Question 1.

Find the error term for the derivative approximation:

$$f''(x_0) \approx \frac{2f(x_0 - h) - 3f(x_0) + f(x_0 + 2h)}{3h^2}.$$

Question 2.

Find the error term for the quadrature method, and state its degree of precision.

$$\int_{x_0}^{x_0+2h} f(x)\, dx \approx \frac{h}{2}\left[3f\left(x_0 + \frac{4}{3}h\right) + f(x_0)\right].$$

Question 3.

Consider the integral $\int_1^7 \cos(x^2)\, dx$

(a) Use the composite Simpson's rule to approximate the value of this integral using $n = 3$ intervals.

(b) Determine the number of intervals n needed to guarantee an error of at most 10^{-4}.

Question 4.

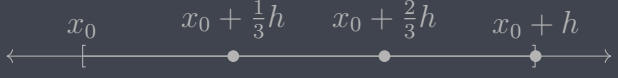
Consider the IVP:

$$2\dot{y} + y = t^4 + 1,\ y(1) = 2.$$

Apply the second degree Taylor method with $h = 0.5$ to this ODE to approximate $y(2)$. Show the details in each step.

Question 5.

Derive an ODE solver based on the stencil and corresponding integration formula.



Formula: $\frac{h}{4}\left(3f\left(x_0 + \frac{1}{3}h\right) + f(x_0 + h)\right) + O(h^4)$