Simulated Annealing

Background Information

What is ***Annealing***?

Annealing is the process of heating a material to a specific temperature to make it easier to work with (Materials like glass, steel, copper, etc). After being heated to a high temperature, the cooling of the material is controlled to allow the material to be reshaped and harden with its new shape.

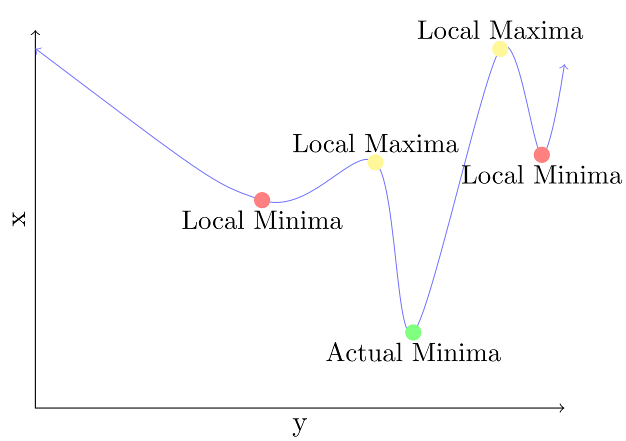
With *Simulated* Annealing, think of our solution canidate as the glass or metal material. At first, the termperature is very high and the solution is very malleable and seceptible to change, whether it be a good or bad change. But as time goes on, it gets harder to change the solution because it cools. *'Changing'* the solution is taking a worse solution, in the context of Simulated Annealing.

Purpose

The purpose of using a Simulated Annealing algorithm is *optimization*. There are many problems that take a factorial amount of time to come up with an optimal solution. With simulated annealing, we can take educated guesses to find a very good solution in polynomial time.

Another reason Simulated Annealing is used is the *Hill Climbing Problem*, or to prevent your solutions from getting stuck in a *local maxima* or *local minima*.

Example function to optimize



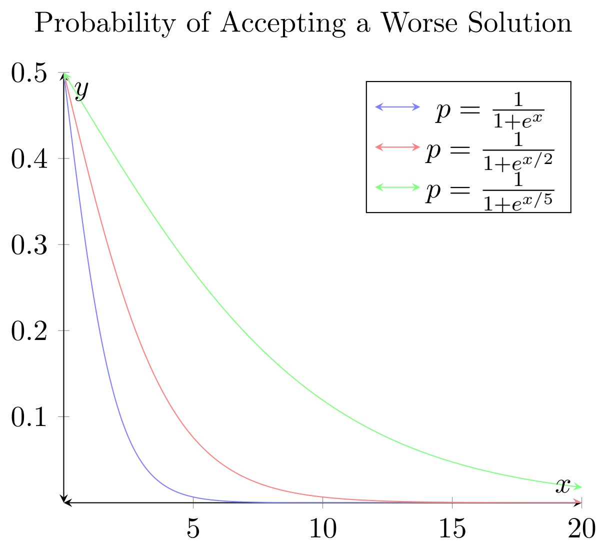
Simulated Annealing Basics

A *Cooling Function* is used to slowly decrease the Temperature for the algorithm. As the temperature decreses, there is less chances of taking a worse solution. In general, the probability will never be more than 50% to take a worse solution, however it can be lower.

$$p = \frac{1}{1 + e^{\Delta E / T_k}}$$

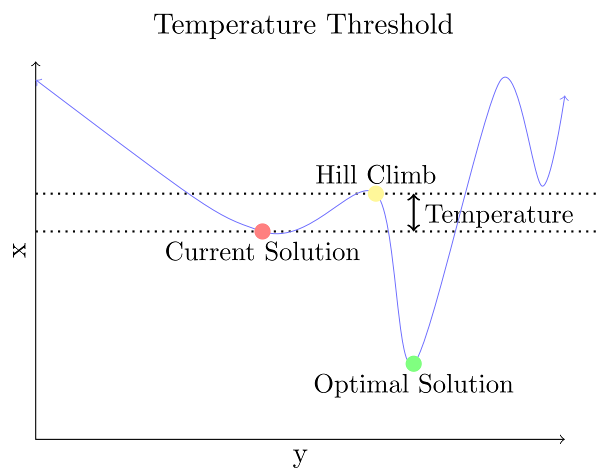
A random number, $0 \leq r \leq 1$ should be generated, and if $p \geq r$ then the new canidate solution will be selected despite being a 'worse' solution.

ΔE is the change in Energy. When simulating annealing, energy is how 'good' a solution is for the respective problem. In the example of the Traveling Salesman Problem, a lower energy would be a lower total weight of all edges in the cycle.



If the solution is marginally worse, and as the temperature decreases, there is a higher chance for x' to be a larger number, which lowers the chances of taking the worse solution.

The Temperature starts off very high, and lowers depending on the cooling algorithm. If the temperature decreases too rapidly, or does not start at a high enough number, canidate solutions can get stuck in a local minima or maxima.



General Simulated Annealing Algorithm

Algorithm 1 Generic Simulated Annealing

Note: The goal is to *Minimize* for this example.
Inputs:
T_0, Initial temperature
s_0, Initial solution/state
E, *Energy* function
J, *Cooling* function
N, *Neighbor* function

Require: $T_0 > 1$

```
T ← T_0
s_k ← s_0
k ← 0
repeat
    s_{k+1} ← N(s_k)                         ▷ Find a neighbor of the current solution
    ΔE ← E(s_{k+1}) − E(s_k)
    if ΔE < 0 then
        s_k ← s_{k+1}                        ▷ Accept the better solution
    else if p ≥ rand(0, 1)*** then
        s_k ← s_{k+1}                        ▷ Accept the worse solution
    else
        s_k ← s_k                            ▷ Keep the previous solution
    end if
    T_k ← J(T_k)                             ▷ The temperature cools
    k + +
until T_k = 1 or s_k satisfies some condition
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*** $p = \dfrac{1}{1 + e^{\Delta E / T_k}}$, probability of accepting the worse solution