128A Discussion 1/27 Notes

- 1.) Attendance / Discussion Grades
  1/3 participation, 2/3 Coding

2.) General Structure - Review, Assignment, HW Questions
 - No HW today, so extra review / Demos

3.) Demos

3.1) Taylor's Theorem

For $k \geq 1$ an integer & $f: \mathbb{R} \to \mathbb{R}$ k-times differentiable,

$$f(x) = \sum_{i=0}^{k} \frac{f^{(i)}(a)}{i!}(x-a)^i + h_k(x)(x-a)^k$$

where $\lim_{x \to a} h_k(x) = 0$ $\quad \left(h_k(x) = \frac{f^{(k+1)}(\xi)}{(k+1)!}(x-a)^{k+1} \right.$

for some ξ between x & a)

- Proof: L'Hopital's Rule

1.) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

$\to e^x = \sum_{n=0}^{2} \frac{x^n}{n!} + h_2(x) = 1 + x + x^2/2 + h_2(x)x^2$

So $\frac{e^x - 1}{x} = 1 + x/2 + h_2(x)x$

and as $x \to 0$, this $\to 1$ $\}$ only needed 1 power of x

2.) $\lim_{x \to 0} \frac{\sqrt{1+\tan(x)} - \sqrt{1+\sin(x)}}{x^3}$ $\to$ want third-order Taylor

$f(x) = \sqrt{1+\tan(x)} - \sqrt{1+\sin(x)}$; $f(0) = 0$

$f'(0) = 0$
$f''(0) = 0$ $\}$ can do by hand, but computer is OK
$f'''(0) = 3/2$
$f^{(4)}(0) = -3$

$$f'(x) = \frac{1+\tan^2(x)}{2\sqrt{\tan(x)+1}} - \frac{\cos(x)}{2\sqrt{1+\sin(x)}}$$

$\vdots$

$f(x) = (3/2)(\frac{1}{6})(x)^3 + h_3(x)x^3 \Rightarrow \lim_{x \to 0} \frac{f(x)}{x^3} = 3/12 = 1/4$

3.2) MatLab Script

- 1.) Syms f(x)

  f(x)= @(x) $(1+\tan(x))^{1/2} - (1+\sin(x))^{1/2}$

  Df(x) = diff(f,x)

  Df(0)=0

  For prev. Problem

- 2.) fdd function

  Compare value

4.) Coding Exercise - MatLab & bCourses

5.) Plotting Demo if Time Permits