Lecture 3:

1

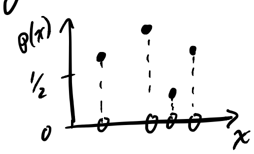
PMF $P_X(x)$ are special.
i.e. Not any function can be a PMF.

PMFs must satisfy

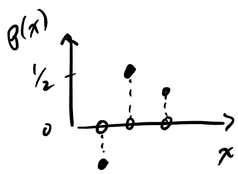
i) sum $P(x)$ over all $x = 1$

ii) $P(x) \geq 0$ for all x.

e.g.



Not a PMF
violates i)



Not a PMF.
violates ii)

x	$p(x)$
π	$\sin^2(22)$
-1000	$\cos^2(22)$

any other x has $p(x)=0$

Yes this $p(x)$ is a PMF of some R.V. X ... $p(x)$ satisfies i) & ii)

Expected value of a R.V.

e.g.
Suppose I told you I rolled a 6-sided die 100 times: $X_1, X_2, \ldots, X_{100}$

number showing on second row.

and got

$$\frac{X_1 + \cdots + X_{100}}{100} = 3.45$$

Then rolled 9900 times more and got

$$\frac{X_1 + \cdots + X_{10,000}}{10,000} = 3.50001$$

Is this what you would expect?
what property of a single Roll implys this?

2

Answer: the fact that the expected value of a single Roll X (denoted $E(X)$ & defined below) is 3.5 implies the long run average of multiple rolls will converge to 3.5

e.g.
create a list of everyones age in the class

$$x_1, x_2, x_3, \ldots, x_{265}$$

x_1: 20 (You), x_2: 40 (me) ...

suppose the average $\bar{x} = 21.45$.
Now randomly choose a person in class. Let X be their age. X is a R.V. but the list is fixed & nonrandom.
what is your best guess of what X will be?
Answer: The best prediction of X is 21.45 (i.e. the average of the list from which X was drawn)
In this case 21.45 is $E(X)$ = "the expected value."

The expected value of a R.V. X, denoted $E(X)$, is just a single number which describes the best prediction of what X will be.

To compute $E(X)$ find the PMF $p_X(x)$ and set

$$E(X) = \left(\text{sum } x\, p_X(x) \text{ over all possible } x \text{ for } X\right)$$

$$= \sum_x x\, p_X(x) \leftarrow \text{short hand.}$$

e.g. roll a 6-sided die let X be the number showing.

$$p_X(x) = \begin{cases} 1/6 & \text{if } x = 1, 2, \dots, 6 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore\ E(X) = 1p_X(1) + 2p_X(2) + 3p_X(3) + \cdots + 6p_X(6)$$

$$= 1\left(\tfrac{1}{6}\right) + 2\left(\tfrac{1}{6}\right) + 3\left(\tfrac{1}{6}\right) + \cdots + 6\left(\tfrac{1}{6}\right)$$

$$= 3.5$$

e.g. Let X be a random draw from the following list:

20, 40, 21, 19, 20, 20.

Find $E(X)$.

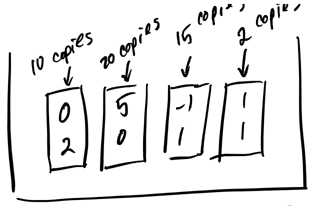
x	$p(x)$	$x\,p(x)$
19	1/6	19 (1/6)
20	1/2	20 (1/2)
21	1/6	21 (1/6)
40	1/6	40 (1/6)
		$23.\overline{3}$ ← This is $E(X)$

same

Note: the list ave is

$$\frac{20+40+21+19+20+20}{6} = 23.\overline{3}$$

e.g.



$10+20+15+2 = 47$ tickets total

Pick a ticket at Random. Let X be the top # & Y be the bottom #.

Define the R.V. $W = X\cdot Y + Y^2$.

Find $E(W)$.

Need the PMF $p_W(w)$.

First find possible w values.

$$0\cdot 2 + 2^2 = 4$$
$$5\cdot 0 + 0^2 = 0$$
$$(-1)\cdot 1 + 1 = 0$$
$$1\cdot 1 + 1 = 2$$

Now PMF

Possible values time the PMF

w	$p_W(w)$	$w\,p_W(w)$
0	$\frac{20+15}{47}$	0
2	$2/47$	$\frac{4}{47}$
4	$\frac{10}{47}$	$\frac{40}{47}$
	1	$\frac{44}{47}$

$$\therefore\ E(W) = \frac{44}{47}.$$