1 | FUNCTIONS AND GRAPHS

Figure 1.1 A portion of the San Andreas Fault in California. Major faults like this are the sites of most of the strongest earthquakes ever recorded. (credit: modification of work by Robb Hannawacker, NPS)

Chapter Outline

Introduction

In the past few years, major earthquakes have occurred in several countries around the world. In January 2010, an earthquake of magnitude 7.3 hit Haiti. A magnitude 9 earthquake shook northeastern Japan in March 2011. In April 2014, an 8.2-magnitude earthquake struck off the coast of northern Chile. What do these numbers mean? In particular, how does a magnitude 9 earthquake compare with an earthquake of magnitude 8.2? Or 7.3? Later in this chapter, we show how logarithmic functions are used to compare the relative intensity of two earthquakes based on the magnitude of each earthquake (see **Example 1.39**).

Calculus is the mathematics that describes changes in functions. In this chapter, we review all the functions necessary to study calculus. We define polynomial, rational, trigonometric, exponential, and logarithmic functions. We review how to evaluate these functions, and we show the properties of their graphs. We provide examples of equations with terms involving these functions and illustrate the algebraic techniques necessary to solve them. In short, this chapter provides the foundation for the material to come. It is essential to be familiar and comfortable with these ideas before proceeding to the formal introduction of calculus in the next chapter.

1.1 | Review of Functions

Learning Objectives

- **1.1.1** Use functional notation to evaluate a function.
- **1.1.2** Determine the domain and range of a function.
- **1.1.3** Draw the graph of a function.
- **1.1.4** Find the zeros of a function.
- **1.1.5** Recognize a function from a table of values.
- **1.1.6** Make new functions from two or more given functions.
- **1.1.7** Describe the symmetry properties of a function.

In this section, we provide a formal definition of a function and examine several ways in which functions are represented—namely, through tables, formulas, and graphs. We study formal notation and terms related to functions. We also define composition of functions and symmetry properties. Most of this material will be a review for you, but it serves as a handy reference to remind you of some of the algebraic techniques useful for working with functions.

Functions

Given two sets A and $B,$ a set with elements that are ordered pairs $(x, y),$ where x is an element of A and y is an element of $B,$ is a relation from A to $B.$ A relation from A to B defines a relationship between those two sets. A function is a special type of relation in which each element of the first set is related to exactly one element of the second set. The element of the first set is called the *input*; the element of the second set is called the *output*. Functions are used all the time in mathematics to describe relationships between two sets. For any function, when we know the input, the output is determined, so we say that the output is a function of the input. For example, the area of a square is determined by its side length, so we say that the area (the output) is a function of its side length (the input). The velocity of a ball thrown in the air can be described as a function of the amount of time the ball is in the air. The cost of mailing a package is a function of the weight of the package. Since functions have so many uses, it is important to have precise definitions and terminology to study them.

Definition

A **function** f consists of a set of inputs, a set of outputs, and a rule for assigning each input to exactly one output. The set of inputs is called the **domain** of the function. The set of outputs is called the **range** of the function.

For example, consider the function $f,$ where the domain is the set of all real numbers and the rule is to square the input. Then, the input $x = 3$ is assigned to the output $3^2 = 9.$ Since every nonnegative real number has a real-value square root, every nonnegative number is an element of the range of this function. Since there is no real number with a square that is negative, the negative real numbers are not elements of the range. We conclude that the range is the set of nonnegative real numbers.

For a general function f with domain $D,$ we often use x to denote the input and y to denote the output associated with $x.$ When doing so, we refer to x as the **independent variable** and y as the **dependent variable**, because it depends on $x.$ Using function notation, we write $y = f(x),$ and we read this equation as "y equals f of $x.$" For the squaring function described earlier, we write $f(x) = x^2.$

The concept of a function can be visualized using **Figure 1.2**, **Figure 1.3**, and **Figure 1.4**.