Question 36.

In Handout #7, we defined differentiability for functions on open sets. Now we give a definition that works over arbitrary sets. For this problem, you will need to read Piazza Post @274 and use Theorem 1.1.

Let $A \subseteq \mathbf{R}^n$ be an arbitrary set, let $f : A \to \mathbf{R}$ be a function, and let $p \in A$ be a point. We say that f is **differentiable** at p if there exists an open neighborhood U of p and a function $\widehat{f} : U \to \mathbf{R}$ such that $\widehat{f}$ is differentiable at p (in the sense of Handout #7) and $\widehat{f}|_{U\cap A} = f|_{U\cap A}$.

(a) Prove that f is differentiable at every point of A if and only if f extends to a differentiable function defined on an open set containing A.

(b) Suppose further that A is closed. Prove that f is differentiable at every point of A if and only if f extends to a differentiable function on $\mathbf{R}^n$.

Question 37.

The following set is called the n**-simplex**:

$$\Delta_n := \{\vec{x} = (x_1, \ldots, x_n) \in \mathbf{R}^n : x_1, \ldots, x_n \geq 0 \text{ and } x_1 + \cdots + x_n \leq 1\}.$$

You can assume, without proof, that Δ_n is Jordan measurable.

Find, with proof, an explicit formula for $\mu(\Delta_n)$ in terms of n.

Proof. □