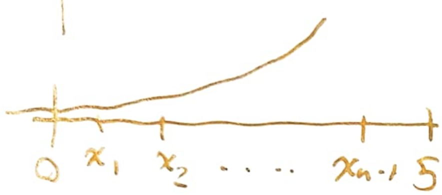
Discussion 4/18

- Definite Integrals

$\to$ Area $= \sum$ width(height)



$\to$ height $= f(x_i)$

width $= x_i - x_{i-1} = \Delta x_i$

Area $\approx \sum_{i=1}^{n} (\Delta x_i) f(x_i)$

"Riemann Sum"

• Goal Area under curve $= \lim_{n \to \infty} \sum_{i=1}^{n} (\Delta x_i) f(x_i)$

for $\Delta x_i \to 0$

i.e. set $\Delta x_i = 1/n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

Ex.) Area under $f(x) = x$ on $[1,4]$

$\rightsquigarrow$ n rectangles $\rightsquigarrow$ ~~[illegible]~~ width $3/n$

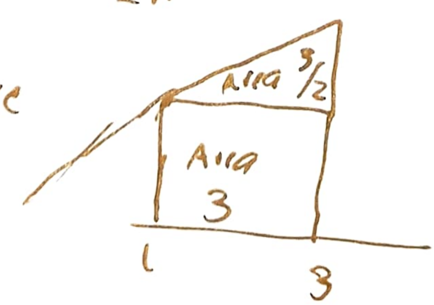
$x_i = 1 + i/n$

$f(x_i) = x_i$

$\sum_{i=1}^{n} \Delta x_i f(x_i) = \sum_{i=1}^{n} (3/n)(1 + i/n) = 3 + \frac{3}{n^2} \sum_{i=1}^{n} i = 3 + \frac{3(n^2+n)}{2n^2}$

$\lim_{n \to \infty} \left(3 + \frac{3(n^2+n)}{2n^2}\right) = 3 + 3/2$

Picture



Problems

1.) Find the area under the graph of $f(x) = 3x^2$ on $[0, 2]$ using Riemann Sums.

B.) what is $F(2) - F(0)$ for $F(x) = x^3$, the antiderivative of f.

2.) Do the same for $f(x) = 3x^3 + 2x$.

B.) $F(x)$ the anti-D. of f, what is $F(2) - F(0)$?

3.) Use right-Endpoints, $\Delta x = 1/2$, to estimate the area under $f(x) = x^4$ on $[0, 2]$.

B.) Compare to $F(2) - F(0)$, $F(x) = \frac{1}{5}x^5$.