

Question 43.

Transverse intersection. Let M, N be two smooth surfaces in \mathbf{R}^3 . We say that M and N intersect transversally if $T_p M \neq T_p N$ for all $p \in M \cap N$.

- (a) Prove that if M, N intersect transversally, then $M \cap N$ is a smooth curve in \mathbf{R}^3 .
- (b) Show by example that the conclusion of (a) fails without the assumption of transverse intersection.

Proof.

(a):

Suppose that M and N intersect transversally. We will show that $M \cap N$ is a smooth 1-manifold in \mathbf{R}^3 .

(b):

Let M be the xy -plane, and let N be the graph of $f(x, y) = x^2 + y^2$. Then $M \cap N$ is simply the origin, which is not a smooth curve.

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