Block Matrix Formalization

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Abstract The point of this article is not to present novel discoveries but rather to make more available information that seems to be very scarcely documented. Block matrices and operations on them are defined in various sources, but very few sources document the proofs behind the workings of block matrices. These proofs are non-trivial and so I will detail here, a theoretical interpretation of block matrices that allows for the same functionality as that which is typically accepted and formalizes the language around the subject.

1 Definitions and Notation

1.1 Definition of a Block Matrix

A block matrix is an array of matrices where all matrices in the i^{th} row have m_i rows and all matrices in the j^{th} column have n_j columns. It follows then that a block matrix can simultaneously be considered an array of matrices and an array of the elements in those matrices.

We will denote a block matrix as a capital letter with a "$*$" in the superscript. The convention will also be that m and n with subscripts will be used to represent the number of rows and columns in the blocks of a block matrix respectively

1.2 Anatomy of a Block Matrix

The ij^{th} block of a block matrix is the ij^{th} entry in the array of matrices and the ij^{th} entry of a block matrix is the ij^{th} entry of the matrix of the elements of the blocks. We will denote the ij^{th} block of a block matrix with the same capital letter as the block matrix with a subscript of ij. That is, if B^* is a block matrix then B_{ij} refers to ij^{th} block of the block matrix. We will denote the ij^{th} entry of a block matrix with the lowercase letter corresponding to the capital letter with a subscript ij. That is, if B^* is a block matrix then b_{ij} will denote the ij^{th} entry in B^*.