Math 54 Final Exam Review Sheet

Ethan Ebbighausen

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1 Systems and Matrices

1.) Start with a system

$$\begin{aligned} 3x + 5y + z &= 5 \\ 2x + 4y + 2z &= 7 \\ x + 2y + 3z &= 9 \end{aligned}$$

Write this system as a vector equation, then as a matrix equation. Solve this via Gaussian elimination. Where are the pivots (and what are pivots)?

2.) What does vector addition look like geometrically? Algebraically? What is the definition of span and what does $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} + \text{span}\{e_1, e_2\}$ look like in $\mathbb{R}^3$?

3.) What are the requirements for Row Echelon Form and Reduced Row Echelon Form?

4.) When does a system have a solution? What critereon do we usually use to show that it fails to have a solution?

5.) What is the definition of linear independence? What are some equivalent versions?

6.) Are $\{\begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 8 \\ 8 \\ 0 \\ 3 \end{bmatrix}\}$ linearly independent? If not, find a linearly independent subset of these vectors.

7.) Without using the properties of invertible matrices, show that a 2x2 matrix A has linearly independent columns iff. $Ax = b$ has a solution for all $b \in \mathbb{R}^2$.

8.) What is a linear transformation (what does linear mean)? When can we represent linear transformations by matrices? How do we compute that matrix?

9.) Let $T \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ and $T \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. Find the matrix of T.

10.) What is the definition of one-to-one? What is the definition of onto? What criterea do we use for a matrix to determine whether it is one-to-one and onto (using pivots)?

11.) Is the matrix in problem 9 either one-to-one or onto? Come up with a couple examples of a one-to-one matrix (that are not all square).

12.) What is the general formula for a 2x2 matrix inverse?

13.) Let

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix}$$
$$B = \begin{bmatrix} 3 & 5 \\ 1 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 9 \\ 3 & 2 \end{bmatrix}$$

What is $(AB)^{-1} + C^T$?

14.) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

compute A^{-1} and use that to solve $Ax = e_1$.

15.) How does the algorithm to compute the inverse matrix work? If you don't remember, look back at elementary matrices.

16.) Do you remember the giant theorem about all the statements that are equivalent to a square matrix being invertible? Write down as many as you can remember and check against theorem 8 on page 121.

17.) If L is a $n \times n$ matrix and the equation $Lx = 0$ has the trivial solution, do the columns of L span $\mathbb{R}^n$.

18.) If the $n \times n$ matrices E and F have $EF = I$, is it true that E and F commute?

2 Bases, Dimension, Rank

1.) What is a subspace of $\mathbb{R}^n$. Give some examples. What is a basis of that subspace?

2.) Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 4 \\ 0 & 1 & 3 \end{bmatrix}$$

Find a basis for the column space of A and the null space of A.

3.) What does it mean to say that a subspace of $\mathbb{R}^n$ has dimension 2. Can you draw a 2-dimensional subspace in $\mathbb{R}^3$? What is a 0-dimensional subspace (and what is the basis for it– it isn't $\{0\}$)?

4.) What is the rank and what is our big theorem involving it? Use that theorem, and no comments about pivots or invertibility, to prove that a square matrix is onto iff. it is one-to-one.

5.) What is the rank of a 4x5 matrix with a 3-D null space?

3 Determinants and Cramer's Rule

4 Vector Spaces and Subspaces

5 Eigen-everything and Diagonalization

6 Orthogonality, Inner Products

7 Symmetric Matrices and Quadratic Forms

8 Second-Order Constant Coefficient Differential Equations

9 Systems of Differential Equations

10 Fourier Series