Lecture 4:

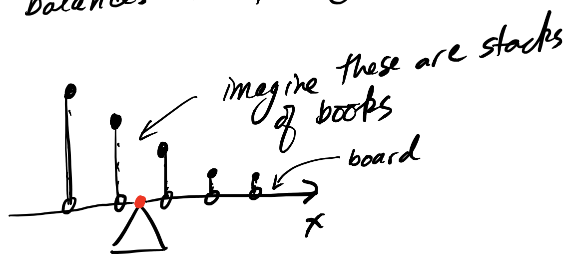
I) Facts about $E(X)$

II) var & std.

III) $E(f(X))$.

Facts about $E(X)$

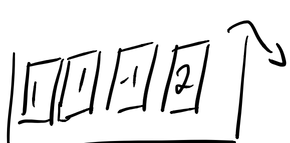
Fact 1: If $P_X(x)$ is the PMF for X then $E(X)$ is the point on the x-axis which "balances" the plot of $P_X(x)$.



$\bullet = E(X) =$ board fulcrum

This fact is useful for visualizing where $E(X)$ is.

Fact 2: When you have a Box Model for a R.V. X, then $E(X)$ can be computed by simply averaging the ticket numbers in the Box.



Pick one at random
Let X be the number showing.

4 tickets

Find $E(X)$ with PMF

x	$p(x)$	$x\,p(x)$
-1	1/4	-1/4
1	1/2	1/2
2	1/4	1/2
		$3/4 = E(X)$

Find $E(X)$ using Fact 2

$$E(X) = \frac{1+1-1+2}{4} = 3/4$$

(easier)

Fact 3: If the R.V. X can only be 0 or 1 then $E(X)$ is just the probability X is 1.

x	$P_X(x)$	$x\,P_X(x)$
0	$1-p$	0
1	p	p
		$p = E(X)$.

Var(X) and sd(X)

* $E(X)$ is the best prediction of X but how good is the best prediction?

* $sd(X)$ = "the standard deviation of X" is a number that tells you how good the prediction $E(X)$ is for X.

* we will also define $var(X)$ = "variance of X" which is useful for computing $sd(X)$.

Def: Let X be a R.V. with PMF $P_X(x)$.

$$sd(X) = \sqrt{var(X)}$$

$$var(X) = E(X^2) - [E(X)]^2.$$

Note: to find $E(X^2)$ you technically need to find the PMF for X^2 (which will usually be different than $P_X(x)$). However there is an easy short cut.

$$E(X^2) = \left(\text{sum } x^2 P_X(x) \text{ over all possible } x \text{ values for } X\right)$$

Note the diffence btwn

$$E(X) = \left(\text{sum } x P_X(x) \text{ over all possible } x \text{ values for } X\right)$$

In fact to compute $E(f(X))$ 3

$f(x)$ is a function

The argument to f is a R.V. X

You can always use the following <u>shortcut</u>.

$$E(f(X)) = \left(\begin{array}{l}\text{sum } f(x)\, p_X(x) \text{ over} \\ \text{all possible } x \text{ values for } X\end{array}\right)$$

i.e.

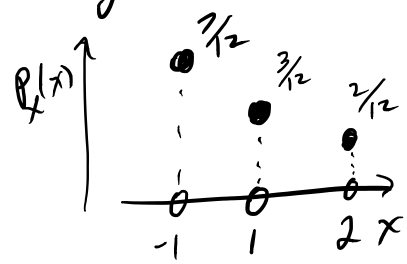
$$E(\cos(X)) = \left(\begin{array}{l}\text{sum } \cos(x)\, p_X(x) \text{ over} \\ \text{all possible } x \text{ values for } X\end{array}\right)$$

$$E(\log(X)) = \left(\begin{array}{l}\text{sum } \log(x)\, p_X(x) \text{ over} \\ \text{all possible } x \text{ values for } X\end{array}\right)$$

$$E\left(e^{-X^2}\right) = \left(\begin{array}{l}\text{sum } e^{-x^2}\, p_X(x) \text{ over} \\ \text{all possible } x \text{ values for } X\end{array}\right)$$

etc.
⋮

e.g. Let X have PMF 4



Find $E(X)$ & $sd(X)$

x	$p_X(x)$	$x\,p_X(x)$	$x^2 p_X(x)$
-1	7/12	-7/12	7/12
1	3/12	3/12	3/12
2	2/12	4/12	8/12
		0	$\frac{18}{12} = \frac{3}{2}$

$\therefore\ E(X) = 0$

$$E(X^2) = \frac{3}{2}$$

$$var(X) = \frac{3}{2} - 0^2 = \frac{3}{2}$$

$$sd(X) = \sqrt{\frac{3}{2}}$$