Lecture 8

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Outline:

- computing $E(f(X,Y))$ from joint PMFs.
- covariance btwn X & Y.

Recall: For any function $f(x)$

$$E\,f(X) = \left(\begin{array}{l}\text{sum } f(x)\,p_X(x) \text{ over} \\ \text{all possible } x \text{ values for } X\end{array}\right)$$

e.g. if X has PMF

x	$p_X(x)$
$\pi/6$	0.2
$\pi/8$	0.4
$\pi/9$	0.4

then

$$E(\cos(X))$$

$$= \cos\left(\tfrac{\pi}{6}\right)0.2 + \cos\left(\tfrac{\pi}{8}\right)0.4 + \cos\left(\tfrac{\pi}{9}\right)0.4$$

For a pair of R.Vs (X,Y) & a function of two arguments $f(x,y)$

$$E(f(X,Y)) = \left(\begin{array}{l}\text{sum } f(x,y)\,p_{X,Y}(x,y) \\ \text{over all possible} \\ \text{pairs } (x,y) \text{ of values} \\ \text{for } (X,Y)\end{array}\right)$$

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e.g. Let (X,Y) have joint PMF

$x \backslash y$	1	2	3	4	5	6
1						1/8
2					1/8	
3				1/8		
4			1/8			
5		1/8				1/8
6	1/8				1/8	

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Find $E(X+Y)$, $E(XY)$, $\text{var}(XY)$.

$E(X+Y) = E(f(X,Y))$, where $f(x,y) = x+y$

$$= \text{sum } f(x,y)\,p(x,y) \text{ over } x,y$$

$$= \text{sum } (x+y)\,p(x,y) \text{ over } x,y.$$

$$= (6+1)\tfrac{1}{8} + (5+2)\tfrac{1}{8} + \cdots + (1+6)\tfrac{1}{8} + (5+6)\tfrac{1}{8} + (6+5)\tfrac{1}{8}$$

$$= 6\cdot 7\cdot\tfrac{1}{8} + 2\cdot 11\cdot\tfrac{1}{8}$$

$$= 8$$

$E(XY) = E(f(X,Y))$, $f(x,y) = xy$

$$= 6\cdot 1\cdot\tfrac{1}{8} + 5\cdot 2\cdot\tfrac{1}{8} + \cdots + 1\cdot 6\cdot\tfrac{1}{8} + 5\cdot 6\tfrac{1}{8} + 6\cdot 5\tfrac{1}{8}$$

$$= 14.5$$

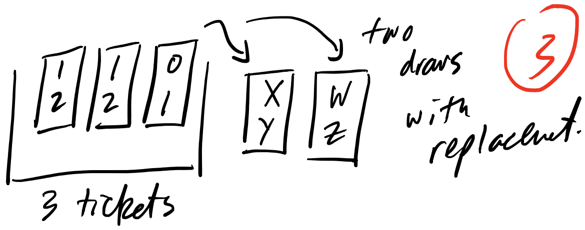
$$\text{var}(XY) = E((XY)^2) - (E(XY))^2$$

$$= E(X^2Y^2) - (14.5)^2$$

$$= 6^2\cdot 1^2\cdot\tfrac{1}{8} + 5^2\cdot 2^2\cdot\tfrac{1}{8} + \cdots + 1^2\cdot 6^2\tfrac{1}{8} + 5^2\cdot 6^2\tfrac{1}{8} + 6^2\cdot 5^2\tfrac{1}{8} - 14.5^2$$

$$= 295 - 14.5^2 = 84.75$$

e.g.



Find $E(XY)$ & $E(XZ)$

Here is the joint PMF $P_{X,Y}(x,y)$

$y \backslash x$	1	0	$P_Y(y)$
1	0	$\frac{1}{3}$	$\frac{1}{3}$
2	$\frac{2}{3}$	0	$\frac{2}{3}$
$P_X(x)$	$\frac{2}{3}$	$\frac{1}{3}$	1

$$\therefore E(XY) = 1 \cdot 2 \cdot \frac{2}{3} + 0 \cdot 1 \cdot \frac{1}{3} = 1$$

For $E(XZ)$ we need $P_{X,Z}(x,z)$.

Since X & Z are indep

$$P_{X,Z}(x,z) = P_X(x)\,P_Z(z)$$
$$= P_X(x)\,\underbrace{P_Y(z)}$$

since Z & Y are identical in terms of randomness.

$$\therefore E(XZ) = 1 \cdot 1 \cdot P_X(1) \cdot P_Y(1)$$
$$+ 1 \cdot 2 \cdot P_X(1) \cdot P_Y(2)$$
$$+ 0 \cdot 1 \cdot P_X(0) \cdot P_Y(1)$$
$$+ 0 \cdot 2 \cdot P_X(0) \cdot P_Y(2)$$
$$= 1 \cdot 1 \cdot \frac{2}{3} \cdot \frac{1}{3} + 1 \cdot 2 \cdot \frac{2}{3} \cdot \frac{2}{3}$$
$$= \frac{2+8}{9} = \frac{10}{9}$$