快读

```
inline bool read(int& a)
{
   int s = 0, w = 1;
   char ch = getchar();
   if(ch==EOF)
       return false;
   while (ch < '0' || ch>'9')
       if (ch == '-')
           w = -1;
        ch = getchar();
   }
   while (ch >= '0' && ch <= '9')
       s = s * 10 + ch - '0';
       ch = getchar();
   }
   a = s * w;
   return true;
}
```

快输

```
void write(int x)
{
    if(x<0)
        putchar('-'), x=-x;
    if(x>9)
        write(x/10);
    putchar(x%10+'0');
    return;
}
```

随机数生成

```
mt19937 eng(time(0));
int randint(int a, int b)
{
    uniform_int_distribution<int> dis(a, b);
    return dis(eng);
}
```

快速幂

时间种子unordered_map

```
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x ^= x << 13;
        x ^= x >> 7;
        x ^= x << 17;
        return x;
    }
    size_t operator () (uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
    chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
unordered_map<uint64_t, int, custom_hash> safe_map;
```

数据结构

树状数组

```
#define lowbit(x) ((x) & -(x))
int tree[N];
void update(int x,int d)
{
    while(x<=N)
    {
        tree[x]+=d;
        x+=lowbit(x);
    }
}
int sum(int x)
{
    int ans=0;
    while(x>0)
    {
        ans+=tree[x];
        x-=lowbit(x);
    }
    return ans;
}
```

二维树状数组

```
#define lowbit(x) ((x) \& -(x))
void add(int x, int y, int d) {
    for (int i = x; i \le n; i += lowbit(i)) {
        for (int j = y; j \leftarrow m; j \leftarrow lowbit(j)) {
             bit[i][j] += d;
        }
    }
}
int query(int x, int y) {
    int ret = 0;
    for (int i = x; i > 0; i = lowbit(i)) {
        for (int j = y; j > 0; j = lowbit(j)) {
             ret += bit[i][j];
        }
    }
    return ret;
}
```

线段树

区间修改查询区间和

```
struct SegmentTree{
    int a[N],tree[N<<2],tag[N<<2];</pre>
    int ls(int p){return p<<1;}</pre>
    int rs(int p){return p<<1|1;}</pre>
    void push_up(int p){
        tree[p]=tree[ls(p)]+tree[rs(p)];
    void build(int p,int pl,int pr){
        tag[p]=0;
        if(pl==pr){
            tree[p]=a[p1];
            return;
        }
        int mid=(pl+pr)>>1;
        build(ls(p),pl,mid);
        build(rs(p),mid+1,pr);
        push_up(p);
    }
    void addtag(int p,int pl,int pr,int d){
        tag[p]+=d;
        tree[p]+=d*(pr-pl+1);
    void push_down(int p,int p1,int pr){
        if(tag[p]){
            int mid=(pl+pr)>>1;
            addtag(ls(p),pl,mid,tag[p]);
            addtag(rs(p),mid+1,pr,tag[p]);
            tag[p]=0;
        }
    }
```

```
void update(int L,int R,int p,int pl,int pr,int d){
        if(L<=p1&&pr<=R){
            addtag(p,pl,pr,d);
            return;
        push_down(p,pl,pr);
        int mid=(pl+pr)>>1;
        if(L<=mid)</pre>
            update(L,R,ls(p),pl,mid,d);
        if(R>mid)
            update(L,R,rs(p),mid+1,pr,d);
        push_up(p);
    }
    int query(int L,int R,int p,int pl,int pr){
        if(L \le pl\&pr \le R)
            return tree[p];
        push_down(p,pl,pr);
        int mid=(pl+pr)>>1;
        int ans=0;
        if(L<=mid)
            ans+=query(L,R,ls(p),pl,mid);
        if(R>mid)
            ans+=query(L,R,rs(p),mid+1,pr);
        return ans;
    }
};
```

区间修改查询区间最值

```
#include <bits/stdc++.h>
using namespace std;
const int maxn=3e5+10;
const int inf =2e9;
struct Node{
    int 1,r,res,tag;
};
struct SegmentTree{
    Node a[maxn*4];
    void tag_init(int i){
        a[i].tag=inf;
    void tag_union(int fa,int i){
        if(a[fa].tag!=inf)a[i].tag=a[fa].tag;
    void tag_cal(int i){
        if(a[i].tag!=inf)a[i].res=a[i].tag;
    void pushdown(int i){
        tag_cal(i);
        if(a[i].1!=a[i].r){
            tag_union(i,i*2);
            tag_union(i,i*2+1);
        }
        tag_init(i);
    void pushup(int i){
        if(a[i].l==a[i].r)return;
```

```
pushdown(i*2);
        pushdown(i*2+1);
        a[i].res=min(a[i*2].res,a[i*2+1].res);
    }
    void build(int i,int l,int r){
        a[i].l=l,a[i].r=r;tag_init(i);
        if(l>=r)return;
        int mid=(1+r)/2;
        build(i*2,1,mid);
        build(i*2+1,mid+1,r);
    }
    void update(int i,int 1,int r,int w){
        pushdown(i);
        if(a[i].r<1||a[i].1>r||1>r)return;
        if(a[i].l>=l&&a[i].r<=r){}
            a[i].tag=w;
            return;
        update(i*2,1,r,w);
        update(i*2+1,1,r,w);
        pushup(i);
    int query(int i,int l,int r){
        pushdown(i);
        if(a[i].r<1||a[i].l>r||l>r) return inf;
        if(a[i].1>=1&&a[i].r<=r){
            return a[i].res;
        return min(query(i*2,1,r),query(i*2+1,1,r));
    }
};
SegmentTree tri;
```

树上问题

树的直径

两次dfs

LCA

倍增

```
vector<int>arc[N];
int deep[N],fa[N][20];
void dfs(int x,int f)
{
    deep[x]=deep[f]+1;
    fa[x][0]=f;
    for(int i=1;(1<<i)<=deep[x];i++)</pre>
        fa[x][i]=fa[fa[x][i-1]][i-1];
    for(auto it:arc[x])
    {
        if(it==f)
            continue;
        dfs(it,x);
    }
}
int LCA(int x,int y)
    if(deep[x]<deep[y])</pre>
        swap(x,y);
    for(int i=19;i>=0;i--)
        if(deep[fa[x][i]]>=deep[y])
            x=fa[x][i];
    if(x==y)
        return x;
    for(int i=19;i>=0;i--)
        if(fa[x][i]!=fa[y][i])
            x=fa[x][i],y=fa[y][i];
    return fa[x][0];
}
```

```
vector<pair<int,int>>arc[N];
int fa[N],ans[N];
bool vis[N];
int find(int x)
{
    return x==fa[x]?fa[x]:fa[x]=find(fa[x]);
}
void tarjan(int x,int f)
{
    vis[x]=true;
    for(auto [it,i]:arc[x])
        if(it==f)
            continue;
        if(!vis[it])
            tarjan(it,x);
            fa[it]=x;
        }
   }
    for(auto [it,i]:arc[x])
        if(it==f)
            continue;
        if(vis[it])
            ans[i]=find(it);
   }
}
```

数学

高斯消元

例题 洛谷 P3389

```
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=105;
double a[N][N];
double eps=1e-7;
int main() {
    ios::sync_with_stdio(false);cin.tie(nullptr);
    int n;
    scanf("%d",&n);
    for(int i=1;i<=n;i++)
    {
        for(int j=1;j<=n+1;j++)
            scanf("%lf",&a[i][j]);
    }
    for(int i=1;i<=n;i++)
    {
</pre>
```

```
int M=0, Mi=0;
        for(int j=i;j<=n;j++)</pre>
        {
             if(fabs(a[j][i])>M)
                 M=fabs(a[j][i]);
                 Mi = j;
             }
        }
        for(int j=i;j<=n+1;j++)
             swap(a[Mi][j],a[i][j]);
        if(fabs(a[i][i])<eps)</pre>
        {
             printf("No Solution\n");
             return 0;
        }
        for(int j=n+1; j>=i; j--)
             a[i][j]/=a[i][i];
        for(int j=1; j \le n; j++)
             if(j==i)
                 continue;
             double temp=a[j][i]/a[i][i];
             for(int k=i;k \le n+1;k++)
                 a[j][k]-=temp*a[i][k];
        }
    }
    for(int i=1;i<=n;i++)</pre>
        printf("%.21f\n",a[i][n+1]);
    return 0;
}
```

GCD

欧几里得算法

```
int gcd(int a,int b)
{
    return b?gcd(b,a%b):a;
}
```

更相损减术

$$\gcd(a,b) = \gcd(b,a-b) = \gcd(a,a-b)$$

```
int gcd(int a,int b)
{
    while(a!=b)
    {
        if(a>b) a=a-b;
        else b=b-a;
    }
    return a;
}
```

LCM

```
int lcm(int a,int b)
{
   return a/gcd(a,b)*b;
}
```

扩展欧几里得

返回d=gcd(a,b);以及ax+by=d的特解x,y

```
11 extend_gcd(11 a,11 b,11 &x,11 &y)
{
    if(b==0){x=1;y=0;return a;}
    11 d=extend_gcd(b,a%b,y,x);
    y-=a/b*x;
    return d;
}
```

逆元

扩展欧几里得

```
11 mod_inverse(11 a,11 m)
{
    11 x,y;
    extend_gcd(a,m,x,y);
    return (x%m+m)%m;
}
```

快速幂

```
11 mod_inverse(ll a,ll m)
{
    return fast_pow(a,m-2,m);
}
```

递推

```
void mod_inverse(ll n,ll p)
{
    inv[1]=1;
    for(int i=1;i<=n;i++)
        inv[i]=(ll)(p-p/i)*inv[p%i]%p;
}</pre>
```

欧拉筛

```
bitset<N>vis;
void get_prime()
{
    for(int i=2;i<N;i++)
    {
        if(!vis[i])
        {
            vis[i]=true;
            p.push_back(i);
            for(int j=2;i*j<N;j++)
                  vis[i*j]=true;
        }
    }
}</pre>
```

筛欧拉函数

```
bool vis[N];
int phi[N];
vector<int>p;
void get_phi()
{
    phi[1]=1;
    for(int i=2;i<N;i++)</pre>
        if(!vis[i])
        {
            vis[i]=true;
            p.push_back(i);
            phi[i]=i-1;
        }
        for(auto p:p)
        {
            if(i*p>=N)
                break;
            vis[i*p]=true;
            if(i\%p==0)
                 phi[i*p]=p*phi[i];
                break;
            phi[i*p]=phi[i]*phi[p];
        }
    }
}
```

筛约数和

```
vector<int>p;
int phi[N],sig[N],num[N];
bool vis[N];
void init()
{
    phi[1]=sig[1]=1;
```

```
for(int i=2;i<N;i++)</pre>
    {
        if(!vis[i])
        {
            vis[i]=true;
            p.push_back(i);
            phi[i]=i-1;
            sig[i]=num[i]=i+1;
        }
        for(auto j:p)
        {
            if(i*j>=N)
                break;
            vis[i*j]=true;
            if(i\%j==0)
                 phi[i*j]=phi[i]*j;
                 num[i*j]=num[i]*j+1;
                 sig[i*j]=sig[i]/num[i]*num[i*j];
                 break;
            }
            phi[i*j]=phi[i]*phi[j];
            num[i*j]=1+j;
            sig[i*j]=sig[i]*sig[j];
        }
    }
}
```

素数判定

Miller Rabin

```
bool is_prime(int x)
{
    if(x<3)
        return x==2;
    if(x\%2==0)
         return false;
    int A[]=\{2,325,9375,28178,450775,9780504,1795265022\},d=x-1,r=0;
    while (d\%2==0)
         d>>=1, r++;
    for(auto a:A)
    {
         int v=qpow(a,d,x);
         if(v \le 1 | | v = x-1)
             continue;
         for(int i=0;i<r;i++)</pre>
             v=(\underline{\quad}int128\_t)v*v%x;
             if(v==x-1&&i!=r-1)
             {
                  v=1;break;
             if(v==1)
                  return false;
         }
         if(v!=1)
```

```
return false;
}
return true;
}
```

质因数分解

Pollard Rho

```
mt19937 eng(time(0));
int randint(int a, int b)
    uniform_int_distribution<int> dis(a, b);
    return dis(eng);
}
int Pollard_Rho(int n){
    if(n==4)
               return 2;
    if(is_prime(n)) return n;
    while(true){
        int c=randint(1,n-1);
        auto f=[=](int x){return ((\underline{int128_t})x*x+c)%n;};
        int t=f(0), r=f(f(0));
        while(t!=r){
            int d=__gcd(abs(t-r),n);
            if(d>1) return d;
            t=f(t), r=f(f(r));
        }
    }
}
```

离散对数

```
11 BSGS(11 a, 11 b, 11 m)
   static unordered_map<11, 11> hs;
   hs.clear();
   ll cur = 1, t = sqrt(m) + 1;
   for (int B = 1; B <= t; ++B)
        (cur *= a) %= m;
        hs[b * cur % m] = B; // 哈希表中存B的值
   }
   11 now = cur; // 此时cur = a^t
   for (int A = 1; A <= t; ++A)
    {
        auto it = hs.find(now);
       if (it != hs.end())
           return A * t - it->second;
        (now *= cur) %= m;
   }
   return -1; // 没有找到, 无解
}
```

```
// 修改版的BSGS,额外带一个系数
11 \text{ BSGS}(11 \text{ a}, 11 \text{ b}, 11 \text{ m}, 11 \text{ k} = 1)
    static unordered_map<11, 11> hs;
    hs.clear();
    ll cur = 1, t = sqrt(m) + 1;
    for (int B = 1; B <= t; ++B)
        (cur *= a) %= m;
        hs[b * cur % m] = B; // 哈希表中存B的值
    }
    11 now = cur * k % m;
    for (int A = 1; A <= t; ++A)
    {
        auto it = hs.find(now);
        if (it != hs.end()) return A * t - it->second;
        (now *= cur) %= m;
    return -INF; // 这里因为要多次加1,要返回更小的负数
}
11 \text{ exBSGS}(11 \text{ a}, 11 \text{ b}, 11 \text{ m}, 11 \text{ k} = 1)
    11 A = a \% = m, B = b \% = m, M = m;
    if (b == 1) return 0;
    11 cur = 1 \% m;
    for (int i = 0; i++)
        if (cur == B) return i;
        cur = cur * A % M;
        11 d = gcd(a, m);
        if (b % d) return -INF;
        if (d == 1) return BSGS(a, b, m, k * a % m) + i + 1;
        k = k * a / d % m, b /= d, m /= d; // 相当于在递归求解exBSGS(a, b / d, m /
d, k * a / d % m)
    }
}
```

组合数

预处理阶乘

```
void init(int n)
{
    fac[0]=1;
    for(int i=1;i<=n;i++)
        fac[i]=fac[i-1]*i%mod;
    rev[n]=qpow(fac[n],mod-2,mod);//n must be less than mod
    for(int i=n;i>=1;i--)
        rev[i-1]=rev[i]*i%mod;
    assert(rev[0]==1);
}
```

```
int C(int n,int m,int p)
{
    if(m>n)
        return 0;
    return fac[n]*rev[m]%p*rev[n-m]%p;
}
int Lucas(int n,int m,int p)
{
    if(m==0)
        return 1;
    return C(n%p,m%p,p)*Lucas(n/p,m/p,p)%p;
}
```

回文串

manacher

p[i]是以i为中心的最长回文串长度

```
int p[N<<1];</pre>
void change(string a)
    S+='$';S+='#';
    for(auto it:a)
    {
        s+=it;s+='#';
    }
    s+='&';
}
void manacher()
    int n=s.length();
    int R=0,C;
    for(int i=1;i<n;i++)</pre>
    {
        if(i<R)
             p[i]=min(p[C]+C-i,p[(C<<1)-i]);</pre>
        else
             p[i]=1;
        while(s[i+p[i]]==s[i-p[i]])
             p[i]++;
        if(p[i]+i>R)
             R=p[i]+i;
             C=i;
        }
    }
}
```

并查集

带权并查集

```
const int N=2e5+5;
int f[N],dis[N];
int find(int x)
    if(x!=f[x])
    {
        int t=f[x];
        f[x]=find(f[x]);
        dis[x]+=dis[t];
    return f[x];
}
bool merge(int a,int b,int d)
{
    int ra=find(a),rb=find(b);
    d+=dis[a]-dis[b];
    if(ra==rb)
        if(d!=0)
            return false;
        return true;
    }
   f[ra]=rb;
    dis[ra]-=d;
    return true;
}
```

字符串

字典树

```
struct Trie{
   int cnt=0,ch[N][26],sz[N];//sz[N]是以这个点结尾的字符串数量
   int newNode(){
       cnt++;
       sz[cnt]=0;
       memset(ch[cnt],0,sizeof(ch[cnt]));
       return cnt;
   }
   void add(string s){
       int now=0;
       for(auto it:s){
           int &c=ch[now][it-'a'];
           if(!c)
               c=newNode();
           now=c;
       sz[now]++;
   }
   int find(string s){
       int now=0;
       for(auto it:s){
```

```
now=ch[now][it-'a'];
    if(!now)
        return 0;
}
return sz[now];
}
```

双哈

```
#define mp make_pair
#define fi first
#define se second
using 11 = long long;
typedef pair<int, int> hashv;
const 11 \mod 1 = 1e9 + 7;
const 11 \mod 2 = 1e9 + 9;
hashv base = mp(13331, 2333);
hashv operator + (hashv a, hashv b) {
    int c1 = a.fi + b.fi, c2 = a.se + b.se;
   if(c1 \ge mod1) c1 -= mod1;
   if(c2 >= mod2) c2 -= mod2;
   return mp(c1, c2);
}
hashv operator - (hashv a, hashv b) {
   int c1 = a.fi - b.fi, c2 = a.se - b.se;
    if(c1 < 0) c1 += mod1;
   if(c2 < 0) c2 += mod2;
    return mp(c1,c2);
}
hashv operator * (hashv a, hashv b) {
   return mp(111*a.fi*b.fi%mod1, 111*a.se*b.se%mod2);
```

计算几何

```
typedef pair<double,double>pll;
pll operator+(pll x,pll y){
    return {x.first+y.first,x.second+y.second};
}
pll operator-(pll x,pll y){
    return {x.first-y.first,x.second-y.second};
}
pll operator*(pll x,double k){
    return {x.first*k,x.second*k};
}
pll operator/(pll x,double k){
    return {x.first/k,x.second/k};
}
double len(pll x){
    return hypot(x.first,x.second);
```

```
double Dot(const pdd &a,const pdd &b){
    return a.first*b.first+a.second*b.second;
}
double Cross(const pdd &a,const pdd &b){
    return a.first*b.second-a.second*b.first;
}
```

实数精度

```
const double pi = acos(-1.0);
                                   //圆周率,精确到15位小数: 3.141592653589793
const double eps = 1e-8;
                                   //偏差值,有时用1e-10,但是要注意精度
int sqn(double x){
                                   //判断x的大小
   if(fabs(x) < eps) return 0;</pre>
                                   //x==0,返回0
   else return x<0?-1:1;
                                   //x<0返回-1, x>0返回1
}
int dcmp(double x, double y){
                                   //比较两个浮点数
   if(fabs(x - y) < eps) return 0; //x==y, 返回0
   else return x<y ?-1:1;
                                   //x<y返回-1, x>y返回1
}
```

点

```
struct Point{
   double x,y;
   Point(){}
   Point(double x,double y):x(x),y(y){}
   Point operator + (Point B){return Point(x+B.x,y+B.y);}
   Point operator - (Point B){return Point(x-B.x,y-B.y);}
   Point operator * (double k){return Point(x*k,y*k);}
   Point operator / (double k){return Point(x/k,y/k);}
   bool operator == (Point B){return sgn(x-B.x)==0 && sgn(y-B.y)==0;}
};
double Distance(Point A, Point B){ return hypot(A.x-B.x,A.y-B.y); }
```

向量

```
typedef Point Vector;
double Dot(Vector A,Vector B){ return A.x*B.x + A.y*B.y; }
double Len(Vector A){return sqrt(Dot(A,A));}
double Len2(Vector A){return Dot(A,A);}
double Angle(Vector A,Vector B){return acos(Dot(A,B)/Len(A)/Len(B));}
double Cross(Vector A,Vector B){return A.x*B.y - A.y*B.x;}
double Area2(Point A,Point B,Point C){ return Cross(B-A, C-A);}
Vector Rotate(Vector A, double rad){ //逆时针旋转rad角度
    return Vector(A.x*cos(rad)-A.y*sin(rad), A.x*sin(rad)+A.y*cos(rad));
}
Vector Normal(Vector A){return Vector(-A.y/Len(A), A.x/Len(A));} //求单位法向量
bool Parallel(Vector A, Vector B){return sgn(Cross(A,B)) == 0;} //返回true表示平行
或重合
```

```
struct Line{
                                 //(1)线上的两个点
   Point p1,p2;
   Line(){}
    Line(Point p1,Point p2):p1(p1),p2(p2){}
    Line(Point p, double angle) { // (4) 根据一个点和倾斜角 angle 确定直
线,0<=angle<pi
        p1 = p;
        if(sgn(angle - pi/2) == 0){p2 = (p1 + Point(0,1));}
        else{p2 = (p1 + Point(1,tan(angle)));}
   Line(double a, double b, double c) \{ // (2) ax+by+c=0
       if(sqn(a) == 0){
            p1 = Point(0, -c/b);
            p2 = Point(1, -c/b);
        }
        else if(sqn(b) == 0){
           p1 = Point(-c/a, 0);
           p2 = Point(-c/a, 1);
        }
        else{
            p1 = Point(0, -c/b);
            p2 = Point(1, (-c-a)/b);
   }
};
typedef Line Segment;
int Point_line_relation(Point p, Line v){
    int c = sgn(Cross(p-v.p1,v.p2-v.p1));
                                 //1: p在v的左边
   if(c < 0)return 1;</pre>
   if(c > 0)return 2;
                                  //2: p在v的右边
   return 0;
                                   //0: p在v上
}
bool Point_on_seq(Point p, Line v) { //点和线段: 0 点不在线段v上; 1 点在线段v上
    return sgn(Cross(p-v.p1, v.p2-v.p1)) == 0 \& sgn(Dot(p - v.p1,p - v.p2)) <=
0;
}
double Dis_point_line(Point p, Line v){
    return fabs(Cross(p-v.p1, v.p2-v.p1))/Distance(v.p1, v.p2);
Point Point_line_proj(Point p, Line v){
    double k = Dot(v.p2-v.p1, p-v.p1)/Len2(v.p2-v.p1);
    return v.p1+(v.p2-v.p1)*k;
Point Point_line_symmetry(Point p, Line v){
    Point q = Point_line_proj(p,v);
    return Point(2*q.x-p.x,2*q.y-p.y);
}
double Dis_point_seg(Point p, Segment v){
    if(sgn(Dot(p-v.p1,v.p2-v.p1))<0 \mid | sgn(Dot(p-v.p2,v.p1-v.p2))<0)
        return min(Distance(p,v.p1),Distance(p,v.p2));
                                       //点的投影在线段上
   return Dis_point_line(p,v);
int Line_relation(Line v1, Line v2){
    if(sgn(Cross(v1.p2-v1.p1,v2.p2-v2.p1)) == 0){
        if(Point_line_relation(v1.p1,v2)==0) return 1; //1 重合
```

```
//0 平行
        else return 0;
   }
                                                       //2 相交
   return 2;
}
Point Cross_point(Point a, Point b, Point c, Point d) { //Line1:ab, Line2:cd
   double s1 = Cross(b-a, c-a);
   double s2 = Cross(b-a,d-a);
                                                  //叉积有正负
   return Point(c.x*s2-d.x*s1,c.y*s2-d.y*s1)/(s2-s1);
}
bool Cross_segment(Point a, Point b, Point c, Point d){
                                                       //Line1:ab, Line2:cd
   double c1 = Cross(b-a,c-a),c2=Cross(b-a,d-a);
   double d1 = Cross(d-c,a-c),d2=Cross(d-c,b-c);
   return sgn(c1)*sgn(c2) < 0 & sgn(d1)*sgn(d2) < 0; //1相交; 0不相交
}
```

多边形

```
int Point_in_polygon(Point pt,Point *p,int n){    //点pt,多边形Point *p
   for(int i = 0; i < n; i++){
                                              //3: 点在多边形的顶点上
       if(p[i] == pt) return 3;
   }
   for(int i = 0; i < n; i++){
                                              //2: 点在多边形的边上
       Line v=Line(p[i],p[(i+1)%n]);
       if(Point_on_seg(pt,v)) return 2;
   }
   int num = 0;
   for(int i = 0; i < n; i++){
       int j = (i+1)\% n;
       int c = sgn(Cross(pt-p[j],p[i]-p[j]));
       int u = sgn(p[i].y - pt.y);
       int v = sgn(p[j].y - pt.y);
       if(c > 0 \&\& u < 0 \&\& v >= 0) num++;
       if(c < 0 \&\& u >= 0 \&\& v < 0) num--;
   return num != 0;
                                               //1: 点在内部; 0: 点在外部
}
double Polygon_area(Point *p, int n){ //Point *p表示多边形
   double area = 0;
   for(int i = 0; i < n; i++)
       area += Cross(p[i],p[(i+1)%n]);
   return area/2;
                                     //面积有正负,返回时不能简单地取绝对值
}
Point Polygon_center(Point *p, int n){ //求多边形重心
   Point ans(0,0);
   if(Polygon_area(p,n)==0) return ans;
   for(int i = 0; i < n; i++)
       ans = ans+(p[i]+p[(i+1)%n])*Cross(p[i],p[(i+1)%n]);
   return ans/Polygon_area(p,n)/6;
}
```

凸包

```
//求下凸包。如果p[i]是右拐弯的,这个点不在凸包上,往回退
   for(int i=0;i<n;i++){</pre>
       while(v>1 & sgn(Cross(ch[v-1]-ch[v-2],p[i]-ch[v-1]))<=0) //把后面ch[v-1]
改成ch[v-2]也行
       ch[v++]=p[i];
   }
   int j=v;
   //求上凸包
   for(int i=n-2; i>=0; i--){
       while(v>j & sgn(Cross(ch[v-1]-ch[v-2],p[i]-ch[v-1]))<=0) //把后面ch[v-1]
改成ch[v-2]也行
       ch[v++]=p[i];
   if(n>1) v--;
                                //返回值v是凸包的顶点数
   return v;
}
                                //输入点是p[], 计算得到的凸包顶点放在ch[]中
Point p[N],ch[N];
```

最近点对

使用前先sort(p,p+n,cmpxy)

```
bool cmpxy(Point A,Point B){return sgn(A.x-B.x)<0||(sgn(A.x-B.x)==0\&sgn(A.y-B.x)=0)
bool cmpy(Point A, Point B){return sgn(A.y-B.y)<0;}</pre>
double Distance(Point A, Point B){ return hypot(A.x-B.x,A.y-B.y); }
double Closest_Pair(int left,int right){
   double dis = INF;
   if(left == right) return dis;
                                           //只剩1个点
   if(left + 1 == right) return Distance(p[left], p[right]);//只剩2个点
   int mid = (left+right)/2;
                                          //分治
   double d1 = Closest_Pair(left,mid);
                                         //求s1内的最近点对
   double d2 = Closest_Pair(mid+1, right); //求s2内的最近点对
   dis = min(d1,d2);
   int k = 0;
   for(int i=left;i<=right;i++)</pre>
                                          //在s1和s2中间附近找可能的最小点对
       if(fabs(p[mid].x - p[i].x) <= dis) //按x坐标来找
           tmp_p[k++] = p[i];
   sort(tmp_p, tmp_p+k, cmpy);
                                  //按y坐标排序,用于剪枝。这里不能按x坐标排序
   for(int i=0;i<k;i++)</pre>
       for(int j=i+1; j< k; j++){
                                                      //剪枝
           if(tmp_p[j].y - tmp_p[i].y >= dis) break;
           dis = min(dis,Distance(tmp_p[i],tmp_p[j]));
   return dis; //返回最小距离
}
```

旋转卡壳

半平面交

```
struct Line{ //半平面的表示
             //直线上一个点
   Point p;
   Vector v;
              //方向向量,它的左边是半平面
   double ang; //极角,从x正半轴旋转到v的角度
   Line(){}:
   Line(Point p, Vector v):p(p),v(v){ang = atan2(v.y, v.x);}
   bool operator < (Line &L){return ang < L.ang;} //用于排序
};
//点p在线L左边,即点p在线L在外面:
bool OnLeft(Line L,Point p){return sgn(Cross(L.v,p-L.p))>0;}
Point Cross_point(Line a, Line b){ //两直线交点
   Vector u=a.p-b.p;
   double t=Cross(b.v,u)/Cross(a.v,b.v);
   return a.p+a.v*t;
}
vector<Point> HPI(vector<Line> L){ //求半平面交,返回凸多边形
   int n=L.size();
   sort(L.begin(),L.end());
                                //将所有半平面按照极角排序。
                                  //指向双端队列的第一个和最后一个元素
   int first,last;
                                 //两个相邻半平面的交点
   vector<Point> p(n);
   vector<Line> q(n);
                                 //双端队列
   vector<Point> ans;
                                  //半平面交形成的凸包
   q[first=last=0]=L[0];
   for(int i=1;i<n;i++){
       //情况1: 删除尾部的半平面
       while(first<last && !OnLeft(L[i], p[last-1])) last--;</pre>
       //情况2: 删除首部的半平面:
       while(first<last && !OnLeft(L[i], p[first])) first++;</pre>
       q[++last]=L[i]; //将当前的半平面加入双端队列尾部
       //极角相同的两个半平面,保留左边:
       if(fabs(Cross(q[last].v,q[last-1].v)) < eps){
       last--:
          if(OnLeft(q[last],L[i].p)) q[last]=L[i];
       }
       //计算队列尾部半平面交点:
       if(first<last) p[last-1]=Cross_point(q[last-1],q[last]);</pre>
   }
   //情况3: 删除队列尾部的无用半平面
   while(first<last && !OnLeft(q[first],p[last-1])) last--;</pre>
   if(last-first<=1) return ans; //空集
   p[last]=Cross_point(q[last],q[first]); //计算队列首尾部的交点。
   for(int i=first;i<=last;i++) ans.push_back(p[i]); //复制。
   return ans; //返回凸多边形
```

员

```
struct Circle{ //圆
              //圆心
   Point c;
               //半径
   double r;
   Circle(){}
   Circle(Point c,double r):c(c),r(r){}
   Circle(double x,double y,double _r){c=Point(x,y);r = _r;}
};
int Point_circle_relation(Point p, Circle C){ //点和圆的关系
   double dst = Distance(p,C.c);
   if(sgn(dst - C.r) < 0) return 0;</pre>
                                      //0 点在圆内
   if(sqn(dst - C.r) == 0) return 1;
                                       //1 圆上
   return 2;
                                        //2 圆外
}
int Line_circle_relation(Line v,Circle C){ //直线和圆的位置关系
   double dst = Dis_point_line(C.c,v);
   if(sgn(dst-C.r) < 0) return 0; //O 直线和圆相交
   if(sgn(dst-C.r) ==0) return 1;
                                   //1 直线和圆相切
   return 2;
                                        //2 直线在圆外
}
int Seg_circle_relation(Segment v,Circle C){ //线段和圆的位置关系
   double dst = Dis_point_seg(C.c,v);
   if(sgn(dst-C.r) < 0) return 0; //0线段在圆内
   if(sgn(dst-C.r) ==0) return 1;
                                    //1线段和圆相切
   return 2;
                                    //2线段在圆外
}
//pa, pb是交点。返回值是交点个数
int Line_cross_circle(Line v, Circle C, Point &pa, Point &pb){ //直线和圆的交点
   if(Line_circle_relation(v, C)==2) return 0;//无交点
   Point q = Point_line_proj(C.c,v);
                                          //圆心在直线上的投影点
   double d = Dis_point_line(C.c,v);
                                          //圆心到直线的距离
   double k = sqrt(C.r*C.r-d*d);
   if(sqn(k) == 0){
                                            //1个交点,直线和圆相切
       pa = q; pb = q; return 1;
   }
   Point n=(v.p2-v.p1)/Len(v.p2-v.p1);
                                           //单位向量
   pa = q + n*k; pb = q - n*k;
                                           //2个交点
   return 2;
}
```

最小圆覆盖

```
Point circle_center(const Point a, const Point b, const Point c){ //圆上三点定圆心 Point center; double al=b.x-a.x, bl=b.y-a.y, cl=(al*al+bl*bl)/2; double a2=c.x-a.x, b2=c.y-a.y, c2=(a2*a2+b2*b2)/2; double d =a1*b2-a2*b1; center.x =a.x+(c1*b2-c2*b1)/d; center.y =a.y+(al*c2-a2*c1)/d; return center; } void min_cover_circle(Point *p, int n, Point &c, double &r){ //最小圆覆盖 random_shuffle(p, p + n); //随机函数,打乱所有点。这一步很重要
```

```
c=p[0]; r=0;
                                      //从第1个点p0开始。圆心为p0,半径为0
   for(int i=1;i<n;i++)</pre>
                                       //扩展所有点
       if(sgn(Distance(p[i],c)-r)>0){
                                      //点pi在圆外部
           c=p[i]; r=0;
                                      //重新设置圆心为pi,半径为0
           for(int j=0;j<i;j++)
                                      //重新检查前面所有的点。
              if(sgn(Distance(p[j],c)-r)>0){ //两点定圆
                  c.x=(p[i].x + p[j].x)/2;
                  c.y=(p[i].y + p[j].y)/2;
                  r=Distance(p[j],c);
                  for(int k=0; k< j; k++)
                      if (sgn(Distance(p[k],c)-r)>0){ //两点不能定圆,就三点定圆
                          c=circle_center(p[i],p[j],p[k]);
                          r=Distance(p[i], c);
                      }
              }
       }
}
```

网络流

二分图匹配

```
vector<int> G[N];
int Nx,Ny,k; //Nx,Ny是两个集合的大小; k是边数
int Mx[N],My[N];
int dx[N],dy[N];
int dis,u,v;
bool used[N];
bool searchP(){
   queue<int> Q;
   dis = INF;
   memset(dx,-1,sizeof(dx));
    memset(dy,-1,sizeof(dy));
    for(int i = 0; i < Nx; ++i)
                     Q.push(i), dx[i] = 0;
    if(Mx[i] == -1)
    while(!Q.empty()){
        int u = Q.front();Q.pop();
        if(dx[u] > dis) break;
        int sz = G[u].size();
        for(int i = 0; i < sz; ++i){
            int v = G[u][i];
            if(dy[v] == -1) {
                dy[v] = dx[u] + 1;
                if(My[v] == -1) dis = dy[v];
                else dx[My[v]] = dy[v] + 1, Q.push(My[v]);
            }
        }
    return dis != INF;
bool DFS(int u){
   int sz = G[u].size();
    for(int i = 0; i < sz; ++i){
        int v = G[u][i];
```

```
if(!used[v] & dy[v] == dx[u] + 1){
            used[v] = true;
            if(My[v] != -1 \&\& dy[v] == dis) continue;
            if(My[v] == -1 \mid\mid DFS(My[v])){
                My[v] = u;
                Mx[u] = v;
                return true;
            }
        }
    }
    return false;
}
int MaxMatch(){
   int res = 0;
    memset(Mx,-1,sizeof(Mx));
    memset(My,-1,sizeof(My));
    while(searchP()){
        memset(used,false,sizeof(used));
        for(int i = 0; i < Nx; ++i)
        if(Mx[i] == -1 \&\& DFS(i)) ++res;
    }
    return res;
}
int main(){
    read(Nx);read(Ny);read(k);
    while(k--){read(u);read(v);if(v \le Ny) G[u-1].push_back(v-1);}
    printf("%d\n",MaxMatch());
}
```

FFT

```
#include<bits/stdc++.h>
using namespace std;
const double PI = acos(-1.0);
const double eps=1e-4;
struct Complex
{
   double x, y;
   Complex(double x = 0.0, double y = 0.0):x(x),y(y){}
   Complex operator-(const Complex \&b) const{return Complex(x - b.x, y - b.y);}
   Complex operator+(const Complex \&b) const{return Complex(x + b.x, y + b.y);}
   Complex operator*(const Complex &b) const{return Complex(x * b.x - y * b.y,
x * b.y + y * b.x);
};
/*
* 进行 FFT 和 IFFT 前的反置变换
* 位置 i 和 i 的二进制反转后的位置互换
*len 必须为 2 的幂
*/
void change(Complex y[], int len)
   int i, j, k;
   for (int i = 1, j = len / 2; i < len - 1; i++)
       if(i < j)
```

```
swap(y[i], y[j]);
        // 交换互为小标反转的元素, i<j 保证交换一次
        // i 做正常的 + 1, j 做反转类型的 + 1, 始终保持 i 和 j 是反转的
        k = len / 2;
        while (j >= k)
            j = j - k;
            k = k / 2;
        }
        if (j < k)
            j += k;
    }
}
 * 做 FFT
 *len 必须是 2^k 形式
 *on == 1 时是 DFT, on == -1 时是 IDFT
void fft(Complex y[], int len, int on)
    change(y, len);
    for (int h = 2; h \leftarrow len; h \leftarrow 1)
        Complex wn(cos(2 * PI / h), sin(on * 2 * PI / h));
        for (int j = 0; j < len; <math>j += h)
        {
            Complex w(1, 0);
            for (int k = j; k < j + h / 2; k++)
            {
                Complex u = y[k];
                Complex t = w * y[k + h / 2];
                y[k] = u + t;
                y[k + h / 2] = u - t;
                w = w * wn;
            }
        }
    }
    if (on == -1)
        for (int i = 0; i < len; i++)
            y[i].x /= len;
const int N = 1e7+5;
Complex a[N],b[N];
int res[N];
int main()
    ios::sync_with_stdio(false);cin.tie(0);
    int n,m;
    cin>>n>>m;
    int len=1;
    while (len < (n+1)*2 | len < (m+1)*2)
        <=1;
    for(int i=0;i<=n;i++)</pre>
        cin>>a[i].x;
    for(int i=0;i <= m;i++)
        cin>>b[i].x;
    fft(a,len,1);
```

```
fft(b,len,1);
for(int i=0;i<len;i++)
    a[i]=a[i]*b[i];
fft(a,len,-1);
for(int i=0;i<len;i++)
    res[i]=(int)(a[i].x+0.5);
for(int i=0;i<=n+m;i++)
    cout<<res[i]<<' ';
cout<<'\n';
return 0;
}</pre>
```

高精度

```
#ifndef __x86_64__
#error Only x86-64 targets are supported
#endif
#include<cstdint>
#include<vector>
#include<string>
#include<iosfwd>
#define __builtin_ia32_adc(x,y,flag) __asm__("addb \%3, \%0\n\t" "adcq \%2,
%1\n\t" "setc %0":"+r"(flag),"+r"(x):"r"(y),"i"(-1):"cc")
struct bigint{// made by dengyaotriangle!
    typedef unsigned long long u64;
    typedef unsigned __int128 u128;
    typedef std::size_t st;
    std::vector<u64> data;
    bigint(){}
    bigint(u64 x):data(x?std::vector<u64>{x}:std::vector<u64>{}){}
    bigint(const std::string &s){
        st pos=s.length();
        int cnt=0;
        u64 val=0;
        while(pos){
            pos--;
            if(cnt==64){
                data.push_back(val);
                val=0; cnt=0;
            val |=(u64)(s[pos]=='1')<<cnt;</pre>
            ++cnt;
        }
        if(cnt&&val)data.push_back(val);
    explicit operator std::string()const{
        if(data.empty())return "0";
        bool t=0;
        std::string ret;
        for(int i=63; i>=0; i--){
            t = (data.back()>>i)&1;
            if(t)ret+='0'|((data.back()>>i)&1);
        }
        st i=data.size()-1;
```

```
while(i){
        i--;
        for(int j=63; j>=0; j--)ret+='0'|((data[i]>>j)&1);
    }
    return ret;
}
explicit operator bool()const{return !data.empty();}
explicit operator u64()const{return data.empty()?0:data[0];}
st digit()const{
    if(data.empty())return 0;
    return (data.size()<<6)-__builtin_clzll(data.back());</pre>
}
bool operator==(const bigint &a)const{return a.data==data;}
bool operator!=(const bigint &a)const{return a.data!=data;}
bool operator<(const bigint &a)const{</pre>
    if(data.size()!=a.data.size())return data.size()<a.data.size();</pre>
    for(st i=data.size();i;){
        if(data[i]!=a.data[i])return data[i]<a.data[i];</pre>
    return 0;
}
bool operator>(const bigint &a)const{return a<(*this);}</pre>
bool operator<=(const bigint &a)const{return !(*this>a);}
bool operator>=(const bigint &a)const{return !(*this<a);}</pre>
bigint &operator<<=(st n){</pre>
    if(data.empty())return *this;
    int w=n\&63; st z=n>>6;
    st i=data.size();
    bool flg=0;
    if(w&&(data.back()>>(64-w)))data.push_back(0),flg=1;
    data.resize(data.size()+z);
    while(i){
        i--:
        if(flg)data[i+z+1] = data[i] >> (64-w);
        data[i+z]=data[i]<<w;</pre>
        flg|=bool(w);
    }
    for(st i=0;i<z;i++)data[i]=0;
    return *this;
bigint &operator>>=(st n){
    int w=n\&63; st z=n>>6, i=0;
    for(;i+z<data.size();i++){</pre>
        if(w&\&i)data[i-1]|=data[i+z]<<(64-w);
        data[i]=data[i+z]>>w;
    while(data.size()>i)data.pop_back();
    while(!data.empty()&&data.back()==0)data.pop_back();
    return *this;
bigint operator<<(st n)const{return bigint(*this)<<=n;}</pre>
bigint operator>>(st n)const{return bigint(*this)>>=n;}
bigint &operator+=(const bigint &a){
    data.resize(std::max(data.size(),a.data.size()));
    bool carry=0;
    for(st i=0;i<data.size();i++){</pre>
        u64 rg=0;
```

```
if(i<a.data.size())rg=a.data[i];</pre>
        __builtin_ia32_adc(data[i],rg,carry);
    }
    if(carry)data.push_back(1);
    return *this;
}
bigint &operator-=(const bigint &a){
    bool carry=1;
    for(st i=0;i<data.size();i++){</pre>
        u64 rg=-1;
        if(i<a.data.size())rg=~a.data[i];</pre>
        __builtin_ia32_adc(data[i],rg,carry);
    while(!data.empty()&&data.back()==0)data.pop_back();
    return *this;
}
bigint &operator++(){return *this+=bigint(1);}
bigint &operator--(){return *this-=bigint(1);}
bigint operator++(int){bigint tmp=*this;++*this;return tmp;}
bigint operator--(int){bigint tmp=*this;--*this;return tmp;}
bigint &operator*=(const bigint &a){
    std::vector<u64> ret(data.size()+a.data.size());
    for(st i=0;i<data.size();i++){</pre>
        u64 carry=0;bool wcarry=0;
        st k=i;
        for(st j=0;j<a.data.size();j++,k++){
            u128 r=data[i]*(u128)a.data[j]+carry;
            u64 cur=r;
            carry=r>>64;
            __builtin_ia32_adc(ret[k],cur,wcarry);
        }
        while(carry||wcarry){
            __builtin_ia32_adc(ret[k],carry,wcarry);
            carry=0; k++;
        }
    }
    while(!ret.empty()&&ret.back()==0)ret.pop_back();
    data=ret;
    return *this;
bigint &operator/=(const bigint &a){
    if(a.digit()>digit()){
        data.clear();
        return *this;
    }
    st z=digit()-a.digit();
    std::vector<u64> ret;
    while(1){
        bigint tmp=a<<z;</pre>
        if(tmp<=*this){</pre>
            *this-=tmp;
            st v1=z>>6;
            if(ret.size()<=v1)ret.resize(v1+1);</pre>
            ret[v1] = (u64)(1) << (z\&63);
        }
        if(!z)break;
        z--;
    }
```

```
data=ret;
        return *this;
    bigint &operator%=(const bigint &a){
        if(a.digit()>digit())return *this;
        st z=digit()-a.digit();
        while(1){
            bigint tmp=a<<z;</pre>
            if(tmp<=*this)*this-=tmp;</pre>
            if(!z)break;
            z--;
        }
        return *this;
    }
    bigint operator+(const bigint &a)const{return bigint(*this)+=a;}
    bigint operator-(const bigint &a)const{return bigint(*this)-=a;}
    bigint operator*(const bigint &a)const{return bigint(*this)*=a;}
    bigint operator/(const bigint &a)const{return bigint(*this)/=a;}
    bigint operator%(const bigint &a)const{return bigint(*this)%=a;}
std::istream &operator>>(std::istream &st,bigint &a){
    std::string s;st>>s;a=bigint(s);return st;
std::ostream &operator<<(std::ostream &st,const bigint &a){</pre>
    return st<<(std::string)(a);</pre>
}
```

BM线性递推#

```
#include<bits/stdc++.h>
using namespace std;
typedef long long 11;
#define int long long
typedef unsigned long long ull;
#define dmp(x) cerr<<"DEBUG"<<__LINE__<<":"<<#x<<" "<<x<endl
const int INF=0x3f3f3f3f;
typedef pair<int,int> pii;
const int mod=1e9+7;
int powmod(int a,int b){
   int res=1;a%=mod;
   assert(b>=0);
   while(b)
        if(b&1) res=res*a%mod;
        a=a*a\%mod;
    return res;
}
int n;
namespace linear_seq{
   const int N=10010;
   int res[N],base[N],_c[N],_md[N];
   vector<int>Md;
   void mul(int *a,int *b,int k){
        for(int i=0; i< k+k; i++) _c[i]=0;
```

```
for(int i=0;i<k;i++)</pre>
         if(a[i])
             for(int j=0; j< k; j++)
                 _{c[i+j]=(_{c[i+j]+a[i]*b[j])mod;}
    for(int i=k+k-1; i>=k; i--)
        if(_c[i])
             for(int j=0;j<(int)Md.size();j++)</pre>
                 _c[i-k+Md[j]]=(_c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
    for(int i=0; i < k; i++)
        a[i]=_c[i];
}
int solve(int n,vector<int>a,vector<int>b){
    int ans=0,pnt=0;
    int k=(int)a.size();
    assert(a.size()==b.size());
    for(int i=0;i< k;i++)
         _{md[k-1-i]=-a[i]};
    _{md[k]=1};
    Md.clear();
    for(int i=0; i < k; i++)
        if(_md[i]!=0)
             Md.push_back(i);
    for(int i=0;i<k;i++)</pre>
         res[i]=base[i]=0;
    res[0]=1;
    while((111<<pnt)<=n)</pre>
         pnt++;
    for(int p=pnt;p>=0;p--){
        mul(res,res,k);
         if((n>>p)\&1){
             for(int i=k-1;i>=0;i--)
                 res[i+1]=res[i];
             res[0]=0;
             for(int j=0;j<(int)Md.size();j++)</pre>
                 res[Md[j]]=(res[Md[j]]-res[k]*_md[Md[j]])%mod;
        }
    }
    for(int i=0; i < k; i++)
        ans=(ans+res[i]*b[i])%mod;
    if(ans<0)
        ans+=mod;
    return ans;
}
vector<int> BM(vector<int> s){
    vector<int> C(1,1),B(1,1);
    int L=0, m=1, b=1;
    for(int i=0;i<(int)s.size();i++){</pre>
        int d=0;
         for(int i=0;i<L+1;i++)</pre>
             d=(d+C[i]*s[n-i])%mod;
        if(d==0)
             ++m;
         else if(2*L <= n){
             vector<int> T=C;
             int c=mod-d*powmod(b,mod-2)%mod;
             while(C.size() < B.size() + m)</pre>
                 C.push_back(0);
             for(int i=0;i<B.size();i++)</pre>
```

```
C[i+m]=(C[i+m]+c*B[i])%mod;
                L=n+1-L; B=T; b=d; m=1;
            }
            else{
                 int c=mod-d*powmod(b,mod-2)%mod;
                while(C.size()<B.size()+m)</pre>
                     C.push_back(0);
                for(int i=0;i<B.size();i++)</pre>
                     C[i+m]=(C[i+m]+c*B[i])%mod;
                ++m;
            }
        }
        return C;
    }
    int gao(vector<int>a,int n){
        vector<int> c=BM(a);
        c.erase(c.begin());
        for(int i=0;i<c.size();i++)</pre>
            c[i]=(mod-c[i])%mod;
        return solve(n,c,vector<int>(a.begin(),a.begin()+(int)c.size()));
    }
signed main() {
    ios::sync_with_stdio(false);cin.tie(0);
    vector<int>v;
    v.push_back(2);
    v.push_back(24);
   v.push_back(96);
    v.push_back(416);
    v.push_back(1536);
    v.push_back(5504);
    v.push_back(18944);
   v.push_back(64000);
    v.push_back(212992);
    v.push_back(702464);
    cout<<li>rear_seq::gao(v,n-1)<<"\n";</pre>
    return 0;
}
```