6 Week 6? i think

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Exercise 6.33. Prove that $[0,1]^2$ is homeomorphic to the closed unit ball $\overline{B}(0,1)$ in $\mathbf{R}^2$.

Proof. We begin by proving a lemma that will help us prove the continuity of the inverse.

Lemma. Let X, Y be compact metric spaces. If $f : X \to Y$ is bijective and continuous, then f^{-1} is continuous.

Let f be bijective and continuous. Thus

Now, we continue to the main result. It is pretty easy to see that the closed box $[0,1]^2$ is homeomorphic to $[-1,1]^2$. We define the function $h : [-1,1]^2 \to \overline{B}(0,1)$ by

$$h(x,y) = \begin{cases} \|(x,y)\|_2^{-1} \left(\cos\left(\arctan\frac{y}{x}\right), \sin\left(\arctan\frac{y}{x}\right)\right), & \text{if } x \neq 0; \\ (0,y), & \text{if } x = 0; \end{cases}$$

□