Question 1.

Find all solutions to the following complex equations.

Proof.

1. $(1+i)\overline{z} = i(2+8i)$.

   Suppose that z is of the form $z = a+bi$, for $a, b \in \mathbb{R}$. Then the equation becomes

   $$(1+i)(a-bi) = i(2+8i) \implies a+b+(a-b)i = -8+2i.$$

   Equating coefficients, we get

   $$a+b = -8 \text{ and } a-b = 2.$$

   Solving the system of equations gives us $a = -3$ and $b = -5$, so $z = -3-5i$.

2. $z^3 = -8i$.

   Suppose that z is of the form $z = re^{i\theta}$, for $r, \theta \in \mathbb{R}$. Then the equation becomes

   $$r^3 e^{3i\theta} = -8i \implies r^3 e^{3i\theta} = 8e^{-i\left(\frac{\pi}{2}+2n\pi\right)}, \text{ for } n \in \mathbb{Z}$$

   Equating the coefficient and exponent gives us

   $$r^3 = 8 \text{ and } 3\theta = \frac{\pi}{2} + 2n\pi \implies r = 2,\ \theta = \frac{\pi}{6} + \frac{2n\pi}{3}$$

3. $e^{\overline{z}} = -2+2i$.

$\square$