Lecture 2

Last time we covered summation Notation $\sum_{i=1}^{n} X_i$ & Notation that distinguishes Random variables from fixed numbers.

Review of Probability Rules from STA 13

illustrate with an example...
roll a 6-sided die.
possible outcomes $\{1,2,3,4,5,6\}$
All equally likely.

Let A & B be events, i.e. collections of outcomes (e.g. $A=\{1,2\}$, $B=\{2,4,6\}$)

$P(A)$ denotes the probability of A.

Rule 1: If all outcomes are equally likely, then

$$P(A) = \frac{\#\text{ of outcomes in } A}{\text{total }\#\text{ of outcomes}}$$

In dice example $P(A) = \frac{2}{6}$

Rule 2: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

In dice example

$$\underset{\{1,2,4,6\}}{P(A \text{ or } B)} = \frac{2}{6} + \frac{3}{6} - \underset{\{2\}}{P(A \text{ and } B)}$$

$$= \frac{2}{6} + \frac{3}{6} - \frac{1}{6}$$

$$= \frac{4}{6}$$

Rule 3: $\underset{A \text{ given } B}{P(A|B)} = \frac{P(A \text{ and } B)}{P(B)}$

In dice example

$$P(A|B) = \frac{P(\{2\})}{1/2} = \frac{1/6}{1/2} = \frac{1}{3}$$

You know the roll is even but Not which number

Rule 4: $P(A \text{ and } B) = P(A|B)P(B)$

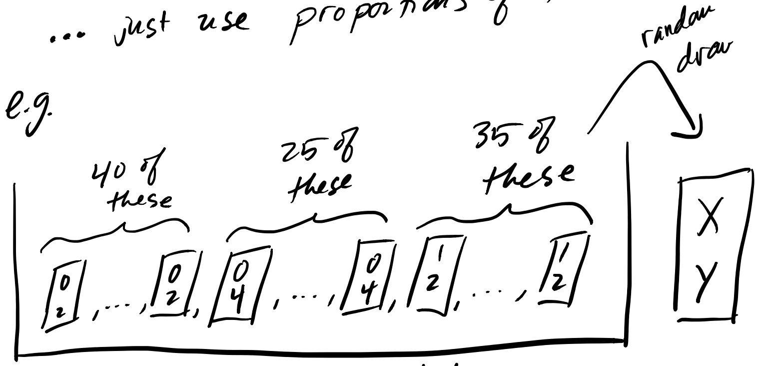
Rule 5: $P(A) = 1 - P(A^c)$

The event A did not happen.

You don't need these Rules if your working with a box Model

... just use proportions of tickets.

e.g.



Box with 100 tickets

$P(X=1)$ = "prob that the random draw has a top number of 1"

$$= \frac{35}{100}$$

$$P(X=0 \text{ and } Y=4) = \frac{25}{100}$$

$$P(X=1 | Y=2) = \frac{35}{40+35}$$

# of tickets which have $X=1$ & $Y=2$

total # of tickets with a Y of 2.

Random variables

Think of a R.V. X as just a name for a number that depends on the outcome of some game of chance or random procedure.

e.g. Let X be the number showing when I roll a 6-sided die.
X could be 1, 2, 3, 4, 5 or 6.

Question:

* How does one characterize or visualize a R.V. X?
* If I have two R.V.s X & Y how can I tell if they are the same in terms of Randomness.
* Is there such thing as a blueprint for a R.V. that I can store on my computer.

Answer: For every R.V. X there exists a function denoted $p_X(x)$ and called the **probability mass function for X** (when X is discrete). PMF for short

This function completely characterizes X, provides a blueprint for making copies and allows visualization of X.

Here is the definition of the PMF $p_X(x)$ for a R.V. X:

$$\underset{\text{The input is some fixed number } x}{p_X(x)} = \underbrace{\text{the probability that } X \text{ is } x}_{\text{The output of the function is this}}$$

e.g. Roll a 6-sided die. Let X be the number showing.
Find the PMF $p_X(x)$.

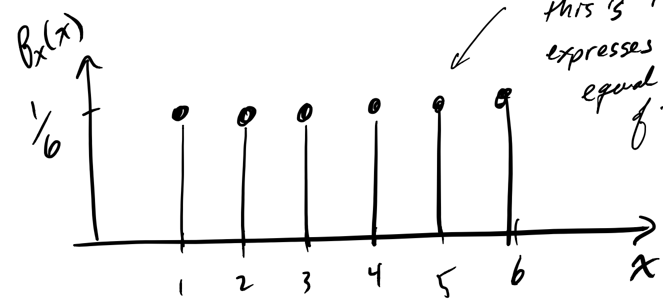
Convenient table format

x	$p_X(x)$	
1	$1/6$	$\leftarrow p_X(1) = P(X=1)$
2	$1/6$	$\leftarrow p_X(2) = P(X=2)$
3	$1/6$	$\vdots$
4	$1/6$	
5	$1/6$	
6	$1/6$	
$\vdots$	0	
x	0	
$\vdots$	$\vdots$	

possible values of X

any other number other than 1, 2, ..., 6

Visualize $p_X(x)$



The fact that this is "Flat" expresses the equal chance of the dice.

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To find the probability of some event involving X just add $P_X(x)$ over all x in the event.

e.g. in the die example suppose you want to find $P(X \le 2)$

This is the event in question an it corresponds to the X values $\{1, 2\}$.

$\therefore P(X \le 2) =$ sum of $P_X(x)$ over x in $\{1,2\}$

$= P_X(1) + P_X(2)$

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}.$

e.g. Flip a coin till you see tails. Let Y denote the number of flips it takes. Find the PMF $P_Y(y)$ & use it to Find $P(Y \le 4)$.

Put Possible Y values here

y	$P_Y(y)$	
1	$\frac{1}{2}$	← $P(Y=1) = P($tail on first flip$)$
2	$\frac{1}{4}$	← $P(Y=2) = P($heads on first but tails on second$)$
3	$\frac{1}{8}$	
4	$\vdots$	
5		
6		
$\vdots$		
n	$(\frac{1}{2})^n$	
$\vdots$		

goes on indefinitely

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Formula for $P_Y(y)$ in "function notation":

$P_Y(y) = (\frac{1}{2})^y$ when y is a positive int.

$P_Y(y) = 0$ when y is not a pos. int.

another way to write the above:

$$P_Y(y) = \begin{cases} (\frac{1}{2})^y & \text{when } y = 1, 2, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

Now

$P(Y \le 4) =$ sum $P_Y(y)$ over all y in $\{1,2,3,4\}$

$= (\frac{1}{2})^1 + (\frac{1}{2})^2 + (\frac{1}{2})^3 + (\frac{1}{2})^4$

$= 0.9375.$

Note: Two R.V.s X & Y have the same randomness if their PMFs are the same.

e.g. all roullette tables have some PMF which describes the randomness in the random numbers they produce. If you are selling roullette tables the PMFs are the only way to guarantee they are operating correctly.