NAME (PRINT): ____________________

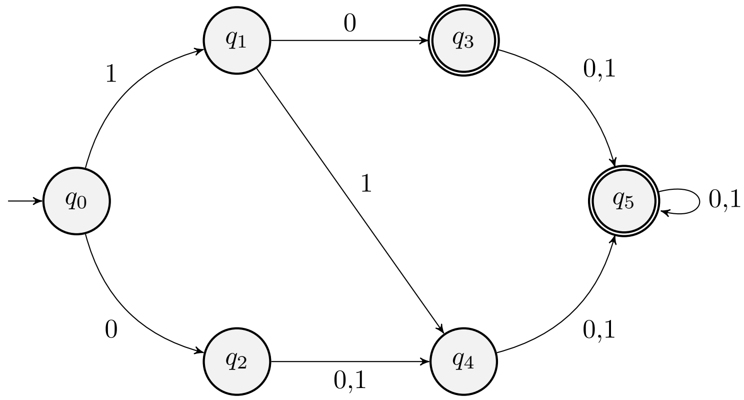
STUDENT NUMBER (PRINT): ____________________

University of Toronto Mississauga
FALL 2024 MOCK FINAL EXAMINATION
Introduction to Theory Computation
Macho Man (mˆ2)
Duration - ~~3 hours~~ 10 minutes
Aids : お前のお母さん

This is a mock exam designed for studying CSC236. Any and all similarities with the Fall 2024 CSC236 final examination are purely coincidence.

If you finish this exam in under 3 hours, you are fully prepared for the final examination.

Q1. (9 points) Consider the DFA below:



a) (1 point) Describe the language accepted by the following DFA.

b) (3 points) Convert this DFA into a minimal NFA (i.e., there is no smaller NFA that accepts this language). Give a brief justification.

c) (5 points) Provide a DFA that accepts the language matched by $(a + ab)^*$. Prove its correctness.

Q2. (6 points)

a) (2 points) State the CLRS version of master theorem. Define all variables and state their conditions.

b) (2 points) Let $f : \mathbb{N} \to \mathbb{R}$ be a nonnegative function. Prove that if $f \in \Theta(n^k)$ for some $k > 0$, then the regularity condition holds true.

c) (2 points) Find the time complexity of a recursive function T defined by

$$T(n) = \begin{cases} 4, & \text{if } n \leq 1; \\ 3T\left(\frac{n}{2}\right) + n^2 \log n, & \text{if } n > 1. \end{cases}$$

Q3. (6 points)

a) (3 points) Let $\Sigma = \{0, 1\}$. Let L be a language on Σ defined by $L = \{1^{n^2} : n \in \mathbb{N}\}$. Prove that L is not a regular language.

b) (3 points) Let L, M be regular languages. Prove that the language $L \cap M$ is regular.

Q4. (6 points)
Let A be a set of functions defined recursively as follows:

- $\sqrt{x} \in A$
- If $f \in A$, then $\frac{1}{f} - f \in A$

a) (4 points) Let $P(f)$ be a predicate on A and suppose you have managed to prove that

- $P(\sqrt{x})$ is true,
- $P(f) \implies P(\frac{1}{f} - f)$ for all $f \in A$.

Prove that $\forall f \in A, P(f)$. You may only assume that the principle of induction holds for the natural numbers, NOT the set A.

b) (2 points) Prove that $\forall f \in A,\ f\left(\frac{1}{2}\right) = \frac{1}{\sqrt{2}}$.

Q5. (7 points)
Consider the program below:

```
def binary_search(x: int, lst: list[int]):
    l = 0, r = len(lst) - 1
    while(r - l > 0):
        mid = (l+r) // 2
        if(lst[mid] == x):
            return mid
        elif(lst[mid] < x):
            r = mid
        else:
            l = mid + 1
    return l
```

a) (2 points) State the preconditions and postconditions of this program.

b) (5 points) Prove that this program is correct.