快读

```cpp
inline bool read(int& a)
{
    int s = 0, w = 1;
    char ch = getchar();
    if(ch==EOF)
        return false;
    while (ch < '0' || ch>'9')
    {
        if (ch == '-')
            w = -1;
        ch = getchar();
    }
    while (ch >= '0' && ch <= '9')
    {
        s = s * 10 + ch - '0';
        ch = getchar();
    }
    a = s * w;
    return true;
}
```

快输

```cpp
void write(int x)
{
    if(x<0)
        putchar('-'),x=-x;
    if(x>9)
        write(x/10);
    putchar(x%10+'0');
    return;
}
```

随机数生成

[a, b]的随机数

```cpp
mt19937 eng(time(0));
int randint(int a, int b)
{
    uniform_int_distribution<int> dis(a, b);
    return dis(eng);
}
```

xor shift

映射到2^{64}

```cpp
const ull mask = std::chrono::steady_clock::now().time_since_epoch().count();
ull shift(ull x){
    x^=x<<13;
    x^=x>>7;
    x^=x<<17;
    x^=mask;
    return x;
}
```

memset

int / long long

"较"的原则：加法不爆

- 极大值: 0x7f
- 较大值: 0x3f
- 较小值: 0xc0
- 极小值: 0x80

float

"较"的原则：保证一定位精度

7f以上一直到be都是-0（很小的>-1.0的负数）

- 极大值: 0x7f
- 较大值: 0x4f
- 较小值: 0xce
- 极小值: 0xfe

double

"较"的原则：保证一定位精度

- 极大值: 0x7f
- 较大值: 0x43
- 较小值: 0xc2
- 极小值: 0xfe

快速幂

```cpp
ll qpow(ll a,ll n,ll m)
{
    ll ans=1;
    while(n)
    {
        if(n&1)
            ans=(__int128_t)ans*a%m;
        a=(__int128_t)a*a%m;
        n>>=1;
    }
    return ans;
}
```

pair哈希

```cpp
struct pair_hash
{
    template <class T1, class T2>
    size_t operator () (pair<T1, T2> const &pair) const
    {
        size_t h1 = hash<T1>()(pair.first);
        size_t h2 = hash<T2>()(pair.second);
        return h1 ^ h2;
    }
};
unordered_set<pair<int,int>,pair_hash>st;
```

时间种子unordered_map

```cpp
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x ^= x << 13;
        x ^= x >> 7;
        x ^= x << 17;
        return x;
    }
    size_t operator () (uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
unordered_map<uint64_t, int, custom_hash> safe_map;
```

数据结构

并查集

普通并查集

```cpp
int f[N];
int find(int x){
    return x==f[x]?x:f[x]=find(f[x]);
}
void merge(int x,int y){
    x=find(x),y=find(y);
    if(x!=y)
        f[x]=y;
}
```

带权并查集

```cpp
const int N=2e5+5;
int f[N],dis[N];
int find(int x)
{
    if(x!=f[x])
```

```
    {
        int t=f[x];
        f[x]=find(f[x]);
        dis[x]+=dis[t];
    }
    return f[x];
}
bool merge(int a,int b,int d)
{
    int ra=find(a),rb=find(b);
    d+=dis[a]-dis[b];
    if(ra==rb)
    {
        if(d!=0)
            return false;
        return true;
    }
    f[ra]=rb;
    dis[ra]-=d;
    return true;
}
```

树状数组

```
inline int lowbit(int x){ return x & (-x); }
int tree[N];
void update(int x,int d){
    while(x<=N){
        tree[x]+=d;
        x+=lowbit(x);
    }
}
int sum(int x){
    int ans=0;
    while(x>0){
        ans+=tree[x];
        x-=lowbit(x);
    }
    return ans;
}
```

二维树状数组

```
#define lowbit(x) ((x) & -(x))
void add(int x, int y, int d) {
    for (int i = x; i <= n; i += lowbit(i)) {
        for (int j = y; j <= m; j += lowbit(j)) {
            bit[i][j] += d;
        }
    }
}
int query(int x, int y) {
    int ret = 0;
    for (int i = x; i > 0; i -= lowbit(i)) {
        for (int j = y; j > 0; j -= lowbit(j)) {
            ret += bit[i][j];
```

```
        }
    }
    return ret;
}
```

线段树

区间修改查询区间和

```
struct SegmentTree{
    int a[N],tree[N<<2],tag[N<<2];
    int ls(int p){return p<<1;}
    int rs(int p){return p<<1|1;}
    void push_up(int p){
        tree[p]=tree[ls(p)]+tree[rs(p)];
    }
    void build(int p,int pl,int pr){
        tag[p]=0;
        if(pl==pr){
            tree[p]=a[pl];
            return;
        }
        int mid=(pl+pr)>>1;
        build(ls(p),pl,mid);
        build(rs(p),mid+1,pr);
        push_up(p);
    }
    void addtag(int p,int pl,int pr,int d){
        tag[p]+=d;
        tree[p]+=d*(pr-pl+1);
    }
    void push_down(int p,int pl,int pr){
        if(tag[p]){
            int mid=(pl+pr)>>1;
            addtag(ls(p),pl,mid,tag[p]);
            addtag(rs(p),mid+1,pr,tag[p]);
            tag[p]=0;
        }
    }
    void update(int L,int R,int p,int pl,int pr,int d){
        if(L<=pl&&pr<=R){
            addtag(p,pl,pr,d);
            return;
        }
        push_down(p,pl,pr);
        int mid=(pl+pr)>>1;
        if(L<=mid)
            update(L,R,ls(p),pl,mid,d);
        if(R>mid)
            update(L,R,rs(p),mid+1,pr,d);
        push_up(p);
    }
    int query(int L,int R,int p,int pl,int pr){
        if(L<=pl&&pr<=R)
            return tree[p];
        push_down(p,pl,pr);
        int mid=(pl+pr)>>1;
```

```
            int ans=0;
            if(L<=mid)
                ans+=query(L,R,ls(p),pl,mid);
            if(R>mid)
                ans+=query(L,R,rs(p),mid+1,pr);
            return ans;
        }
};
```

区间修改查询区间最值

```cpp
#include <bits/stdc++.h>
using namespace std;
const int maxn=3e5+10;
const int inf =2e9;
struct Node{
    int l,r,res,tag;
};
struct SegmentTree{
    Node a[maxn*4];
    void tag_init(int i){
        a[i].tag=inf;
    }
    void tag_union(int fa,int i){
        if(a[fa].tag!=inf)a[i].tag=a[fa].tag;
    }
    void tag_cal(int i){
        if(a[i].tag!=inf)a[i].res=a[i].tag;
    }
    void pushdown(int i){
        tag_cal(i);
        if(a[i].l!=a[i].r){
            tag_union(i,i*2);
            tag_union(i,i*2+1);
        }
        tag_init(i);
    }
    void pushup(int i){
        if(a[i].l==a[i].r)return;
        pushdown(i*2);
        pushdown(i*2+1);
        a[i].res=min(a[i*2].res,a[i*2+1].res);
    }
    void build(int i,int l,int r){
        a[i].l=l,a[i].r=r;tag_init(i);
        if(l>=r)return;
        int mid=(l+r)/2;
        build(i*2,l,mid);
        build(i*2+1,mid+1,r);
    }
    void update(int i,int l,int r,int w){
        pushdown(i);
        if(a[i].r<l||a[i].l>r||l>r)return;
        if(a[i].l>=l&&a[i].r<=r){
            a[i].tag=w;
            return;
        }
```

```
        update(i*2,l,r,w);
        update(i*2+1,l,r,w);
        pushup(i);
    }
    int query(int i,int l,int r){
        pushdown(i);
        if(a[i].r<l||a[i].l>r||l>r)return inf;
        if(a[i].l>=l&&a[i].r<=r){
            return a[i].res;
        }
        return min(query(i*2,l,r),query(i*2+1,l,r));
    }
};
SegmentTree tri;
```

区间取 $max(x, a_i)$ 或 $min(x, a_i)$ (吉老师线段树)

```
#define lson l, m, rt << 1
#define rson m + 1, r, rt << 1 | 1
typedef long long LL;
typedef unsigned long long uLL;
LL z = 1;
const int N = 1e6 + 5;
struct SegmentBeats{//以下板子中都是(1,n,1,L,R) 先大区间、区间标号，然后当前区间左右界
    int v1[N * 4], cnt[N * 4], tag[N * 4], v2[N * 4]; // v1是最大值，v2是次大值，tag
是最值标记，cnt 是最小值的个数
    LL sum[N * 4];
    void pushup(int rt){
        if(v1[rt << 1] > v1[rt << 1 | 1]){
            v1[rt] = v1[rt << 1];
            cnt[rt] = cnt[rt << 1];
            v2[rt] = max(v2[rt << 1], v1[rt << 1 | 1]);
        }
        else if(v1[rt << 1] < v1[rt << 1 | 1]){
            v1[rt] = v1[rt << 1 | 1];
            cnt[rt] = cnt[rt << 1 | 1];
            v2[rt] = max(v1[rt << 1], v2[rt << 1 | 1]);
        }
        else{
            v1[rt] = v1[rt << 1];
            cnt[rt] = cnt[rt << 1] + cnt[rt << 1 | 1];
            v2[rt] = max(v2[rt << 1], v2[rt << 1 | 1]);
        }
        sum[rt] = sum[rt << 1] + sum[rt << 1 | 1];
    }
    void build(int l, int r, int rt){
        tag[rt] = -1;
        if(l == r){
            int x;
            cin>>x;
            v1[rt] = sum[rt] = x, v2[rt] = -1, cnt[rt] = 1;//次大值初始化为 -1
            return;
        }
        int m = l + r >> 1;
        build(lson);
        build(rson);
```

```
            pushup(rt);
        }
        void addtag(int rt, int c){
            if(v1[rt] <= c) return;
            sum[rt] -= z * cnt[rt] * (v1[rt] - c);//更新 sum
            v1[rt] = tag[rt] = c;//更新最值
        }
        void pushdown(int rt){
            if(tag[rt] != -1){
                addtag(rt << 1, tag[rt]);//给左儿子加 tag
                addtag(rt << 1 | 1, tag[rt]);//给右儿子加 tag
                tag[rt] = -1;
            }
        }
        void update(int l, int r, int rt, int a, int b, int c){
            if(v1[rt] <= c) return;
            if(l >= a && r <= b && v2[rt] < c){//该区间的最小值都变成 c
                addtag(rt, c);
                return;
            }
            pushdown(rt);
            int m = l + r >> 1;
            if(a <= m) update(lson, a, b, c);
            if(b > m) update(rson, a, b, c);
            pushup(rt);
        }
        int query_max(int l, int r, int rt, int a, int b){
            if(l >= a && r <= b) return v1[rt];
            int m = l + r >> 1, ans = 0;
            pushdown(rt);
            if(a <= m) ans = max(ans, query_max(lson, a, b));
            if(b > m) ans = max(ans, query_max(rson, a, b));
            return ans;
        }
        LL query_sum(int l, int r, int rt, int a, int b){
            if(l >= a && r <= b) return sum[rt];
            int m = l + r >> 1;
            LL ans = 0;
            pushdown(rt);
            if(a <= m) ans += query_sum(lson, a, b);
            if(b > m) ans += query_sum(rson, a, b);
            return ans;
        }
};
```

ODT

在随机数据下以 $O(n \log n \log n)$ 的速度实现

- 区间加
- 区间赋值
- 求区间第k大值
- 求区间n次方和

```
struct node
{
    ll l, r;
```

```cpp
    mutable ll v;  // 这里mutable要写不然可能会CE
    node(ll l, ll r, ll v) : l(l), r(r), v(v) {} // 构造函数
    bool operator<(const node &o) const { return l < o.l; } // 重载小于运算符
};
set<node> tree;
auto split(ll pos)// <l,r,v> -> <l,pos-1,v>, <pos,r,v>
{
    auto it = tree.lower_bound(node(pos, 0, 0)); // 寻找左端点大于等于pos的第一个节点
    if (it != tree.end() && it->l == pos) // 如果已经存在以pos为左端点的节点，直接返回
        return it;
    it--; // 否则往前数一个节点
    ll l = it->l, r = it->r, v = it->v;
    tree.erase(it); // 删除该节点
    tree.insert(node(l, pos - 1, v)); // 插入<l,pos-1,v>和<pos,r,v>
    return tree.insert(node(pos, r, v)).first; // 返回以pos开头的那个节点的迭代器
                                               // insert默认返回值是一个pair，第一个
成员是我们要的
}
void assign(ll l, ll r, ll v)//区间赋值 <l,r> a_i -> v
{
    auto end = split(r + 1), begin = split(l); // 顺序不能颠倒，否则可能RE
    tree.erase(begin, end); // 清除一系列节点
    tree.insert(node(l, r, v)); // 插入新的节点
}
void add(ll l, ll r, ll v)//区间加 <l,r> a_i -> a_i+v
{
    auto end = split(r + 1);
    for (auto it = split(l); it != end; it++)
        it->v += v;
}
ll kth(ll l, ll r, ll k)//求区间第k大 <l,r>
{
    auto end = split(r + 1);
    vector<pair<ll, ll>> v; // 这个pair里存节点的值和区间长度
    for (auto it = split(l); it != end; it++)
        v.push_back(make_pair(it->v, it->r - it->l + 1));
    sort(v.begin(), v.end()); // 直接按节点的值的大小排下序
    for (int i = 0; i < v.size(); i++) // 然后挨个丢出来，直到丢出k个元素为止
    {
        k -= v[i].second;
        if (k <= 0)
            return v[i].first;
    }
}
ll sum_of_pow(ll l, ll r, ll x, ll y)//求区间x次方和 模y
{
    ll tot = 0;
    auto end = split(r + 1);
    for (auto it = split(l); it != end; it++)
        tot = (tot + qpow(it->v, x, y) * (it->r - it->l + 1)) % y; // qpow自己写一
下
    return tot;
}
```

图论

树的直径

两次dfs

任意一点出发找到最远点A，A一定在直径上，再从A出发找到最远点B，B即为直径

```cpp
vector<pair<int,int>>arc[N];
void dfs(int x,int f,int d)
{
    dist[x]=d;
    fa[x]=fa;
    for(auto [it,i]:arc[x])
    {
        if(it==f)
            continue;
        dfs(it,x,d+i);
    }
}
```

树形dp

```cpp
int dp[N],M=0;
bool vis[N];
void Dfs(int x,int f)
{
    vis[x]=true;
    for(auto [it,i]:arc[x])
    {
        if(vis[it]||it==f)
            continue;
        Dfs(it,x);
        M=max(M,dp[x]+dp[it]+i);
        dp[x]=max(dp[x],dp[it]+i);
    }
}
```

LCA

倍增

树上倍增

```cpp
vector<int>arc[N];
int deep[N],fa[N][20];
void dfs(int x,int f)
{
    deep[x]=deep[f]+1;
    fa[x][0]=f;
    for(int i=1;i<=19;i++)
        fa[x][i]=fa[fa[x][i-1]][i-1];
    for(auto it:arc[x])
    {
        if(it==f)
            continue;
        dfs(it,x);
    }
}
```

```
}
int LCA(int x,int y)
{
    if(deep[x]<deep[y])
        swap(x,y);
    for(int i=19;i>=0;i--)
        if(deep[fa[x][i]]>=deep[y])
            x=fa[x][i];
    if(x==y)
        return x;
    for(int i=19;i>=0;i--)
        if(fa[x][i]!=fa[y][i])
            x=fa[x][i],y=fa[y][i];
    return fa[x][0];
}
```

tarjan

离线之后并查集找LCA

```
vector<pair<int,int>>arc[N];
int fa[N],ans[N];
bool vis[N];
int find(int x)
{
    return x==fa[x]?fa[x]:fa[x]=find(fa[x]);
}
void tarjan(int x,int f)
{
    vis[x]=true;
    for(auto [it,i]:arc[x])
    {
        if(it==f)
            continue;
        if(!vis[it])
        {
            tarjan(it,x);
            fa[it]=x;
        }
    }
    for(auto [it,i]:arc[x])
    {
        if(it==f)
            continue;
        if(vis[it])
            ans[i]=find(it);
    }
}
```

最短路

Floyd

时间复杂度$O(n^3)$，空间复杂度$O(n^2)$

```
for (k = 1; k <= n; k++)
    for (x = 1; x <= n; x++)
        for (y = 1; y <= n; y++)
            f[x][y] = min(f[x][y], f[x][k] + f[k][y]);
```

Bellman-ford

对于边(u,v)，松弛操作对应$dis(v) = min(dis(v), dis(u) + w(u,v))$

最短路存在的情况下，最多经过$n - 1$次松弛操作，时间复杂度为$O(nm)$

可以用于判图中是否有负环，如果从s点没跑出负环，只能说明从s点出发不能抵达负环，并不能说明图中没有负环

可以建立一个超级源点，向图上每一个节点连一个权值为0的边，对超级源点执行$Bellman - ford$

```
int dis[N];
bool bellmanford(int n,int s){//图的点数为n,出发点为s
    memset(dis,63,sizeof(dis));
    dis[s]=0;
    bool flag=false;
    for(int i=1;i<=n;i++){
        flag=false;
        for(int j=1;j<=n;j++){
            if(dis[j]==inf)
                continue;
            for(auto [it,w]:G[j]){
                if(dis[it]>dis[j]+w){
                    dis[it]=dis[j]+w;
                    flag=true;
                }
            }
        }
        if(!flag)
            break;
    }
    return flag;
}
```

SPFA

```
int dis[N],cnt[N];
bool vis[N];
queue<int>q;
bool spfa(int n,int s){
    memset(dis,63,sizeof(dis));
    dis[s]=0;vis[s]=true;
    q.push(s);
    while(q.size()){
        int tmp=q.front();
        q.pop();
        vis[tmp]=false;
```

```cpp
        for(auto [it,w]:G[tmp]){
            if(dis[it]>dis[tmp]+w){
                dis[it]=dis[tmp]+w;
                cnt[it]=cnt[tmp]+1;
                if(cnt[it]>=n)
                    return false;
                if(!vis[it]){
                    q.push(it);
                    vis[it]=true;
                }
            }
        }
    }
    return true;
}
```

Dijkstra

优先队列实现

复杂度$O(m \log m)$

```cpp
int dis[N];
bool vis[N];
priority_queue<pii,vector<pii>,greater<pii>>pq;//{距离，点}
void Dijkstra(int n,int s){
    memset(dis,63,sizeof(dis));
    memset(vis,false,sizeof(vis));
    while(pq.size())
        pq.pop();
    dis[s]=0;
    pq.push({0,s});
    while(pq.size()){
        pii tmp=pq.top();
        pq.pop();
        if(vis[tmp.second])
            continue;
        vis[tmp.second]=true;
        for(auto [it,w]:G[tmp.second]){
            if(dis[it]>dis[tmp.second]+w){
                dis[it]=dis[tmp.second]+w;
                pq.push({dis[it],it});
            }
        }
    }
}
```

暴力实现

复杂度$O(n^2)$

```cpp
int dis[N];
bool vis[N];
void Dijkstra(int n,int s){
    memset(dis,0x3f,sizeof(dis));
    dis[s]=0;
    for(int i=1;i<=n;i++){
```

```
        int k=0,m=1e15;
        for(int j=1;j<=n;j++){
            if(!vis[j]&&dis[j]<m){
                k=j;m=dis[j];
            }
        }
        vis[k]=1;
        for(auto [it,w]:G[k])
            dis[it]=min(dis[it],dis[k]+w);
    }
}
```

差分约束

n个变量$x_1, x_2, \ldots, x_n$以及m个约束条件$x_i - x_j \leq c_k$

约束是否有解，如果有解，给出一组解

$x_i - x_j \leq c_k \iff x_i \leq x_j + c_k$，类比单源最短路中的三角形不等式 $dist[y] \leq dist[x] + z$

将j向i连长度为c_k的有向边，设超级源点0，向每个点连一条权为0的有向边，跑spfa，若图中有负环，则无解；否则$x_i = dist[i]$就是一组解

缩点

将强连通分量缩为一个点，原图变为DAG

Tarjan缩点

$num[N], low[N]$

- num值：dfs时这个点的时间戳
- low值：能返回的最远祖先的时间戳

相同low值的属于一个SCC，在dfs的同时把点按SCC分开

复杂度$O(n + m)$

```cpp
const int N=1e4+5;
int a[N];//点权
vector<int>G[N];
int low[N],num[N],dfn,id[N];
int cnt,v[N];
stack<int>st;
void dfs(int x){
    low[x]=num[x]=++dfn;
    st.push(x);
    for(auto it:G[x]){
        if(!num[it]){
            dfs(it);
            low[x]=min(low[x],low[it]);
        }
        else if(!id[it])
            low[x]=min(low[x],num[it]);
    }
    if(low[x]==num[x]){
        cnt++;
```

```
            while(true){
                int tmp=st.top();
                st.pop();
                v[cnt]+=a[tmp];
                id[tmp]=cnt;
                if(x==tmp)
                    break;
            }
        }
    }
}
void Tarjan(int n){
    dfn=cnt=0;
    memset(low,0,sizeof(low));
    memset(num,0,sizeof(num));
    memset(id,0,sizeof(id));
    while(st.size())
        st.pop();
    for(int i=1;i<=n;i++)
        if(!num[i])
            dfs(i);
}
```

Kosaraju缩点

1. 原图的反图（边的方向取反）的连通性不变
2. 按原图的dfs的逆序开始dfs反图，可以将强连通分量挖出来

复杂度$O(n+m)$

```
const int N=1e4+5;
int a[N];
vector<int>G[N],rG[N];
vector<int>S;
bool vis[N];
int cnt,id[N];
void dfs1(int x){
    if(vis[x])
        return;
    vis[x]=true;
    for(auto it:G[x])
        dfs1(it);
    S.push_back(x);
}
int d[N],v[N];
void dfs2(int x){
    if(id[x])
        return;
    id[x]=cnt;
    v[cnt]+=a[x];
    for(auto it:rG[x])
        dfs2(it);
}
void Korasaju(int n){
    memset(vis,false,sizeof(vis));
    memset(id,0,sizeof(id));
    cnt=0;
    S.clear();
```

```
    for(int i=1;i<=n;i++)
        dfs1(i);
    reverse(S.begin(),S.end());
    for(auto it:S){
        if(!id[it]){
            cnt++;
            dfs2(it);
        }
    }
}
```

2-SAT

n个集合，每个集合两个元素，已知若干个$< a,b >$，表示a与b矛盾（a,b属于不同集合），从每个集合选一个元素，判断能否选n个两两不矛盾的元素

可以变为布尔方程，选a则必选b，则连$a \to b$的有向边，在图上缩点之后判断是否有一个集合中的两个数在一个SCC里

树链剖分

重链剖分

- $id[x]$: x点的dfs序
- $rk[x]$: dfs序为x的节点
- $top[x]$: x所在重链的顶部节点

```
int sz[N],top[N],rk[N],id[N],son[N],fa[N],deep[N];
vector<int>G[N];
void dfs1(int x,int f){
    sz[x]=1;
    fa[x]=f;
    deep[x]=deep[f]+1;
    for(auto it:G[x]){
        if(it==f)
            continue;
        dfs1(it,x);
        sz[x]+=sz[it];
        if(!son[x]||sz[son[x]]<sz[it])
            son[x]=it;
    }
}
void dfs2(int x,int topx){
    top[x]=topx;
    id[x]=++num;
    rk[num]=x;
    if(!son[x])
        return;
    dfs2(son[x],topx);
    for(auto it:G[x]){
        if(it!=fa[x]&&it!=son[x])
            dfs2(it,it);
    }
}
```

```
dfs1(root,0);
dfs2(root,root);
```

树上区间修改/查询

```cpp
struct SegmentTree{
    int a[N],tree[N<<2],tag[N<<2];
    int ls(int p){return p<<1;}
    int rs(int p){return p<<1|1;}
    void push_up(int p){
        tree[p]=tree[ls(p)]+tree[rs(p)];
        tree[p]%=mod;
    }
    void build(int p,int pl,int pr){
        tag[p]=0;
        if(pl==pr){
            tree[p]=a[rk[pl]];
            return;
        }
        int mid=(pl+pr)>>1;
        build(ls(p),pl,mid);
        build(rs(p),mid+1,pr);
        push_up(p);
    }
    void addtag(int p,int pl,int pr,int d){
        tag[p]+=d;
        tree[p]+=d*(pr-pl+1);
        tree[p]%=mod;
    }
    void push_down(int p,int pl,int pr){
        if(tag[p]){
            int mid=(pl+pr)>>1;
            addtag(ls(p),pl,mid,tag[p]);
            addtag(rs(p),mid+1,pr,tag[p]);
            tag[p]=0;
        }
    }
    void update(int L,int R,int p,int pl,int pr,int d){
        if(L<=pl&&pr<=R){
            addtag(p,pl,pr,d);
            return;
        }
        push_down(p,pl,pr);
        int mid=(pl+pr)>>1;
        if(L<=mid)
            update(L,R,ls(p),pl,mid,d);
        if(R>mid)
            update(L,R,rs(p),mid+1,pr,d);
        push_up(p);
    }
    int query(int L,int R,int p,int pl,int pr){
        if(L<=pl&&pr<=R)
            return tree[p];
        push_down(p,pl,pr);
        int mid=(pl+pr)>>1;
        int ans=0;
        if(L<=mid)
```

```
                ans+=query(L,R,ls(p),pl,mid);
            if(R>mid)
                ans+=query(L,R,rs(p),mid+1,pr);
            return ans;
        }
}Tr;
void add_range(int x,int y,int d){
    while(top[x]!=top[y]){
        if(deep[top[x]]<deep[top[y]])
            swap(x,y);
        Tr.update(id[top[x]],id[x],1,1,n,d);
        x=fa[top[x]];
    }
    if(deep[x]>deep[y])
        swap(x,y);
    Tr.update(id[x],id[y],1,1,n,d);
}
int query_range(int x,int y){
    int ans=0;
    while(top[x]!=top[y]){
        if(deep[top[x]]<deep[top[y]])
            swap(x,y);
        ans+=Tr.query(id[top[x]],id[x],1,1,n);
        ans%=mod;
        x=fa[top[x]];
    }
    if(deep[x]>deep[y])
        swap(x,y);
    ans+=Tr.query(id[x],id[y],1,1,n);
    return ans%mod;
}
void add_tree(int x,int d){
    Tr.update(id[x],id[x]+sz[x]-1,1,1,n,d);
}
int query_tree(int x){
    return Tr.query(id[x],id[x]+sz[x]-1,1,1,n)%mod;
}
```

树上启发式合并(DSU on tree)

常用于不带修子树询问

对于节点i

- 递归轻儿子，消除递归的影响
- 递归重儿子，不消除递归的影响
- 统计所有轻儿子对答案的影响
- 更新该节点答案
- 删除所有轻儿子对答案的影响

主题框架：

```
void dfs(int x,int fa,int opt){
    for(auto it:G[x]){
        if(it==fa)
            continue;
        if(it!=son[x])
            dfs(it,x,0);//暴力统计轻边的贡献
    }
    if(son[x])  dfs(son[x],x,1);//统计重儿子的贡献，不消除影响
    add(x);//暴力统计所有轻儿子的贡献
    ans[x]=NowAns;//更新答案
    if(!opt)    delet(x);//需要删除贡献的话就删掉
}
```

示例代码:

```
int c[N],sz[N],son[N],cnt[N],sum,Mx,Son,ans[N];
vector<int>G[N];
void dfs1(int x,int f){
    sz[x]=1;
    for(auto it:G[x]){
        if(it==f)
            continue;
        dfs1(it,x);
        sz[x]+=sz[it];
        if(!son[x]||sz[son[x]]<sz[it])
            son[x]=it;
    }
}
void add(int x,int f,int val){
    cnt[c[x]]+=val;
    if(cnt[c[x]]>Mx){
        Mx=cnt[c[x]];
        sum=c[x];
    }
    else if(cnt[c[x]]==Mx)
        sum+=c[x];
    for(auto it:G[x]){
        if(it==f||it==Son)
            continue;
        add(it,x,val);
    }
}
void dfs2(int x,int f,int opt){
    for(auto it:G[x]){
        if(it==f||it==son[x])
            continue;
        dfs2(it,x,0);
    }
    if(son[x]){
        dfs2(son[x],x,1);
        Son=son[x];
    }
    add(x,f,1);Son=0;
    ans[x]=sum;
    if(!opt){
        add(x,f,-1);
```

```
        Mx=sum=0;
    }
}
```

网络流

二分图匹配

```cpp
vector<int> G[N];
int Nx,Ny,k;  //Nx,Ny是两个集合的大小；k是边数

int Mx[N],My[N];
int dx[N],dy[N];
int dis,u,v;
bool used[N];
bool searchP(){
    queue<int> Q;
    dis = INF;
    memset(dx,-1,sizeof(dx));
    memset(dy,-1,sizeof(dy));
    for(int i = 0;i < Nx;++i)
    if(Mx[i] == -1)    Q.push(i), dx[i] = 0;
    while(!Q.empty()){
        int u = Q.front();Q.pop();
        if(dx[u] > dis) break;
        int sz = G[u].size();
        for(int i = 0;i < sz;++i){
            int v = G[u][i];
            if(dy[v] == -1) {
                dy[v] = dx[u] + 1;
                if(My[v] == -1) dis = dy[v];
                else dx[My[v]] = dy[v] + 1, Q.push(My[v]);
            }
        }
    }
    return dis != INF;
}
bool DFS(int u){
    int sz = G[u].size();
    for(int i = 0;i < sz;++i){
        int v = G[u][i];
        if(!used[v] && dy[v] == dx[u] + 1){
            used[v] = true;
            if(My[v] != -1 && dy[v] == dis) continue;
            if(My[v] == -1 || DFS(My[v])){
                My[v] = u;
                Mx[u] = v;
                return true;
            }
        }
    }
    return false;
}
```

```cpp
int MaxMatch(){
    int res = 0;
    memset(Mx,-1,sizeof(Mx));
    memset(My,-1,sizeof(My));
    while(searchP()){
        memset(used,false,sizeof(used));
        for(int i = 0;i < Nx;++i)
        if(Mx[i] == -1 && DFS(i)) ++res;
    }
    return res;
}
int main(){
    read(Nx);read(Ny);read(k);
    while(k--){read(u);read(v);if(v<=Ny) G[u-1].push_back(v-1);}
    printf("%d\n",MaxMatch());
}
```

树哈希

$$f(S) = (c + \sum_{x \in S} g(x)) \mod m$$

一般取$c = 1$，g为整数到整数的映射

```cpp
const ull mask = std::chrono::steady_clock::now().time_since_epoch().count();
ull shift(int x){
    x^=x<<13;
    x^=x>>7;
    x^=x<<17;
    x^=mask;
    return x;
}
ull Hash[N];
set<ull>st;
void dfs(int x,int f){
    Hash[x]=1;
    for(auto it:G[x]){
        if(it==f)
            continue;
        dfs(it,x);
        Hash[x]+=shift(Hash[it]);
    }
    st.insert(Hash[x]);
}
```

凯莱公式

完全图K_n有n^{n-2}棵生成树

生成森林连成生成树的方案数

n个点m条边的带标号无向图有k个连通块，加$k-1$条边连通，每个连通块的点数为
$s_i(i = 1, 2, \cdots, k)$

方案数：$n^{k-2} \cdot \Pi_{i=1}^{k} s_i$

数学

常见数论函数

欧拉函数

$$\varphi(x) = x \cdot \Pi(1 - \frac{1}{p_i})$$

性质

$$\phi(x) = \sum_{d|n} \frac{\mu(d)}{d}$$

费马小定理

$$p \in Prim \Rightarrow a^{p-1} \equiv 1 \mod p$$

欧拉定理

$$(a, m) = 1 \Rightarrow a^{\phi(m)} \equiv 1 \mod m$$

扩展欧拉定理

$$a^b \equiv a^{b \mod \phi(m) + \phi(m)} \mod m \ (b \geq \phi(m))$$

高斯消元

例题 洛谷 P3389

```
namespace Gauss{
    const int N=1e3+5;
    double eps=1e-7;
    bool solve(double a[N][N],int n){//n+1行为 Ax=b 矩阵方程右侧向量，运算后结果在 n+1
行
        for(int i=1;i<=n;i++){
            double M=0;
            int Mi=0;
            for(int j=i;j<=n;j++){
                if(fabs(a[j][i])>M){
                    M=fabs(a[j][i]);
                    Mi=j;
                }
            }
            for(int j=i;j<=n+1;j++)
                swap(a[Mi][j],a[i][j]);
            if(fabs(a[i][i])<eps){//无解
                return false;
            }
            for(int j=n+1;j>=i;j--)
                a[i][j]/=a[i][i];
            for(int j=1;j<=n;j++){
                if(j==i)
                    continue;
                double temp=a[j][i]/a[i][i];
                for(int k=i;k<=n+1;k++)
                    a[j][k]-=temp*a[i][k];
            }
```

```
        }
        return true;
    }
}
```

GCD

欧几里得算法

```
int gcd(int a,int b)
{
    return b?gcd(b,a%b):a;
}
```

更相损减术

$$\gcd(a, b) = \gcd(b, a - b) = \gcd(a, a - b)$$

```
int gcd(int a,int b)
{
    while(a!=b)
    {
        if(a>b)  a=a-b;
        else     b=b-a;
    }
    return a;
}
```

LCM

```
int lcm(int a,int b)
{
    return a/gcd(a,b)*b;
}
```

扩展欧几里得

返回$d = gcd(a, b)$;以及$ax + by = d$的特解x_0, y_0

通解:

$$x = \frac{c}{d}x_0 + \frac{b}{d}t, \; y = \frac{c}{d}y_0 - \frac{a}{d}t$$

```
ll extend_gcd(ll a,ll b,ll &x,ll &y)
{
    if(b==0){x=1;y=0;return a;}
    ll d=extend_gcd(b,a%b,y,x);
    y-=a/b*x;
    return d;
}
```

逆元

扩展欧几里得

```
ll mod_inverse(ll a,ll m)
{
    ll x,y;
    extend_gcd(a,m,x,y);
    return (x%m+m)%m;
}
```

快速幂

$$a^{-1} \equiv a^{p-2} \mod p$$

```
ll mod_inverse(ll a,ll m)
{
    return qpow(a,m-2,m);
}
```

递推

求$1-n$的所有逆元

```
void mod_inverse(ll n,ll p)
{
    inv[1]=1;
    for(int i=1;i<=n;i++)
        inv[i]=(ll)(p-p/i)*inv[p%i]%p;
}
```

埃氏筛法

```
bitset<N>vis;
void get_prime(int n)
{
    for(int i=2;i<=n;i++)
    {
        if(!vis[i])
        {
            vis[i]=true;
            p.push_back(i);
            for(int j=i*i;j<=n;j+=i)
                vis[j]=1;
        }
    }
}
```

欧拉筛

筛素数

```cpp
bool vis[N];
vector<int>p;
void init(int n){
    for(int i=2;i<=n;i++){
        if(!vis[i])
            p.push_back(i);
        for(auto it:p){
            if(1ll*i*it>n)
                break;
            vis[i*it]=1;
            if(i%it==0)
                break;
        }
    }
}
```

筛欧拉函数

```cpp
bool vis[N];
int phi[N];
vector<int>p;
void get_phi()
{
    phi[1]=1;
    for(int i=2;i<N;i++)
    {
        if(!vis[i])
        {
            vis[i]=true;
            p.push_back(i);
            phi[i]=i-1;
        }
        for(auto p:p)
        {
            if(i*p>=N)
                break;
            vis[i*p]=true;
            if(i%p==0)
            {
                phi[i*p]=p*phi[i];
                break;
            }
            phi[i*p]=phi[i]*phi[p];
        }
    }
}
```

筛约数和

```cpp
vector<int>p;
int phi[N],sig[N],num[N];
bool vis[N];
void init()
{
```

```cpp
        phi[1]=sig[1]=1;
        for(int i=2;i<N;i++)
        {
            if(!vis[i])
            {
                vis[i]=true;
                p.push_back(i);
                phi[i]=i-1;
                sig[i]=num[i]=i+1;
            }
            for(auto j:p)
            {
                if(i*j>=N)
                    break;
                vis[i*j]=true;
                if(i%j==0)
                {
                    phi[i*j]=phi[i]*j;
                    num[i*j]=num[i]*j+1;
                    sig[i*j]=sig[i]/num[i]*num[i*j];
                    break;
                }
                phi[i*j]=phi[i]*phi[j];
                num[i*j]=1+j;
                sig[i*j]=sig[i]*sig[j];
            }
        }
    }
```

素数判定

Miller Rabin

复杂度$O(k\log n)$

```cpp
bool is_prime(int x)
{
    if(x<3)
        return x==2;
    if(x%2==0)
        return false;
    int A[]={2,325,9375,28178,450775,9780504,1795265022},d=x-1,r=0;
    while(d%2==0)
        d>>=1,r++;
    for(auto a:A)
    {
        int v=qpow(a,d,x);
        if(v<=1||v==x-1)
            continue;
        for(int i=0;i<r;i++)
        {
            v=(__int128_t)v*v%x;
            if(v==x-1&&i!=r-1)
            {
                v=1;break;
            }
            if(v==1)
```

```
                return false;
        }
        if(v!=1)
            return false;
    }
    return true;
}
```

质因数分解

Pollard Rho

找出一个约数的时间复杂度$O(n^{\frac{1}{4}})$

```cpp
mt19937_64 rnd(time(0));
namespace Pollard_Rho
{
    #define ldb long double
    long long mul(long long x, long long y, long long mod)
    {
        return ((x * y - (long long)((ldb)x / mod * y) * mod) + mod) % mod;
    }
    long long gcd(long long a, long long b)
    {
        return (b == 0 ? a : gcd(b, a % b));
    }
    long long ksm(long long a, long long b, long long mod)
    {
        long long ans = 1; a %= mod;
        while (b) {if (b & 1)ans = mul(ans, a, mod); b >>= 1; a = mul(a, a,
mod);}
        return ans;
    }
    int pr[15] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
    bool Miller_Rabin(long long n)
    {
        if (n == 2 || n == 3)return 1;
        if (n % 2 == 0 || n == 1)return 0;
        long long d = n - 1;
        int s = 0;
        while (d % 2 == 0)s ++, d >>= 1;
        for (int i = 0; i <= 11; i ++)
        {
            if (pr[i] >= n)break;
            long long a = pr[i];
            long long x = ksm(a, d, n);
            long long y = 0;
            for (int j = 0; j <= s - 1; j ++)
            {
                y = mul(x, x, n);
                if (y == 1 && x != 1 && x != (n - 1))return 0;
                x = y;
            }
            if (y != 1)return 0;
        }
        return 1;
    }
```

```cpp
    long long Pollard_Rho(long long n)
    {
        long long now, pre, g;
        while (true)
        {
            now = pre = rnd() % (n - 1) + 1;
            g = 1;
            long long c = rnd() % (n - 1) + 1;
            for (int i = 1, fst = 1 ;; i ++)
            {
                now = (mul(now, now, n) + c) % n;
                g = mul(g, abs(now - pre), n);
                if (now == pre || !g)break;
                if (!(i & 127) || i == fst)
                {
                    g = gcd(g, n);
                    if (g > 1)return g;
                    if (i == fst)pre = now, fst <<= 1;
                }
            }
        }
    }
    void Find(long long n, map<long long, long long>& _P, int c = 1)
    {
        if (n == 1)return ;
        if (Miller_Rabin(n))
        {
            _P[n] += c;
            return;
        }
        long long p = Pollard_Rho(n);
        int cnt = 0;
        while (!(n % p))
        {
            n /= p, cnt ++;
        }
        Find(p, _P, cnt * c);
        Find(n, _P, c);
    }
}
```

离散对数

bsgs

对$a, b \in Z^+$，可以以$O(\sqrt{m})$的复杂度内求解

$$a^x \equiv b \mod m$$

其中$(a, m) = 1$，解$0 \leq x < m$，m不一定是素数

取$x = A\lceil \sqrt{m} \rceil - B$，其中$0 \leq A, B \leq \lceil \sqrt{m} \rceil$，有$a^{A\lceil \sqrt{m} \rceil - B} \equiv b \mod m$

$\iff a^{A\lceil \sqrt{m} \rceil} \equiv ba^B \mod m$

同时枚举左右两边，用hashmap存，可以$O(\sqrt{m})$的复杂度内解决

```cpp
ll BSGS(ll a, ll b, ll m)
{
    static unordered_map<ll, ll> hs;
    hs.clear();
    ll cur = 1, t = sqrt(m) + 1;
    for (int B = 1; B <= t; ++B)
    {
        (cur *= a) %= m;
        hs[b * cur % m] = B; // 哈希表中存B的值
    }
    ll now = cur; // 此时cur = a^t
    for (int A = 1; A <= t; ++A)
    {
        auto it = hs.find(now);
        if (it != hs.end())
            return A * t - it->second;
        (now *= cur) %= m;
    }
    return -1; // 没有找到，无解
}
```

扩展bsgs

a, m不一定互质

```cpp
// 修改版的BSGS，额外带一个系数
ll BSGS(ll a, ll b, ll m, ll k = 1)
{
    static unordered_map<ll, ll> hs;
    hs.clear();
    ll cur = 1, t = sqrt(m) + 1;
    for (int B = 1; B <= t; ++B)
    {
        (cur *= a) %= m;
        hs[b * cur % m] = B; // 哈希表中存B的值
    }
    ll now = cur * k % m;
    for (int A = 1; A <= t; ++A)
    {
        auto it = hs.find(now);
        if (it != hs.end()) return A * t - it->second;
        (now *= cur) %= m;
    }
    return -INF; // 这里因为要多次加1，要返回更小的负数
}
ll exBSGS(ll a, ll b, ll m, ll k = 1)
{
    ll A = a %= m, B = b %= m, M = m;
    if (b == 1) return 0;
    ll cur = 1 % m;
    for (int i = 0;; i++)
    {
        if (cur == B) return i;
        cur = cur * A % M;
        ll d = gcd(a, m);
        if (b % d) return -INF;
        if (d == 1) return BSGS(a, b, m, k * a % m) + i + 1;
```

```
        k = k * a / d % m, b /= d, m /= d; // 相当于在递归求解exBSGS(a, b / d, m /
d, k * a / d % m)
    }
}
```

组合数

$$\binom{n}{m} = C_n^m = \frac{P_n^m}{P_m} = \frac{n!}{m!(n-m)!}$$

组合恒等式

$$C_n^k = C_n^{m-k}$$

$$C_{n+1}^k = C_n^k + C_n^{k-1}$$

$$(C_n^0)^2 + (C_n^1)^2 + (C_n^2)^2 + \ldots + (C_n^n)^2 = C_{2n}^n = \frac{(2n)!}{(n!)^2}$$

$$C_{-n}^k = \frac{(-n)(-n-1)(-n-2)\ldots(-n-k+1)}{k!} = (-1)^k C_{n+k-1}^k$$

预处理阶乘

```
void init(int n)
{
    fac[0]=1;
    for(int i=1;i<=n;i++)
        fac[i]=fac[i-1]*i%mod;
    rev[n]=qpow(fac[n],mod-2,mod);//n must be less than mod
    for(int i=n;i>=1;i--)
        rev[i-1]=rev[i]*i%mod;
    assert(rev[0]==1);
}
```

Lucas

$$p \in Prim, \ C_n^m \equiv C_{n/p}^{m/p} \cdot C_{n\%p}^{m\%p} \mod p$$

模数较小，但组合数很大

推论:

$$m, n \in Z^+, p \in Prim, \ C_n^m \equiv \Pi_{i=0}^k C_{n_i}^{m_i}$$

$$m = m_k p^k + \cdots + m_1 p + m_0, \ n = n_k p^k + \cdots + n_1 p + n_0$$

```
int C(int n,int m,int p)
{
    if(m>n)
        return 0;
    return fac[n]*rev[m]%p*rev[n-m]%p;
}
int Lucas(int n,int m,int p)
{
    if(m==0)
        return 1;
    return C(n%p,m%p,p)*Lucas(n/p,m/p,p)%p;
}
```

Wilson定理

$$p \in Prim, \ (p-1)! \equiv -1 \mod p$$

中国剩余定理

$$x \equiv a_1 \mod m_1$$
$$x \equiv a_2 \mod m_2$$
$$\vdots$$
$$x \equiv a_k \mod m_k$$

其中m_i, m_j两两互质

设:

$$M = \Pi_{i=1}^k m_i, \ M_i = \frac{M}{m_i}, \ M_i^{-1} \cdot M_i \equiv 1 \mod m_i$$

方程组在模M意义下有唯一解

$$x \equiv \sum_{i=1}^k a_i M_i M_i^{-1} \mod M$$

升幂引理(LTE)

$v_p(n)$为n的标准分解中质因数p的幂次，即$v_p(n)$满足$p^{v_p(n)} \mid n$且$p^{v_p(n)+1} \nmid n$

以下设$p \in Prim, x, y \in Z, p \nmid x, p \nmid y, n \in Z^+$

- 第一部分: $p \in Prim, (n, p) = 1$

    1. 若$p \mid (x - y)$，则

    $$v_p(x^n - y^n) = v_p(x - y)$$

    2.

    3. 若$p \mid (x + y), n$奇，则

    $$v_p(x^n + y^n) = v_p(x + y)$$

- 第二部分: p奇素数

    1. 若$p \mid (x - y)$，则

$$v_p(x^n - y^n) = v_p(x - y) + v_p(n)$$

2. 若$p \mid (x + y)$，则对奇数n有

$$v_p(x^n + y^n) = v_p(x + y) + v_p(n)$$

- 第三部分：$p = 2$且$p \mid (x - y)$

    1. 对奇数n有

$$v_p(x^n - y^n) = v_p(x - y)$$

    2. 对偶数n有

$$v_p(x^n - y^n) = v_p(x - y) + v_p(x + y) + v_p(n) - 1$$

类欧几里得

$$f(x) = \frac{ax + b}{c}, \text{求} x \in [0, n] \text{且} x \in Z \text{时}, f(x) \text{下的整点个数之和}$$

$$f(a, b, c, n) = \sum_{i=0}^{n} \lfloor \frac{ai + b}{c} \rfloor$$

时间复杂度$O(\log n)$

```cpp
ll f(ll a, ll b, ll c, ll n) {
    if (!a) return b / c * (n + 1);
    if (a >= c || b >= c)
        return f(a % c, b % c, c, n) + (a / c) * n * (n + 1) / 2 + (b / c) * (n
+ 1);
    ll m = (a * n + b) / c;
    return n * m - f(c, c - b - 1, a, m - 1);
}
```

阶

定义

若满足$a^n \equiv 1 \mod m$的最小正整数n存在，这个n称为a模m的阶，记作$n = \delta_m(a)$或$ord_m(a)$

性质

- $a, a^2, \ldots, a^{\delta_m(a)}$模$m$两两不同余
- 若$a^n \equiv 1 \mod m$，则$\delta_m(a) \mid n$
- $a^p \equiv a^q \Rightarrow p \equiv q \mod \delta_m(a)$
- $m \in N^*, a, b \in Z, (a, m) = (b, m) = 1$，则

$$\delta_m(ab) = \delta_m(a)\delta_m(b) \iff (\delta_m(a), \delta_m(b)) = 1$$

- $k \in N, m \in N^*, a \in Z, (a, m) = 1$，则

$$\delta_m(a^k) = \frac{\delta_m(a)}{(\delta_m(a), k)}$$

原根

定义

若$(g, m) = 1$且$\delta_m(g) = \phi(m)$，则称g为模m的原根

性质

若一个数m有原根，则它原根的个数为$\phi(\phi(m))$

原根存在定理

一个数m存在原根当且仅当$m = 2, 4, p^\alpha, 2p^\alpha$，其中$p$为奇素数，$\alpha \in N^*$

莫比乌斯反演

莫比乌斯函数

$$\mu(n) = \begin{cases} 1 & n = 1 \\ 0 & n\text{含有平方因子} \\ (-1)^k & k\text{为}n\text{的本质不同质因子个数} \end{cases}$$

性质

- 积性函数

- $$\sum_{d|n} \mu(d) = \begin{cases} 1 & n = 1 \\ 0 & n \neq 1 \end{cases} \iff \sum_{d|n} \mu(d) = \varepsilon(n) = [n == 1],\ \mu * 1 = \varepsilon$$

- $$[gcd(i, j) == 1] = \sum_{d|gcd(i,j)} \mu(d)$$

莫比乌斯变换

设$f(n), g(n)$为数论函数

$$f(n) = \sum_{d|n} g(d) \Rightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})$$

$f(n)$称为$g(n)$的莫比乌斯变换，$g(n)$称为$f(n)$的莫比乌斯逆变换（反演）

$$f(n) = \sum_{n|d} g(d) \Rightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$$

BM线性递推

```cpp
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
#define int long long
typedef unsigned long long ull;
#define dmp(x) cerr<<"DEBUG"<<__LINE__<<":"<<#x<<" "<<x<<endl
const int INF=0x3f3f3f3f;
typedef pair<int,int> pii;
const int mod=1e9+7;
int powmod(int a,int b){
    int res=1;a%=mod;
```

```cpp
        assert(b>=0);
        while(b)
        {
            if(b&1) res=res*a%mod;
            a=a*a%mod;
        }
        return res;
}
int n;
namespace linear_seq{
    const int N=10010;
    int res[N],base[N],_c[N],_md[N];
    vector<int>Md;
    void mul(int *a,int *b,int k){
        for(int i=0;i<k+k;i++)  _c[i]=0;
        for(int i=0;i<k;i++)
            if(a[i])
                for(int j=0;j<k;j++)
                    _c[i+j]=(_c[i+j]+a[i]*b[j])%mod;
        for(int i=k+k-1;i>=k;i--)
            if(_c[i])
                for(int j=0;j<(int)Md.size();j++)
                    _c[i-k+Md[j]]=(_c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
        for(int i=0;i<k;i++)
            a[i]=_c[i];
    }
    int solve(int n,vector<int>a,vector<int>b){
        int ans=0,pnt=0;
        int k=(int)a.size();
        assert(a.size()==b.size());
        for(int i=0;i<k;i++)
            _md[k-1-i]=-a[i];
        _md[k]=1;
        Md.clear();
        for(int i=0;i<k;i++)
            if(_md[i]!=0)
                Md.push_back(i);
        for(int i=0;i<k;i++)
            res[i]=base[i]=0;
        res[0]=1;
        while((1ll<<pnt)<=n)
            pnt++;
        for(int p=pnt;p>=0;p--){
            mul(res,res,k);
            if((n>>p)&1){
                for(int i=k-1;i>=0;i--)
                    res[i+1]=res[i];
                res[0]=0;
                for(int j=0;j<(int)Md.size();j++)
                    res[Md[j]]=(res[Md[j]]-res[k]*_md[Md[j]])%mod;
            }
        }
        for(int i=0;i<k;i++)
            ans=(ans+res[i]*b[i])%mod;
        if(ans<0)
            ans+=mod;
        return ans;
    }
```

```cpp
        vector<int> BM(vector<int> s){
            vector<int> C(1,1),B(1,1);
            int L=0,m=1,b=1;
            for(int i=0;i<(int)s.size();i++){
                int d=0;
                for(int i=0;i<L+1;i++)
                    d=(d+C[i]*s[n-i])%mod;
                if(d==0)
                    ++m;
                else if(2*L<=n){
                    vector<int> T=C;
                    int c=mod-d*powmod(b,mod-2)%mod;
                    while(C.size()<B.size()+m)
                        C.push_back(0);
                    for(int i=0;i<B.size();i++)
                        C[i+m]=(C[i+m]+c*B[i])%mod;
                    L=n+1-L;B=T;b=d;m=1;
                }
                else{
                    int c=mod-d*powmod(b,mod-2)%mod;
                    while(C.size()<B.size()+m)
                        C.push_back(0);
                    for(int i=0;i<B.size();i++)
                        C[i+m]=(C[i+m]+c*B[i])%mod;
                    ++m;
                }
            }
            return C;
        }
        int gao(vector<int>a,int n){
            vector<int> c=BM(a);
            c.erase(c.begin());
            for(int i=0;i<c.size();i++)
                c[i]=(mod-c[i])%mod;
            return solve(n,c,vector<int>(a.begin(),a.begin()+(int)c.size()));
        }
}
signed main() {
    ios::sync_with_stdio(false);cin.tie(0);
    vector<int>v;
    v.push_back(2);
    v.push_back(24);
    v.push_back(96);
    v.push_back(416);
    v.push_back(1536);
    v.push_back(5504);
    v.push_back(18944);
    v.push_back(64000);
    v.push_back(212992);
    v.push_back(702464);
    cin>>n;
    cout<<linear_seq::gao(v,n-1)<<"\n";
    return 0;
}
```

斐波那契数

$$F_n = F_{n-1} + F_{n-2},\ F_0 = 0, F_1 = 1$$

性质

- $$F_{n-1}F_{n+1} - F_n^2 = (-1)^n$$

- $$F_{n+k} = F_k F_{n+1} + F_{k-1} F_n$$

- $$F_{2n} = F_n(F_{n+1} + F_{n-1})$$

- $$a \mid b \iff F_a \mid F_b$$

- $$gcd(F_m, F_n) = F_{gcd(m,n)}$$

卡特兰(Catalan)数

$$H_n = \frac{C_{2n}^n}{n+1},\ n \geq 2, n \in N^+,\ H_0 = H_1 = 1$$

$$H_n = \begin{cases} \sum_{i=1}^n H_{i-1}H_{n-i} & n \geq 2, n \in N^+ \\ 1 & n = 0, 1 \end{cases}$$

$$H_n = \frac{H_{n-1}(4n-2)}{n+1}$$

$$H_n = C_{2n}^n - C_{2n}^{n-1}$$

封闭形式

$$H(x) = \frac{1 - \sqrt{1-4x}}{2x} = \sum_{n \geq 0} C_{2n}^n \frac{1}{n+1} x^n$$

典型问题

- 有$2n$个人排成一行进入剧场。入场费 5 元。其中只有n个人有一张 5 元钞票，另外n人只有 10 元钞票，剧院无其它钞票，问有多少种方法使得只要有 10 元的人买票，售票处就有 5 元的钞票找零？
- 一位大城市的律师在她住所以北n个街区和以东n个街区处工作。每天她走$2n$个街区去上班。如果他从不穿越（但可以碰到)从家到办公室的对角线，那么有多少条可能的道路？
- 在圆上选择$2n$个点，将这些点成对连接起来使得所得到的n条线段不相交的方法数？
- 对角线不相交的情况下，将一个凸多边形区域分成三角形区域的方法数？
- 一个栈（无穷大）的进栈序列为$1, 2, \ldots, n$，有多少个不同的出栈序列？
- n个结点可构造多少个不同的二叉树？
- n个$+1$和n个-1构成$2n$项 $a_1, a_2, \ldots, a_{2n}$，其部分和满足
  $a_1 + a_2 + \cdots + a_k \geq 0 (k = 1, 2, 3, \ldots, 2n)$，序列个数为？

斯特林数

第二类斯特林数(斯特林子集数)

$S(n, k)$表示将n个两两不同的元素，划分为k个互不区分的非空子集的方案数

$$S(n, k) = S(n - 1, k - 1) + k \cdot S(n - 1, k), \ S(n, 0) = [n == 0]$$

$$S(n, m) = \sum_{i=0}^{m} \frac{(-1)^{m-i} i^n}{i!(m-i)!}$$

第一类斯特林数(斯特林轮换数)

$s(n, k)$表示将n个两两不同的元素，划分为k个互不区分的非空轮换的方案数

$$s(n, k) = s(n - 1, k - 1) + (n - 1) \cdot s(n - 1, k), \ s(n, 0) = [n == 0]$$

字符串

KMP

前缀数组

```cpp
vector<int> prefix_function(string s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i - 1];
        while (j > 0 && s[i] != s[j]) j = pi[j - 1];
        if (s[i] == s[j]) j++;
        pi[i] = j;
    }
    return pi;
}
```

模式匹配

```cpp
vector<int> find_occurrences(string text, string pattern) {
    string cur = pattern + '#' + text;
    int sz1 = text.size(), sz2 = pattern.size();
    vector<int> v;
    vector<int> lps = prefix_function(cur);
    for (int i = sz2 + 1; i <= sz1 + sz2; i++) {
        if (lps[i] == sz2)
        v.push_back(i - 2 * sz2);
    }
    return v;
}
```

回文串

manacher

p[i]是以i为中心的最长回文串长度

```cpp
int p[N<<1];
void change(string a)
{
    s+='$';s+='#';
```

```cpp
    for(auto it:a)
    {
        s+=it;s+='#';
    }
    s+='&';
}
void manacher()
{
    int n=s.length();
    int R=0,C;
    for(int i=1;i<n;i++)
    {
        if(i<R)
            p[i]=min(p[C]+C-i,p[(C<<1)-i]);
        else
            p[i]=1;
        while(s[i+p[i]]==s[i-p[i]])
            p[i]++;
        if(p[i]+i>R)
        {
            R=p[i]+i;
            C=i;
        }
    }
}
```

字典树

```cpp
struct Trie{//maxL是字符串总长
    int cnt=0,ch[maxL][26],sz[maxL],Cnt[maxL];//sz[maxL]是以这个点结尾的字符串数量
    int newNode(){
        cnt++;
        sz[cnt]=0;
        memset(ch[cnt],0,sizeof(ch[cnt]));
        return cnt;
    }
    void add(string s){
        int now=0;
        for(auto it:s){
            int &c=ch[now][it-'a'];
            if(!c)
                c=newNode();
            now=c;
            Cnt[now]++;
        }
        sz[now]++;
    }
    int find(string s){
        int now=0;
        for(auto it:s){
            now=ch[now][it-'a'];
            if(!now)
                return 0;
        }
        return sz[now];
    }
};
```

双哈

```cpp
typedef pair<int, int> hashv;
const ll mod1 = 1e9 + 7;
const ll mod2 = 1e9 + 9;
hashv base = make_pair(13331, 2333);
hashv operator + (hashv a, hashv b) {
    int c1 = a.first + b.first, c2 = a.second + b.second;
    if(c1 >= mod1) c1 -= mod1;
    if(c2 >= mod2) c2 -= mod2;
    return make_pair(c1, c2);
}
hashv operator - (hashv a, hashv b) {
    int c1 = a.first - b.first, c2 = a.second - b.second;
    if(c1 < 0) c1 += mod1;
    if(c2 < 0) c2 += mod2;
    return make_pair(c1,c2);
}
hashv operator * (hashv a, hashv b) {
    return make_pair(1ll*a.first*b.first%mod1, 1ll*a.second*b.second%mod2);
}
```

博弈论

Nim游戏

简介

n堆物品，每堆有a_i个，两个玩家轮流取走任意一堆的任意个物品，但不能不取，取走最后一个物品的获胜

Nim和

定义Nim和$= a_1 \oplus a_2 \oplus \cdots \oplus a_n$

当且仅当Nim和为0时，状态为必败状态，否则为必胜状态

SG函数

mex函数

值为不属于集合S中的最小非负整数

$$mex(S) = \min\{x\} \ (x \notin S, x \in N)$$

SG函数

设状态x的后继为$y_1, y_2, \ldots, y_k$，

$$SG(x) = mex\{SG(y_1), SG(y_2), \ldots, SG(y_k)\}$$

对于由n个有向图游戏组成的组合游戏，设起点分别为$s_1, s_2, \ldots, s_n$，当且仅当
$SG(s_1) \oplus SG(s_2) \oplus \cdots \oplus SG(s_n) \neq 0$时，这个游戏是先手必胜的，同时，这是一个组合游戏的游戏状态x的SG的

打表SG函数

记忆化搜索或者dp

多项式

常见的幂级数展开

$$e^x = 1 + x + \frac{1}{2!}x^2 + \cdots + \frac{1}{n!}x^n + \ldots$$

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!}x^2 + \cdots + \frac{a(a-1)\ldots(a-n+1)}{n!}x^n + \ldots$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \cdots + \frac{(-1)^n}{(2n)!}x^{2n} + \ldots$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} + \ldots$$

$$\frac{1}{1+x} = 1 - x + x^2 + \cdots + (-1)^n x^n + \ldots$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots + \frac{(-1)^{n-1}}{n}x^n + \ldots$$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 + \cdots + (-1)^n x^{2n} + \ldots$$

生成函数

$$F(x) = \sum_n a_n k_n(x)$$

$k_n(x)$为核函数

- 普通生成函数：$k_n(x) = x^n$
- 指数生成函数：$k_n(x) = \frac{x^n}{n!}$
- 狄利克雷生成函数：$k_n(x) = \frac{1}{n^x}$

计算方式

通常在封闭形式和展开形式间转换进行计算

1. 对于任意多项式$P(x), Q(x)$，生成函数$\frac{P(x)}{Q(x)}$的展开式都可以用待定系数法求出

   当对分母进行因式分解但有重根时，每有一个重根就要多一个分式

   如$G(x) = \frac{1}{(1-x)(1-2x)^2} \Rightarrow G(x) = \frac{c_0}{1-x} + \frac{c_1}{1-2x} + \frac{c_2}{(1-2x)^2}$

2. 牛顿二项式定理

$$C_r^k = \frac{r(r-1)(r-2)\ldots(r-k+1)}{k}, \ k \in N, r \in R$$

$$(1+x)^\alpha = \sum_{n \geq 0} C_n^\alpha x^n$$

3. 推论:

$$\frac{1}{(1-x)^n} = \sum_{k=0}^{\infty} C_{n+k-1}^k x^k$$

4.

快速傅里叶变换(FFT)

以$O(n \log n)$的速度计算两个n度多项式乘法

```cpp
namespace FFT{
    #define el '\n'
    #define rep(i, a, b) for (int i = (a); i <= (b); i++)
    #define lop(i, a, b) for (int i = (a); i < (b); i++)
    #define dwn(i, a, b) for (int i = (a); i >= (b); i--)
    #define ceil(a, b) (a + (b - 1)) / b
    #define db double

    constexpr int N = 4e6 + 10, M = 4e6 + 10, B = 66, md = 1e9 + 7;
    const double PI = acos(-1.0), eps = 1e-8;

    int T, n, m;

    struct Complex
    {
        double x, y;
        Complex(){}
        Complex(double x,double y):x(x),y(y){}
        Complex operator+(const Complex &t) const
        {
            return {x + t.x, y + t.y};
        }
        Complex operator-(const Complex &t) const
        {
            return {x - t.x, y - t.y};
        }
        Complex operator*(const Complex &t) const
        {
            return {x * t.x - y * t.y, x * t.y + y * t.x};
        }

    } a[N], b[N];

    int rev[N], bit, tot, res[N];
    void fft(Complex a[], int inv)
    {
        for (int i = 0; i < tot; i++)
        {
            if (i < rev[i])
                swap(a[i], a[rev[i]]); //只需要交换一次就行了，交换两次等于没有换
        }
        for (int mid = 1; mid < tot; mid <<= 1)
        {
            auto w1 = Complex({cos(PI / mid), inv * sin(PI / mid)});
```

```
            for (int i = 0; i < tot; i += mid * 2)
            {
                auto wk = Complex({1, 0});              //初始为w(0,mid),定义为
w(k,mid)
                for (int j = 0; j < mid; j++, wk = wk * w1) //单位根递推式
                {
                    auto x = a[i + j], y = wk * a[i + j + mid];
                    a[i + j] = x + y, a[i + j + mid] = x - y;
                }
            }
        }
    }

    void workFFT(int n, int m)
    {// a[0, n], b[0, m]
        while ((1 << bit) < n + m + 1)
            bit++;
        tot = 1 << bit;
        for (int i = 0; i < tot; i++)
            rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (bit - 1));
        //递推(bit<<1)在bit之前，就已经被算出rev,最后一位是否为1
        fft(a, 1), fft(b, 1);
        for (int i = 0; i < tot; i++)
            a[i] = a[i] * b[i]; //点表示法直接运算
        fft(a, -1);//逆变换，点表示法转换为多项式表示法
        for (int i = 0; i <= n + m; i++)
            res[i]  = (int)(a[i].x / tot + 0.5); //向上去整
    }
}
```

快速数论变换(NTT)

知乎板子:

```
const int MOD = 998244353, G = 3;
int rev[MAXN * 3];
void ntt(int F[], int N, int sgn)
{
    int bit = __lg(N);
    for (int i = 0; i < N; ++i) // 蝴蝶变换
    {
        rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (bit - 1));
        if (i < rev[i]) swap(F[i], F[rev[i]]);
    }
    for (int l = 1, t = 1; l < N; l <<= 1, t++) // 枚举合并前的区间长度
    {
        int step = qpow(G, ((MOD - 1) >> t) * sgn + MOD - 1);
        for (int i = 0; i < N; i += l << 1) // 遍历起始点
            for (int k = i, cur = 1; k < i + l; ++k, cur = (ll)cur * step % MOD)
            {
                int g = F[k], h = (ll)F[k + l] * cur % MOD;
                F[k] = (g + h) % MOD, F[k + l] = ((g - h) % MOD + MOD) % MOD;
            }
    }
    if (sgn == -1) // 不用再在外部除以N
```

```
    {
        int invN = qpow(N, MOD - 2);
        for (int i = 0; i < N; ++i)
            F[i] = (ll)F[i] * invN % MOD;
    }
}
```

tarjen板子:

```cpp
#include <bits/stdc++.h>

#define fp(i, a, b) for (int i = (a), i##_ = (b) + 1; i < i##_; ++i)
#define fd(i, a, b) for (int i = (a), i##_ = (b) - 1; i > i##_; --i)
#define file(s) freopen(s".in","r",stdin),freopen(s".out","w",stdout)
using namespace std;
const int maxn = 2e5 + 5, P = 998244353;
using arr = int[maxn];
using ll = int64_t;
/*-------------------------------------------------------------------------*/
class Cipolla {
    int P, I2{};
    using pll = pair<ll, ll>;
#define X first
#define Y second
    ll mul(ll a, ll b) const { return a * b % P; }
    pll mul(pll a, pll b) const { return {(a.X * b.X + I2 * a.Y % P * b.Y) % P,
(a.X * b.Y + a.Y * b.X) % P}; }
    template<class T> T POW(T a, int b, T x) { for (; b; b >>= 1, a = mul(a, a))
if (b & 1) x = mul(x, a); return x; }
public:
    Cipolla(int p = 0) : P(p) {}
    pair<int, int> sqrt(int n) {
        int a = rand(), x;
        if (!(n %= P))return {0, 0};
        if (POW(n, (P - 1) >> 1, (int)1) == P - 1) return {-1, -1};
        while (POW(I2 = ((ll) a * a - n + P) % P, (P - 1) >> 1, (int)1) == 1) a
= rand();
        x = (int) POW(pll{a, 1}, (P + 1) >> 1, {1, 0}).X;
        if (2 * x > P) x = P - x;
        return {x, P - x};
    }
#undef X
#undef Y
};
/*-------------------------------------------------------------------------*/
#define ADD(a, b) (((a) += (b)) >= P ? (a) -=P : 0) // (a += b) %= P
#define SUB(a, b) (((a) -= (b)) < 0 ? (a) += P: 0)  // ((a -= b) += P) %= P
#define MUL(a, b) ((ll) (a) * (b) % P)
//vector<int> getInv(int L) {
//    vector<int> inv(L); inv[1] = 1;
//    fp(i, 1, L - 1) inv[i] = MUL((P - P / i), inv[P % i]);
//    return inv;
//}
//auto inv = getInv(maxn); // NOLINT
int POW(ll a, int b = P - 2, ll x = 1) { for (; b; b >>= 1, a = a * a % P) if (b
& 1) x = x * a % P; return x; }
//int INV(int a) { return a < maxn ? inv[a] : POW(a); }
```

```cpp
namespace NTT {
    const int g = 3;
    vector<int> Omega(int L) {
        int wn = POW(g, P / L);
        vector<int> w(L); w[L >> 1] = 1;
        fp(i, L / 2 + 1, L - 1) w[i] = MUL(w[i - 1], wn);
        fd(i, L / 2 - 1, 1) w[i] = w[i << 1];
        return w;
    }
    auto W = Omega(1 << 21); // NOLINT
    void DIF(int *a, int n) {
        for (int k = n >> 1; k; k >>= 1)
            for (int i = 0, y; i < n; i += k << 1)
                fp(j, 0, k - 1)
                    y = a[i + j + k], a[i + j + k] = MUL(a[i + j] - y + P, W[k +
j]), ADD(a[i + j], y);
    }
    void IDIT(int *a, int n) {
        for (int k = 1; k < n; k <<= 1)
            for (int i = 0, x, y; i < n; i += k << 1)
                fp(j, 0, k - 1)
                    x = a[i + j], y = MUL(a[i + j + k], W[k + j]),
                    a[i + j + k] = x - y < 0 ? x - y + P : x - y, ADD(a[i + j],
y);
        int Inv = P - (P - 1) / n;
        fp(i, 0, n - 1) a[i] = MUL(a[i], Inv);
        reverse(a + 1, a + n);
    }
}
namespace Polynomial {
    using Poly = std::vector<int>;

    // mul/div int
    Poly &operator*=(Poly &a, int b) { for (auto &x : a) x = MUL(x, b); return
a; }
    Poly operator*(Poly a, int b) { return a *= b; }
    Poly operator*(int a, Poly b) { return b * a; }
    Poly &operator/=(Poly &a, int b) { return a *= POW(b); }
    Poly operator/(Poly a, int b) { return a /= b; }

    // Poly add/sub
    Poly &operator+=(Poly &a, Poly b) {
        a.resize(max(a.size(), b.size()));
        fp(i, 0, b.size() - 1) ADD(a[i], b[i]);
        return a;
    }
    Poly operator+(Poly a, Poly b) { return a += b; }
    Poly &operator-=(Poly &a, Poly b) {
        a.resize(max(a.size(), b.size()));
        fp(i, 0, b.size() - 1) SUB(a[i], b[i]);
        return a;
    }
    Poly operator-(Poly a, Poly b) { return a -= b; }

    // Poly mul
    void DFT(Poly &a) { NTT::DIF(a.data(), a.size()); }
    void IDFT(Poly &a) { NTT::IDIT(a.data(), a.size()); }
    int norm(int n) { return 1 << (32 - __builtin_clz(n - 1)); }
```

```cpp
void norm(Poly &a) { if (!a.empty()) a.resize(norm(a.size()), 0); }
Poly &dot(Poly &a, Poly &b) {
    fp(i, 0, a.size() - 1) a[i] = MUL(a[i], b[i]);
    return a;
}
Poly operator*(Poly a, Poly b) {
    int n = a.size() + b.size() - 1, L = norm(n);
    if (a.size() <= 8 || b.size() <= 8) {
        Poly c(n);
        fp(i, 0, a.size() - 1) fp(j, 0, b.size() - 1)
            c[i + j] = (c[i + j] + (ll) a[i] * b[j]) % P;
        return c;
    }
    a.resize(L), b.resize(L);
    DFT(a), DFT(b), dot(a, b), IDFT(a);
    return a.resize(n), a;
}

// Poly inv
Poly Inv2k(Poly a) { // a.size() = 2^k
    int n = a.size(), m = n >> 1;
    if (n == 1) return {POW(a[0])};
    Poly b = Inv2k(Poly(a.begin(), a.begin() + m)), c = b;
    b.resize(n), DFT(a), DFT(b), dot(a, b), IDFT(a);
    fp(i, 0, n - 1) a[i] = i < m ? 0 : P - a[i];
    DFT(a), dot(a, b), IDFT(a);
    return move(c.begin(), c.end(), a.begin()), a;
}
Poly Inv(Poly a) {
    int n = a.size();
    norm(a), a = Inv2k(a);
    return a.resize(n), a;
}

// Poly div/mod
Poly operator/(Poly a,Poly b){
    int k = a.size() - b.size() + 1;
    if (k < 0) return {0};
    reverse(a.begin(), a.end());
    reverse(b.begin(), b.end());
    b.resize(k), a = a * Inv(b);
    a.resize(k), reverse(a.begin(), a.end());
    return a;
}
pair<Poly, Poly> operator%(Poly a, const Poly& b) {
    Poly c = a / b;
    a -= b * c, a.resize(b.size() - 1);
    return {c, a};
}

// Poly sqrt
Poly Sqrt(Poly a) {
    int n = a.size(), k = norm(n);
    Poly b = {(new Cipolla(P))->sqrt(a[0]).first}, c;
    a.resize(k * 2, 0);
    for (int L = 2; L <= k; L <<= 1) {
        b.resize(2 * L, 0), c = Poly(a.begin(), a.begin() + L) * Inv(b);
        fp(i, 0, 2 * L - 1) b[i] = MUL(b[i] + c[i], (P + 1) / 2);
```

```
        }
        return b.resize(n), b;
    }

    // Poly calculus
    void Derivative(Poly &a) {
        fp(i, 1, a.size() - 1) a[i - 1] = MUL(i, a[i]);
        a.pop_back();
    }
}
```

计算几何

```
typedef pair<double,double>pll;
pll operator+(pll x,pll y){
    return {x.first+y.first,x.second+y.second};
}
pll operator-(pll x,pll y){
    return {x.first-y.first,x.second-y.second};
}
pll operator*(pll x,double k){
    return {x.first*k,x.second*k};
}
pll operator/(pll x,double k){
    return {x.first/k,x.second/k};
}
double len(pll x){
    return hypot(x.first,x.second);
}
double Dot(const pdd &a,const pdd &b){
    return a.first*b.first+a.second*b.second;
}
double Cross(const pdd &a,const pdd &b){
    return a.first*b.second-a.second*b.first;
}
```

实数精度

```
const double pi = acos(-1.0);          //圆周率，精确到15位小数：3.141592653589793
const double eps = 1e-8;               //偏差值，有时用1e-10，但是要注意精度
int sgn(double x){                     //判断x的大小
    if(fabs(x) < eps)  return 0;       //x==0，返回0
    else return x<0?-1:1;              //x<0返回-1，x>0返回1
}
int dcmp(double x, double y){          //比较两个浮点数
    if(fabs(x - y) < eps)   return 0;  //x==y，返回0
    else return x<y ?-1:1;             //x<y返回-1，x>y返回1
}
```

点

```cpp
struct Point{
    double x,y;
    Point(){}
    Point(double x,double y):x(x),y(y){}
    Point operator + (Point B){return Point(x+B.x,y+B.y);}
    Point operator - (Point B){return Point(x-B.x,y-B.y);}
    Point operator * (double k){return Point(x*k,y*k);}
    Point operator / (double k){return Point(x/k,y/k);}
    bool operator == (Point B){return sgn(x-B.x)==0 && sgn(y-B.y)==0;}
};
double Distance(Point A, Point B){ return hypot(A.x-B.x,A.y-B.y); }
```

向量

```cpp
typedef Point Vector;
double Dot(Vector A,Vector B){ return A.x*B.x + A.y*B.y; }
double Len(Vector A){return sqrt(Dot(A,A));}
double Len2(Vector A){return Dot(A,A);}
double Angle(Vector A,Vector B){return acos(Dot(A,B)/Len(A)/Len(B));}
double Cross(Vector A,Vector B){return A.x*B.y - A.y*B.x;}
double Area2(Point A,Point B,Point C){ return Cross(B-A, C-A);}
Vector Rotate(Vector A, double rad){  //逆时针旋转rad角度
    return Vector(A.x*cos(rad)-A.y*sin(rad), A.x*sin(rad)+A.y*cos(rad));
}
Vector Normal(Vector A){return Vector(-A.y/Len(A), A.x/Len(A));} //求单位法向量
bool Parallel(Vector A, Vector B){return sgn(Cross(A,B)) == 0;}  //返回true表示平行
或重合
```

线

```cpp
struct Line{
    Point p1,p2;                    //（1）线上的两个点
    Line(){}
    Line(Point p1,Point p2):p1(p1),p2(p2){}
    Line(Point p,double angle){    //（4）根据一个点和倾斜角 angle 确定直
线,0<=angle<pi
        p1 = p;
        if(sgn(angle - pi/2) == 0){p2 = (p1 + Point(0,1));}
        else{p2 = (p1 + Point(1,tan(angle)));}
    }
    Line(double a,double b,double c){     //（2）ax+by+c=0
        if(sgn(a) == 0){
            p1 = Point(0,-c/b);
            p2 = Point(1,-c/b);
        }
        else if(sgn(b) == 0){
            p1 = Point(-c/a,0);
            p2 = Point(-c/a,1);
        }
        else{
            p1 = Point(0,-c/b);
            p2 = Point(1,(-c-a)/b);
        }
    }
```

```cpp
};
typedef Line Segment;
int Point_line_relation(Point p, Line v){
    int c = sgn(Cross(p-v.p1,v.p2-v.p1));
    if(c < 0)return 1;              //1：p在v的左边
    if(c > 0)return 2;              //2：p在v的右边
    return 0;                       //0：p在v上
}
bool Point_on_seg(Point p, Line v){ //点和线段：0 点不在线段v上；1 点在线段v上
    return sgn(Cross(p-v.p1, v.p2-v.p1)) == 0 && sgn(Dot(p - v.p1,p - v.p2)) <=
0;
}
double Dis_point_line(Point p, Line v){
    return fabs(Cross(p-v.p1,v.p2-v.p1))/Distance(v.p1,v.p2);
}
Point Point_line_proj(Point p, Line v){
    double k = Dot(v.p2-v.p1,p-v.p1)/Len2(v.p2-v.p1);
    return v.p1+(v.p2-v.p1)*k;
}
Point Point_line_symmetry(Point p, Line v){
    Point q = Point_line_proj(p,v);
    return Point(2*q.x-p.x,2*q.y-p.y);
}
double Dis_point_seg(Point p, Segment v){
    if(sgn(Dot(p- v.p1,v.p2-v.p1))<0 || sgn(Dot(p- v.p2,v.p1-v.p2))<0)
        return min(Distance(p,v.p1),Distance(p,v.p2));
    return Dis_point_line(p,v);           //点的投影在线段上
}
int Line_relation(Line v1, Line v2){
    if(sgn(Cross(v1.p2-v1.p1,v2.p2-v2.p1)) == 0){
        if(Point_line_relation(v1.p1,v2)==0) return 1;  //1 重合
        else return 0;                               //0 平行
    }
    return 2;                                        //2 相交
}
Point Cross_point(Point a,Point b,Point c,Point d){  //Line1:ab,  Line2:cd
    double s1 = Cross(b-a,c-a);
    double s2 = Cross(b-a,d-a);                   //叉积有正负
    return Point(c.x*s2-d.x*s1,c.y*s2-d.y*s1)/(s2-s1);
}
bool Cross_segment(Point a,Point b,Point c,Point d){   //Line1:ab,  Line2:cd
    double c1 = Cross(b-a,c-a),c2=Cross(b-a,d-a);
    double d1 = Cross(d-c,a-c),d2=Cross(d-c,b-c);
    return sgn(c1)*sgn(c2) < 0 && sgn(d1)*sgn(d2) < 0;  //1相交；0不相交
}
```

多边形

```cpp
int Point_in_polygon(Point pt,Point *p,int n){  //点pt，多边形Point *p
    for(int i = 0;i < n;i++){              //3：点在多边形的顶点上
        if(p[i] == pt)  return 3;
    }
    for(int i = 0;i < n;i++){              //2：点在多边形的边上
        Line v=Line(p[i],p[(i+1)%n]);
        if(Point_on_seg(pt,v)) return 2;
    }
    int num = 0;
```

```cpp
    for(int i = 0;i < n;i++){
        int j = (i+1)% n;
        int c = sgn(Cross(pt-p[j],p[i]-p[j]));
        int u = sgn(p[i].y - pt.y);
        int v = sgn(p[j].y - pt.y);
        if(c > 0 && u < 0 && v >=0) num++;
        if(c < 0 && u >=0 && v < 0) num--;
    }
    return num != 0;                           //1：点在内部；0：点在外部
}
double Polygon_area(Point *p, int n){    //Point *p表示多边形
    double area = 0;
    for(int i = 0;i < n;i++)
        area += Cross(p[i],p[(i+1)%n]);
    return area/2;                       //面积有正负，返回时不能简单地取绝对值
}
Point Polygon_center(Point *p, int n){        //求多边形重心
    Point ans(0,0);
    if(Polygon_area(p,n)==0) return ans;
    for(int i = 0;i < n;i++)
        ans = ans+(p[i]+p[(i+1)%n])*Cross(p[i],p[(i+1)%n]);
    return ans/Polygon_area(p,n)/6;
}
```

凸包

```cpp
//Convex_hull()求凸包。凸包顶点放在ch中，返回值是凸包的顶点数
int Convex_hull(Point *p,int n,Point *ch){
    n = unique(p,p+n)-p;      //去除重复点
    sort(p,p+n);              //对点排序：按x从小到大排序，如果x相同，按y排序
    int v=0;
    //求下凸包。如果p[i]是右拐弯的，这个点不在凸包上，往回退
    for(int i=0;i<n;i++){
        while(v>1 && sgn(Cross(ch[v-1]-ch[v-2],p[i]-ch[v-1]))<=0) //把后面ch[v-1]
改成ch[v-2]也行
            v--;
        ch[v++]=p[i];
    }
    int j=v;
    //求上凸包
    for(int i=n-2;i>=0;i--){
        while(v>j && sgn(Cross(ch[v-1]-ch[v-2],p[i]-ch[v-1]))<=0) //把后面ch[v-1]
改成ch[v-2]也行
            v--;
        ch[v++]=p[i];
    }
    if(n>1) v--;
    return v;                          //返回值v是凸包的顶点数
}
Point p[N],ch[N];                       //输入点是p[]，计算得到的凸包顶点放在ch[]中
```

最近点对

使用前先sort(p,p+n,cmpxy)

```cpp
bool cmpxy(Point A,Point B){return sgn(A.x-B.x)<0||(sgn(A.x-B.x)==0&&sgn(A.y-B.y)<0);}
bool cmpy(Point A,Point B){return sgn(A.y-B.y)<0;}
double Distance(Point A, Point B){ return hypot(A.x-B.x,A.y-B.y); }
double Closest_Pair(int left,int right){
    double dis = INF;
    if(left == right) return dis;            //只剩1个点
    if(left + 1 == right) return Distance(p[left], p[right]);//只剩2个点
    int mid = (left+right)/2;                //分治
    double d1 = Closest_Pair(left,mid);      //求s1内的最近点对
    double d2 = Closest_Pair(mid+1,right);   //求s2内的最近点对
    dis = min(d1,d2);
    int k = 0;
    for(int i=left;i<=right;i++)             //在s1和s2中间附近找可能的最小点对
        if(fabs(p[mid].x - p[i].x) <= dis)   //按x坐标来找
            tmp_p[k++] = p[i];
    sort(tmp_p,tmp_p+k,cmpy);                //按y坐标排序，用于剪枝。这里不能按x坐标排序
    for(int i=0;i<k;i++)
        for(int j=i+1;j<k;j++){
            if(tmp_p[j].y - tmp_p[i].y >= dis)  break;    //剪枝
            dis = min(dis,Distance(tmp_p[i],tmp_p[j]));
        }
    return dis;   //返回最小距离
}
```

旋转卡壳

```cpp
double Rotating_Calipers(Point *p,int n){
    double ans=0;
    int j=2;
    for(int i=0;i<n;i++){
        while(fabs(Cross(p[j]-p[i],p[j]-p[(i+1)%n]))<fabs(Cross(p[(j+1)%n]-p[i],p[(j+1)%n]-p[(i+1)%n])))
            j=(j+1)%n;
        ans=max(ans,max(Distance(p[i],p[j]),Distance(p[(i+1)%n],p[j])));
    }
    return ans;
}
```

半平面交

```cpp
struct Line{      //半平面的表示
    Point p;         //直线上一个点
    Vector v;        //方向向量，它的左边是半平面
    double ang;      //极角，从x正半轴旋转到v的角度
    Line(){};
    Line(Point p, Vector v):p(p),v(v){ang = atan2(v.y, v.x);}
    bool operator < (Line &L){return ang < L.ang;}      //用于排序
};
//点p在线L左边，即点p在线L在外面:
bool OnLeft(Line L,Point p){return sgn(Cross(L.v,p-L.p))>0;}
Point Cross_point(Line a,Line b){     //两直线交点
```

```cpp
    Vector u=a.p-b.p;
    double t=Cross(b.v,u)/Cross(a.v,b.v);
    return a.p+a.v*t;
}
vector<Point> HPI(vector<Line> L){        //求半平面交，返回凸多边形
    int n=L.size();
    sort(L.begin(),L.end());               //将所有半平面按照极角排序。
    int first,last;                        //指向双端队列的第一个和最后一个元素
    vector<Point> p(n);                    //两个相邻半平面的交点
    vector<Line> q(n);                     //双端队列
    vector<Point> ans;                     //半平面交形成的凸包
    q[first=last=0]=L[0];
    for(int i=1;i<n;i++){
        //情况1：删除尾部的半平面
        while(first<last && !OnLeft(L[i], p[last-1])) last--;
        //情况2：删除首部的半平面：
        while(first<last && !OnLeft(L[i], p[first]))  first++;
        q[++last]=L[i];        //将当前的半平面加入双端队列尾部
        //极角相同的两个半平面，保留左边：
        if(fabs(Cross(q[last].v,q[last-1].v)) < eps){
        last--;
            if(OnLeft(q[last],L[i].p)) q[last]=L[i];
        }
        //计算队列尾部半平面交点：
        if(first<last) p[last-1]=Cross_point(q[last-1],q[last]);
    }
    //情况3：删除队列尾部的无用半平面
    while(first<last && !OnLeft(q[first],p[last-1])) last--;
    if(last-first<=1) return ans;      //空集
    p[last]=Cross_point(q[last],q[first]);   //计算队列首尾部的交点。
    for(int i=first;i<=last;i++)  ans.push_back(p[i]);     //复制。
    return ans;                        //返回凸多边形
}
```

圆

```cpp
struct Circle{ //圆
    Point c;          //圆心
    double r;         //半径
    Circle(){}
    Circle(Point c,double r):c(c),r(r){}
    Circle(double x,double y,double _r){c=Point(x,y);r = _r;}
};
int Point_circle_relation(Point p, Circle C){ //点和圆的关系
    double dst = Distance(p,C.c);
    if(sgn(dst - C.r) < 0) return 0;          //0  点在圆内
    if(sgn(dst - C.r) ==0) return 1;          //1  圆上
    return 2;                                  //2  圆外
}
int Line_circle_relation(Line v,Circle C){ //直线和圆的位置关系
    double dst = Dis_point_line(C.c,v);
    if(sgn(dst-C.r) < 0) return 0;     //0  直线和圆相交
    if(sgn(dst-C.r) ==0) return 1;     //1  直线和圆相切
    return 2;                           //2  直线在圆外
}
int Seg_circle_relation(Segment v,Circle C){ //线段和圆的位置关系
    double dst = Dis_point_seg(C.c,v);
```

```cpp
        if(sgn(dst-C.r) < 0) return 0;          //0线段在圆内
        if(sgn(dst-C.r) ==0) return 1;          //1线段和圆相切
        return 2;                               //2线段在圆外
}
//pa，pb是交点。返回值是交点个数
int Line_cross_circle(Line v,Circle C,Point &pa,Point &pb){  //直线和圆的交点
        if(Line_circle_relation(v, C)==2)  return 0;//无交点
        Point q = Point_line_proj(C.c,v);           //圆心在直线上的投影点
        double d = Dis_point_line(C.c,v);           //圆心到直线的距离
        double k = sqrt(C.r*C.r-d*d);
        if(sgn(k) == 0){                            //1个交点，直线和圆相切
            pa = q; pb = q; return 1;
        }
        Point n=(v.p2-v.p1)/ Len(v.p2-v.p1);        //单位向量
        pa = q + n*k;  pb = q - n*k;
        return 2;                                   //2个交点
}
```

最小圆覆盖

```cpp
Point circle_center(const Point a, const Point b, const Point c){  //圆上三点定圆心
        Point center;
        double a1=b.x-a.x, b1=b.y-a.y, c1=(a1*a1+b1*b1)/2;
        double a2=c.x-a.x, b2=c.y-a.y, c2=(a2*a2+b2*b2)/2;
        double d =a1*b2-a2*b1;
        center.x =a.x+(c1*b2-c2*b1)/d;
        center.y =a.y+(a1*c2-a2*c1)/d;
        return center;
}
void min_cover_circle(Point *p, int n, Point &c, double &r){  //最小圆覆盖
        random_shuffle(p, p + n);                   //随机函数，打乱所有点。这一步很重要
        c=p[0]; r=0;                                //从第1个点p0开始。圆心为p0，半径为0
        for(int i=1;i<n;i++)                        //扩展所有点
            if(sgn(Distance(p[i],c)-r)>0){          //点pi在圆外部
                c=p[i]; r=0;                        //重新设置圆心为pi，半径为0
                for(int j=0;j<i;j++)                //重新检查前面所有的点。
                    if(sgn(Distance(p[j],c)-r)>0){  //两点定圆
                        c.x=(p[i].x + p[j].x)/2;
                        c.y=(p[i].y + p[j].y)/2;
                        r=Distance(p[j],c);
                        for(int k=0;k<j;k++)
                            if (sgn(Distance(p[k],c)-r)>0){  //两点不能定圆，就三点定圆
                                c=circle_center(p[i],p[j],p[k]);
                                r=Distance(p[i], c);
                            }
                    }
            }
}
```

三维计算几何

```cpp
struct Point{
        double x,y,z;
        Point(){};
        Point(double x,double y,double z):x(x),y(y),z(z){};
        Point operator - (Point t){
```

```cpp
            return Point(x-t.x,y-t.y,z-t.z);
        }
        Point operator + (Point t){
            return Point(x+t.x,y+t.y,z+t.z);
        }
        Point operator / (double k){
            return Point(x/k,y/k,z/k);
        }
        double operator * (Point t){
            return x*t.x+y*t.y+z*t.z;
        }
        Point operator ^ (Point t){
            return Point(y*t.z-z*t.y,z*t.x-x*t.z,x*t.y-y*t.x);
        }
        double len(){
            return sqrt(x*x+y*y+z*z);
        }
        Point unit(){
            return *this/len();
        }
};
```

高精度

```cpp
#ifndef __x86_64__
#error Only x86-64 targets are supported
#endif
#include<cstdint>
#include<vector>
#include<string>
#include<iosfwd>
#define __builtin_ia32_adc(x,y,flag) __asm__("addb   %3, %0\n\t" "adcq   %2,
%1\n\t" "setc   %0":"+r"(flag),"+r"(x):"r"(y),"i"(-1):"cc")

struct bigint{// made by dengyaotriangle!
    typedef unsigned long long u64;
    typedef unsigned __int128 u128;
    typedef std::size_t st;
    std::vector<u64> data;
    bigint(){}
    bigint(u64 x):data(x?std::vector<u64>{x}:std::vector<u64>{}){}
    bigint(const std::string &s){
        st pos=s.length();
        int cnt=0;
        u64 val=0;
        while(pos){
            pos--;
            if(cnt==64){
                data.push_back(val);
                val=0;cnt=0;
            }
            val|=(u64)(s[pos]=='1')<<cnt;
            ++cnt;
```

```cpp
        }
        if(cnt&&val)data.push_back(val);
    }
    explicit operator std::string()const{
        if(data.empty())return "0";
        bool t=0;
        std::string ret;
        for(int i=63;i>=0;i--){
            t|=(data.back()>>i)&1;
            if(t)ret+='0'|((data.back()>>i)&1);
        }
        st i=data.size()-1;
        while(i){
            i--;
            for(int j=63;j>=0;j--)ret+='0'|((data[i]>>j)&1);
        }
        return ret;
    }
    explicit operator bool()const{return !data.empty();}
    explicit operator u64()const{return data.empty()?0:data[0];}
    st digit()const{
        if(data.empty())return 0;
        return (data.size()<<6)-__builtin_clzll(data.back());
    }
    bool operator==(const bigint &a)const{return a.data==data;}
    bool operator!=(const bigint &a)const{return a.data!=data;}
    bool operator<(const bigint &a)const{
        if(data.size()!=a.data.size())return data.size()<a.data.size();
        for(st i=data.size();i;){
            i--;
            if(data[i]!=a.data[i])return data[i]<a.data[i];
        }
        return 0;
    }
    bool operator>(const bigint &a)const{return a<(*this);}
    bool operator<=(const bigint &a)const{return !(*this>a);}
    bool operator>=(const bigint &a)const{return !(*this<a);}
    bigint &operator<<=(st n){
        if(data.empty())return *this;
        int w=n&63;st z=n>>6;
        st i=data.size();
        bool flg=0;
        if(w&&(data.back()>>(64-w)))data.push_back(0),flg=1;
        data.resize(data.size()+z);
        while(i){
            i--;
            if(flg)data[i+z+1]|=data[i]>>(64-w);
            data[i+z]=data[i]<<w;
            flg|=bool(w);
        }
        for(st i=0;i<z;i++)data[i]=0;
        return *this;
    }
    bigint &operator>>=(st n){
        int w=n&63;st z=n>>6,i=0;
        for(;i+z<data.size();i++){
            if(w&&i)data[i-1]|=data[i+z]<<(64-w);
            data[i]=data[i+z]>>w;
```

```cpp
        }
        while(data.size()>i)data.pop_back();
        while(!data.empty()&&data.back()==0)data.pop_back();
        return *this;
    }
    bigint operator<<(st n)const{return bigint(*this)<<=n;}
    bigint operator>>(st n)const{return bigint(*this)>>=n;}
    bigint &operator+=(const bigint &a){
        data.resize(std::max(data.size(),a.data.size()));
        bool carry=0;
        for(st i=0;i<data.size();i++){
            u64 rg=0;
            if(i<a.data.size())rg=a.data[i];
            __builtin_ia32_adc(data[i],rg,carry);
        }
        if(carry)data.push_back(1);
        return *this;
    }
    bigint &operator-=(const bigint &a){
        bool carry=1;
        for(st i=0;i<data.size();i++){
            u64 rg=-1;
            if(i<a.data.size())rg=~a.data[i];
            __builtin_ia32_adc(data[i],rg,carry);
        }
        while(!data.empty()&&data.back()==0)data.pop_back();
        return *this;
    }
    bigint &operator++(){return *this+=bigint(1);}
    bigint &operator--(){return *this-=bigint(1);}
    bigint operator++(int){bigint tmp=*this;++*this;return tmp;}
    bigint operator--(int){bigint tmp=*this;--*this;return tmp;}
    bigint &operator*=(const bigint &a){
        std::vector<u64> ret(data.size()+a.data.size());
        for(st i=0;i<data.size();i++){
            u64 carry=0;bool wcarry=0;
            st k=i;
            for(st j=0;j<a.data.size();j++,k++){
                u128 r=data[i]*(u128)a.data[j]+carry;
                u64 cur=r;
                carry=r>>64;
                __builtin_ia32_adc(ret[k],cur,wcarry);
            }
            while(carry||wcarry){
                __builtin_ia32_adc(ret[k],carry,wcarry);
                carry=0;k++;
            }
        }
        while(!ret.empty()&&ret.back()==0)ret.pop_back();
        data=ret;
        return *this;
    }
    bigint &operator/=(const bigint &a){
        if(a.digit()>digit()){
            data.clear();
            return *this;
        }
        st z=digit()-a.digit();
```

```cpp
        std::vector<u64> ret;
        while(1){
            bigint tmp=a<<z;
            if(tmp<=*this){
                *this-=tmp;
                st v1=z>>6;
                if(ret.size()<=v1)ret.resize(v1+1);
                ret[v1]|=(u64)(1)<<(z&63);
            }
            if(!z)break;
            z--;
        }
        data=ret;
        return *this;
    }
    bigint &operator%=(const bigint &a){
        if(a.digit()>digit())return *this;
        st z=digit()-a.digit();
        while(1){
            bigint tmp=a<<z;
            if(tmp<=*this)*this-=tmp;
            if(!z)break;
            z--;
        }
        return *this;
    }
    bigint operator+(const bigint &a)const{return bigint(*this)+=a;}
    bigint operator-(const bigint &a)const{return bigint(*this)-=a;}
    bigint operator*(const bigint &a)const{return bigint(*this)*=a;}
    bigint operator/(const bigint &a)const{return bigint(*this)/=a;}
    bigint operator%(const bigint &a)const{return bigint(*this)%=a;}
};
std::istream &operator>>(std::istream &st,bigint &a){
    std::string s;st>>s;a=bigint(s);return st;
}
std::ostream &operator<<(std::ostream &st,const bigint &a){
    return st<<(std::string)(a);
}
```

对拍

```cpp
#pragma GCC optimize(2)
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
#define int long long
typedef unsigned long long ull;
#define dmp(x) cerr<<"DEBUG"<<__LINE__<<":"<<#x<<" "<<x<<endl
const ll INF=0x3f3f3f3f3f3f3f3fLL;
typedef pair<int,int> pii;
signed main() {
    ios::sync_with_stdio(false);cin.tie(0);
    while(true){
        system("gene.exe>data.txt");
        system("A.exe<data.txt>out.txt");
        system("B.exe<data.txt>ans.txt");
        if(system("fc out.txt ans.txt")){// Windows
```

```
            break;
        }
        // if(system("diff out.txt ans.txt")){// Linux
        //      break;
        // }
    }
    return 0;
}
```