $P_X(x)$ over all x in the event.

e.g. in the die example suppose you want to find $P(\underbrace{X \leq 2})$

This is the event in question an it corresponds to the X values $\{1, 2\}$.

$$\therefore P(X \leq 2) = \text{sum of } P_X(x) \text{ over } x \text{ in } \{1,2\}$$
$$= P_X(1) + P_X(2)$$
$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}.$$

e.g. Flip a coin till you see tails. Let Y denote the number of flips it takes. Find the PMF $P_Y(y)$ & use it to Find $P(Y \leq 4)$.

Put possible Y values here

y	$P_Y(y)$	
1	$\frac{1}{2}$	← $P(Y=1) = P(\text{tail on first flip})$
2	$\frac{1}{4}$	← $P(Y=2) = P(\text{heads on first but tails on second})$
3	$\frac{1}{8}$	
4	$\vdots$	
5		
6		
$\vdots$		
n	$(\frac{1}{2})^n$	
$\vdots$		

goes on indefinitely

Formula for $P_Y(y)$ in "function notation":

$P_Y(y) = (\frac{1}{2})^y$ when y is a positive int.

$P_Y(y) = 0$ when y is not a pos. int.

another way to write the above:

$$P_Y(y) = \begin{cases} (\frac{1}{2})^y & \text{when } y = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

Now

$$P(Y \leq 4) = \text{sum } P_Y(y) \text{ over all } y \text{ in } \{1,2,3,4\}$$
$$= (\tfrac{1}{2})^1 + (\tfrac{1}{2})^2 + (\tfrac{1}{2})^3 + (\tfrac{1}{2})^4$$
$$= 0.9375.$$

Note: Two R.V.s X & Y have the same randomness if their PMFs are the same.

e.g. all roullette tables have some PMF which describes the randomness in the random numbers they produce. If you are selling roullette tables the PMFs are the only way to guarantee they are operating correctly.