Question 30.

Let $U \subseteq \mathbf{R}^n$ be an open set in $\mathbf{R}^n$, and let K be a compact subset of U. Prove that there exists an *infinitely differentiable* function $\varphi : \mathbf{R}^n \to [0, 1]$ such that $\varphi(p) = 1$ for all $p \in K$, and $\varphi(p) = 0$ for all $p \in \mathbf{R}^n \setminus U$. This is called a **bump function** supported on U.
(For a function $f : U \to Y$, the n**th total derivative** $f^{(n)}$ is defined as follows: for $n = 0$, we set $f^{(0)} = f$; for $n \geq 1$, if $f^{(n-1)}$ is totally differentiable, we set $f^{(n)} = (f^{(n-1)})'$. We say that f is **infinitely differentiable** if $f^{(n)}$ exists for all $n \geq 0$.)