Math 54 Discussion Notes: 8/28

Intro.

Me. I'm a second year math Ph.D. This is my second time teaching math 54, so I have some insight to the process. I focus on Harmonic Analysis - later in the course I'll point out some cool facts about related topics in ODEs & Fourier Series.

Contact -

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~D Please. emergencies, extensions, and quiz make-ups only. Put regrades through gradescope.

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~D I post the notes I use in discussion.

Today - Mini-lecture/worksheet

Wednesday - Quiz + Solutions

Monday - Holiday, HW due (usually HW & Ex.)

DSP - if you need extra time on quizzes, email me ASAP so we can set up times

☆ Please don't use Chat GPT during section, just ask for help. Your grade will do better this way!

Mini Lecture

1.) Systems of Equations may be represented in matrix form

$$3x + y = 1$$
$$x + 3y = 4$$

$$\longrightarrow \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 4 \end{bmatrix}$$

Solve via Row Operations:

1.) multiply row by nonzero constant
2.) add / subtract rows
3.) Swap rows

> Recall, a system may have **1 Solution**, none, or infinitely many

e.g.)

$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1/3 & 1/3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1/3 & 1/3 \\ 0 & 8/3 & 11/3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1/3 & 1/3 \\ 0 & 1 & 11/8 \end{bmatrix} \text{(A)}$$

$$\sim \begin{bmatrix} 1 & 0 & 1/3 - 11/24 \\ 0 & 1 & 11/8 \end{bmatrix} \text{(B)} \rightarrow \begin{array}{l} x = -3/24 = -1/8 \\ y = 11/8 \end{array}$$

2.) Echelon Form:

1.) All Zero rows on the bottom
2.) In each row, zeros to left, then some nonzero Pivot like Step (A) or (B)
3.) Each pivot has Zeroes below it, & pivot in next row is to the right

3.) Reduced Echelon Form — Step (B)

→ Echelon form, but each pivot is 1 and Zeroes above & below the pivot.

e.g.

$$\begin{bmatrix} 1 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & 3 & 3 \end{bmatrix}$$

ex.) How many 2×2 matrices with entries 0 or 2

are there?

$$2^4 = 16$$

How many are in echelon form?

forms →
$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, ~~

6!

Recall: A system is consistent iff. there are no pivots in the final column!

Worksheet 1: Some Hints

1.) A.) Think about pivot requirements (the staircase)

   B.) Depends on Answer to A)

   C/D.) If in echelon form, can nonzero numbers be below the diagonal (again, staircase/pivot rule)

2.) Is it Consistent?

3.) Find all pivot locations and count from there
     ⤷ Roughly 6 cases

4.) Put in a matrix, row reduce, how do we get a pivot in the last Column?

5.) A.) Think about pivot requirements like in 1

   B.) ... like A)

   C.) Yes - the long explanation, but pivots=1 gives this

6.) A.) How many pivots can we have? All zero rows at the bottom.

   B.) Zero rows vs. Columns

7.) what about non-pivot entries?