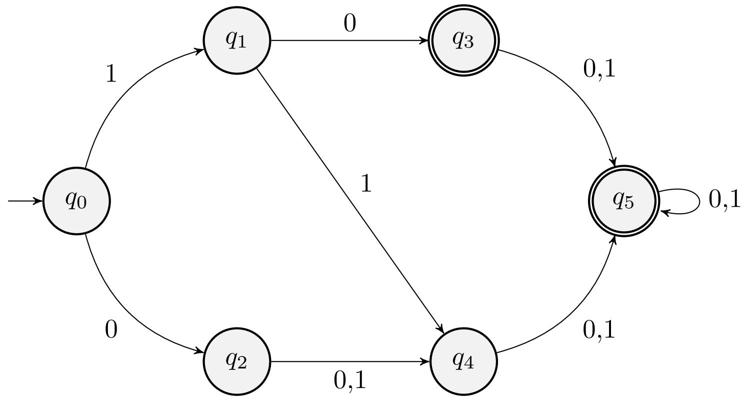
NAME (PRINT): ______________________________

STUDENT NUMBER (PRINT): ______________________________

University of Toronto Mississauga
FALL 2024 MOCK FINAL EXAMINATION
Introduction to Theory Computation
Macho Man (mˆ2)
Duration - ~~3 hours~~ 10 minutes
Aids : お前のお母さん

Q1. (9 points) Consider the DFA below:



a) (1 point) Describe the language accepted by the following DFA.

b) (3 points) Convert this DFA into a minimal NFA (i.e., there is no smaller NFA that accepts this language). Give a brief justification.

c) (5 points) Provide a DFA that accepts the language matched by $(a + ab)^*$. Prove its correctness.

Q2. (6 points)

a) (2 points) State the CLRS version of master theorem. Define all variables and state their conditions.

b) (2 points) Let $f : \mathbb{N} \to \mathbb{R}$ be a nonnegative function. Prove that if $f \in \Theta(n^k)$ for some $k > 0$, then the regularity condition holds true.

c) (2 points) Find the time complexity of a recursive function T defined by

$$T(n) = \begin{cases} 4, & \text{if } n \leq 1; \\ 3T\left(\frac{n}{2}\right) + n^2 \log n, & \text{if } n > 1. \end{cases}$$

Q3. (6 points)

a) (3 points) Let $\Sigma = \{0, 1\}$. Let L be a language on Σ defined by $L = \{1^{n^2} : n \in \mathbb{N}\}$. Prove that L is not a regular language.

b) (3 points) Let L, M be regular languages. Prove that the language $L \cap M$ is regular.

Q4.
Recall that a well-ordered set is a set that has a minimal element.

a) Under the assumption that the principle of simple induction holds for natural numbers, prove that any well-ordered countable set also has the principle of simple induction.

b) s