ECM1416 Forumlas

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Contents

1 Linear Algeobra

1.1 Vectors

1.1.1 Vector Norm

$$||v|| = \sqrt{\sum v_i^2} \tag{1}$$

1.1.2 Inner Product

$$u \cdot v = |u||v| \cos(\theta) \tag{2}$$

$$u \cdot v = u^T \cdot v = \sum_{i=1}^{n} u_i v_i \tag{3}$$

1.1.3 Rotating Vector

$$v' = \begin{bmatrix} v_x \cos(\theta) - v_y \sin(\theta) \\ v_x \sin(\theta) + v_y \cos(\theta) \end{bmatrix} \tag{4}$$

1.2 Matracies

1.2.1 Matrix Product $(A = (m \times n), B = (n \times p))$

$$A \cdot B = \begin{bmatrix} \Sigma_{i=1}^{n} a_{1i} b_{i1} & \cdots & \Sigma_{i=1}^{n} a_{1i} b_{ip} \\ \vdots & \ddots & \vdots \\ \Sigma_{i=1}^{n} a_{ni} b_{i1} & \cdots & \Sigma_{i=1}^{n} a_{ni} b_{ip} \end{bmatrix} \tag{5}$$

1.2.2 Rotation Matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{6}$$

1.2.3 Scale Matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \lambda_1 x \\ \lambda_2 y \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{7}$$

1.2.4 Shear Matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x + \lambda y \\ y \end{bmatrix} = \begin{bmatrix} 1 & \lambda \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ \mu x + y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \mu & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{9}$$

1.2.5 Transpose Properties

$$(A + B)^T = A^T + B^T \tag{10}$$

$$(AB)^T = A^T B^T \tag{11}$$

1.2.6 Identity Matrix

$$v = I_n v, \forall v \in \mathbb{R}^n \tag{12}$$

1.2.7 Determinant of a 2×2 Matrix

$$\det(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = ad - bc \tag{13}$$

1.2.8 Determinant of a $n \times n$ Matrix

$$\det(A) = \sum_{k=1}^{n} (-1)^{i+k} a_{ik} M_{ik} = \sum_{k=1}^{n} (-1)^{j+k} a_{kj} M_{kj} \tag{14}$$

1.2.9 Inverse of a 2×2 Matrix

$$A^-1 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{15}$$

1.2.10 Inverse of a $n \times n$ Matrix

$$A^-1 = \frac{1}{\det(A)} C^T = \frac{1}{\det(A)} \begin{bmatrix} C_{11} & C_{21} & \cdots & C_{n1} \\ C_{21} & C_{22} & \cdots & C_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ C_{1n} & C_{2n} & \cdots & C_{nn} \end{bmatrix} \tag{16}$$

$$C_{ij} = (-1)^{i+j} M_{ij} \tag{17}$$

1.2.11 Properties of an Inverse

$$(A^{-1})^{-1} = A \tag{18}$$

$$(A^T)^{-1} = (A^{-1})^T \tag{19}$$

$$(AB)^{-1} = B^{-1}A^{-1} \tag{20}$$

$$(kA)^{-1} = k^{-1}A^{-1} \tag{21}$$

1.2.12 Solving a System of Linear Equations

$$Ax = b \tag{22}$$

$$x = A^{-1}b \tag{23}$$

2 Differentation

2.1 Matrix Calculus

2.1.1 Gradient

$$\nabla f = \begin{bmatrix} f_{x_1} \\ f_{x_2} \\ \vdots \\ f_{x_n} \end{bmatrix} \tag{24}$$

2.1.2 Hessian

$$H_f = \begin{bmatrix} f_{x_1x_1} & f_{x_1x_2} & \cdots & f_{x_1x_n} \\ f_{x_2x_1} & f_{x_2x_2} & \cdots & f_{x_2x_n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{x_mx_1} & f_{x_mx_2} & \cdots & f_{x_mx_n} \end{bmatrix} \tag{25}$$

2.1.3 Jacobian

$$D_f = \begin{bmatrix} \nabla f_1 \\ \nabla f_2 \\ \vdots \\ \nabla f_n \end{bmatrix} = \begin{bmatrix} f_{1x_1} & f_{1x_2} & \cdots & f_{1x_n} \\ f_{2x_1} & f_{2x_2} & \cdots & f_{2x_n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{nx_1} & f_{nx_2} & \cdots & f_{nx_n} \end{bmatrix} \tag{26}$$

2.2 Taylor Expantion

$$f(x) \approx \sum_{i=0}^{n} \frac{f^i(a)(x-a)^i}{i!} \tag{27}$$

2.3 Maclorian Expantion (a = 0)

2.3.1 $\sin(x)$

$$\sin(x) \approx \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \tag{28}$$

2.3.2 $\cos(x)$

$$\cos(x) \approx \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!} \tag{29}$$

2.3.3 e^x

$$e^x \approx \sum_{k=0}^{n} \frac{x^k}{k!} \tag{30}$$

2.3.4 $\ln|1+x|$

$$\ln|1+x| \approx \sum_{k=0}^{n} (-1)^k \frac{x^{k+1}}{k+1} \tag{31}$$

2.3.5 $c(\frac{1}{1-x})$

$$c(\frac{1}{1-x}) \approx c\sum_{k=0}^{n} x^k \tag{32}$$

2.4 Solving ODEs

2.4.1 Intergrating Factor

$$\frac{dy}{dx} + P(x)y = Q(x) \tag{33}$$

$$\text{I.F} = e^{\int P(x)dx} \tag{34}$$

$$\frac{d}{dx}(e^{\int P(x)dx}y) = Q(x)e^{\int P(x)dx} \tag{35}$$

$$y = \frac{\int Q(x)e^{\int P(x)dx}dx}{e^{\int P(x)dx}} \tag{36}$$

2.4.2 Seperation of Variables

$$\frac{dy}{dx} = g(x)h(y) \tag{37}$$

$$\frac{dy}{dx}\frac{1}{h(y)} = g(x) \tag{38}$$

$$\int \frac{1}{h(y)} dy = \int g(x) dx \tag{39}$$

2.4.3 Eulers Method

$$y_{n+1} = y_n + hf(x_n, y + n) \tag{40}$$

$$x_{n+1} = x_n + h \tag{41}$$

3 Probability

3.0.1 Given That

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{42}$$

3.0.2 Independence

$$P(A \cap B) = P(A)P(B) \tag{43}$$

3.0.3 Law of Total Probability

$$P(A) = \sum_n P(A \cap B_n) = \sum_n P(A|B_n)P(B_n) = \sum_n P(B_n|A)P(A) \tag{44}$$

3.0.4 Bayes Theorm

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{45}$$

4 Markov Chains