Discussion 9/2

•) 1.) Notes from the quiz

- Show your work!
- if you want to take the quiz in another section, you need to tell me ahead of time. I did have 1 person reach out, several others did not. It will be a 0 from now on if I don't okay it.

- Organize your work, box answers

2.) Make-ups - See ~~Ed~~ Announcement.

Mini-Lecture:

1.3:

- a vector is a matrix w/ 1 column:

We can add vectors Component-wise

$$\begin{bmatrix}2\\3\end{bmatrix} + \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix}$$

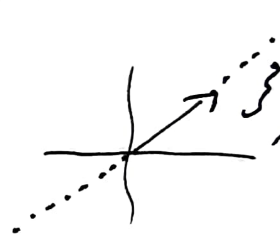
and multiply by Scalars $\quad 2 \cdot \begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}4\\6\end{bmatrix}$

- Combining vectors allows us to write equations

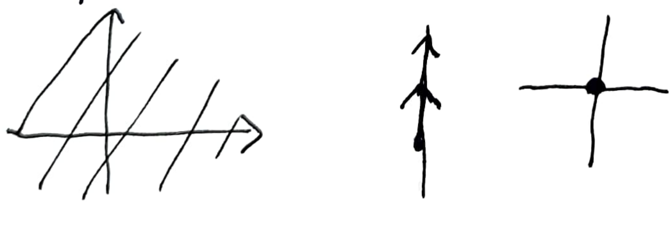
$$x_1 \begin{bmatrix}1\\2\end{bmatrix} + x_2 \begin{bmatrix}2\\3\end{bmatrix} = \begin{bmatrix}4\\5\end{bmatrix} \iff \begin{matrix} x_1 + 2x_2 = 4 \\ 2x_1 + 3x_2 = 5 \end{matrix}$$

$$\text{Span}\{v_1, v_2, \dots, v_p\} = \{x \in \mathbb{R}^n \mid x = c_1 v_1 + \dots + c_p v_p \text{ for some } c_1 \dots c_p \in \mathbb{R}\}$$

-> Span of 1 vector is a line:
(or, if $v=0$, it is just 0)



-> Span of 2 vectors is a plane, a line, or 0



1.4:

- Given a matrix A & vector x, we may multiply

e.g.)
$$\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}\begin{bmatrix}4\\5\end{bmatrix} = \begin{bmatrix}4\cdot2+3\cdot5\\4\cdot1+2\cdot5\end{bmatrix} = \begin{bmatrix}23\\14\end{bmatrix}$$

$A = (a_{ij})$ is $m \times n$
$v = (v_j)$ is $n \times 1$

-> Av is a vector of length m with $(Av)_k = \sum_{j=1}^{n} a_{kj} v_j$

→ If the vector is of variables, we may write a system as $Ax = b$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \iff \begin{matrix} x_1 + 2x_2 = 4 \\ 3x_1 + 4x_2 = 5 \end{matrix}$$

$$\iff x_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

therefore, a solution to $Ax = b$ exists $\iff$ b is a linear combination of columns of A.

$\iff$ If A has columns (a_j), $b \in \text{span}\{a_1, \dots a_n\}$

~~[illegible]~~

1.5 - Solution Sets

- If x_1 solves $Ax_1 = 0$ and x_2 solves $Ax_2 = b$,

$$A(x_1 + x_2) = Ax_1 + Ax_2 = 0 + b = b$$

- Recall that $Ax = 0$ has a nonzero ($x \neq 0$, x <u>nontrivial</u>) solution iff. we have at least 1 free variable
- Solving $Ax = b$ for 1 solution and $Ax = 0$ for all solutions then gives $Ax = b$ for all solutions

e.g.) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ may be solved by $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \vec{0}$ by $\{(-t, +t) \mid t \in \mathbb{R}\}$

So we have a solution set $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \end{bmatrix} \,\middle|\, t \in \mathbb{R} \right\}$

Worksheet Hints

1.) Set up a System $Ax=b$ (for any choice of $b \in \mathbb{R}^3$)

2.) Row operations both Column space...

3.) Count pivots

4.) A.) How do we get no solutions? (Pivot in last column) If we can change the last column, can we get infinitely many solutions?

B.) How many solutions do we have to $Ax=0$?

5.) Expand linear combos

6.) Find A so $A\left(c_1\begin{bmatrix} \vdots \end{bmatrix} + c_2\begin{bmatrix} \vdots \end{bmatrix}\right) = 0$

(and these are the only such vectors)

via a system.

$$A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \begin{aligned} a-b+c+3d&=0 \\ 2a+b-c+5d&=0 \ \ldots \end{aligned}$$

7.) A.) Try $A+B=I$

B.) $A+B=0$?

C.) 2 cases