STA290

Winter 2014

Selected presentation materials

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Chapter 1

Orthogonal Projections for Fixed Vectors

For any two fixed vectors z_1, z_2 that are of equal dimension,

$$\langle z_1, z_2 \rangle = z_1 \cdot z_2 = z_1^T z_2$$

Suppose M is an m-dimensional linear space that is a subset of $\mathbb{R}^d$. Consequently, define $M^\perp$ as the orthogonal compliment of M that has dimension $d - m$ which can be defined with ϕ_k as mutually orthogonal vectors for $k = 1, ..., d$ such that

$$M = span\{\phi_1, ..., \phi_m\}$$
$$M^\perp = span\{\phi_{m+1}, ..., \phi_d\}$$

Any vector $z \in \mathbb{R}^d$ can be uniquely represented as $x_1 + x_2 = z$ where $x_1 \in M$ and $x_2 \in M^\perp$, which is to say

$$M \oplus M^\perp = \mathbb{R}^d$$

For any $y \in \mathbb{R}^d$ notice that

$$y = \sum_{k=1}^{d} \langle y, \phi_k \rangle \phi_k$$
$$||y||^2 = \langle y, y \rangle = \sum_{k=1}^{d} \langle y, \phi_k \rangle^2$$

and the projection of y onto M is defined as $P_M y = \text{argmin}_{w \in M} ||w - y||^2$, which can be expressed as

$$\boxed{P_M y = \sum_{k=1}^{m} \langle y, \phi_k \rangle \phi_k} \tag{1.1}$$

The following relation holds for the space M and its orthogonal compliment $M^{\perp}$

$$\boxed{P_M y \perp P_{M^{\perp}} y \qquad i.e. \qquad P_M y \perp (y - P_M y)}$$

Chapter 2

Projections for Gaussian Random Vectors

Assume a Gaussian setting where we consider Y a dimension $d \times 1$ vector of values and U as an orthogonal "rotation" matrix of dimension $d \times d$, where $I = U^T U$.

$$Y \sim N\left(0, \sigma^2 I_d\right) \quad \Rightarrow \quad UY \sim N\left(0, \sigma^2 I_d\right)$$

Several consequential results are expressed from the properties of fixed vectors with the orthogonal $\phi_1, ..., \phi_d$ representation where $M = span\{\phi_1, ..., \phi_m\}$.

$$\boxed{\begin{pmatrix} \langle Y, \phi_1 \rangle \\ \vdots \\ \langle Y, \phi_d \rangle \end{pmatrix} \sim N(0, \sigma^2 I_d)} \tag{2.1}$$

$$\boxed{||P_M Y||^2 = \sum_{k=1}^{m} \langle Y, \phi_k \rangle^2 \quad \sim \quad \sigma^2 \chi^2_m} \tag{2.2}$$

$$\boxed{P_M Y \text{ is independent of } (Y - P_M Y)} \tag{2.3}$$