Question 1.

Use the general Principle of Inclusion-Exclusion to obtain a formula involving a summation for the number of ways to place n identical balls into k labeled bins so that no bin is left empty, where n and k are positive integers. Note that you may need to consider different possible cases for n, k as necessary; make sure you show all your steps.

Proof. Label the bins from 1 to k. Let A_i be the ways to place the n balls into the k bins such that bin i has no balls. We want to count

$$\left|A_1^C \cap \cdots \cap A_n^C\right|$$

□