Question 42.

Let $M \subseteq \mathbf{R}^N$ be a smooth n-manifold (with or without boundary!).

(a) Show that if $n < N$, then M is a *Lebesgue null set*.

(b) Show that if $n = N$ and M is closed and its boundary is nonempty, then ∂M coincides with the usual topological boundary (as defined on Handout #2).

(c) Show that if M is compact and its boundary is nonempty, then M is Jordan measurable.

Proof.
(a):
We begin by proving a lemma:
Lemma: A countable union of sets with Jordan measure 0 is a Lebesgue null set.
Let $E = \bigcup_{i \geq 0} E_i$, where $\mu(E_i) = 0$.

$\square$