Nonlinear model predictive control of PVTOL aircraft under state and input constraints

Ankit Gupta
Control and Dynamics Lab
Electrical Department
VJTI Mumbai
ankitgupta47@gmail.com

Manas Mejari
Control and Dynamics Lab
Electrical Department
VJTI Mumbai
manas.mejari@gmail.com

Vishwanath Ketkar
Control and Dynamics Lab
Electrical Department
VJTI Mumbai
vishwanath91@gmail.com

Mandar Datar
Control and Dynamics Lab
Electrical Department
VJTI Mumbai
mandar.k.datar@gmail.com

N.M.Singh
Control and Dynamics Lab
Electrical Department
VJTI Mumbai
nmsingh59@gmail.com

ABSTRACT
The paper presents a nonlinear model predictive control (NMPC) strategy for stabilization and trajectory tracking control of planar vertical Take-off and landing (PVTOL) aircraft. PVTOL system is considered as a benchmark for investigating dynamics and control related issues for unmanned aerial vehicles (UAVs). Control problem of such systems is made challenging due to their under-actuated nature and nonlinear dynamics. We propose a nonlinear MPC methodology to effectively deal with such nonlinearities. Also, the proposed control strategy takes into account state (roll angle) as well as input thrust constraints. Validation of control law is performed through MATLAB simulations for various operating conditions and coupling scenarios.

Categories and Subject Descriptors
G.1.6 [**Optimization**]: Constrained optimization, Convex programming, Quadratic programming methods, I.2.8 [**Problem Solving, Control Methods, and Search**]: Control theory, Dynamic programming, I.2.9 [**Robotics**]: Autonomous vehicles.

General Terms
Algorithms, Performance, Design, Theory.

Keywords
NMPC, PVTOL aircraft, UAV, under actuated system, QP.

1. INTRODUCTION
PVTOL aircraft systems are benchmark system used in controls community to study dynamics of aircrafts with capabilities of vertical take-off and landing. The dynamics for PVTOL system was taken from [2]. The system has three degrees of freedom and two control inputs. Its dynamics is nonlinear, underactuated and non-minimum phase because of input coupling. Control of such system became topic of interest for researchers worldwide. This has led to various control strategy proposed for PVTOL system. Many aircrafts like helicopter and Harrier Jump jet use PVTOL technique for takeoff and landing. The key advantage of PVTOL capability is that it enables an aircraft to hover over an area, allowing for longer loiter time, and allow the craft to operate from a diverse array of airfields, using less space to get airborne. Various control strategy has been proposed on PVTOL system for stabilization and tracking problem by different researcher over the years. In [2] for bounded tracking and asymptotic stability an approximate input-output linearization technique for non-minimum phase system using feedback is proposed. In [15] has used same technique for approximation purpose and proposed a back-stepping approach for control. A robust optimal control strategy is proposed in [9]. Generally the problems related to control can be broadly divided into two categories, first is stabilizing control and second is tracking control problem. Paper [1], [3], [5], [7] gives the controller design for stabilizing PVTOL Aircraft. A global stabilizing controller is proposed in [6]. In [4] stabilizing controller is proposed considering the PVTOL is under cross wind. Similarly there are many control strategy in literature proposed for trajectory tracking problem like a [8], [16]. In [17] a nonlinear optimal control strategy is proposed for trajectory tracking. A variety of different control techniques have been applied on PVTOL system to exploit their benefits. An IDA-PBC technique for stabilization is proposed in [10], and a sliding mode control technique is used in [12]. A passivity based controller is proposed in [14]. A robust optimal control strategy is proposed in [9]. A multirate digital control scheme for a PVTOL system is proposed in [18]. In [3] a global discrete-time fast predictive control is proposed, complete problem was divided into two quadratic programing problem first problem focuses only on controlling roll angle of PVTOL and other focuses on translation of PVTOL from initial point to final point.

In this paper we focus on NMPC technique. The main advantage of this technique is we can frame the whole problem in one QP problem. Almost all systems today have to work under certain state and input constraints, also the system are multi-input and multi-output. Many nonlinear control strategies are available today like feedback linearization, constructive Lyapunov-based

AIR'13, July 04 - 06 2013, Pune, India

http://dx.doi.org/10.1145/2506095.2506123

methods lead to very elegant solutions, but they depend on complicated design procedures that do not scale well to complex systems and they are not developed in order to handle constraints in a systematic manner. NMPC technique on other employs solving finite time optimal control problem continuously to obtain optimal solution, Because of its optimal solution and ability to handle constraints together NMPC is one of the most preferred control technique. Also NMPC technique can easily handle system with multi-input multi-output. Initially MPC technique was applied only to system with slow dynamical behavior but because of presence of high end solvers and high speed processors this technique can be applied to system with faster dynamics.

The main contribution of this paper is to synthesize a control law for PVTOL system based on NMPC control strategy. The paper considers PVTOL as a highly constrained system with bounds on input thrust generated by the engine of aircraft and also there are state constraints like aircraft should not roll during transition period. We have considered gravity as constant disturbance acting on the complete system.

The outline of this paper is as follows: The non-linear model for PVTOL system is discussed in Section 2. General MPC theory is discussed in Section 3. NMPC controller design is proposed in Section 4. Section 5 is dedicated for simulation of PVTOL under NMPC. In Section 6 concluding remarks is given for proposed strategy.

2. DYNAMICAL MODEL OF THE PVTOL AIRCRAFT

The nonlinear dynamics of PVTOL system in [2] is used in this paper. From figure it can be seen that aircraft has three degree of freedom along (x, y, θ) and system is under actuated with two inputs 'u_1' and 'u_2'.The proposed system is a non-minimum phase system.

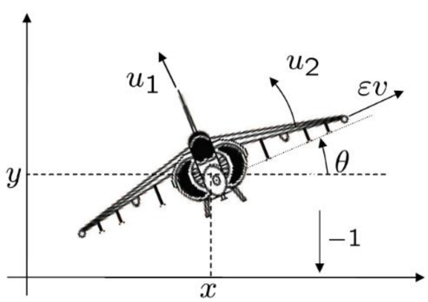

Figure 1. PVTOL aircraft [3]

The dynamics of the PVTOL aircraft system is given below,

$$\dot{x}_1 = x_2 \tag{1}$$

$$\dot{x}_2 = -u_1 \sin(x_5) + \varepsilon \cos(x_5) u_2 \tag{2}$$

$$\dot{x}_3 = x_4 \tag{3}$$

$$\dot{x}_4 = u_1 \cos(x_5) + \varepsilon \sin(x_5) u_2 - 1 \tag{4}$$

$$\dot{x}_5 = x_6 \tag{5}$$

$$\dot{x}_6 = u_2 \tag{6}$$

It can be written in the form,

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}) + d \tag{7}$$

Where,

$\boldsymbol{x} = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T = [x \quad \dot{x} \quad y \quad \dot{y} \quad \theta \quad \dot{\theta}]^T$, $\boldsymbol{u} = [u_1 \quad u_2]^T$

And $d = [0 \quad 0 \quad 0 \quad -1 \quad 0 \quad 0]^T$.

Where, x and y represents the position of center of mass of PVTOL aircraft, while the 'θ' represents the roll angle of aircraft with respect to horizontal axis. The term '-1' in the dynamics represents the normalized acceleration due to gravity (g=9.81m/sec^2). Here 'u_1' and 'u_2' represent the normalized thrust required by aircraft for upward motion and rolling moment of the aircraft. 'ε' represents the coupling between lateral motion of aircraft and rolling moment (0<ε<1). Our aim here is to move the aircraft to a predefined point and to stabilize the roll angle 'θ' at zero. As in [3] to achieve desired stabilization point roll angle 'θ' must evolve positively and negatively as required before saturating at $\theta = 0$ since control input u_1 is always positive, so when ε = 0 'u_2' will not be able to effect the translation dynamics, hence to achieve required translation 'θ' must evolve positively and negatively or else it will only have vertical motion.

3. MODEL PREDICTIVE CONTROL

At each sampling time, starting at the current state, an open-loop optimal control problem is solved over a finite horizon. The computed optimal input signal is applied to the process.The states of the system is updated at each sampling period and a new optimization problem is solved at each sampling interval over a shifted horizon. This is known as the model predictive control.

Problem 1:

$$\mathrm{J}\big(x(k), u(k)\big) = \min_{u_k} (x_N - r_N)^T P (x_N - r_N) + \sum_{k=0}^{N-1} (x_k - r_k)^T Q (x_k - r_k) + u_k^T R \, u_k$$

Subj to: $x(k+1) = A\,x(k) + Bu(k) + d$

$$where\ k = 0,1..N-1$$

$$x_k \in \mathcal{X}, u_k \in \mathcal{U}\ where\ k = 0,1..N-1$$

$$x_N \in \mathcal{X}_f$$

As shown in **Problem 1** the MPC control problem of a system involves an objective function $\mathrm{J}\big(x(k), u(k)\big)$ which is to be minimized or maximized, set of constraints $(\mathcal{X}, \mathcal{U})$ which should be satisfied at every instant and $[A, B, d]$ is linear dynamics of system to be controlled, also $P, Q \geq 0$ and $R > 0$. At all sampling instant **Problem 1** is solved with updated variables. r_N is desired terminal position of PVTOLs center of mass, while r_k is determines the reference trajectory given to PVTOL.

4. CONTROLLER DESIGN

Here we use Batch Approach as given in [13]. Consider a linear system with constant disturbance model below,

$$\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + B\boldsymbol{u}(k) + d.$$

Here 'd' is constant disturbance acting on the system.

'A' and 'B' can be obtained from (7)

$$A = \frac{\partial f}{\partial \boldsymbol{x}} \text{ And } B = \frac{\partial f}{\partial \boldsymbol{u}}$$

First we write the equality constraints explicitly to express all future states $\boldsymbol{x}_1, \boldsymbol{x}_2 \ldots \boldsymbol{x}_N$ as a function of the future inputs

$\boldsymbol{u}_1, \boldsymbol{u}_2 \dots \boldsymbol{u}_{N-1}$ and then we eliminate all intermediate states by successive substitution to obtain,

$$\underbrace{\begin{bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \vdots \\ \boldsymbol{x}_N \end{bmatrix}}_{\mathcal{X}} = \underbrace{\begin{bmatrix} I \\ A \\ A^2 \\ \vdots \\ \vdots \\ A^N \end{bmatrix}}_{S^x} \boldsymbol{x}(0)$$

$$+ \underbrace{\begin{bmatrix} 0 & \cdots & \cdots & 0 \\ B & \cdots & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & 0 \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix}}_{S^u} \underbrace{\begin{bmatrix} \boldsymbol{u}_0 \\ \vdots \\ \vdots \\ \vdots \\ \boldsymbol{u}_{N-1} \end{bmatrix}}_{\mathcal{U}_0} + \underbrace{\begin{bmatrix} 0 \\ I \\ I + A \\ I + A + A^2 \\ \vdots \\ \vdots \\ I + A + \cdots A^{N-1} \end{bmatrix}}_{S^d} d$$

Here all future states are explicit functions of the present state $\boldsymbol{x}(0)$, disturbance $'d'$ and the future inputs $\boldsymbol{u}_1, \boldsymbol{u}_2 \dots \boldsymbol{u}_{N-1}$ only. By defining the appropriate quantities we can rewrite this expression compactly as,

$$\mathcal{X} = S^x\, \boldsymbol{x}(0) + S^u \mathcal{U}_0 + S^d \mathrm{d} \tag{8}$$

Using the same notation the objective function in **Problem 1** can be rewritten as

$$\mathrm{J}(\boldsymbol{x}(0), \mathcal{U}_0) = (\mathcal{X} - r)^T \bar{Q} (\mathcal{X} - r) + \mathcal{U}_0^T \bar{R}\, \mathcal{U}_0 \tag{9}$$

Here,

$$\bar{Q} = \text{blockdiag}\{Q,\ Q\ \dots P\},\ \bar{Q} \geq 0, \text{ and}$$

$$\bar{R} = \text{blockdiag}\{\ R,\ R\ \dots R\},\ \bar{R} > 0,$$

$'r'$ is given reference trajectory or terminal position

From (8) and (9),

$$\mathrm{J}(\boldsymbol{x}(0), \mathcal{U}_0) = \mathcal{U}_0^T H \mathcal{U}_0 + F \mathcal{U}_0 + constant\ terms \tag{10}$$

Where,

$$H = {S^u}^T \bar{Q} S^u + \bar{R}$$

$$F = 2(\boldsymbol{x}(0)^T {S^x}^T + d^T {S^d}^T - r^T) \bar{Q} S^u$$

Problem 2:

$$\mathrm{J}(\boldsymbol{x}(0), \mathcal{U}_0) = \min_{\mathcal{U}_0} \mathcal{U}_0^T H \mathcal{U}_0 + F \mathcal{U}_0$$

$$\text{Subj to: } V\mathcal{U}_0 \leq W$$

Here, $'V'$ matrix is constraint depended matrix. Because $\bar{R} > 0$, also $H > 0$. Thus $\mathrm{J}(\boldsymbol{x}(0), \mathcal{U}_0)$ is a positive definite quadratic function of $\mathcal{U}_0$. Therefore, its minimum can be found by Quadratic programming technique on **Problem 2**. The above strategy returns the sequence of input $\boldsymbol{u}_1, \boldsymbol{u}_2 \dots \boldsymbol{u}_{N-1}$. Only first sample of the sequence is used as input to the system and others are discarded. The whole process is repeated for next sampling instant.

The NMPC approach involves linearizing system at every sampling instant and forming discrete LTI system. Solving **Problem 2** for each sampling time returns optimal input for next sampling time. The system is linearized on next instant using current states and input. The recursive application of above process leads to NMPC behavior. The above process returns sub-optimal solution for control.

The whole process is summarized in below algorithm.

NMPC Algorithm:

1) Obtain measurements of the states of the system (at k=0 use $\boldsymbol{x} = \boldsymbol{x}_0$ and $\boldsymbol{u} = \boldsymbol{u}_0$). Linearize the system model using current state and input value to obtain matrices A and B.
2) Compute an optimal input sequence by minimizing given cost function over a certain prediction horizon using Quadratic Programming (i.e. Solve **Problem 2**).
3) Implement the first part of the optimal input sequence obtained earlier.
4) Continue with step 1 until end time is reached.

5. SIMULATION

Simulation of PVTOL under proposed controller was performed using MATLAB to verify the results.

We have assumed following constraints on system,

a) $0 \leq u_1 \leq 4$
b) $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

Here we have considered two cases for simulation purpose.

5.1 Stabilization of PVTOL at desired point

The proposed scheme uses both the inputs to achieve desired position. We have taken initial condition as $x = 0.1, y = 0.7\ and\ \theta = 0\ radians$ and desired stabilization point is $x = 5, y = 6\ and\ \theta = 0\ radians$.For simulation purpose we considered two cases in first ε = 0 and in second ε = 1. Fig 2-4 Shows trajectory in 'x' and 'y' direction and roll angle of PVTOL during transition for ε = 0. Fig 5-6 shows behavior of controller when ε = 0. It can be seen that the PVTOL reaches the desired position easily within 25 sec satisfying all the constraints.

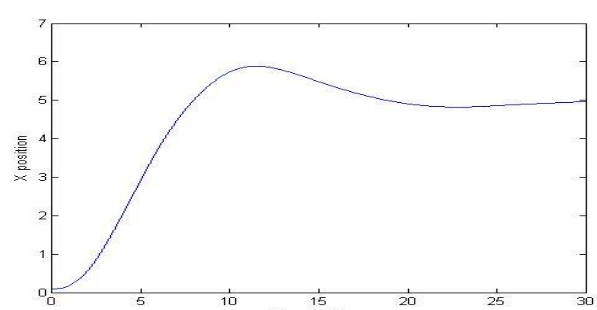

Figure 2. Trajectory in x direction (ε = 0)

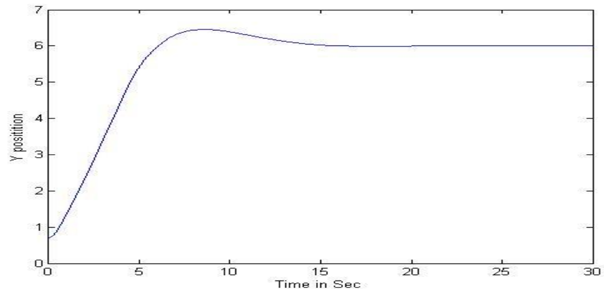

Figure 3. Trajectory in y direction (ε = 0)

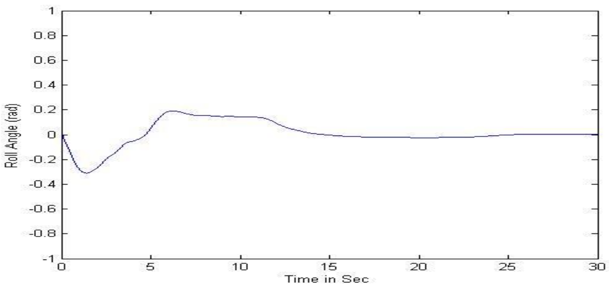

Figure 4. Roll angle (ε = 0)

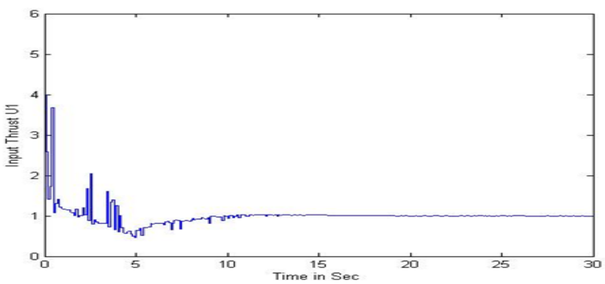

Figure 5. Input Thrust (ε = 0)

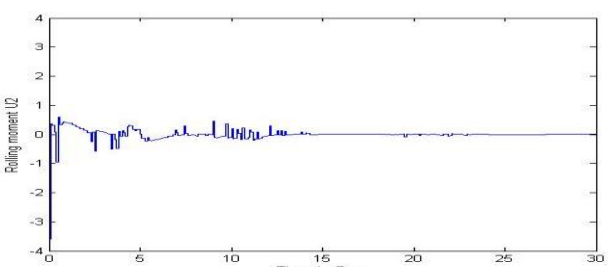

Figure 6. Rolling Moment (ε = 0)

For ε = 1, Fig 7-9 Shows translation and roll angle of PVTOL during transition. Fig 10-11 shows behavior of control inputs.

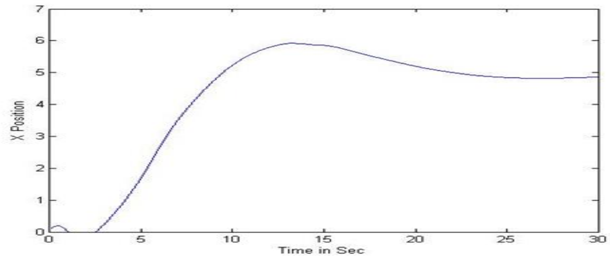

Figure 7. Trajectory in x direction (ε = 1)

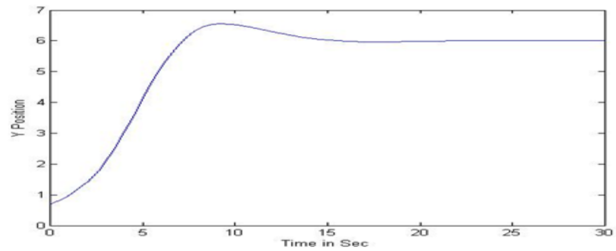

Figure 8. Trajectory in y direction (ε = 1)

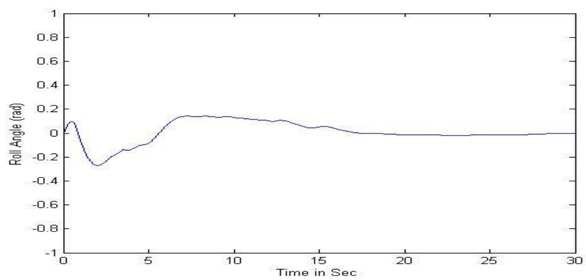

Figure 9. Roll angle (ε = 1)

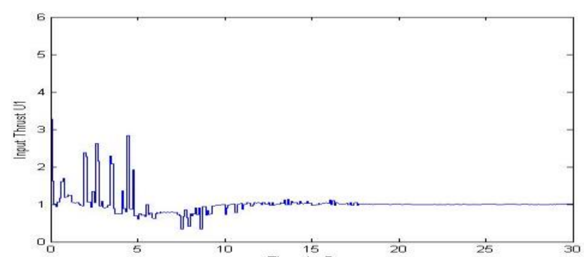

Figure 10. Input Thrust (ε = 1)

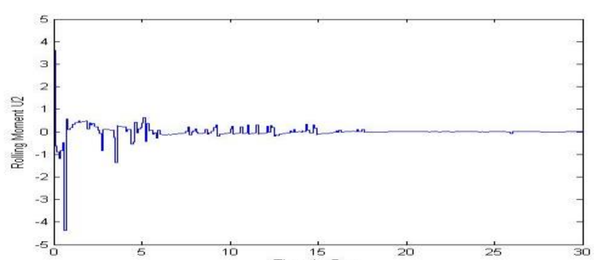

Figure 11. Rolling Moment (ε = 1)

Here also PVTOL reaches the desired position within 28 sec satisfying all the constraints. Also in both the cases it can be seen that reaching desired point in 'x' direction requires more time as compared to 'y' since there is no direct control available for achieving required 'x' position it is achieved through combination of thrust and rolling moment. Required 'y' position can be directly achieved through input thrust

5.2 Tracking a given Path

A reference path is given for tracking purpose. Fig 12 shows the path tracking ability of PVTOL using proposed controller. Here we have given a circular path of radius =3 as a reference path to PVTOL. The initial position for center of mass of PVTOL is taken as (0.707, 0.707). The following equation is used for reference path

$$x = 3\cos(0.1t) + 4$$
$$y = 3\sin(0.1t) + 4$$

It can be seen that PVTOL is able to track the given path using the mentioned controller. Also all constrains are satisfied during tracking.

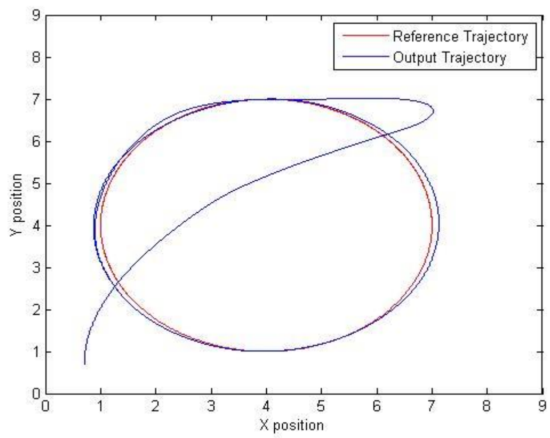

Figure 12. Path Tracking

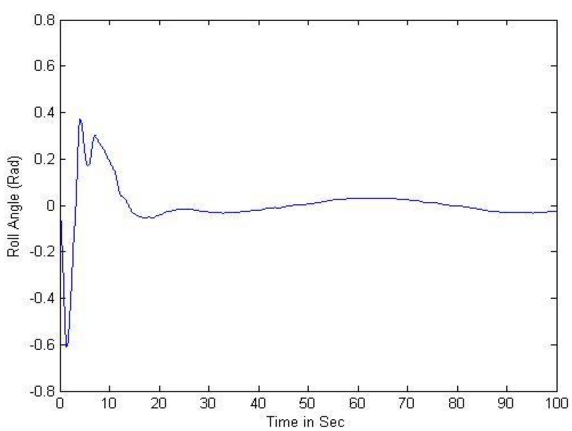

Figure 13. Roll Angle for Tracking

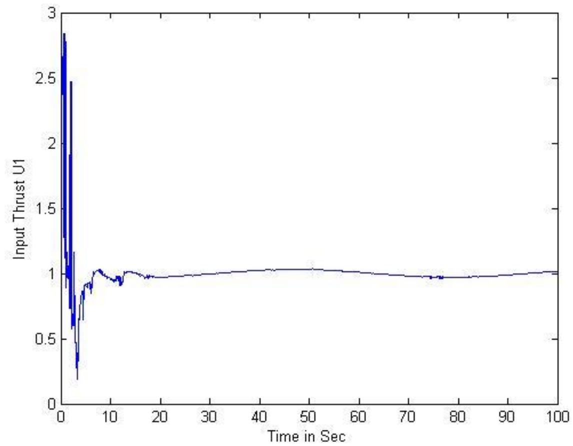

Figure 14. Input Thrust for Tracking

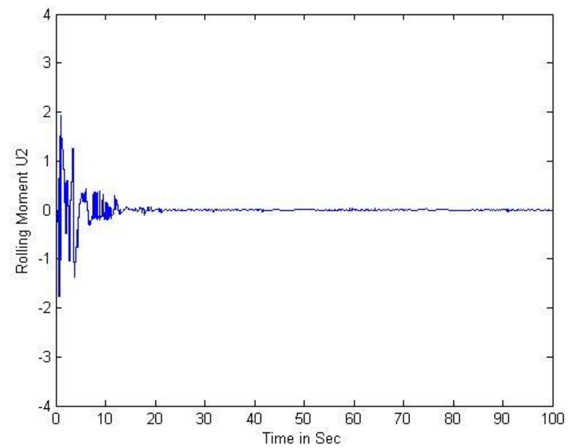

Figure 15. Rolling Moment for Tracking

6. CONCLUSION

In this paper we have proposed a NMPC controller for PVTOL aircraft because of its ability to handle constraints and MIMO systems. From simulation results it can be verified that PVTOL under proposed controller can easily be stabilized at a given point. Also PVTOL can track a given reference path while satisfying the bounds on input thrust and rolling angle. The control strategy is found to be effective with or without coupling. Thus the PVTOL system is stabilizable using the NMPC control.

7. ACKNOWLEDGEMENT

We acknowledge support of Centre of Excellence (CoE) in Complex and Non-linear Dynamical Systems.

8. REFERENCES

[1] Michael T. Frye, Shihong Ding, Chunjiang Qian, and Shihua Li. 2009. Global Finite-Time Stabilization of a PVTOL Aircraft by Output Feedback. *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference*. Page: 2831- 2836.

[2] John Hauser, Shankar Sastry, George Meyer.1992. Nonlinear control design for slightly non-minimum phase systems: Application to V/STOL aircraft. *Automatica* (1992), *Volume* 28, *Issue* 4, Pages: 665-679.
[3] Ahmed Chemori, Nicolas Marchand. 2008. Global discrete-time stabilization of the PVTOL aircraft based on fast predictive control. *Proceedings of the 17th IFAC World Congress* (2008), *Volume* 17, *Part* 1, Pages: 1747-1752.
[4] Munoz Laura E, Santos Omar, Castillo Pedro. 2010. Robust nonlinear real-time control strategy to stabilize a PVTOL aircraft in crosswind. *IEEE/RSJ International Conference on Intelligent Robots and Systems* (2010), Pages: 1606-1611.
[5] Wood R, Cazzolato B. 2007. An Alternative Nonlinear Control Law for the Global Stabilization of the PVTOL Vehicle. *IEEE Transactions on Automatic* Control (2007), vol.52, no.7, Pages: 1282-1287.
[6] Olfati-Saber R. 2000. Global configuration stabilization for the VTOL aircraft with strong input coupling. *Proceedings of the 39th IEEE Conference on Decision and Control* (2000), vol.4 Pages: 3588-3589.
[7] Turker Turker, Tugce Oflaz, Haluk Gorgun and Galip Cansever. 2012. A Stabilizing Controller for PVTOL Aircraft. *American Control Conference* (*2012) Pages*: 909-913.
[8] Nielsen C, Consolini L, Maggiore M, Tosques M.2008. Path following for the PVTOL: A set stabilization approach. 47th *IEEE Conference on Decision and* Control (2008). Pages: 584-589.
[9] Feng Lin, Zhang W, Brandt R.D.1999. Robust hovering control of a PVTOL aircraft. *IEEE Transactions on Control Systems Technology*, vol.7, no.3, Pages: 343-351.
[10] Acosta, J.A, Ortega R, Astolfi A, Mahindrakar A.D.2005. Interconnection and damping assignment passivity-based control of mechanical systems with underactuation degree one. *IEEE Transactions on Automatic Control* (2005), vol.50, no.12, Pages: 1936-1955.
[11] Wood R, Cazzolato B, Halim D.2005. A Global Non-linear Control Design for a PVTOL Vehicle with Aerodynamics. *44th IEEE Conference on Decision and Control, 2005 and 2005 European Control Conference*, Pages: 7478-7483.
[12] Cardenas R, Aguilar L.T.2011. Output feedback sliding mode control of a PVTOL including actuators dynamics. *IEEE International Conference on Control Applications (CCA)*, 2011, Pages: 1482-1486.
[13] F. Borrelli, A. Bemporad, M. Morari.2011. Predictive Control for linear and hybrid systems. *Cambridge February 2011*.
[14] Venkatesh C, Mehra R, Kazi F, Singh N.M.2013. Passivity based controller for underactuated PVTOL system. *IEEE International Conference on Electronics, Computing and Communication Technologies* (2013), Pages: 1-5.
[15] R. Sepulchre, M. Jankovic, and P. Kokotovic.1997. Constructive Nonlinear Control, ser. Communications and control engineering. *London; NewYork: Springer, 1997*, IEEE Standard C57.19.100-1995, Aug. 1995.
[16] Gruszka A, Malisoff M, Mazenc F.2011. On tracking for the PVTOL model with bounded feedbacks. *American Control Conference (ACC)*, Pages: 1428-1433.
[17] Benosman M, Liao F, Lum K.-Y.2007. Output trajectory tracking for the PVTOL aircraft through control allocation. *IEEE International Conference on Control Applications* (2007), Pages: 880-885.
[18] Benvenuti L, Di Giamberardino P, Farina L.1996. Trajectory tracking for a PVTOL aircraft: a comparative analysis. *Proceedings of the 35th IEEE Conference on Decision and Control* (1996), vol.2, Pages: 1563-1568.
[19] Di Giamberardino P, Djemai M.1994. Digital control of a PVTOL aircraft. *Proceedings of the 33rd IEEE Conference on Decision and Control* (1994), vol.4, Pages: 3844-3849.
[20] Notarstefano G, Hauser J, Frezza R.2005. Trajectory Manifold Exploration for the PVTOL aircraft. 44th IEEE conference on Decision and Control (2005) and (2005) European Control Conference, Pages: 5848-5853.
[21] Pucci D, Hamel T, Morin Pascal, Samson Claude.2011. Nonlinear control of PVTOL vehicles subjected to drag and lift. 50th IEEE Conference on Decision and Control and European Control Conference (2011), Pages: 6177-6183.
[22] Martinez S, Cortes J, Bullo F.2003. Analysis and design of oscillatory control systems. IEEE Transactions on Automatic Control (2003), vol.48, no.7, Pages: 1164-1177.