biegeleisen_project

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print(AustraliaG)

0.0.2 MATG 511 - Computational Methods for Analytics 0.0.3 Final Project 0.0.4 Preparing the data set for DMD [1]: import pandas as pd import numpy as np import matplotlib.pyplot as plt data = pd.read_csv('greenhouse_gas_inventory_data_data.csv') # Uses pandas to read the dataset in a dataframe [2]: AustraliaC = data.iloc[0:25,2] # The carbon dioxide values for Australia are on the third column and the first \Box →through 25th rows AustraliaC = np.array(AustraliaC) # Turns the carbon dioxide values for Australia into a numpy array AustraliaC = np.flip(AustraliaC) # Flips the values of the numpy array from reverse-chronological order \leftrightarrow (2014-1990) to chronological order (1990-2014) # Repeat the above process for the greenhouse gas values AustraliaG = data.iloc[1074:1099,2] # The greenhouse gas for Australia are on the third column and the 1075th →through 1100th rows AustraliaG = np.array(AustraliaG) # Turns the greenhouse gas values for Australia into a numpy array AustraliaG = np.flip(AustraliaG) # Flips the values of the numpy array from reverse-chronological order \rightarrow (2014-1990) to chronological order (1990-2014) print(AustraliaC)

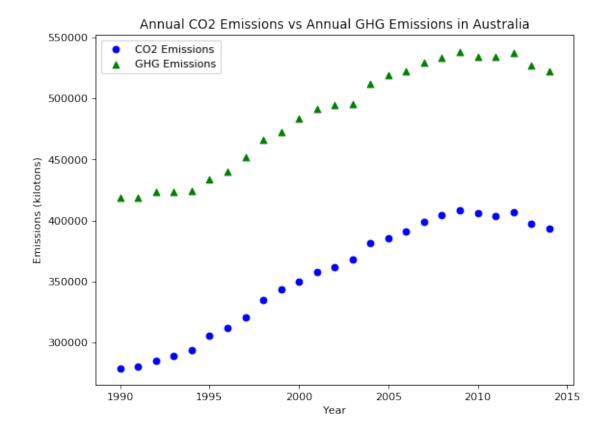
```
[278265.89894077 279741.63901186 284766.09271784 289142.26768133 293830.70914119 305162.54354873 311914.81982423 320439.11681939 334328.1426466 343713.90694777 349885.43310893 357653.3298993 361861.38789603 368345.97742511 381519.26159278 385581.13280647 391134.10090945 398816.45354355 404237.82821408 408448.47899963 406200.99318434 403705.52831399 406462.8477036 396913.93653029 393126.94699429]
[418623.04956265 418674.05749386 423080.10819125 423764.93256425 424092.95937256 433478.65082585 439803.96222825 451721.71781467 466382.11247181 472245.9907776 483445.82666482 491441.69593714 494740.10077673 495154.0859668 511710.24202082 518850.75946238 522517.19166329 529842.87034248 533693.50820738 537889.89262362 533917.43629539 534089.79868471 537377.5714528 526882.66726659 522397.09071135]
```

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[3]: # Plotting Australia's carbon dioxide value array and greenhouse gas value array

x = np.linspace(1990, 2014, num = 25) # Creates a numpy array containing all of
the values of x for the plot

plt.figure(figsize=(8, 6), dpi=80)
plt.scatter(x, AustraliaC, marker = 'o', color = 'blue', label = 'CO2 Emissions')
plt.scatter(x, AustraliaG, marker = 'a', color = 'green', label = 'GHG
Emissions')

plt.title('Annual CO2 Emissions vs Annual GHG Emissions in Australia')
plt.xlabel('Year')
plt.ylabel('Emissions (kilotons)')
plt.legend()
plt.show()
```



Repeat the above process for every country or area in the dataset:

```
[4]: AustriaC = data.iloc[25:50,2]
    AustriaC = np.array(AustriaC)
    AustriaC = np.flip(AustriaC)
    AustriaG = data.iloc[1099:1124,2]
    AustriaG = np.array(AustriaG)
    AustriaG = np.flip(AustriaG)
[5]: BelarusC = data.iloc[50:75,2]
    BelarusC = np.array(BelarusC)
    BelarusC = np.flip(BelarusC)
    BelarusG = data.iloc[1124:1149,2]
    BelarusG = np.array(BelarusG)
    BelarusG = np.flip(BelarusG)
[6]: BelgiumC = data.iloc[75:100,2]
    BelgiumC = np.array(BelgiumC)
    BelgiumC = np.flip(BelgiumC)
    BelgiumG = data.iloc[1149:1174,2]
```

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BelgiumG = np.array(BelgiumG)
     BelgiumG = np.flip(BelgiumG)
 [7]: BulgariaC = data.iloc[100:125,2]
     BulgariaC = np.array(BulgariaC)
     BulgariaC = np.flip(BulgariaC)
     # Bulgaria does not have greenhouse gas values in the data set so it cannot be_{f U}
      \rightarrow included
 [8]: CanadaC = data.iloc[125:150,2]
     CanadaC = np.array(CanadaC)
     CanadaC = np.flip(CanadaC)
     CanadaG = data.iloc[1174:1199,2]
     CanadaG = np.array(CanadaG)
     CanadaG = np.flip(CanadaG)
 [9]: CroatiaC = data.iloc[150:175,2]
     CroatiaC = np.array(CroatiaC)
     CroatiaC = np.flip(CroatiaC)
     CroatiaG = data.iloc[1199:1224,2]
     CroatiaG = np.array(CroatiaG)
     CroatiaG = np.flip(CroatiaG)
[10]: CyprusC = data.iloc[175:200,2]
     CyprusC = np.array(CyprusC)
     CyprusC = np.flip(CyprusC)
     CyprusG = data.iloc[1224:1249,2]
     CyprusG = np.array(CyprusG)
     CyprusG = np.flip(CyprusG)
[11]: Czech_RepublicC = data.iloc[200:225,2]
     Czech_RepublicC = np.array(Czech_RepublicC)
     Czech_RepublicC = np.flip(Czech_RepublicC)
     Czech_RepublicG = data.iloc[1249:1274,2]
     Czech_RepublicG = np.array(Czech_RepublicG)
     Czech_RepublicG = np.flip(Czech_RepublicG)
[12]: DenmarkC = data.iloc[225:250,2]
     DenmarkC = np.array(DenmarkC)
     DenmarkC = np.flip(DenmarkC)
     DenmarkG = data.iloc[1274:1299,2]
     DenmarkG = np.array(DenmarkG)
     DenmarkG = np.flip(DenmarkG)
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[13]: EstoniaC = data.iloc[250:275,2]
     EstoniaC = np.array(EstoniaC)
     EstoniaC = np.flip(EstoniaC)
     EstoniaG = data.iloc[1299:1324,2]
     EstoniaG = np.array(EstoniaG)
     EstoniaG = np.flip(EstoniaG)
[14]: European_UnionC = data.iloc[275:300,2]
     European_UnionC = np.array(European_UnionC)
     European_UnionC = np.flip(European_UnionC)
     European_UnionG = data.iloc[1324:1349,2]
     European_UnionG = np.array(European_UnionG)
     European_UnionG = np.flip(European_UnionG)
[15]: FinlandC = data.iloc[300:325,2]
     FinlandC = np.array(FinlandC)
     FinlandC = np.flip(FinlandC)
     FinlandG = data.iloc[1349:1374,2]
     FinlandG = np.array(FinlandG)
     FinlandG = np.flip(FinlandG)
[16]: FranceC = data.iloc[325:350,2]
     FranceC = np.array(FranceC)
     FranceC = np.flip(FranceC)
     FranceG = data.iloc[1374:1399,2]
     FranceG = np.array(FranceG)
     FranceG = np.flip(FranceG)
[17]: GermanyC = data.iloc[350:375,2]
     GermanyC = np.array(GermanyC)
     GermanyC = np.flip(GermanyC)
     GermanyG = data.iloc[1399:1424,2]
     GermanyG = np.array(GermanyG)
     GermanyG = np.flip(GermanyG)
[18]: GreeceC = data.iloc[375:400,2]
     GreeceC = np.array(GreeceC)
     GreeceC = np.flip(GreeceC)
     GreeceG = data.iloc[1424:1449,2]
     GreeceG = np.array(GreeceG)
     GreeceG = np.flip(GreeceG)
[19]: HungaryC = data.iloc[400:425,2]
     HungaryC = np.array(HungaryC)
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HungaryC = np.flip(HungaryC)
     HungaryG = data.iloc[1449:1474,2]
     HungaryG = np.array(HungaryG)
     HungaryG = np.flip(HungaryG)
[20]: IcelandC = data.iloc[425:450,2]
     IcelandC = np.array(IcelandC)
     IcelandC = np.flip(IcelandC)
     IcelandG = data.iloc[1474:1499,2]
     IcelandG = np.array(IcelandG)
     IcelandG = np.flip(IcelandG)
[21]: IrelandC = data.iloc[450:475,2]
     IrelandC = np.array(IrelandC)
     IrelandC = np.flip(IrelandC)
     IrelandG = data.iloc[1499:1524,2]
     IrelandG = np.array(IrelandG)
     IrelandG = np.flip(IrelandG)
[22]: ItalyC = data.iloc[475:500,2]
     ItalyC = np.array(ItalyC)
     ItalyC = np.flip(ItalyC)
     ItalyG = data.iloc[1524:1549,2]
     ItalyG = np.array(ItalyG)
     ItalyG = np.flip(ItalyG)
[23]: JapanC = data.iloc[500:525,2]
     JapanC = np.array(JapanC)
     JapanC = np.flip(JapanC)
     JapanG = data.iloc[1549:1574,2]
     JapanG = np.array(JapanG)
     JapanG = np.flip(JapanG)
[24]: LatviaC = data.iloc[525:550,2]
     LatviaC = np.array(LatviaC)
     LatviaC = np.flip(LatviaC)
     LatviaG = data.iloc[1574:1599,2]
     LatviaG = np.array(LatviaG)
     LatviaG = np.flip(LatviaG)
[25]: LiechtensteinC = data.iloc[550:575,2]
     LiechtensteinC = np.array(LiechtensteinC)
     LiechtensteinC = np.flip(LiechtensteinC)
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# Liechtenstein does not have greenhouse gas values in the data set so it cannot_{\sqcup}
      →be included
[26]: LithuaniaC = data.iloc[575:600,2]
     LithuaniaC = np.array(LithuaniaC)
     LithuaniaC = np.flip(LithuaniaC)
     LithuaniaG = data.iloc[1599:1624,2]
     LithuaniaG = np.array(LithuaniaG)
     LithuaniaG = np.flip(LithuaniaG)
[27]: LuxembourgC = data.iloc[600:625,2]
     LuxembourgC = np.array(LuxembourgC)
     LuxembourgC = np.flip(LuxembourgC)
     # Luxembourg does not have greenhouse gas values in the data set so it cannot be_{f L}
      \rightarrow included
[28]: MaltaC = data.iloc[625:650,2]
     MaltaC = np.array(MaltaC)
     MaltaC = np.flip(MaltaC)
     MaltaG = data.iloc[1624:1649,2]
     MaltaG = np.array(MaltaG)
     MaltaG = np.flip(MaltaG)
[29]: # Monaco is not included because its data set covers 1990-2013 instead of
      →1990-2014
[30]: NetherlandsC = data.iloc[674:699,2]
     NetherlandsC = np.array(NetherlandsC)
     NetherlandsC = np.flip(NetherlandsC)
     NetherlandsG = data.iloc[1673:1698,2]
     NetherlandsG = np.array(NetherlandsG)
     NetherlandsG = np.flip(NetherlandsG)
[31]: New_ZealandC = data.iloc[699:724,2]
     New_ZealandC = np.array(New_ZealandC)
     New_ZealandC = np.flip(New_ZealandC)
     New_ZealandG = data.iloc[1698:1723,2]
     New_ZealandG = np.array(New_ZealandG)
     New_ZealandG = np.flip(New_ZealandG)
[32]: NorwayC = data.iloc[724:749,2]
     NorwayC = np.array(NorwayC)
     NorwayC = np.flip(NorwayC)
     NorwayG = data.iloc[1723:1748,2]
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NorwayG = np.array(NorwayG)
     NorwayG = np.flip(NorwayG)
[33]: PolandC = data.iloc[749:774,2]
     PolandC = np.array(PolandC)
     PolandC = np.flip(PolandC)
     PolandG = data.iloc[1748:1773,2]
     PolandG = np.array(PolandG)
     PolandG = np.flip(PolandG)
[34]: PortugalC = data.iloc[774:799,2]
     PortugalC = np.array(PortugalC)
     PortugalC = np.flip(PortugalC)
     PortugalG = data.iloc[1773:1798,2]
     PortugalG = np.array(PortugalG)
     PortugalG = np.flip(PortugalG)
[35]: RomaniaC = data.iloc[799:824,2]
     RomaniaC = np.array(RomaniaC)
     RomaniaC = np.flip(RomaniaC)
     RomaniaG = data.iloc[1798:1823,2]
     RomaniaG = np.array(RomaniaG)
     RomaniaG = np.flip(RomaniaG)
[36]: Russian_FederationC = data.iloc[824:849,2]
     Russian_FederationC = np.array(Russian_FederationC)
     Russian_FederationC = np.flip(Russian_FederationC)
     Russian_FederationG = data.iloc[1823:1848,2]
     Russian_FederationG = np.array(Russian_FederationG)
     Russian_FederationG = np.flip(Russian_FederationG)
[37]: SlovakiaC = data.iloc[849:874,2]
     SlovakiaC = np.array(SlovakiaC)
     SlovakiaC = np.flip(SlovakiaC)
     SlovakiaG = data.iloc[1848:1873,2]
     SlovakiaG = np.array(SlovakiaG)
     SlovakiaG = np.flip(SlovakiaG)
[38]: SloveniaC = data.iloc[874:899,2]
     SloveniaC = np.array(SloveniaC)
     SloveniaC = np.flip(SloveniaC)
     SloveniaG = data.iloc[1873:1898,2]
     SloveniaG = np.array(SloveniaG)
     SloveniaG = np.flip(SloveniaG)
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[39]: SpainC = data.iloc[899:924,2]
     SpainC = np.array(SpainC)
     SpainC = np.flip(SpainC)
     # Spain does not have greenhouse gas values in the data set so it cannot be_{f \sqcup}
      \rightarrow included
[40]: SwedenC = data.iloc[924:949,2]
     SwedenC = np.array(SwedenC)
     SwedenC = np.flip(SwedenC)
     SwedenG = data.iloc[1898:1923,2]
     SwedenG = np.array(SwedenG)
     SwedenG = np.flip(SwedenG)
[41]: SwitzerlandC = data.iloc[949:974,2]
     SwitzerlandC = np.array(SwitzerlandC)
     SwitzerlandC = np.flip(SwitzerlandC)
     SwitzerlandG = data.iloc[1923:1948,2]
     SwitzerlandG = np.array(SwitzerlandG)
     SwitzerlandG = np.flip(SwitzerlandG)
[42]: TurkeyC = data.iloc[974:999,2]
     TurkeyC = np.array(TurkeyC)
     TurkeyC = np.flip(TurkeyC)
     TurkeyG = data.iloc[1948:1973,2]
     TurkeyG = np.array(TurkeyG)
     TurkeyG = np.flip(TurkeyG)
[43]: UkraineC = data.iloc[999:1024,2]
     UkraineC = np.array(UkraineC)
     UkraineC = np.flip(UkraineC)
     UkraineG = data.iloc[1973:1998,2]
     UkraineG = np.array(UkraineG)
     UkraineG = np.flip(UkraineG)
[44]: United_KingdomC = data.iloc[1024:1049,2]
     United_KingdomC = np.array(United_KingdomC)
     United_KingdomC = np.flip(United_KingdomC)
     # United Kingdom does not have greenhouse gas values in the data set so it_{f \sqcup}
      \rightarrow cannot be included
[45]: United_StatesC = data.iloc[1049:1074,2]
     United_StatesC = np.array(United_StatesC)
     United_StatesC = np.flip(United_StatesC)
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United_StatesG = data.iloc[1998:2023,2]
                United_StatesG = np.array(United_StatesG)
                United_StatesG = np.flip(United_StatesG)
[46]: X = np.array([AustraliaC[0:24],AustriaC[0:24],BelarusC[0:24],BelgiumC[0:
                    \rightarrow24], CanadaC[0:24], CroatiaC[0:24],
                                                              CyprusC[0:24],Czech_RepublicC[0:24],DenmarkC[0:24],EstoniaC[0:
                    →24],European_UnionC[0:24],
                                                              FinlandC[0:24], FranceC[0:24], GermanyC[0:24], GreeceC[0:24]
                    \rightarrow24], HungaryC[0:24], IcelandC[0:24],
                                                              IrelandC[0:24],ItalyC[0:24],JapanC[0:24],LatviaC[0:
                    \rightarrow24],LithuaniaC[0:24],MaltaC[0:24],
                                                             \label{lem:netherlandsC[0:24],New_ZealandC[0:24],NorwayC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC[0:24],PolandC
                    \rightarrow24], PortugalC[0:24],
                                                              RomaniaC[0:24],Russian_FederationC[0:24],SlovakiaC[0:
                    \rightarrow24],SloveniaC[0:24],SwedenC[0:24],
                                                              SwitzerlandC[0:24], TurkeyC[0:24], UkraineC[0:24], United_StatesC[0:
                   →24]])
                # X is a matrix of carbon dioxide values for each country or area in the dataset
                 # Each row is for a country or area
                # Each column is for a year from 1990 to 2013
                print(X)
```

```
[[2.78265899e+05 2.79741639e+05 2.84766093e+05 2.89142268e+05
 2.93830709e+05 3.05162544e+05 3.11914820e+05 3.20439117e+05
 3.34328143e+05 3.43713907e+05 3.49885433e+05 3.57653330e+05
 3.61861388e+05 3.68345977e+05 3.81519262e+05 3.85581133e+05
 3.91134101e+05 3.98816454e+05 4.04237828e+05 4.08448479e+05
 4.06200993e+05 4.03705528e+05 4.06462848e+05 3.96913937e+05]
 [6.22971656e+04 6.59035133e+04 6.04322493e+04 6.07907577e+04
 6.11891025e+04 6.42021592e+04 6.76672627e+04 6.74558927e+04
 6.70472507e+04 6.56601336e+04 6.62747433e+04 7.02994711e+04
 7.21265914e+04 7.80239539e+04 7.83889974e+04 7.95885019e+04
 7.69349515e+04 7.42681107e+04 7.40662558e+04 6.76827251e+04
 7.25318261e+04 7.03271670e+04 6.76986586e+04 6.79571211e+04]
 [1.00438095e+05 9.36491661e+04 8.70608103e+04 7.55517553e+04
 6.24022791e+04 5.60059394e+04 5.72513557e+04 6.09718163e+04
 5.74263296e+04 5.50339521e+04 5.36868575e+04 5.37838477e+04
 5.38437044e+04 5.48451458e+04 5.84576476e+04 5.96647391e+04
 6.22293989e+04 6.15638817e+04 6.38461038e+04 5.95962054e+04
 6.16324927e+04 6.11284006e+04 6.06368514e+04 6.15109872e+04]
 [1.19982503e+05 1.23130653e+05 1.22217920e+05 1.21156965e+05
 1.24426886e+05 1.25519442e+05 1.29010472e+05 1.23495743e+05
 1.29737090e+05 1.24208072e+05 1.26315210e+05 1.25649371e+05
 1.26015195e+05 1.27523990e+05 1.28640483e+05 1.25118017e+05
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1.22319594e+05 1.18027409e+05 1.20362610e+05 1.07282544e+05
1.14155387e+05 1.04945714e+05 1.00931596e+05 1.01744737e+05]
[4.63493332e+05 4.54246138e+05 4.68546459e+05 4.67607237e+05
4.82691671e+05 4.97224837e+05 5.10619682e+05 5.25120011e+05
5.33794454e+05 5.50183063e+05 5.72290386e+05 5.64617020e+05
5.69862146e+05 5.87306679e+05 5.85414861e+05 5.79225336e+05
5.73479270e+05 5.96041322e+05 5.77444930e+05 5.42829459e+05
5.55003760e+05 5.58152723e+05 5.63423613e+05 5.73094039e+05]
[2.33900752e+04 1.72689103e+04 1.66071345e+04 1.70812280e+04
1.63421819e+04 1.69928004e+04 1.75878985e+04 1.86664670e+04
1.93723876e+04 2.02156532e+04 1.97891218e+04 2.09431100e+04
2.20311267e+04 2.33552211e+04 2.30052349e+04 2.34518458e+04
2.36849350e+04 2.49553759e+04 2.37153843e+04 2.19656353e+04
2.11837093e+04 2.06144405e+04 1.87763813e+04 1.83594998e+04]
[4.62100718e+03 5.10708289e+03 5.48297487e+03 5.73367410e+03
5.97725710e+03 5.84804379e+03 6.15613213e+03 6.28703166e+03
6.58029615e+03 6.84765912e+03 7.09596438e+03 6.96946618e+03
7.16038846e+03 7.55008193e+03 7.77690940e+03 7.96198399e+03
8.15249231e+03 8.45866426e+03 8.62039558e+03 8.37239659e+03
8.00491202e+03 7.69666781e+03 7.16097188e+03 6.44961081e+03
[1.61662019e+05 1.46047340e+05 1.41565720e+05 1.35588566e+05
1.29188734e+05 1.29767166e+05 1.32234072e+05 1.28732639e+05
1.23696517e+05 1.15475260e+05 1.25841969e+05 1.25541536e+05
1.22593400e+05 1.26151649e+05 1.26876916e+05 1.24590138e+05
1.25923402e+05 1.26933163e+05 1.21780716e+05 1.13970029e+05
1.15757369e+05 1.14113147e+05 1.09841329e+05 1.06477089e+05]
[5.43064333e+04 6.48748139e+04 5.89603115e+04 6.10386712e+04
6.49887809e+04 6.19141610e+04 7.51108410e+04 6.54578348e+04
6.11173416e+04 5.84564158e+04 5.41489050e+04 5.58451472e+04
5.53396717e+04 6.04939617e+04 5.49169106e+04 5.13072445e+04
5.91624996e+04 5.44290852e+04 5.09356715e+04 4.85922114e+04
4.89934174e+04 4.39707893e+04 3.95244020e+04 4.15370358e+04]
[3.66660273e+04 3.36623725e+04 2.41878276e+04 1.88109639e+04
1.96264459e+04 1.79537360e+04 1.86661360e+04 1.82236043e+04
1.66843031e+04 1.55184906e+04 1.51413877e+04 1.54938610e+04
1.50202842e+04 1.68332143e+04 1.70550742e+04 1.63307338e+04
1.57195289e+04 1.87556479e+04 1.72645200e+04 1.41530052e+04
1.78337796e+04 1.84499309e+04 1.73046372e+04 1.95663684e+04]
[4.46220514e+06 4.40372316e+06 4.25992281e+06 4.17710651e+06
4.16132132e+06 4.20448086e+06 4.30479328e+06 4.21652473e+06
4.20766603e+06 4.13900299e+06 4.16514544e+06 4.23203686e+06
4.20886012e+06 4.30610394e+06 4.31456087e+06 4.29058667e+06
4.29798816e+06 4.24421313e+06 4.14962430e+06 3.81430526e+06
3.93329052e+06 3.78976364e+06 3.72896503e+06 3.64638953e+06]
[5.69508482e+04 5.51871795e+04 5.42643793e+04 5.63332680e+04
6.16963853e+04 5.81262475e+04 6.40389222e+04 6.26980868e+04
5.93514981e+04 5.88729308e+04 5.69733505e+04 6.24426401e+04
6.49464682e+04 7.25225902e+04 6.88218129e+04 5.69123149e+04
```

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6.82019060e+04 6.65317208e+04 5.84616330e+04 5.56355828e+04
6.38280182e+04 5.64787456e+04 5.09832729e+04 5.18346967e+04]
[4.00652624e+05 4.25042921e+05 4.14955077e+05 3.94334834e+05
3.94536962e+05 4.01571225e+05 4.14013070e+05 4.07475112e+05
4.29051500e+05 4.20506322e+05 4.17116545e+05 4.17864706e+05
4.13569248e+05 4.22312884e+05 4.23168636e+05 4.28557610e+05
4.17662679e+05 4.08508590e+05 4.01878865e+05 3.83957624e+05
3.94235674e+05 3.68183438e+05 3.70491154e+05 3.69985595e+05]
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9.39207328e+05 9.38148093e+05 9.58368725e+05 9.30768561e+05
9.22778540e+05 8.95336786e+05 8.99284993e+05 9.15634440e+05
8.99161567e+05 9.00377585e+05 8.86344630e+05 8.65958513e+05
8.77422457e+05 8.50795721e+05 8.53229941e+05 7.88422438e+05
8.32259203e+05 8.12483220e+05 8.17031440e+05 8.35792632e+05]
[8.34037200e+04 8.33903547e+04 8.49490258e+04 8.42630079e+04
8.64258565e+04 8.69806458e+04 8.91344254e+04 9.38398699e+04
9.86631942e+04 9.79797784e+04 1.03019665e+05 1.05407308e+05
1.05051466e+05 1.09126568e+05 1.09571061e+05 1.13219822e+05
1.11777744e+05 1.14127554e+05 1.10255766e+05 1.03969784e+05
9.70350828e+04 9.41029440e+04 9.07105742e+04 8.29106260e+04]
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6.15683746e+04 6.13544620e+04 6.29502532e+04 6.15499125e+04
6.10877638e+04 6.15931410e+04 5.83368071e+04 5.99957345e+04
5.89682278e+04 6.17668396e+04 6.02316620e+04 6.03295050e+04
5.97517311e+04 5.84190392e+04 5.73251623e+04 5.16306900e+04
5.21088928e+04 5.02728651e+04 4.67749448e+04 4.39308782e+04]
[2.10602607e+03 2.04293698e+03 2.17261936e+03 2.29450464e+03
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2.45023426e+03 2.65539473e+03 2.72893749e+03 2.71029473e+03
2.79590731e+03 2.79272313e+03 2.83702901e+03 2.79731459e+03
2.93927660e+03 3.23689534e+03 3.56720926e+03 3.52189071e+03
3.38355222e+03 3.29856769e+03 3.29057643e+03 3.30179631e+03]
[3.27685268e+04 3.35084420e+04 3.33441024e+04 3.35749717e+04
3.46964857e+04 3.57187486e+04 3.73351081e+04 3.86755940e+04
4.05721759e+04 4.23020879e+04 4.51233354e+04 4.74827208e+04
4.59527302e+04 4.55543225e+04 4.60424022e+04 4.79522044e+04
4.74059667e+04 4.74931267e+04 4.71755403e+04 4.19963247e+04
4.15582751e+04 3.78951954e+04 3.80316187e+04 3.70494238e+04]
[4.36204250e+05 4.35938614e+05 4.35535835e+05 4.29158009e+05
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4.54967633e+05 4.59840642e+05 4.65175073e+05 4.71124277e+05
4.73504059e+05 4.89595508e+05 4.92746256e+05 4.90914263e+05
4.86594067e+05 4.78237711e+05 4.67933593e+05 4.18881077e+05
4.28879681e+05 4.16499534e+05 3.89340763e+05 3.62063649e+05]
[1.15599362e+06 1.16447435e+06 1.17461901e+06 1.16771472e+06
1.22893669e+06 1.24249402e+06 1.25526420e+06 1.25305731e+06
1.21825776e+06 1.25308215e+06 1.27429799e+06 1.25738456e+06
1.29439920e+06 1.29949061e+06 1.29843389e+06 1.30593882e+06
```

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1.21297023e+06 1.26186294e+06 1.29618636e+06 1.31150915e+06]
[1.97942945e+04 1.80211952e+04 1.42446534e+04 1.19661455e+04
1.03991411e+04 9.14135352e+03 9.21638399e+03 8.67754581e+03
8.30059528e+03 7.70904999e+03 7.06994204e+03 7.48050261e+03
7.50874324e+03 7.70377151e+03 7.70946076e+03 7.79040393e+03
8.28920668e+03 8.61025540e+03 8.18497125e+03 7.44328601e+03
8.52408723e+03 7.80765941e+03 7.52112253e+03 7.35322044e+03
[3.58128937e+04 3.79086125e+04 2.12078874e+04 1.63471718e+04
1.57654148e+04 1.50225441e+04 1.56768310e+04 1.50862858e+04
1.59015176e+04 1.34221511e+04 1.18019649e+04 1.25079667e+04
1.26144250e+04 1.26052382e+04 1.31854934e+04 1.39468843e+04
1.43030327e+04 1.56240763e+04 1.49632547e+04 1.27271200e+04
1.36189127e+04 1.39190155e+04 1.39755232e+04 1.29878175e+04]
[1.86047365e+03 2.04124609e+03 2.17463447e+03 2.25206277e+03
2.30277028e+03 2.31321609e+03 2.32918170e+03 2.33195473e+03
2.33806279e+03 2.41855556e+03 2.41428381e+03 2.55060406e+03
2.46421223e+03 2.66613433e+03 2.60474483e+03 2.73631293e+03
2.77631431e+03 2.83164226e+03 2.82240617e+03 2.68452129e+03
2.69051804e+03 2.78589657e+03 2.86863813e+03 2.47607443e+03]
[1.62497560e+05 1.71081474e+05 1.70975243e+05 1.71497891e+05
1.75008703e+05 1.73196297e+05 1.82573583e+05 1.75487971e+05
1.76617169e+05 1.71228110e+05 1.72059812e+05 1.77037402e+05
1.76462401e+05 1.79867889e+05 1.81546257e+05 1.77540473e+05
1.72772893e+05 1.73178420e+05 1.76174418e+05 1.70796157e+05
1.82529787e+05 1.69792137e+05 1.65665024e+05 1.65475884e+05]
[2.54197717e+04 2.61080645e+04 2.80997000e+04 2.76972273e+04
2.78790592e+04 2.81434576e+04 2.93682650e+04 3.13634526e+04
2.99275014e+04 3.15231861e+04 3.23559570e+04 3.45817228e+04
3.47682782e+04 3.64971710e+04 3.60636633e+04 3.76454343e+04
3.75191047e+04 3.66513715e+04 3.76740518e+04 3.48064021e+04
3.50172414e+04 3.43665827e+04 3.56210260e+04 3.50942867e+04]
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3.84347652e+04 3.84649067e+04 4.15867086e+04 4.16831334e+04
4.19390379e+04 4.26917564e+04 4.21946192e+04 4.35303488e+04
4.26558748e+04 4.39067097e+04 4.43200049e+04 4.35482303e+04
4.39137448e+04 4.58428095e+04 4.48930942e+04 4.32120925e+04
4.58338921e+04 4.49468694e+04 4.45539657e+04 4.43090560e+04
[3.78782539e+05 3.75684157e+05 3.65865160e+05 3.66427661e+05
3.62012959e+05 3.63885778e+05 3.77308596e+05 3.68415853e+05
3.39145830e+05 3.29380898e+05 3.19120405e+05 3.15274094e+05
3.07535642e+05 3.20230170e+05 3.24279450e+05 3.23372578e+05
3.36500579e+05 3.36268111e+05 3.29338171e+05 3.15937966e+05
3.34026155e+05 3.33713651e+05 3.26597791e+05 3.22440487e+05]
[4.58090714e+04 4.74068425e+04 5.18505657e+04 5.05021226e+04
5.13032041e+04 5.51343295e+04 5.23484688e+04 5.54542111e+04
5.99762099e+04 6.76128119e+04 6.64113299e+04 6.61137557e+04
6.99544945e+04 6.46972992e+04 6.74581591e+04 7.00345294e+04
```

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5.35838859e+04 5.21506534e+04 5.02624224e+04 4.84955505e+04]
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1.06813978e+05 9.04433508e+04 9.51909034e+04 1.00377987e+05
1.00655313e+05 1.05737481e+05 1.04575078e+05 1.02414252e+05
1.06175991e+05 1.04540351e+05 1.01035421e+05 8.52882940e+04
8.07957740e+04 8.61636343e+04 8.46612305e+04 7.40833582e+04]
[2.58972531e+06 2.43318765e+06 2.01317548e+06 1.90957784e+06
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1.47658893e+06 1.51265193e+06 1.50418273e+06 1.54250006e+06
1.53120021e+06 1.56851014e+06 1.57518086e+06 1.59393334e+06
1.65328919e+06 1.65313906e+06 1.68499982e+06 1.57670397e+06
1.66259226e+06 1.71722195e+06 1.72748910e+06 1.66664442e+06]
[6.18375668e+04 5.34474400e+04 4.89791258e+04 4.64675492e+04
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4.40169870e+04 4.31813145e+04 4.11550115e+04 4.34680934e+04
4.15935332e+04 4.20605502e+04 4.25769589e+04 4.25810225e+04
4.23204845e+04 4.07252998e+04 4.12363240e+04 3.74334334e+04
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1.59804778e+04 1.53530714e+04 1.54601542e+04 1.63847561e+04
1.65263127e+04 1.62893859e+04 1.66391711e+04 1.69383243e+04
1.71298020e+04 1.72723792e+04 1.82197304e+04 1.63282103e+04
1.63634298e+04 1.63482391e+04 1.58001872e+04 1.51487254e+04]
[5.75465433e+04 5.77556347e+04 5.74820647e+04 5.74259514e+04
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5.87334064e+04 5.58612192e+04 5.47304244e+04 5.56867457e+04
5.66318962e+04 5.72035991e+04 5.64288031e+04 5.38592057e+04
5.36976559e+04 5.29117267e+04 5.08315768e+04 4.72674467e+04
5.30577346e+04 4.91335869e+04 4.65596400e+04 4.48991519e+04]
[4.41155341e+04 4.61448095e+04 4.59747025e+04 4.35562764e+04
4.26133037e+04 4.33684572e+04 4.40883349e+04 4.29716245e+04
4.45223714e+04 4.43545431e+04 4.35459597e+04 4.50022788e+04
4.33739856e+04 4.45879803e+04 4.51496671e+04 4.57780895e+04
4.53511720e+04 4.33603857e+04 4.46769440e+04 4.35090866e+04
4.50270645e+04 4.09731492e+04 4.22415536e+04 4.31816594e+04
[1.46750637e+05 1.52335174e+05 1.58571546e+05 1.67443152e+05
1.64623553e+05 1.78812409e+05 1.94751779e+05 2.08480070e+05
2.08365344e+05 2.06388620e+05 2.32549190e+05 2.17469503e+05
2.26365412e+05 2.40902463e+05 2.53172078e+05 2.79130741e+05
3.02195955e+05 3.33166419e+05 3.23536545e+05 3.10580701e+05
3.20356659e+05 3.38094408e+05 3.63126007e+05 3.54961042e+05]
[6.93024767e+05 6.21305236e+05 5.77064260e+05 4.98504839e+05
4.09721674e+05 3.80871284e+05 3.44347189e+05 3.34208702e+05
3.09207663e+05 2.84109569e+05 2.71429774e+05 2.90774710e+05
2.81939350e+05 2.85720480e+05 2.96753732e+05 3.07341483e+05
```

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2.87113606e+05 3.01273502e+05 2.95706064e+05 2.87436404e+05]
      [5.11509505e+06 5.06487975e+06 5.17027435e+06 5.28475862e+06
      5.37749222e+06 5.44159923e+06 5.63011372e+06 5.70499687e+06
      5.74467220e+06 5.81897239e+06 5.99243804e+06 5.89446294e+06
      5.93573878e+06 5.98228917e+06 6.09697836e+06 6.12274661e+06
       6.04239361e+06 6.12165386e+06 5.92320138e+06 5.48832028e+06
       5.68875601e+06 5.55950766e+06 5.34922095e+06 5.50255071e+06]]
[47]: | Xbar = np.array([AustraliaC[1:25], AustriaC[1:25], BelarusC[1:25], BelgiumC[1:
      \rightarrow25], CanadaC[1:25], CroatiaC[1:25],
                       CyprusC[1:25],Czech_RepublicC[1:25],DenmarkC[1:25],EstoniaC[1:
      \rightarrow25], European_UnionC[1:25],
                       FinlandC[1:25],FranceC[1:25],GermanyC[1:25],GreeceC[1:
      \rightarrow25], Hungary C[1:25], Iceland C[1:25],
                       IrelandC[1:25],ItalyC[1:25],JapanC[1:25],LatviaC[1:
      \rightarrow25],LithuaniaC[1:25],MaltaC[1:25],
                       NetherlandsC[1:25], New_ZealandC[1:25], NorwayC[1:25], PolandC[1:
      \rightarrow25], PortugalC[1:25],
                       RomaniaC[1:25],Russian_FederationC[1:25],SlovakiaC[1:
      \rightarrow25],SloveniaC[1:25],SwedenC[1:25],
                       SwitzerlandC[1:25], TurkeyC[1:25], UkraineC[1:
      →25],United_StatesC[1:25]])
     # Xbar is a matrix of carbon dioxide values for each country or area in the
      \rightarrow dataset
     # Each row is for a country or area
     # Each column is for a year from 1991 to 2014
     print(Xbar)
    [[2.79741639e+05 2.84766093e+05 2.89142268e+05 2.93830709e+05
       3.05162544e+05 3.11914820e+05 3.20439117e+05 3.34328143e+05
      3.43713907e+05 3.49885433e+05 3.57653330e+05 3.61861388e+05
```

3.25586622e+05 3.30087251e+05 3.17063844e+05 2.69598422e+05

```
[2.79741639e+05 2.84766093e+05 2.89142268e+05 2.93830709e+05 3.05162544e+05 3.11914820e+05 3.20439117e+05 3.34328143e+05 3.43713907e+05 3.49885433e+05 3.57653330e+05 3.61861388e+05 3.68345977e+05 3.81519262e+05 3.85581133e+05 3.91134101e+05 3.98816454e+05 4.04237828e+05 4.08448479e+05 4.06200993e+05 4.03705528e+05 4.06462848e+05 3.96913937e+05 3.93126947e+05] [6.59035133e+04 6.04322493e+04 6.07907577e+04 6.11891025e+04 6.42021592e+04 6.76672627e+04 6.74558927e+04 6.70472507e+04 6.56601336e+04 6.62747433e+04 7.02994711e+04 7.21265914e+04 7.80239539e+04 7.83889974e+04 7.95885019e+04 7.69349515e+04 7.42681107e+04 7.40662558e+04 6.76827251e+04 7.25318261e+04 7.03271670e+04 6.76986586e+04 6.79571211e+04 6.42625640e+04] [9.36491661e+04 8.70608103e+04 7.55517553e+04 6.24022791e+04 5.60059394e+04 5.72513557e+04 6.09718163e+04 5.74263296e+04 5.50339521e+04 5.36868575e+04 5.37838477e+04 5.38437044e+04 5.48451458e+04 5.84576476e+04 5.96647391e+04 6.22293989e+04
```

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6.15638817e+04 6.38461038e+04 5.95962054e+04 6.16324927e+04
6.11284006e+04 6.06368514e+04 6.15109872e+04 6.06156457e+04]
[1.23130653e+05 1.22217920e+05 1.21156965e+05 1.24426886e+05
1.25519442e+05 1.29010472e+05 1.23495743e+05 1.29737090e+05
1.24208072e+05 1.26315210e+05 1.25649371e+05 1.26015195e+05
1.27523990e+05 1.28640483e+05 1.25118017e+05 1.22319594e+05
1.18027409e+05 1.20362610e+05 1.07282544e+05 1.14155387e+05
1.04945714e+05 1.00931596e+05 1.01744737e+05 9.63254132e+04
[4.54246138e+05 4.68546459e+05 4.67607237e+05 4.82691671e+05
4.97224837e+05 5.10619682e+05 5.25120011e+05 5.33794454e+05
5.50183063e+05 5.72290386e+05 5.64617020e+05 5.69862146e+05
5.87306679e+05 5.85414861e+05 5.79225336e+05 5.73479270e+05
5.96041322e+05 5.77444930e+05 5.42829459e+05 5.55003760e+05
5.58152723e+05 5.63423613e+05 5.73094039e+05 5.74099776e+05]
[1.72689103e+04 1.66071345e+04 1.70812280e+04 1.63421819e+04
1.69928004e+04 1.75878985e+04 1.86664670e+04 1.93723876e+04
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2.33552211e+04 2.30052349e+04 2.34518458e+04 2.36849350e+04
2.49553759e+04 2.37153843e+04 2.19656353e+04 2.11837093e+04
2.06144405e+04 1.87763813e+04 1.83594998e+04 1.76073224e+04]
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6.84765912e+03 7.09596438e+03 6.96946618e+03 7.16038846e+03
7.55008193e+03 7.77690940e+03 7.96198399e+03 8.15249231e+03
8.45866426e+03 8.62039558e+03 8.37239659e+03 8.00491202e+03
7.69666781e+03 7.16097188e+03 6.44961081e+03 6.87855531e+03]
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1.29767166e+05 1.32234072e+05 1.28732639e+05 1.23696517e+05
1.15475260e+05 1.25841969e+05 1.25541536e+05 1.22593400e+05
1.26151649e+05 1.26876916e+05 1.24590138e+05 1.25923402e+05
1.26933163e+05 1.21780716e+05 1.13970029e+05 1.15757369e+05
1.14113147e+05 1.09841329e+05 1.06477089e+05 1.01159122e+05]
[6.48748139e+04 5.89603115e+04 6.10386712e+04 6.49887809e+04
6.19141610e+04 7.51108410e+04 6.54578348e+04 6.11173416e+04
5.84564158e+04 5.41489050e+04 5.58451472e+04 5.53396717e+04
6.04939617e+04 5.49169106e+04 5.13072445e+04 5.91624996e+04
5.44290852e+04 5.09356715e+04 4.85922114e+04 4.89934174e+04
4.39707893e+04 3.95244020e+04 4.15370358e+04 3.74606565e+04
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1.55184906e+04 1.51413877e+04 1.54938610e+04 1.50202842e+04
1.68332143e+04 1.70550742e+04 1.63307338e+04 1.57195289e+04
1.87556479e+04 1.72645200e+04 1.41530052e+04 1.78337796e+04
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[4.40372316e+06 4.25992281e+06 4.17710651e+06 4.16132132e+06
4.20448086e+06 4.30479328e+06 4.21652473e+06 4.20766603e+06
4.13900299e+06 4.16514544e+06 4.23203686e+06 4.20886012e+06
4.30610394e+06 4.31456087e+06 4.29058667e+06 4.29798816e+06
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[5.51871795e+04 5.42643793e+04 5.63332680e+04 6.16963853e+04
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5.88729308e+04 5.69733505e+04 6.24426401e+04 6.49464682e+04
7.25225902e+04 6.88218129e+04 5.69123149e+04 6.82019060e+04
6.65317208e+04 5.84616330e+04 5.56355828e+04 6.38280182e+04
5.64787456e+04 5.09832729e+04 5.18346967e+04 4.75977335e+04]
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4.01571225e+05 4.14013070e+05 4.07475112e+05 4.29051500e+05
4.20506322e+05 4.17116545e+05 4.17864706e+05 4.13569248e+05
4.22312884e+05 4.23168636e+05 4.28557610e+05 4.17662679e+05
4.08508590e+05 4.01878865e+05 3.83957624e+05 3.94235674e+05
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[1.01411972e+06 9.65790835e+05 9.55977599e+05 9.39207328e+05
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8.95336786e+05 8.99284993e+05 9.15634440e+05 8.99161567e+05
9.00377585e+05 8.86344630e+05 8.65958513e+05 8.77422457e+05
8.50795721e+05 8.53229941e+05 7.88422438e+05 8.32259203e+05
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1.09126568e+05 1.09571061e+05 1.13219822e+05 1.11777744e+05
1.14127554e+05 1.10255766e+05 1.03969784e+05 9.70350828e+04
9.41029440e+04 9.07105742e+04 8.29106260e+04 7.96282744e+04]
[6.93927256e+04 6.20501264e+04 6.31749784e+04 6.15683746e+04
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6.15931410e+04 5.83368071e+04 5.99957345e+04 5.89682278e+04
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5.02728651e+04 4.67749448e+04 4.39308782e+04 4.35734088e+04]
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4.23020879e+04 4.51233354e+04 4.74827208e+04 4.59527302e+04
4.55543225e+04 4.60424022e+04 4.79522044e+04 4.74059667e+04
4.74931267e+04 4.71755403e+04 4.19963247e+04 4.15582751e+04
3.78951954e+04 3.80316187e+04 3.70494238e+04 3.65590686e+04]
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4.59840642e+05 4.65175073e+05 4.71124277e+05 4.73504059e+05
4.89595508e+05 4.92746256e+05 4.90914263e+05 4.86594067e+05
```

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[1.16447435e+06 1.17461901e+06 1.16771472e+06 1.22893669e+06
1.24249402e+06 1.25526420e+06 1.25305731e+06 1.21825776e+06
1.25308215e+06 1.27429799e+06 1.25738456e+06 1.29439920e+06
1.29949061e+06 1.29843389e+06 1.30593882e+06 1.28517784e+06
1.31979919e+06 1.23545581e+06 1.16260619e+06 1.21297023e+06
1.26186294e+06 1.29618636e+06 1.31150915e+06 1.26549061e+06]
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7.70904999e+03 7.06994204e+03 7.48050261e+03 7.50874324e+03
7.70377151e+03 7.70946076e+03 7.79040393e+03 8.28920668e+03
8.61025540e+03 8.18497125e+03 7.44328601e+03 8.52408723e+03
7.80765941e+03 7.52112253e+03 7.35322044e+03 7.15871804e+03]
[3.79086125e+04 2.12078874e+04 1.63471718e+04 1.57654148e+04
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1.34221511e+04 1.18019649e+04 1.25079667e+04 1.26144250e+04
1.26052382e+04 1.31854934e+04 1.39468843e+04 1.43030327e+04
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2.66613433e+03 2.60474483e+03 2.73631293e+03 2.77631431e+03
2.83164226e+03 2.82240617e+03 2.68452129e+03 2.69051804e+03
2.78589657e+03 2.86863813e+03 2.47607443e+03 2.48353679e+03]
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1.71228110e+05 1.72059812e+05 1.77037402e+05 1.76462401e+05
1.79867889e+05 1.81546257e+05 1.77540473e+05 1.72772893e+05
1.73178420e+05 1.76174418e+05 1.70796157e+05 1.82529787e+05
1.69792137e+05 1.65665024e+05 1.65475884e+05 1.57789582e+05]
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3.15231861e+04 3.23559570e+04 3.45817228e+04 3.47682782e+04
3.64971710e+04 3.60636633e+04 3.76454343e+04 3.75191047e+04
3.66513715e+04 3.76740518e+04 3.48064021e+04 3.50172414e+04
3.43665827e+04 3.56210260e+04 3.50942867e+04 3.56169853e+04
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3.84649067e+04 4.15867086e+04 4.16831334e+04 4.19390379e+04
4.26917564e+04 4.21946192e+04 4.35303488e+04 4.26558748e+04
4.39067097e+04 4.43200049e+04 4.35482303e+04 4.39137448e+04
4.58428095e+04 4.48930942e+04 4.32120925e+04 4.58338921e+04
4.49468694e+04 4.45539657e+04 4.43090560e+04 4.38694511e+04]
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3.29380898e+05 3.19120405e+05 3.15274094e+05 3.07535642e+05
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```

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6.76128119e+04 6.64113299e+04 6.61137557e+04 6.99544945e+04
6.46972992e+04 6.74581591e+04 7.00345294e+04 6.54654197e+04
6.27911478e+04 6.05500402e+04 5.80041970e+04 5.35838859e+04
5.21506534e+04 5.02624224e+04 4.84955505e+04 4.83536903e+04]
[1.43586608e+05 1.32732909e+05 1.24819100e+05 1.22610777e+05
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9.04433508e+04 9.51909034e+04 1.00377987e+05 1.00655313e+05
1.05737481e+05 1.04575078e+05 1.02414252e+05 1.06175991e+05
1.04540351e+05 1.01035421e+05 8.52882940e+04 8.07957740e+04
8.61636343e+04 8.46612305e+04 7.40833582e+04 7.40464676e+04]
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1.51265193e+06 1.50418273e+06 1.54250006e+06 1.53120021e+06
1.56851014e+06 1.57518086e+06 1.59393334e+06 1.65328919e+06
1.65313906e+06 1.68499982e+06 1.57670397e+06 1.66259226e+06
1.71722195e+06 1.72748910e+06 1.66664442e+06 1.67156865e+06]
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4.31813145e+04 4.11550115e+04 4.34680934e+04 4.15935332e+04
4.20605502e+04 4.25769589e+04 4.25810225e+04 4.23204845e+04
4.07252998e+04 4.12363240e+04 3.74334334e+04 3.83859425e+04
3.78808531e+04 3.58678963e+04 3.53951549e+04 3.33874287e+04]
[1.40072826e+04 1.40093986e+04 1.43039937e+04 1.46487255e+04
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1.62893859e+04 1.66391711e+04 1.69383243e+04 1.71298020e+04
1.72723792e+04 1.82197304e+04 1.63282103e+04 1.63634298e+04
1.63482391e+04 1.58001872e+04 1.51487254e+04 1.34899800e+04]
[5.77556347e+04 5.74820647e+04 5.74259514e+04 5.98769988e+04
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5.58612192e+04 5.47304244e+04 5.56867457e+04 5.66318962e+04
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4.43545431e+04 4.35459597e+04 4.50022788e+04 4.33739856e+04
4.45879803e+04 4.51496671e+04 4.57780895e+04 4.53511720e+04
4.33603857e+04 4.46769440e+04 4.35090866e+04 4.50270645e+04
4.09731492e+04 4.22415536e+04 4.31816594e+04 3.92650526e+04]
[1.52335174e+05 1.58571546e+05 1.67443152e+05 1.64623553e+05
1.78812409e+05 1.94751779e+05 2.08480070e+05 2.08365344e+05
2.06388620e+05 2.32549190e+05 2.17469503e+05 2.26365412e+05
2.40902463e+05 2.53172078e+05 2.79130741e+05 3.02195955e+05
```

```
3.33166419e+05 3.23536545e+05 3.10580701e+05 3.20356659e+05
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     [6.21305236e+05 5.77064260e+05 4.98504839e+05 4.09721674e+05
      3.80871284e+05 3.44347189e+05 3.34208702e+05 3.09207663e+05
      2.84109569e+05 2.71429774e+05 2.90774710e+05 2.81939350e+05
      2.85720480e+05 2.96753732e+05 3.07341483e+05 3.25586622e+05
      3.30087251e+05 3.17063844e+05 2.69598422e+05 2.87113606e+05
      3.01273502e+05 2.95706064e+05 2.87436404e+05 2.47561221e+05]
     [5.06487975e+06 5.17027435e+06 5.28475862e+06 5.37749222e+06
      5.44159923e+06 5.63011372e+06 5.70499687e+06 5.74467220e+06
      5.81897239e+06 5.99243804e+06 5.89446294e+06 5.93573878e+06
      5.98228917e+06 6.09697836e+06 6.12274661e+06 6.04239361e+06
      6.12165386e+06 5.92320138e+06 5.48832028e+06 5.68875601e+06
      5.55950766e+06 5.34922095e+06 5.50255071e+06 5.55600658e+06]]
[48]: Y = np.array([AustraliaG[0:24],AustriaG[0:24],BelarusG[0:24],BelgiumG[0:
      \rightarrow24], CanadaG[0:24], CroatiaG[0:24],
                    CyprusG[0:24],Czech_RepublicG[0:24],DenmarkG[0:24],EstoniaG[0:
      →24], European_UnionG[0:24],
                   FinlandG[0:24], FranceG[0:24], GermanyG[0:24], GreeceG[0:24]
      \rightarrow24], HungaryG[0:24], IcelandG[0:24],
                    IrelandG[0:24],ItalyG[0:24],JapanG[0:24],LatviaG[0:
      \rightarrow24], LithuaniaG[0:24], MaltaG[0:24],
                    NetherlandsG[0:24], New_ZealandG[0:24], NorwayG[0:24], PolandG[0:
      \rightarrow24], PortugalG[0:24],
                    RomaniaG[0:24],Russian_FederationG[0:24],SlovakiaG[0:
      \rightarrow24],SloveniaG[0:24],SwedenG[0:24],
                    SwitzerlandG[0:24], TurkeyG[0:24], UkraineG[0:24], United_StatesG[0:
      →24]])
     # Y is a matrix of greenhouse gas values for each country or area in the dataset
     # Each row is for a country or area
     # Each column is for a year from 1990 to 2013
     print(Y)
    [[4.18623050e+05 4.18674057e+05 4.23080108e+05 4.23764933e+05
      4.24092959e+05 4.33478651e+05 4.39803962e+05 4.51721718e+05
      4.66382112e+05 4.72245991e+05 4.83445827e+05 4.91441696e+05
      4.94740101e+05 4.95154086e+05 5.11710242e+05 5.18850759e+05
      5.22517192e+05 5.29842870e+05 5.33693508e+05 5.37889893e+05
      5.33917436e+05 5.34089799e+05 5.37377571e+05 5.26882667e+05]
     [7.88446273e+04 8.26369035e+04 7.59309958e+04 7.59882270e+04
      7.65034853e+04 7.98130345e+04 8.30088063e+04 8.24743920e+04
      8.17708396e+04 8.01067172e+04 8.04293434e+04 8.43808069e+04
```

8.61295464e+04 9.20184811e+04 9.18358942e+04 9.28104872e+04 8.99810213e+04 8.72411545e+04 8.71011347e+04 8.01906514e+04

```
8.49460197e+04 8.26266380e+04 7.98971151e+04 8.00425418e+04]
[1.33457162e+05 1.25601053e+05 1.16588686e+05 1.03951525e+05
8.82063567e+04 8.03697823e+04 8.23751942e+04 8.62825059e+04
8.26688316e+04 7.94978421e+04 7.79597250e+04 7.79218276e+04
7.66838679e+04 7.91587188e+04 8.36882996e+04 8.55960987e+04
8.94496554e+04 8.89206419e+04 9.23643490e+04 8.92236563e+04
9.11835494e+04 9.16717270e+04 9.09251146e+04 9.30372454e+04]
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1.54340393e+05 1.48062444e+05 1.49213017e+05 1.47626133e+05
1.47240365e+05 1.47579070e+05 1.48971405e+05 1.44803082e+05
1.41046708e+05 1.36451339e+05 1.38749626e+05 1.25811802e+05
1.33258406e+05 1.22833396e+05 1.18761338e+05 1.19375302e+05]
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7.52124481e+05 7.66107895e+05 7.69691185e+05 7.53897578e+05
7.46308214e+05 7.66021807e+05 7.43519659e+05 7.02098420e+05
7.18178080e+05 7.22862507e+05 7.29475889e+05 7.37423135e+05]
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2.74200966e+04 2.87828213e+04 2.88759848e+04 2.92857516e+04
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7.79955926e+03 8.05908339e+03 8.32688573e+03 8.28254807e+03
8.51908140e+03 8.91458309e+03 9.14437142e+03 9.30798056e+03
9.56754719e+03 9.91323462e+03 1.00932700e+04 9.87218369e+03
9.57165722e+03 9.29616313e+03 8.76936603e+03 8.02009847e+03
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1.45622966e+05 1.48967238e+05 1.49733015e+05 1.47600477e+05
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7.80335456e+04 7.55202702e+04 7.10079297e+04 7.27571164e+04
7.22601099e+04 7.72126404e+04 7.10726799e+04 6.65946810e+04
7.43213577e+04 6.96611796e+04 6.59637634e+04 6.30666578e+04
 6.34998367e+04 5.82443623e+04 5.34084070e+04 5.54350429e+04]
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2.17131633e+04 1.99347906e+04 2.06317527e+04 2.02621584e+04
1.87058235e+04 1.73585005e+04 1.70618526e+04 1.74730230e+04
1.69300907e+04 1.87823647e+04 1.90554875e+04 1.82976747e+04
1.76864656e+04 2.08169353e+04 1.93746240e+04 1.61819654e+04
```

```
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5.20537470e+06 5.14622626e+06 5.03630424e+06 4.67279193e+06
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8.07390685e+04 7.92127392e+04 7.13245754e+04 6.76702912e+04
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5.75083631e+05 5.62048359e+05 5.58215973e+05 5.58213521e+05
5.52270726e+05 5.58762916e+05 5.56057007e+05 5.59398806e+05
5.47645150e+05 5.38725788e+05 5.31834047e+05 5.11294614e+05
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3.93690101e+03 3.91859270e+03 3.94012142e+03 3.89768784e+03
4.45403499e+03 4.71481831e+03 5.14033616e+03 4.85577231e+03
4.73035143e+03 4.52022086e+03 4.55040348e+03 4.53466283e+03]
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6.60032456e+04 6.71417469e+04 6.93249552e+04 7.13939184e+04
6.93712338e+04 6.96137987e+04 6.89326740e+04 7.03963261e+04
6.97951672e+04 6.90831959e+04 6.84510017e+04 6.28593798e+04
```

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1.39033114e+06 1.39270215e+06 1.38959319e+06 1.39710182e+06
1.37786194e+06 1.41279507e+06 1.32716853e+06 1.25100358e+06
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1.23731196e+04 1.16345689e+04 1.15262094e+04 1.14509296e+04]
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2.09859501e+04 2.12030139e+04 2.19844112e+04 2.30361383e+04
2.33784488e+04 2.53523511e+04 2.43945307e+04 1.99359191e+04
2.07551347e+04 2.12544575e+04 2.11126764e+04 1.98508290e+04]
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2.52414375e+03 2.61024691e+03 2.62580577e+03 2.76912479e+03
2.68748314e+03 2.89302628e+03 2.84549961e+03 2.99916443e+03
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3.09880571e+03 3.21170609e+03 3.32449702e+03 2.95395832e+03]
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2.35646031e+05 2.22262485e+05 2.20250174e+05 2.20117418e+05
2.18138866e+05 2.18174535e+05 2.19308110e+05 2.14444895e+05
2.09389008e+05 2.08328352e+05 2.07659631e+05 2.01898385e+05
2.13760312e+05 2.00033683e+05 1.95295944e+05 1.95039408e+05]
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7.96169981e+04 8.20215878e+04 8.16488273e+04 8.36662879e+04
8.36967494e+04 8.11578794e+04 8.10206461e+04 7.86247003e+04
7.89422909e+04 7.89415209e+04 8.08903639e+04 8.02983251e+04]
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5.44990166e+04 5.54764362e+04 5.48833750e+04 5.61450028e+04
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5.50716861e+04 5.69663456e+04 5.54409768e+04 5.26998529e+04
```

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4.11535544e+05 4.11240339e+05 4.03146506e+05 3.84946329e+05
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7.71952342e+04 8.51949945e+04 8.39308396e+04 8.38433293e+04
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7.13335981e+04 6.97742214e+04 6.76812488e+04 6.58848578e+04]
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1.78107904e+05 1.85010511e+05 1.87475719e+05 1.75492327e+05
1.55283126e+05 1.36767728e+05 1.42404667e+05 1.49257929e+05
1.48022889e+05 1.53271396e+05 1.51877025e+05 1.49329228e+05
1.51431555e+05 1.48708180e+05 1.43548869e+05 1.25528056e+05
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2.22967467e+06 2.26139154e+06 2.27449794e+06 2.33406689e+06
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2.57441201e+06 2.58430906e+06 2.61646355e+06 2.48198779e+06
2.60248343e+06 2.66517926e+06 2.70092524e+06 2.64306460e+06]
[7.42715107e+04 6.46445332e+04 5.92042798e+04 5.56892829e+04
5.30532990e+04 5.44055771e+04 5.47060659e+04 5.43969380e+04
5.29938430e+04 5.17004256e+04 4.97124777e+04 5.21276656e+04
5.01447587e+04 5.05865760e+04 5.13151625e+04 5.12873750e+04
5.12069150e+04 4.94050731e+04 5.00887365e+04 4.55576290e+04
4.64828738e+04 4.56040184e+04 4.31755885e+04 4.27924789e+04]
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2.01598957e+04 1.98782472e+04 2.01984457e+04 2.05227384e+04
2.06949725e+04 2.08318366e+04 2.15235484e+04 1.95953672e+04
1.96187649e+04 1.96264640e+04 1.90354209e+04 1.83137815e+04]
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7.01607817e+04 7.06013604e+04 6.95824418e+04 6.69683027e+04
6.69234245e+04 6.53726536e+04 6.31211225e+04 5.90527638e+04
 6.49970874e+04 6.09871477e+04 5.75783591e+04 5.59395233e+04]
[5.37199849e+04 5.56397238e+04 5.53284188e+04 5.26531435e+04
5.16210235e+04 5.24787556e+04 5.31862855e+04 5.19120054e+04
5.34927539e+04 5.32362713e+04 5.25034062e+04 5.40828663e+04
5.24811427e+04 5.36872737e+04 5.43087416e+04 5.49875709e+04
5.46097558e+04 5.26799643e+04 5.42100330e+04 5.29008178e+04
```

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5.44980863e+04 5.04204744e+04 5.17596721e+04 5.26412178e+04]
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      2.71755933e+05 2.71357955e+05 2.96810835e+05 2.78784946e+05
      2.87094568e+05 3.03694425e+05 3.16893705e+05 3.45231795e+05
      3.71320553e+05 4.03392486e+05 3.93100908e+05 3.82513419e+05
      3.95282510e+05 4.15868814e+05 4.47452331e+05 4.38819584e+05]
      [9.37954204e+05 8.53207752e+05 7.95446961e+05 7.05376576e+05
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      4.64400031e+05 4.37037554e+05 4.13923442e+05 4.33832546e+05
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      4.52089763e+05 4.55267445e+05 4.42172375e+05 3.84651788e+05
      4.01929093e+05 4.21635993e+05 4.09531353e+05 4.01066966e+05]
      [6.39714449e+06 6.33093789e+06 6.44867715e+06 6.56641586e+06
      6.65029660e+06 6.74852893e+06 6.94983800e+06 6.99999910e+06
      7.06678816e+06 7.09044261e+06 7.25897312e+06 7.14089914e+06
      7.18532814e+06 7.22497403e+06 7.36997003e+06 7.37877550e+06
      7.31614446e+06 7.42220796e+06 7.21641507e+06 6.77622977e+06
      6.98545705e+06 6.86539790e+06 6.64301058e+06 6.79997930e+06]]
[49]: | Ybar = np.array([AustraliaG[1:25], AustriaG[1:25], BelarusG[1:25], BelgiumG[1:
      \rightarrow25], CanadaG[1:25], CroatiaG[1:25],
                       CyprusG[1:25],Czech_RepublicG[1:25],DenmarkG[1:25],EstoniaG[1:
      \rightarrow25], European_UnionG[1:25],
                       FinlandG[1:25], FranceG[1:25], GermanyG[1:25], GreeceG[1:25]
      \rightarrow25], Hungary G[1:25], Iceland G[1:25],
                       IrelandG[1:25],ItalyG[1:25],JapanG[1:25],LatviaG[1:
      \rightarrow25], LithuaniaG[1:25], MaltaG[1:25],
                       NetherlandsG[1:25], New_ZealandG[1:25], NorwayG[1:25], PolandG[1:25]
      \rightarrow25], PortugalG[1:25],
                       RomaniaG[1:25],Russian_FederationG[1:25],SlovakiaG[1:
      \rightarrow25],SloveniaG[1:25],SwedenG[1:25],
                       SwitzerlandG[1:25], TurkeyG[1:25], UkraineG[1:
      →25],United_StatesG[1:25]])
     # Ybar is a matrix of greenhouse gas values for each country or area in the
      \rightarrow dataset
     # Each row is for a country or area
     # Each column is for a year from 1991 to 2014
     print(Ybar)
    [[4.18674057e+05 4.23080108e+05 4.23764933e+05 4.24092959e+05
      4.33478651e+05 4.39803962e+05 4.51721718e+05 4.66382112e+05
```

4.72245991e+05 4.83445827e+05 4.91441696e+05 4.94740101e+05 4.95154086e+05 5.11710242e+05 5.18850759e+05 5.22517192e+05 5.29842870e+05 5.33693508e+05 5.37889893e+05 5.33917436e+05

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0.0.5 DMD for the data set of carbon dioxide values

```
[50]: # Note: C is added to the end of the variable names to distinguish them from the variable names when

# DMD is utilized for the greenhouse gas values

# The comments will refer to each variable name without the C at the end of each name

from scipy.linalg import diagsvd

UC, sigma_arrayC, V_tC = np.linalg.svd(X)

# np.linalg.svd() calculates the corresponding values of U, the array of sigma_values, and V transpose for X

VC = V_tC.T # V is the transpose of V transpose

SigmaC = diagsvd(sigma_arrayC, 37, 24)

# Creates Sigma as a 37x24 rectangular diagonal matrix whose diagonal entries are the entries of the sigma array

# Sigma is known to be 37x24 because it has the same dimensions as X, which is au

→ 37x24 matrix
```

```
[51]: rC = np.linalg.matrix_rank(X) # r is the rank of X
     print(rC)
    24
[52]: UrC = UC[:,0:24] # Ur is a matrix containing the first r columns of U
     SigmarC = SigmaC[0:24,0:24] # Sigmar is a matrix containing the first r rows and
      → the first r columns of Sigma
     VrC = VC[:,0:24] # Vr is a matrix containing the first r columns of V
     Vr_starC = VrC.conj().T # Vr_star is the conjugate transpose of Vr
     Xr = np.dot(np.dot(UrC,SigmarC),Vr_starC)
     \# Xr = (Ur)(Sigmar)(Vr\_star)
[53]: Xplus = np.linalg.pinv(X) # Calculates Xplus, which is the pseudo-inverse of X
     Ur_starC = UrC.conj().T # Ur_star is the conjugate transpose of Ur
     ArC = np.dot(np.dot(np.dot(Ur_starC, Xbar), Xplus), UrC)
     \# Ar = (Ur\_star)(Xbar)(Xplus)(Ur)
     # Ar can also be found using the following: ArC = np.dot(np.dot(np.
      →dot(Ur_star, Xbar), VrC), np. linalg.inv(SigmarC))
     # In the above line, Ar = (Ur_star)(Xbar)(Vr)(Sigmar's inverse)
     eigenvalue_arrayC, WC = np.linalg.eig(ArC)
     # Determines the eigenvalues of Ar and places them as entries in the eigenvalue
      \hookrightarrow array
     # Also determines the eigenvectors of Ar and places each eigenvector in the \Box
      →array W as a column
[54]: PhiC = np.dot(np.dot(np.dot(Xbar, VrC), np.linalg.inv(SigmarC)), WC)
     # Phi = (Xbar)(Vr)(Sigmar's inverse)(W)
     BC = np.dot(np.linalg.pinv(PhiC),X[:,0])
     # B = (Phi's pseudo-inverse)(X's first column)
     np.shape(BC)
[54]: (24,)
[55]: def discreteC(k):
         xveck = np.zeros(37) # Initializes x vector k as a zero array
         for j in range(1,rC):
             xveck = xveck + np.dot(np.dot(PhiC[:,j],eigenvalue_arrayC[j]**(k -u)
      \rightarrow 1)), BC[j])
              # For j in range of 1 to r:
              # x vector k = (x \text{ vector } k) + (Phi's jth column)((eigenvalue array's jth_u)
      \rightarrow entry)^(k - 1))(B's jth entry)
         return xveck
```

0.0.6 DMD for the data set of greenhouse gas values

```
[57]: # Note: G is added to the end of the variable names to distinguish them from the
      →variable names when
     # DMD is utilized for the carbon dioxide values
     # The comments will refer to each variable name without the G at the end of each
      \rightarrow name
     from scipy.linalg import diagsvd
     UG, sigma_arrayG, V_tG = np.linalg.svd(Y)
     # np.linalq.svd() calculates the corresponding values of U, the array of sigma_{\sqcup}
      \rightarrow values, and V transpose for Y
     VG = V_tG.T # V is the transpose of V transpose
     SigmaG = diagsvd(sigma_arrayG, 37, 24)
     # Creates Sigma as a 37x24 rectangular diagonal matrix whose diagonal entries_
      →are the entries of the sigma array
     # Sigma is known to be 37x24 because it has the same dimensions as Y, which is a_{\sf L}
      \rightarrow 37x24 matrix
[58]: rG = np.linalg.matrix_rank(Y) # r is the rank of Y
     print(rG)
```

24

```
[59]: UrG = UG[:,0:24] # Ur is a matrix containing the first r columns of U

SigmarG = SigmaG[0:24,0:24] # Sigmar is a matrix containing the first r rows and

the first r columns of Sigma

VrG = VG[:,0:24] # Vr is a matrix containing the first r columns of V

Vr_starG = VrG.conj().T # Vr_star is the conjugate transpose of Vr

Yr = np.dot(np.dot(UrG,SigmarG),Vr_starG)

# Xr = (Ur)(Sigmar)(Vr_star)
```

```
[60]: Yplus = np.linalg.pinv(Y) # Calculates Yplus, which is the pseudo-inverse of Y
     Ur_starG = UrG.conj().T # Ur_star is the conjugate transpose of Ur
     ArG = np.dot(np.dot(np.dot(Ur_starG, Ybar), Yplus), UrG)
     \# Ar = (Ur\_star)(Ybar)(Yplus)(Ur)
     # Ar can also be found using the following: ArG = np.dot(np.dot(np.
      →dot(Ur_starG, Ybar), VrG), np.linalg.inv(SigmarG))
     # In the above line, Ar = (Ur\_star)(Ybar)(Vr)(Sigmar's inverse)
     eigenvalue_arrayG, WG = np.linalg.eig(ArG)
     # Determines the eigenvalues of Ar and places them as entries in the eigenvalue
      \rightarrow array
     # Also determines the eigenvectors of Ar and places each eigenvector in the \Box
      →array W as a column
[61]: PhiG = np.dot(np.dot(np.dot(Ybar, VrG), np.linalg.inv(SigmarG)), WG)
     # Phi = (Ybar)(Vr)(Sigmar's inverse)(W)
     BG = np.dot(np.linalg.pinv(PhiG),Y[:,0])
     # B = (Phi's pseudo-inverse)(Y's first column)
[62]: def discreteG(k):
         yveck = np.zeros(37) # Initializes y vector k as a zero array
         for j in range(1,rG):
             yveck = yveck + np.dot(np.dot(PhiG[:,j],eigenvalue_arrayG[j]**(k -__
      \rightarrow 1)), BG[j])
              # For j in range of 1 to r:
              # y vector k = (x \text{ vector } k) + (Phi's jth \text{ column})((eigenvalue array's jth_u))
      \rightarrow entry)^(k - 1))(B's jth entry)
         return yveck
[63]: def continuousG(k):
         yveck = np.zeros(37) # Initializes y vector k as a zero array
         for j in range(1,rG):
             yveck = yveck + np.dot(np.dot(PhiG[:,j],np.exp(np.
      →log(eigenvalue_arrayG[j])/(1/(k - 1)))),BG[j])
              # For j in range of 1 to r:
              # y vector k = (y \ vector \ k) + (Phi's jth \ column)(e**(omega's jth_u))
      \rightarrow entry))(B's jth entry)
              # omega's jth entry = log(eigenvalue array's jth entry)/deltat
              # deltat = 1/(k - 1)
         return yveck
```

0.0.7 Carbon dioxide DMD visualizations

```
[64]: plt.figure(figsize = (12,8), dpi = 80)

plt.scatter(np.linspace(1,37,num=37),X[:,20], s = 100, marker = 's', color = 'green', label = '2010 CO2 Data Set')

plt.scatter(np.linspace(1,37,num=37),discreteC(20), s = 100, marker = 'o', color = 'blue', label = '2010 CO2 Discrete DMD')

plt.scatter(np.linspace(1,37,num=37),continuousC(20), s = 100, marker = 'o', color = 'red', label = '2010 CO2 Continuous DMD')

# Plots the 2010 carbon dioxide emissions from the data set, the 2010 carbon dioxide discrete DMD column,

# and the 2010 carbon dioxide continuous DMD column

plt.xlabel('Country / Area')

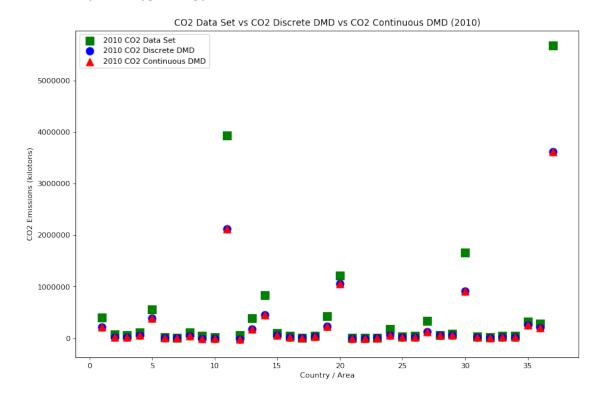
plt.ylabel('CO2 Emissions (kilotons)')

plt.title('CO2 Data Set vs CO2 Discrete DMD vs CO2 Continuous DMD (2010)')

plt.legend()

plt.show()
```

C:\Users\Ethan\Anaconda3\lib\site-packages\numpy\core\numeric.py:591:
ComplexWarning: Casting complex values to real discards the imaginary part
return array(a, dtype, copy=False, order=order, subok=True)



```
[65]: DMDmatrixC = np.
      →array([discreteC(0),discreteC(1),discreteC(2),discreteC(3),discreteC(4),discreteC(5),discrete
      -discreteC(7), discreteC(8), discreteC(9), discreteC(10), discreteC(11), discreteC(12), discreteC(13)
      →discreteC(14), discreteC(15), discreteC(16), discreteC(17), discreteC(18), discreteC(19),
      →discreteC(20),discreteC(21),discreteC(22),discreteC(23),discreteC(24),discreteC(25),
                            discreteC(26),discreteC(27),
      →discreteC(28),discreteC(29),discreteC(30),discreteC(31),
                           discreteC(32),discreteC(33), discreteC(34)])
     DMDmatrixC = DMDmatrixC.T
     # Turns DMDmatricC into a 35x37 matrix
     # Creates a 2D numpy array containing the first 35 columns of the discrete {\tt DMD_U}
      →function for carbon dioxide values
     # Each column corresponds to a year of carbon dioxide emissions in each country \Box
      →or area from 1990 to 2024
[66]: plt.figure(figsize = (12,8), dpi = 80)
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixC[:,25], s = 100, marker = 's',__

→color = 'green', label = '2015')
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixC[:,26], s = 100, marker = 'o',__

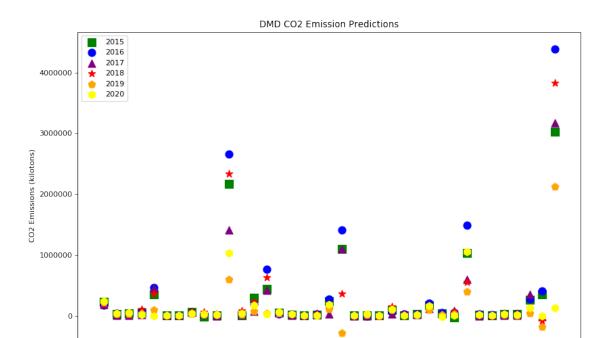
color = 'blue', label = '2016')
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixC[:,27], s = 100, marker = '^',__

→color = 'purple', label = '2017')
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixC[:,28], s = 100, marker = '*', u

→color = 'red', label = '2018')
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixC[:,29], s = 100, marker = 'p',__

→color = 'orange', label = '2019')
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixC[:,30], s = 100, marker = 'h', u

→color = 'yellow', label = '2020')
     # Plots the 2015-2020 estimates for carbon dioxide emission values from each
      →country or area
     plt.xlabel('Country / Area')
     plt.ylabel('CO2 Emissions (kilotons)')
     plt.title('DMD CO2 Emission Predictions')
     plt.legend()
     plt.show()
```



```
[67]: UNdataC = np.

array([[402995,413370],[66704,67402],[58966,60509],[100229,100244],[565577,558189],[17997,182],

[6903,7308],[104785,106543],[36625,38472],[15891,17494],[3510586,3489056],[44346,47926],

[344390,346790],[797078,801753],[74962,71373],[46665,47578],[3536,3490],[38444,39928],

[355483,350323],[1223666,1204319],[7334,7264],[13142,13157],[1758,1420],[165030,165522],

[35837,34463],[44664,44032],[310526,321182],[52205,50285],[77788,75052],[1671895,1668070],

[33897,33997],[13599,14400],[43386,42568],[38739,39205],[380858,402821],[223580,235156],

[5420804,5310861]])

# This is an array containing the 2015 and 2016 data on carbon dioxide emissions,

afor the same countries and areas

# in the matrix X. This data was retrieved from: http://data.un.org/Data.aspx?

ad=GHG8f=seriesID%3aCO2

[68]: plt.figure(figsize = (12,8), dpi = 80)

plt.scatter(np.linspace(1,37,num=37),DMDmatrixC[:,25], s = 100, marker = 's',u

acolor = 'green', label = '2015 CO2 DMD')
```

Country / Area

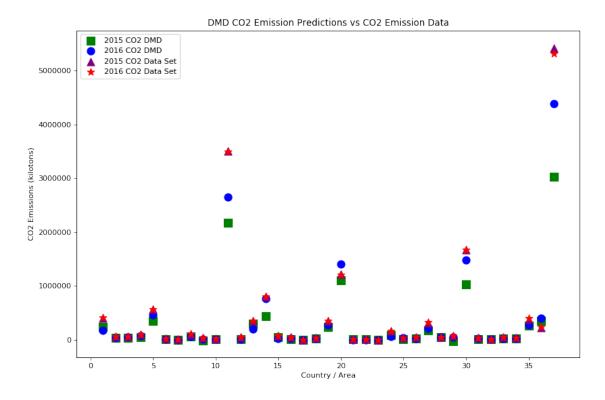
25

30

```
plt.scatter(np.linspace(1,37,num=37),DMDmatrixC[:,26], s = 100, marker = 'o', which is a color = 'blue', label = '2016 CO2 DMD')

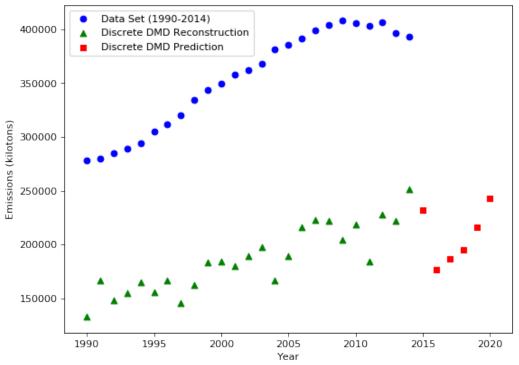
# Plots the 2015-2016 estimates for carbon dioxide emission values from each which executive or area

plt.scatter(np.linspace(1,37,num=37),UNdataC[:,0], s = 100, marker = 'o', color which executive is a color which
```

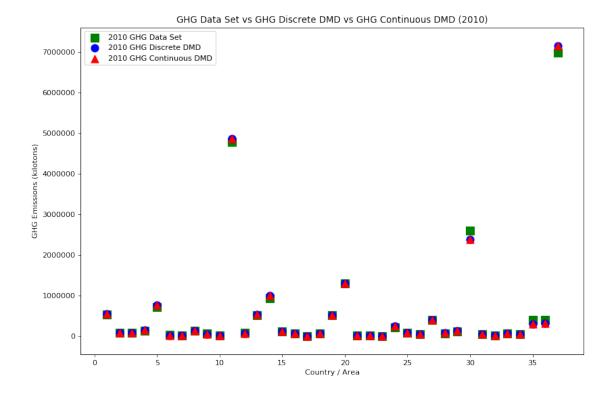


```
plt.scatter(x, AustraliaC, marker = 'o', color = 'blue', label = 'Data Setu
 plt.scatter(x, DMDmatrixC[0,0:25], marker = '^', color = 'green', label = __
→'Discrete DMD Reconstruction')
plt.scatter(x1, DMDmatrixC[0,25:31], marker = 's', color = 'red', label =
→ 'Discrete DMD Prediction')
# Plots the Australia carbon dioxide emission levels data set, the discrete DMD_
→reconstruction of Australia's carbon
# dioxide data set from 1990 to 2014, and the discrete DMD predictions for \Box
→ Australia's carbon dioxide emission levels
# from 2015 to 2020
plt.title('Annual CO2 Emissions vs CO2 DMD Reconstruction vs CO2 DMD Predictions,
 →in Australia')
plt.xlabel('Year')
plt.ylabel('Emissions (kilotons)')
plt.legend()
plt.show()
```

Annual CO2 Emissions vs CO2 DMD Reconstruction vs CO2 DMD Predictions in Australia



0.0.8 Greenhouse gas DMD visualizations



```
[71]: DMDmatrixG = np.

array([discreteG(0),discreteG(1),discreteG(2),discreteG(3),discreteG(4),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),discreteG(5),
```

```
-discreteG(7), discreteG(8), discreteG(9), discreteG(10), discreteG(11), discreteG(12), discreteG(13
      →discreteG(14), discreteG(15), discreteG(16), discreteG(17), discreteG(18), discreteG(19),
      →discreteG(20),discreteG(21),discreteG(22),discreteG(23),discreteG(24),discreteG(25),
                             discreteG(26),discreteG(27),_
      \rightarrowdiscreteG(28),discreteG(29),discreteG(30),discreteG(31),
                            discreteG(32),discreteG(33), discreteG(34)])
     DMDmatrixG = DMDmatrixG.T
     # Turns DMDmatricG into a 35x37 matrix
     # Creates a 2D numpy array containing the first 35 columns of the discrete DMD_{f \sqcup}
      → function for greenhouse gas values
     # Each column corresponds to a year of greenhouse gas emissions in each country_{f \sqcup}
      →or area from 1990 to 2024
[72]: plt.figure(figsize = (12,8), dpi = 80)
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixG[:,25], s = 100, marker = 's',__

→color = 'green', label = '2015')
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixG[:,26], s = 100, marker = 'o',__

→color = 'blue', label = '2016')
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixG[:,27], s = 100, marker = '^',u

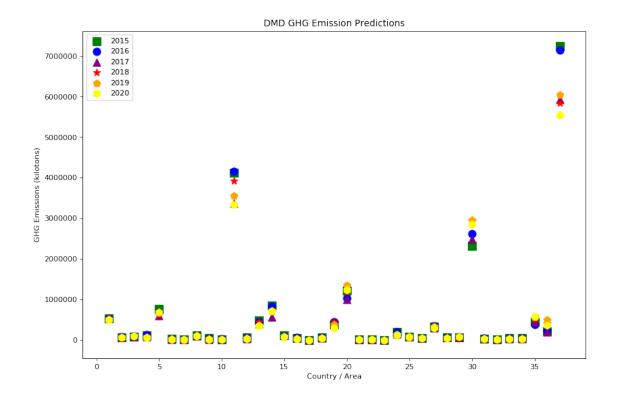
color = 'purple', label = '2017')

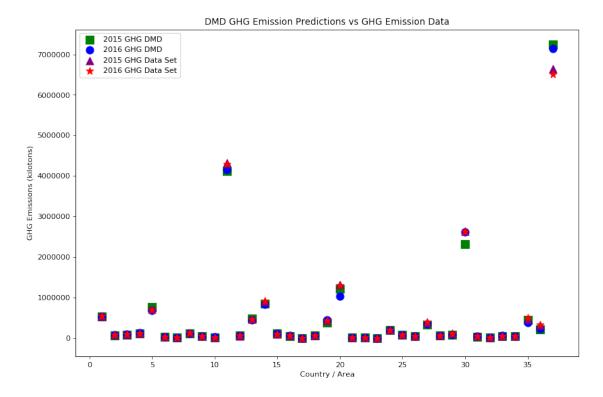
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixG[:,28], s = 100, marker = '*',__

→color = 'red', label = '2018')
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixG[:,29], s = 100, marker = 'p',__

color = 'orange', label = '2019')
     plt.scatter(np.linspace(1,37,num=37),DMDmatrixG[:,30], s = 100, marker = 'h', u

color = 'yellow', label = '2020')
     # Plots the 2015-2020 estimates for greenhouse gas emission values from each
      \rightarrow country or area
     plt.xlabel('Country / Area')
     plt.ylabel('GHG Emissions (kilotons)')
     plt.title('DMD GHG Emission Predictions')
     plt.legend()
     plt.show()
```





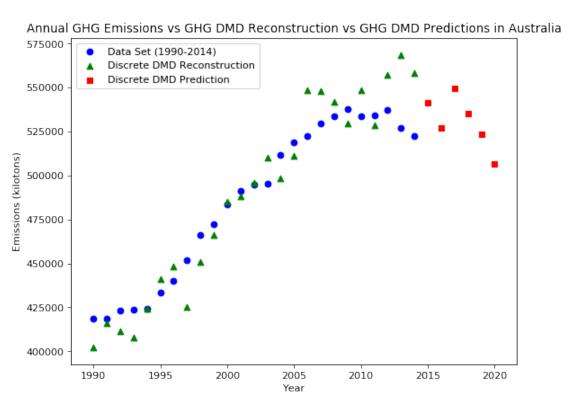
```
plt.scatter(x1, DMDmatrixG[0,25:31], marker = 's', color = 'red', label = 'DMD Prediction')

# Plots the Australia greenhouse gas emission levels data set, the discrete DMD preconstruction of Australia's

# greenhouse gas data set from 1990 to 2014, and the discrete DMD predictions → for Australia's greenhouse gas

# emission levels from 2015 to 2020

plt.title('Annual GHG Emissions vs GHG DMD Reconstruction vs GHG DMD Predictions → in Australia')
plt.xlabel('Year')
plt.ylabel('Emissions (kilotons)')
plt.legend()
plt.show()
```



[]: